

A "Basic" Mathematics Guide for High Schoolers

Stasya (Discord: stasssiee)

2024

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Chapter 1

Proof

1.1 Proof

Introduction to Proof

In this section we will working with these topics:

- Consequence and Equivalence
- Proof by Exhaustion
- Proof by Deduction
- Disproof by Counter-Example
- Proof by Contradiction

When we look at consequence, we essentially say that " a implies b ", or:

$$a \rightarrow b$$

If the arrow points the other way, we say that " b implies a ", or:

$$a \leftarrow b$$

Let's say that statement a states that p is a prime number > 2 .

Let's say that statement b states that p is an odd number.

For these statements, we see that a does imply b , so we can write that

$$a \rightarrow b$$

The other way however does not work, since because p is an odd number, it does not imply that p is a prime number.

However, if this was true, we can write that a implies b and b implies a , or:

$$a \leftrightarrow b$$

which is sometimes written as " a if and only b " or " a iff b ".

Let's show a logical equivalence. Let a be the statement n^2 is odd and b be the statement n is odd.

We know that when n^2 is odd, that n is odd when we list out the odd squared numbers. We can see the converse is true as well in this statement since every time a number n is squared, we are given an odd number, therefore:

$$a \leftrightarrow b$$

Proof by Exhaustion

Proof by Deduction

Disprove by Counter-Example

Proof by Contradiction

Chapter 2

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The Laws of Indices

2.2 Surds

Simplifying Surds

Rationalising the Denominator

Problem Solving

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The Difference of Two Squares

Factorising Quadratics

Sketching Quadratics from Factorised Form

Completing the Square

Sketching Quadratics from Completed Square Form

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Using the Discriminant

Using the Quadratic Formula

Sketching Quadratics Using the Quadratic Formula

Sketching Quadratic Using a Calculator

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The Substitution Method

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Finding the Distance between Two Points

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Sketching Linear Graphs

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Intersections of Lines

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SOHCAHTOA

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Introducing e

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Sketching $y = \ln(x + a)$

Sketching $y = \ln(x + a) + b$

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The Laws of Logarithms

The Natural Logarithm

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