AP Calculus BC Notes

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Contents

1 Limits and Continuity

1.1 Limits and Their Properties

Properties of Limits:

If L, M, c, and k are real numbers and $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$, then

1. Sum Rule: $\lim_{x\to c}(f(x) + g(x)) = L + M$
2. Difference Rule: $\lim_{x\to c}(f(x) - g(x)) = L - M$
3. Product Rule: $\lim_{x\to c}(f(x) \cdot g(x)) = L \cdot M$
4. Quotient Rule: $\lim_{x\to c}\left(\frac{f(x)}{g(x)}\right) = \frac{L}{M}$ where $M \neq 0$
5. Constant Multiple Rule: $\lim_{x\to c}(k \cdot f(x)) = k \cdot L$
6. Power Rule: If r and s are integers, $s \neq 0$, then $\lim_{x\to c}(f(x))^{r/s} = L^{r/s}$ provided that $L^{r/s}$ is a real number.
7. Limit of a Composite Function Rule: If f and g are functions such that $\lim_{x\to c} g(x) = L$ and $\lim_{x\to c} f(x) = f(L)$, then $\lim_{x\to c} f(g(x)) = f(\lim_{x\to c} g(x)) = f(L)$.

Example

Given that $\lim_{x\to a} f(x) = 2$ and $\lim_{x\to a} g(x) = 3$, find the limit if it exists.

(a) $\lim_{x\to a}(5g(x)) = 15$

(b) $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{2}{3}$

Two special trig limits:

$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$

$\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta} = 0$

Example

$\lim_{x\to 0} \frac{\sin(5x)}{4x}$

This can be written as $\frac{1}{4}\lim_{x\to 0} \frac{\sin(5x)}{x} = \frac{5}{4}$

Example

$\lim_{x\to 0} \frac{2x+\sin x}{x}$

We can split it up, $\lim_{x\to 0} \frac{2x}{x} + \lim_{x\leq 0} \frac{\sin x}{x} = 2 + 1 = 3$

Theorem 1.1: Squeeze Theorem

If $h(x) \leq f(x) \leq g(x)$ for all x in an open interval containing c, except possible at c itself, and if $\lim_{x\to c} h(x) = L$ and $\lim_{x\to c} g(x) = L$, then $\lim_{x\to c} f(x) = L$.

Example

If $2 \leq f(x) \leq x^2 + 2$ for all x, find $\lim_{x \to 0} f(x)$.

The answer is 2.

1.2 Continuity

Definition: Continuity

A function f is said to be continuous at $x = c$ if and only if:

1. $f(c)$ defined
2. $\lim_{x \to c} f(x)$ exists
3. $f(c) = \lim_{x \to c} f(x)$

Essentially, no gaps or holes.

Exercise Sketch a function f so that $f(c)$ is not defined.

Exercise Sketch a function f so that $\lim_{x \to c} f(x)$ does not exist.

Exercise Sketch a function where $f(c)$ is defined and $\lim_{x \to c} f(x)$ exists but $\lim_{x \to c} f(x) \neq f(c)$

Exercise Sketch a function where f is continuous at $x = c$

If a function f is not continuous at $x = c$, the discontinuity may be one of three types:

1. point discontinuity (essentially a hole)
2. jump discontinuity (gap)
3. asymptotic discontinuity (asymptote)

A point discontinuity is said to be a removable discontinuity because the function can be redefined at the point in such as way as to make the function continuous there. Jump discontinuites and asymptotic discontinuites are non-removable because the functions which contain them cannot be redefined at a point to make them continuous.

Example

Find the value(s) of x at which the given function is discontinuous. Identify each value as a point, jump, or asymptotic discontinuity. Identify each value as a removable or non-removable discontinuity. If it is removable, redefine the function at that value so that it will be continuous.

(a) $f(x) = \frac{x^2 - x - 6}{x - 3}$

Factoring this gives gives $x = 3$ a hole, so the function is redefined as $f(x) = x + 2$. There is a removable discontinuity in this.

(b) $f(x) = \frac{1}{x - 3}$

This is asymptotic at $x = 3$.

Exercise Do the same for the example above with the function $f(x) = \begin{cases} x + 2 \text{ if } x < 1 \\ 2 - x \text{ if } x > 1 \end{cases}$

Example

Find k so that f will be continuous at $x = 2$ given $f(x) = \begin{cases} x + 3 \text{ if } x \leq 2 \\ kx + 6 \text{ if } x > 2 \end{cases}$

Since we know that $x+3 = kx+6$ when $x = 2$, plug in 2 for x, to get $-1/2 = k$.

1.3 Intermediate Value Theorem

Since a person's height is a continuous function of time, if a child had a height of 58 inches at age 11 and a height of 64 inches at age 14, then the child's height took on every value between 58 and 64 inches for some time between the age of 11 and the age of 14. This is a simple example of an important theorem in calculus called the Intermediate Value Theorem.

Theorem 1.2: Intermediate Value Theorem

If

1. $f(x)$ is continuous on the closed interval $[a, b]$
2. if $f(a) \neq f(b)$
3. if k is between $f(a)$ and $f(b)$

then there exists a number c between a and b for which $f(c) = k$.

In other words, a function $y = f(x)$ that is continuous on a closed interval $[a, b]$ takes on every value between $f(a)$ and $f(b)$. If k is between $f(a)$ and $f(b)$, then there is at least one value c in (a, b) for which $f(c) = k$.

Example

Determine if the Intermediate Value Theorem holds for the given values of k. If the theorem holds, find a number c for which $f(c) = k$. If the theorem does not hold, give the reason.

$f(x) = \frac{1}{x-2}$, $[a, b] = \left[2\frac{1}{2}, 7\right], k = \frac{1}{4}$

We have $\frac{1}{x-2} = \frac{1}{4}$, so $x = 6$.

Exercise Do the same for $f(x) = x^2 + 5x - 6, [a, b] = [-1, 2], k = 4$

2 Differentiation

2.1 Derivatives

Rates of change play a role whenever we study the relationship between two changing quantities. A familiar example is velocity, which is the rate of change of position with respect to time.

If an object is traveling in a straight line, the average velocity over a given time interval from t_1 to t_2 is defined as

$$\text{Average velocity} = \frac{\text{change in position}}{\text{change in time}} = \frac{\Delta s}{\Delta t} = \frac{s(t_2)-s(t_1)}{t_2-t_1}$$

For example, if a car travels 200 miles in 4 hours, then its average velocity during this 4-hour period is 50 miles per hour. At any given moment, the car may be going faster or slower than the average.

We cannot define instantaneous velocity as a ratio because we would have to divide by the length of the time interval, which is zero. However we can estimate the instantaneous velocity by computing the average velocity over successively smaller intervals and then determine the limit of the average velocity as the lengths of the time intervals get closer and closer to zero.

If we look at a graph of the position of an object, a secent line can be drawn through two arbitrary points $(t_1, s(t_1))$ and $(t_2, s(t_2))$. The average velocity of the object would be the slope of the secant line through these two points. If we move the secant line so that t_2 gets closer and closer to t_1, the secant line gets closer and closer to becoming a tangent line to the graph at $(t_1, s(t_1))$. The slope of the tangent line is the instantaneous velocity of the object when the time is t_1.

Velocity is only one of many examples of a rate of change. Our same reasoning applies to any quantity y that depends on a variable x. The average rate of change of y with respect to x over the interval x_1 to x_2 is the ratio

$$\text{Average rate of change} = \frac{\text{change in y}}{\text{change in x}} = \frac{\Delta y}{\Delta x} = \frac{y(x_2)-y(x_1)}{x_2-x_1}$$

The instantaneous rate of change at $x = x_1$ is the limit of the average rates of change as x_2 gets closer and closer to x_1. We must consider values of x_2 on both the left side of x_1 and on the right side of x_1 as we take the limit.

An expression such as $\frac{s(t_2)-s(t_1)}{t_2-t_1}$ or $\frac{f(x)-f(c)}{x-c}$ is called a difference quotient.

The instantaneous rate of change of a function is called its derivative and is defined as follows.

Definition

The derivative of $f(x)$ at $x = c$ is the limit of the difference quotients (if it exists):

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

When the limit exists, we say that f is differentiable at $x = c$. An equivalent definition of the derivative is called the alternative form of the definition of the derivative.

$$f'(c) = \lim_{x\to c} \frac{f(x) - f(c)}{x - c}$$

Example

A diver jumps from a diving board that is 32 feet above the water. The position of the diver from the water is given by $s(t) = -16t^2 + 16t + 32$, where t is measured in seconds.

(a) Find the average velocity at which the diver is moving for the time interval $[1, 1.5]$.

This is the slope of the secant line between those two values which is equal to -24 ft/sec.

(b) Estimate the instantaneous velocity by finding the average velocity betewen the intervals $[0.9, 1]$, $[0.99, 1]$, $[0.999, 1]$, $[1, 1.1]$

We can see that between $[0.999, 1]$, the slope is -15.984, for $[1, 1.001]$ it is -16.016. It is reasonable to assume that the diver is moving -16 ft/sec at $t = 1$ second.

Example

Given $f(x) = x^2 - 4x$

(a) Graph the function and draw a tangent line at $x = 3$. Do you expect $f'(3)$ to be positive or negative?

Graph the line please. We expect $f'(3) > 0$

(b) Compute $f'(3)$ by using the definition of the derivative and the alternative form of the derivative.

Limit definition:

$$\frac{(x+h)^2 - 4(x+h) - (x^2 - 4x)}{h}$$

$$\frac{2xh + h^2 - 4h}{h}$$

$$\frac{2xh + h^2 - 4h}{h} = 2x + h - 4$$

Taking the limit of this gives $f'(x) = 2x - 4$, so $f'(3) = 2$.

The alternative form is up to the reader.

(c) Write the equation of the tangent line to the graph of $y = f(x)$ at $x = 3$. Leave your equation in point-slope form, $y - y_1 = m(x - x_1)$.

This is just $y + 3 = 2(x - 3)$

2.2 More on Derivatives

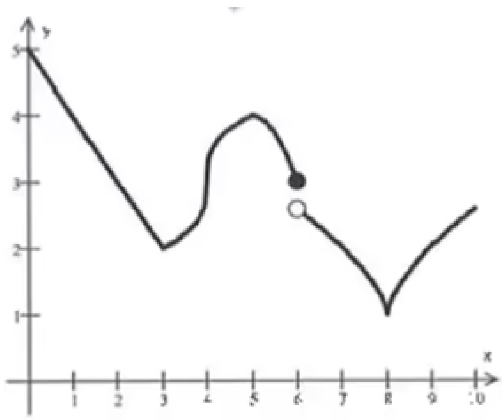
Example

List the x-values for which the function appears to be: (a) not continuous

Remember this is where there are holes or gaps are where the function will not be continuous, so $x = 6$.

(b) not differentiable

$x = 3, 4, 6, 8$, because these are sharp turns, vertical tangents, or places where the function is not continuous.



Example

At which labeled points is the slope of the graph:

(a) zero

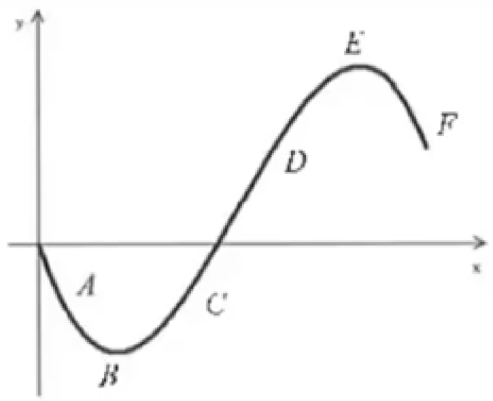
This is horizontal tangent lines. B and E.

(b) positive

C, D

(c) negative

A, F



Example

Let f be a function which satisfies $f(2+h) - f(2) = 5h - 3h^2 + 9h^3$ for all real numbers h. Find $f'(2)$.

We can find it by writing $f'(c) = \lim_{h \to 0} \frac{f(c+h)-f(c)}{h}$, so $f'(2) = \lim_{h \to 0} \frac{f(2+h)-f(2)}{h}$.

And we know these values, plug them in, and we get that $f'(2) = 5$.

Example

Let f be a function which satisfies the property $f(x+y) = f(x) - 4f(y) + 9xy$ for all real numbers x and y and suppose that $\lim_{h \to 0} \frac{f(h)}{h} = 2$. Use the definition of the derivative to find $f'(x)$.

Using the limit definition of a derivative, we can simplify the limit to end up being $\frac{-4f(h)+9xh}{h}$, and the limit of this gives $f'(x) = -8 + 9x$.

Exercise Given $\lim_{x \to 5} \frac{f(x)-f(5)}{x-5} = 3$. Which of the following must be true, might be true, or can never be true?

1. $f'(5) = 3$
2. $f'(5) = 0$
3. $f(5) = 3$
4. f is continuous at $x = 0$
5. f is continuous at $x = 5$
6. $\lim_{x \to 5} f(x) = f(5)$

2.3 Derivatives Using Data

Example

Water is flowing into a tank over a 24-hour period. The amount of water in the tank is modeled by a differentiable function W for $0 \leq t \leq 24$, where t is measured in hours and $W(t)$ is measured in gallons. Values of $W(t)$ at selected values of time t are shown in the table below.

t (hours)	0	4	8	12	16	20	24
$W(t)$ (gallons)	150	184	221	257	294	327	357

(a) Use the data in the table to find $W(8)$. Using appropriate units, explain the meaning of your answer.

There are 221 gallons of water in the tank when $t = 8$.

(b) Use the data in the table to find $W^{-1}(257)$. Using appropriate units, explain the meaning of your answer.

There are 257 gallons of water in the tank when $t = 12$.

(c) Use the data in the table to find an approximation for $W'(15)$. Show the computations that lead to your answer. Using appropriate units, explain the meaning of your answer.

The secant line that contains $t = 15$ is 37/4 gallons/hr. At $t = 15$, the rate of change of the amount of water in the tank is $\approx \frac{37}{4}$ gallons/hr.

(d) Use the data in the table to find the average rate of change of $W(t)$ over the time period $4 \leq t \leq 20$ hours. Show the computations that lead to your answer.

143/16 gallons per hour

(e) For $0 < t < 24$ must there be a time t when the tank contains 265 gallons of water? Justify your answer.

IVT, yes, the function is continuous becase it's differentiable.

Exercise Continuing the following above FRQ:

- A model for the amount of water in the tank is given by $A(t) = \frac{1}{225}(-t^2 + 30t^2 + 1800t + 33750)$, where $A(t)$ is measured in gallons and t is measured in hours. Find $A'(15)$.
- Use the model given in the above question to find the average rate of change of $A(t)$ over the time period $4 \leq t \leq 20$ hours.

2.4 Basic Differentiation Rules

The power rule is $\frac{d}{dx}[x^n] = nx^{n-1}$.

Exercise Find the derivative of $f(x) = x^3$

Exercise Find the derivative of $5x^3$

Example

(a) Find the derivative of $y = \sqrt{x}$

This can be written as $x^{1/2}$, so the derivative is $\frac{1}{2}x^{-1/2}$.

(b) Find the derivative of $f(x) = 5$

This is equal to $5x^0$, the derivative is therefore 0.

Exercise Find the derivative of $g(t) = \frac{1}{t^4}$

If c is a constant, then $\frac{d}{dx}[c] = 0$

$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Exercise Find the derivative of $f(x) = 4x^3 - 5x^2 + 7x + 3$

Example

Find the derivative of $y = \frac{5x^5 - 3x^4 + 4}{x}$.

Rewrite this as $5x^4 - 3x^3 + \frac{4}{x}$ and take the derivative of this to get $20x^3 - 9x^2 - 4x^{-2}$.

Example

Determine the point(s) at which the given function has a horizontal tangent line. $f(x) = x^3 - 12x$

This is where the slope is 0, of $f'(x) = 0$.

So just plug in $0 = 3x^2 - 12$, and get the x values.

Plug those x values in to get y values, the points are $(2, -16)$ and $(-2, 16)$.

The derivative of $\sin x$ is $\cos x$, the derivative of $\cos x$ is $-\sin x$.

Exercise Find the derivative of $f(x) = 6x^2 + 5\cos x$

Example

$f(x) = x^3 - 3x^2 + 4$.

(a) Find the average rate of change of f on $[1, 5]$.

The slope, so 13.

(b) Find the instantaneous rate of change of f at $x = 3$.

The derivative, you should get 9.

Example

Find k so that the function $f(x) = x^2 + kx$ will be tangent to the line $y = 2x - 9$.

Let $x^2 + kx = 2x - 9$, so $k = 2 - 2x$.

Then we can have $x^2 + (2 - 2x)x = 2x - 9$ and $-x^2 = -9$. $x = \pm 3$, so $k = -4$ or $k = 8$.

Exercise Sketch the graph of a function f such that $f'(x) < 0$ for all x and the rate of change of the function is increasing.

Example

Find a and b so that f is differentiable everywhere.

$$f(x) = \begin{cases} ax^2 + 1, x \leq 2 \\ bx - 3, x > 2 \end{cases}$$

We have $ax^2 + 1 = bx - 3$ and when $x = 2$ we get $4a + 1 = 2b - 3$.

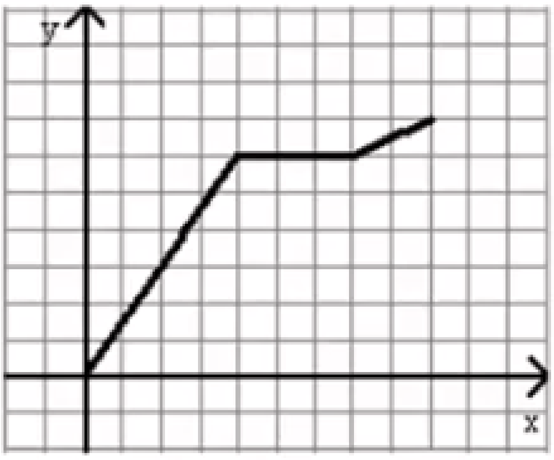
Solving for b we get $2ax$, so plugging this in we get $4a = b$. We have everything now.

Solve for a to get $a = 1$, and $b = 4$.

Exercise At time $t = 0$, a diver jumps from a diving board that is 32 ft above the water with an initial velocity of 16 ft/sec. Use the position function $s(t) = -16t^2 + v_0 t + s_0$, where v_0 is initial velocity and s_0 is initial velocity.

- (a) When the diver hit the water?
- (b) Find the instantaneous velocity when $t = 1$ seconds.
- (C) Find the average velocity on the interval $[1, 2]$.

Exercise The graph of a position function is shown. Sketch the velocity function on $(0, 9)$.



Position function

2.5 Product and Quotient Rules

Product Rule: $d[f(x) \cdot g(x)] = fg' + gf'$

Example

Find y' if $y = x^3 \cos x - 2 \sin x$.

Do the product rule on the first term and then just do the derivative of the second term.

$y' = x^3(-\sin x) + \cos x(3x^2) - 2\cos x$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{gf' - fg'}{g^2}$.

Exercise Find the derivative of $y = \frac{\sin x}{x^3}$.

Example

Find the derivative of $f(x) = \frac{x^2+2}{\sqrt[4]{x}}$.

Don't do the quotient rule. The derivative is $\frac{7}{4}x^{3/4} - \frac{2}{4}x^{-5/4}$

If you have a quotient in which the numerator is a constant, it's easier to rewrite it as a function with a negative exponent and use the power rule.

Exercise Find the derivative of $y = \frac{9}{5x^2}$

You can also use the quotient rule to derive the derivative of $\tan x$: you get $\sec^2 x$.

Likewise a similar process can be used to find the derivative of $\sec x$: you get $\tan x \sec x$.

The derivative of $\cot x$ is $-\csc x$ and the derivative of $\csc x$ is $-\csc x \cot x$.

Exercise Find the derivative of $f(x) = 3x^2 \tan x$.

Exercise Given $f(x) = g(x)h(x)$. Find $f'(2)$ if $g(2) = 3, g'(2) = -4, h(2) = -1$, and $h'(2) = 5$.

2.6 Chain Rule

In algebra, you learned how to find a composition of two functions $f(x)$ and $g(x)$, which was written symbolically as $f(g(x))$.

In Calculus, we use the chain rule when we need to find the derivative of a composition of functions.

If $y = (u)^n$, then $y' = n(u)^{n-1}\frac{du}{dx}$.

Example

Find the derivative of $f(x) = (4x^3 + 3x - 2)^5$.

Chain Rule: $5(4x^3 + 3x - 2)^4(12x^2 + 3)$.

Exercise Find the derivative of $y = (x^4 + 2)^(2/3)$

Exercise Find the derivative of $y = \frac{-7}{(2x-3)^2}$

Example

Find the derivative of $g(x) = \frac{5x}{\sqrt[3]{x^2+2}}$

You have to do the quotient rule as well as the chain rule. You end up with

$\frac{(x^2+2)^{1/3}(5)-5(\frac{1}{3}(x^2+2)^{-2/3}(2x))}{((x^2+2)^{1/3})^2}$

Here are some more exercises for chain rule.

Exercise Find the derivative of $y = \cos(3x)$.

Exercise Find the derivative of $f(x) = sin(5x^2)$.

Exercise Find the derivative of $y = \csc(7x^5)$.

Example

Find the derivative of $y = \cos^4(5x)$.

This can be written as $(\cos(5x))^4$ and we can chain rule this.

We get $4(\cos(5x))^3(-\sin(5x)) \cdot 5$.

Exercise Find the derivative of $f(x) = \sec^2(4x^3 + 5)$.

Exercise Find the derivative of $y = \tan(\sin x)$

Exercise Find $h'(3)$ if $h(x) = f(g(x))$ if $f(3) = 2, g(3) = 4, f(4) = -6, f'(3) = -7, g'(3) = -5, f'(4) = 8$.

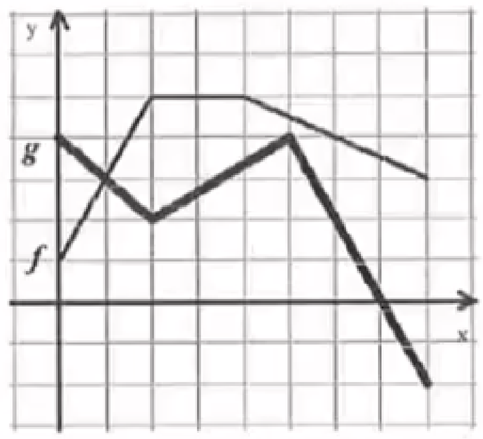
Exercise Find the second derivative of $f(x) = 4(x^2 - 5)^3$.

Exercise Find the point(s) where f has a horizontal tangent, when $f(x) = \frac{x}{\sqrt{2x-1}}$.

Example

Use the graphs of f and g to find the following, if they exist.

(a) $h(x) = f(x)g(x)$. Find $h'(1)$



$h'(1) = f(1)g'(1) + g(1)f'(1) = 3.$

(b) $k(x) = \frac{f(x)}{g(x)}$. Find $k'(6)$.

Use the quotient rule to get $9/4$.

Exercise These next two parts are a continuation of the above.

- (c) $p(x) = f(g(x))$. Find $p'(6)$.
- (d) $t(x) = g(f(x))$. Find $t'(7)$.

2.7 Implicit Differentiation

Examples of explicitly defined functions are

- $y = x^2 - 5x + 7$
- $f(x) = \sqrt{x^2 + 5}$

Examples of implicitly defined functions:

- $x^2 + xy - y^2 = 3$
- $\cos x \sin y = \frac{\sqrt{3}}{4}$
- $y^3 + y^2 - x^2 = -4$

Example

Given $y^3 + y^2 - x^2 = -4$

(a) Differentiate with respect to t. Since you must apply the Chain Rule, each derivative will a d/dt as part of the derivative.

This will end up being $3y^2\frac{dy}{dt} + 2y\frac{dy}{dt} - 2x\frac{dx}{dt} = 0$

(b) Differentiate with respect to w.

The exact same, but with w on the bottom. $ey^2\frac{dy}{dw} + 2y\frac{dy}{dw} - 2x\frac{dx}{dw} = 0$

(c) Now differentiate with respect to x

You get $3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 2x = 0$.

You can see above you can find $\frac{dy}{dx}$ easily from this, it is $\frac{2x}{3y^2+2y}$.

Exercise Find the derivative of $x^2 + xy - y^2 = 3$

Exercise Find the derivative of $\cos x \sin y = \frac{\sqrt{3}}{4}$

Example

Consider the curve given by $y^3 + y^2 - 5y - x^2 = -4$.

(a) Find $\frac{dy}{dx}$

You should get $y' = \frac{2x}{3y^2+2y-5}$

(b) Write the equation of the tangent line to the curve at the point $(1, -3)$.

Plug in what you have to get $y + 3 = \frac{1}{8}(x - 1)$.

(c) Find the coordinates of the point(s) on the curve where the line tangent to the curve is vertical.

The Bottom has to equal 0 of the derivative, and the top cannot be 0.

$3y^2 + 2y - 5$ will give you points for y values. Find your points from this.

3 Applications of Differentiation

3.1 Definition of the Derivative Meets Derivative Rules

3.2 Related Rates

3.3 Extrema on an Interval

3.4 Mean Value Theorem and Rolle's Theorem

3.5 Increasing and Decreasing Functions and the First Derivative Test

3.6 Concavity and the Second Derivative

3.7 Second Derivative Test

3.8 Graphs of f, f', and f"

4 Particle Motion and Integration

4.1 Particle Motion

4.2 Optimization

4.3 Antiderivatives and Indefinite Integrals

4.4 Integration using U-Substitution

4.5 Integration and Area Under a Curve

4.6 Riemann Sums and the Definite Integral

4.7 Fundamental Theorem of Calculus

4.8 U-Substitution with Definite Integrals

4.9 Another Kind of U-Substitution

4.10 Fundamental Theorem of Calculus

4.11 Average Value of a Continuous Function

5 Integration with Data, Functions Defined by Integrals, and Natural Logs

5.1 Integration Using Data

5.2 Second Fundamental Theorem of Calculus

5.3 Natural Logs and Differentiation

5.4 The Natural Log Function and Integration

5.5 Derivatives of Inverse Functions

5.6 Exponential Functions

5.7 Bases other than e

5.8 Inverse Trig Functions and Differentiation

5.9 Inverse Trig Integration

6 Differential Equations

6.1 Differential Equations

6.2 Euler's Method

6.3 Exponential Growth and Decay

7 Area between Curves, Volume, and Arc Length

7.1 Area Between Two Curves

7.2 Volume with Known Cross Sections

7.3 Volume: The Disc Method

7.4 Arc Length

8 Techniques of Integration

8.1 Integration by Parts

8.2 Integration by Partial Fractions

8.3 Logistic Growth

9 Improper Integrals, Sequences and Series, Taylor Polynomials, and Taylor Series

9.1 Improper Integrals

9.2 nth Term Test

9.3 Geometric Series and Telescopic

9.4 Integral Test and p-Series

9.5 Comparison of Series

9.6 Alternating Series

9.7 Ratio Test

9.8 Taylor Polynomials

9.9 Radius and Interval of Convergence

9.10 Taylor Series

9.11 Lagrange Error Bound

9.12 More on Error

10 Parametrics, Vectors, and Polar

10.1 Parametric Equations: The Basics

10.2 Vectors and Motion along a Curve

10.3 Polar Coordinates and Polar Graphs

10.4 Area Bounded by a Polar Curve

10.5 Notes on Polar

10.6 More on Polar Graphs