AP Calculus AB Notes

Stasya

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1 Limits and Continuity

1.1 Introduction to Limits

Let's start with an example.

Given the function $f(x) = \frac{x^2-1}{x-1}$, find $f(1)$.

We will end up getting 0/0. This is a difficulty! 0/0 is called indeterminate, so we need another way to answer this.

Let's start by approaching $x = 1$ from the left and approach $x = 1$ from the right. If we put this in tabular data, we end up approaching 2.

Now we can see that as x gets close to 1, then the function $f(x) = \frac{x^2-1}{x-1}$ gets close to 2.

We are now faced with an interesting situation: When $x = 1$, the answer is undefined, but we can see that it is going to be 2.

We want to give the answer "2", but we can't, so instead mathematicians say exactly what is going on by using the special world "limit".

$$\text{The limit of } \frac{x^2-1}{x-1} \text{ is 2.}$$

Symbolically, this is written as $lim_{x\to 1}\frac{x^2-1}{x-1} = 2$. It is a special way of saying, "ignore what happens when we get there, but as we get closer and closer, the answer gets closer and closer to 2".

Definition

If when the x values are approaching $x = c$ from either side $f(x)$ becomes arbitrarily close to a single number $y = L$, then the limit $f(x)$ as x approaches c is L.

$$\lim_{x\to c} f(x) = L$$

Limits can also be used even when we know the value when we get there. Nobody said they are only for difficult functions.

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