AP Physics C: Electricity & Magnetism Notes

anastasia

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1 Electric Charges, Fields and Gauss's Law

Brief Calculus Review

The derivative of a function at some point characterizes the rate of change of the function at that point; The rate of change of the function is basically the slope at that point.

Because the derivative is a slope, the notation can be written as

$$f'(x) = \frac{\mathrm{d}x}{\mathrm{d}t}$$

There are some derivative rules to know.

- $\frac{\mathrm{d}}{\mathrm{d}x} = 0$
- $\frac{\mathrm{d}}{\mathrm{d}x}(x) = C$
- $\frac{\mathrm{d}}{\mathrm{d}x}x^n = nx^{n-1}$
- $\frac{\mathrm{d}}{\mathrm{d}x}(f(x) \pm g(x)) = \frac{\mathrm{d}}{\mathrm{d}x}f(x) \pm \frac{\mathrm{d}}{\mathrm{d}x}g(x)$

An integral is simply finding the area under a curve.

The integral notation is:

$$f(x) = \int f'(x)\mathrm{d}x$$

In physics, we use the definite integral, where the area is found over an interval $[a, b]$.

The notation for this is:

$$A = \int_b^a f'(x)\mathrm{d}x = f(b) - f(a)$$

There are some integration rules to know as well.

- $\int \mathrm{d}x = x + C$
- $\int x^n \mathrm{d}x = \frac{1}{n+1}x^{n+1}, n \neq -1$
- $\int f(x) \pm g(x)\mathrm{d}x = \int f(x)\mathrm{d}x \pm \int g(x)\mathrm{d}x$

A differential equation is an equation involving one or more derivatives of an unknown function. The order of a differential equation is defined to be the order of the highest derivative it contains.

All differential equations are considered to be separable and can be solved by integration. This process is called separation of parts.

Much like derivatives there are set of integrals that don't follow the basic power rule of integration. There are some special integral rules.

- $\int e^{ax}\mathrm{d}x = \frac{1}{a}e^{ax} + C$
- $\int \frac{\mathrm{d}x}{x+n} = \ln|x+a| + C$
- $\int \cos(ax)\mathrm{d}x = \frac{1}{a}\sin(ax)$
- $\int \sin(ax)\mathrm{d}x = -\frac{1}{a}\cos(ax)$

Integration by substitution is a way of undoing the derivative's chain rule. You need the integral to look like this:

$$\int f(g(x))g'(x)\mathrm{d}x = \int f(u)\mathrm{d}u$$

Here are some special derivatives:

- $\frac{\mathrm{d}}{\mathrm{d}x}e^{ax} = ae^{ax}$
- $\frac{\mathrm{d}}{\mathrm{d}x}\ln ax = \frac{1}{x}$
- $\frac{\mathrm{d}}{\mathrm{d}x}\sin ax = a\cos ax$
- $\frac{\mathrm{d}}{\mathrm{d}x}\cos ax = -a\sin ax$

All derivatives on the formula sheet are written using the chain rule.

The chain rule follows this general rule:

$$f(g(x)) = f'(g(x)) \cdot g'(x)$$

A vector is a quantity that has both magnitude and direction. The length of the line shows its magnitude and the arrowhead points in the direction. To add vectors, place the tip of the first vector to the tail of the second vector. The resultant is the arrow drawn from the tail of the first vector to the tip of the second vector.

The goal of subtracting vectors is to turn it into addition by finding the inverse of the second vector. Basically:

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

When adding or subtracting vectors algebraically, the first thing you need to do is to resolve the vectors into components.

- $A_x = A\cos\theta$
- $A_y = A\sin\theta$

Once all the vectors are broken down, you can add the horizontal and vertical components. This will give you the horizontal and vertical components of the resultant.

To find the magnitude of the resultant, you can find the hypotenuse:

$$R = \sqrt{R_x^2 + R_y^2}$$

.

The direction can be found from:

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

There are times when you need to "scale" up or down a vector. To do so, you multiply the magnitude of a vector, but not the direction, by a scalar.

A unit vector has a magnitude of 1 and a direction that goes along one of the axes.

The dot product is the process of multiplying two vectors and getting a scalar answer in return. There are two ways to find this:

- $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$
- $\vec{a} \cdot \vec{b} = |a||b|\cos\theta$

The second method also helps determine if the vectors are orthogonal, or perpendicular to each other.

The cross product is the process of multiplying two vectors and getting a vector in return. The answer is a vector that is at a right angle to the two original vectors. The magnitude of the cross product equals the area of the parallelogram with the two original vectors as sides.

The cross product is zero in length when the original vectors point in the same or opposite directions. It reaches maximum length when the original vectors are at right angles to each other.

There are two ways to calculate the cross product.

The first is:

$$\vec{a}\text{x}\vec{b} = [|a||b|\sin\theta]\hat{n}$$

This method does not give you the direction of the vector.

The second way is to use a set of formulas to find the components:

- $C_x = a_y b_2 - a_2 b_y$
- $C_y = a_2 b_x - a_x b_2$
- $C_z = a_x b_y - a_y b_x$

The direction is determined by the right hand rule. Your index finger points in the direction of vector a, your middle points in the direction of b, and your thumb points in the direction of the answer.

1.1 Electric Charge and Electric Force

Electric charge is a fundamental property of all matter.

Charge is scalar value, which means it has no direction, and is described as either positive or negative.

The magnitude of charge on a single electron is the elementary charge which is $e = 1.6 \times 10^{-19}$ C (coulomb). The coulomb is the unit of charge.

Coulomb's Law describes the electrostatic force between two charges objects. The equation for this is:

$$F_E = \frac{kq_1q_2}{r^2}$$

This equation is similar to the universal gravitation formula. Note that r can be written sometimes as d, it is the distance between the centers. k is the electrostatic constant and is equal to 9×10^9N·m^2/C^2. k is sometimes written as $\frac{1}{4\pi\epsilon_0}$.

The direction of the electrostatic force depends on the signs. Opposite charges attract and like charges repel. Electrostatic force can also cause other forces like tension, friction, and normal force.

Electrostatic force can be attractive (different signs) or repulsive (same signs), while gravitational force, which is similar, can only be attractive.

The electrostatic force has a much larger magnitude than gravitational force, but gravitational force acts on a larger scale in that the electrostatic force works at a microscropic scale, while gravitational force will be on a planetary scale.

Free space (a region where there is no electromagnetic or gravitational fields) has a constant value of electric (or vacuum) permittivity which is equal to $\epsilon_0 = 8.85 \times 10^{-12}$C^2/(N·m^2).

Example

Point charges Q$_1$ = 2.0μC and Q$_2$ = -4.0μC are located at $\vec{r_1} = (4.0\hat{i} - 2.0\hat{j} + 5.0\hat{k})$m and $\vec{r_2} = (8.0\hat{i} + 5.0\hat{j} - 9.0\hat{k})$m. What is the force of Q$_2$ on Q$_1$?

We have the equation

$$F_E = \frac{kq_1q_2}{r^2}$$

We first have to find the distance between the charges. We can use the distance formula for this:

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z^1)^2}$$
$$r = \sqrt{(8-4)^2 + (5+2)^2 + (-9-5)^2}$$
$$r = 16.2\text{m}$$

Now we can plug this into the equation.

$$F_E = \frac{kq_1q_2}{r^2}$$
$$F_E = \frac{(9 \times 10^9)(2 \times 10^{-6})(-4 \times 10^{-6})}{(16.2)^2}$$
$$F_E = -2.74 \times 10^{-4}\text{N}$$

Note that since both point charges have opposite signs, they will try and attract each other, which means the resulting force calculated will be negative.

Exercise Two small spheres have charges of $2Q$ and $3Q$. When their centers are separated by a distance d, they exert an electrostatic force of magnitude F_0 on each other. What is the magnitude of the force exerted on two other spheres that have charges of $2Q$ and $9Q$ when their centers are a distance $2d$ apart?

Exercise Two point charges, both with charge Q, exert an electrostatic force of magnitude F_0 on each other when they are a distance d apart. What is the magnitude of the electrostatic force on two other point charges, each with a charge $2Q$, that are a distance $\frac{d}{3}$ apart?

1.2 Conservation of Electric Charge and the Process of Charging

The net charge or charge distribution of a system can change in response to the prescence of, or changes in, the net charge or charge distribution of other systems. For example, the net charge can change due to friction or contact between systems.

Induced charge separation occurs when electrostatic force between two systems alters the distribution or charges within the systems, resulting in the polarization of one or both systems. Induced charge separation can only occur in neutral systems.

Any change to a system's net charge is due to a transfer of charge between the system and its surroundings. Most of the time, this is the result of a transfer of electrons.

An application of this is grounding, which involves electricially connecting a charged object to a much larger and approximately neutral system (such as the Earth).

Example

There are two identical metal spheres on insulating stands. Sphere 1 has a charge of -1.02×10^{-16}C. Sphere 2 has a deficit of 841 electrons. The two spheres are brought together so that they touch each other. They are then separated again so that they are no longer touching. What charge does each sphere have after they have touched? Consider the new charge on Sphere 2. Does this correspond to a deficit or an excess of electrons? How many electrons is the deficit/excess?

First, we must find the charge on Sphere 2. Because the problem states that there is a deficit of electrons, the charge is positive.

The charge on Sphere 2 is:
$$841 \times (1.6 \times 10^{-19}) = 1.35 \times 10^{-16}\text{C}$$

Now we find the net charge on the system by adding it to the charge on Sphere 1:

$$1.35 \times 10^{-16} + (-1.02 \times 10^{-16}) = 3.3 \times 10^{-17}\text{C}$$

Now we simply divide this number by two since the charge is divded evenly between the two spheres:

$$\frac{3.3 \times 10^{-17}\text{C}}{2} = 1.6 \times 10^{-17}\text{C}$$

This answers the first part of the question.

The new charge is positive, which means there is an deficit of electrons. To find the amount of electrons in this deficit we must convert to electrons:

$$1.65 \times 10^{-17}\text{C} \times \frac{1e}{1.6 \times 10^{-19}\text{C}} = 103 \text{ electrons}$$

Exercise A student has a positively charged insulating cube and a neutral conducting sphere. The student can experiment by performing any of the following actions.

1. Touch the cube to the sphere.
2. Bring the cube near, but not touching the sphere.
3. Move the cube away from the sphere.
4. Connect the sphere to the ground.
5. Disconnect the sphere from ground.

Which actions, and in what order, could the student perform so that the sphere becomes negatively charged?

Exercise Two identical conducting spheres have different charges. One sphere has a charge of $+1$ nC, and the other sphere has a charge of -3 nC. The two spheres are brought into contact and then separated. What is the magnitude of charge that is transferred between the spheres? Write an accurate description of how to calculate the magnitude of the transferred charge.

Exercise A positively charged glass rod if brought near but does not touch a metal sphere. A wire is then used to ground the sphere while the charged rod remains close to it. The grounding wire is disconnected and then the charged rod is moved away from the sphere. What is the final net charge on the sphere and provide an explanation.

1.3 Electric Fields

Electric fields may originate from charged particles.

The electric field at a given point is the ratio of the electric force exerted on a test charge at the point to the charge and the charge on the test charge itself.

Mathematically this is:

$$E = \frac{F_E}{q}[N/C]$$

Another way of writing this is:

$$F_E = qE$$

The E-field points away from an isolated positive charge towards an isolated negative charge. Therefore, if the test charge is negative, the electric force points opposite the direction of the electric field.

Example

(a) Find the direction and magnitude of an electric field that exerts a 4.80×10^{-17} N westward force on an electron.

(b) What magnitude and direction force does this field exert on a proton?

For part (a), we use the formula:

$$E = \frac{F_E}{q} = \frac{4.80 \times 10^{-17}\text{ N}}{1.6 \times 10^{-19}\text{ C}} = 300\text{ N/C East}$$

For part (b), the proton and electron have the same charge, so they experience the same electric field. The force and the E-field because of the positive charge results in the force pointing in the same direction, so 300 N/C West.

Exercise Two spheres having charges of $+3Q$ and $-Q$ are separated by a distance d between their centers. What is the magnitude of the electric field at a location halfway between the centers of the two spheres?

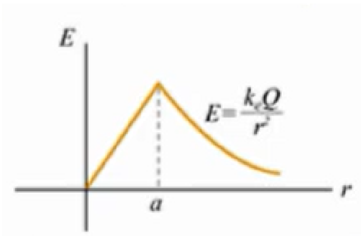
1.4 Electrostatic Equilibrium

Many problems on the FRQ section will involve conductors and insulators.

A conductor is an object or type of material that allows the flow or charge (electric current) in one or more directions. An insulator is a material in which electric current does not flow freely.

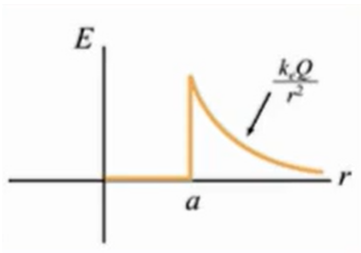
Electrostatic equilibrium occurs when there is no net motion of charge in an insulator or conductor.

When an insulator is in equilibrium, the excess charge of an insulator is distributed throughout the interior of the insulator as well as the surface. The electric field within the insulator may have a nonzero value.



In this, the peak is the surface of the insulator.

When in equilibrium, the excess charge on a conductor lies on the surface of the conductor, making the electric field equal to zero inside. The electric field is perpendicular to the surface of the conductor.



Example

A particle of charge 2.0×10^{-8}C experiences an upward force of magnitude 4.0×10^{-6}N when it is placed in a particular point in an electric field.

(a) What is the electric field at that point?

(b) If a charge $q = -1.0 \times 10^{-8}$C is placed there, what is the force on it?

For part A:

$$E = \frac{F_E}{q} = \frac{4.0 \times 10^{-6}\text{N}}{2.0 \times 10^{-8}\text{C}} = 200\text{N/C upward}$$

For part B:

$$F_E = gE = (1.0 \times 10^{-8}\text{C})(200\text{N/C}) = 2.0 \times 10^{-6}\text{N downwards}$$

1.5 Electric Fields of Charge Distributions

The expressions for the electric field of specified charge distributions can be found using integration and the principle of superposition.

Mathematically:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2}\hat{r}$$

Symmetry considerations of certain charge distributions can simplify analysis of the electric field resulting from those charge distributions.

Some common distributions are: Infinite Wire, Finite Wire, Semicircle (Arc), Ring of Charge

There are three densities - linear, area, and volume.

Linear:

$$\lambda = \frac{Q}{L} \longrightarrow \lambda = \frac{dQ}{dL} \longrightarrow dQ = \lambda dL$$

Area:

$$\sigma = \frac{Q}{A} \longrightarrow \sigma = \frac{dQ}{dA} \longrightarrow dQ = \sigma dA$$

Volume:

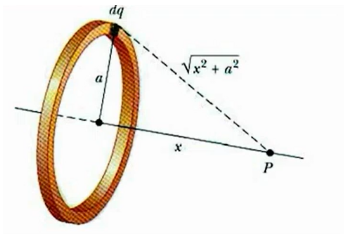
$$\rho = \frac{Q}{V} \longrightarrow \rho = \frac{dQ}{dV} \longrightarrow DQ = \rho dV$$

At a point charge:

$$E_{PC} = \frac{kQ}{r^2} \therefore E = \frac{1}{4\pi\epsilon_0}\int \frac{dQ}{r^2}\hat{r}$$

Example

Derive an expression for the electric field at the center of a ring of charge with radius a a distance P away from the center.



Note that $\sqrt{x^2+a^2}$, a, and x are constant.

Looking at point P, there will be an infinite amount of electric fields caused by dq at point P.

All the y-components will cancel, so we will integrate through x.

$$E_{net} = dE_x = dE\cos\theta$$
$$E_x = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2}$$

We want to substitute $\frac{x}{\sqrt{x^2+a^2}}$ for $\cos\theta$.

$$\begin{aligned}E_x &= \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2}\cos\theta \\ &= \frac{1}{4\pi\epsilon_0}\int \frac{dq}{\sqrt{x^2+a^2}}\cdot\frac{x}{\sqrt{x^2+a^2}} \\ &= \frac{x}{4\pi\epsilon_0(x^2+a^2)^{3/2}}\int dQ \\ &= \frac{qx}{4\pi\epsilon_0(x^2+a^2)^{3/2}}\end{aligned}$$

This is the expression for the ring of charge.

Exercise Two charged rings are concentric and have radii R and $2R$. Point P is located on a line that passes through the common center of the rings and is perpendicular to the plane of the rings. Each ring is uniformly charged, initially with charges $+Q$ and $-Q$ on the ring of radius $2R$ and the ring of radius R, respectively. If each ring is given the opposite charge from what it had initially, what happens to the electric field magnitude E at point P? What evidence or reasoning supports the claim?

1.6 Electric Flux

Flux describes the amount of a given quantity that passes through a given area.

For an electric field that is constant across an area, the electric flux through the area is defined as:

$$\phi_E = \vec{E}\cdot\vec{A} = EA\cos\theta$$

The direction of the area vector is defined as perpendicular to the plane of the surface and outward from closed surface.

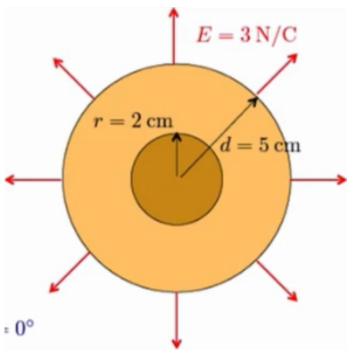
The sign of the flux is given by the dot product of the electric field vector and the area vector.

The total electric flux passing through a surface is defined by the surface integral of the electric field over the surface:

$$\phi_E = \int \vec{E}\cdot \mathrm{d}\vec{A} = EA$$

Example

A sphere of radius $r = 2$ cm creates an electric field $E = 3$N/C at a distance $d = 5$ cm from the center of the sphere. What is the electric flux through the surface of the sphere drawn at a distance $d = 5$cm?



The surface area of a sphere is $4\pi r^2$, so

$$\phi_E = \int \vec{E}\cdot \mathrm{d}\vec{A} = EA = E(4\pi r^2) = (3)(4\pi(0.05)^2) = 0.094\text{N}\cdot\text{m}^2/\text{C}$$

Exercise A positive point charge is located at the center of a sphere of variable radius r. Draw a graph that indicates the electric flux through the surface of the sphere as a function of its radius.

1.7 Gauss's Law

Gauss's law relates electric flux to a Gaussian surface to the charge enclosed by that surface:

$$\phi_E = \frac{q_{\text{enc}}}{\epsilon_0} = \oint \vec{E} \cdot \mathrm{d}\vec{A} = EA$$

A gaussian surface is a three-dimensional, closed surface.

The total electric flux through the surface is independent of the size of the Gaussian surface if the amount of enclosed charge remains constant.

Surfaces are constructed such that the electric field generated by the enclosed charge is either perpendicular or parallel to different regions of the Gaussian surface.

If a function of charge density is given for a charge distribution, the total charge can be determined by integrating the charge density of the length (1D), area (2D), or volume (3D).

Maxwell's equations are the collection of equations that fully describe electromagnetism. The first of these is Gauss's Law.

Example

A spherical cloud of charge radius R contains a total charge $+Q$ with a nonuniform charge density that varies according to the equation:

$$\rho(r) = \rho_0 \left(1 - \frac{r}{R}\right) \text{ for } r \leq R \text{ and}$$
$$\rho = 0 \text{ for } r > R,$$

where r is the distance from the center of the cloud. Express all algebraic answers in terms of Q, R, and fundamental constants. Determine the magnitude E of the electric field when $r > R$.

$$\frac{q_{\text{enc}}}{\epsilon_0} = \int \vec{E} \cdot \mathrm{d}\vec{A} = EA = E(4\pi r^2)$$
$$\implies E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2}$$

Exercise A solid insulating sphere of radius 10 cm has a net charge of 40 nC distributed uniformly throughout its volume. What is the electric field at a distance of 2.0 cm from the cneter of the sphere?

Exercise A very long solid insulating cylinder of radius R has a charge uniformly distributed over its volume, such that its overall linear charge is λ. What is the magnitude of the electric field at a perpendicular distance $r \leq R$ from the cylinder's central axis?

2 Electric Potential

2.1 Electric Potential Energy

The electric potential energy of a system of two point charges equals the amount of work required for an external force to bring point charges to their current positions from infinitely far away.

The general form of the electric potential energy between two charged objects is given by the equation:

$$U = \frac{kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1q_2}{r}$$

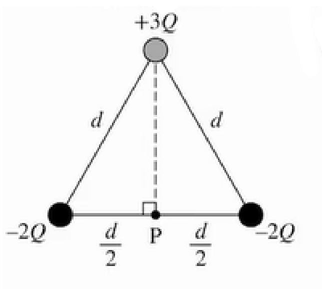
The total electric potential energy of a system can be determined by finding the sum of the electric potential energies of the individual interactions between each pair of charged objects in the system.

When there are opposite signs, the U value will decrease when close together.

When there are same signs, the U value will increase when close together.

Example

Derive an expression for the work required to assemble the charges in the configuration shown.



Remember that

$$W = \Delta U = \sum U$$

So we have:

$$\frac{kq_1q_2}{d} + \frac{kq_1q_3}{d} + \frac{kq_2q_3}{d} = k\left[\frac{(-2Q)(3Q)}{d} + \frac{(-2Q)(-2Q)}{d} + \frac{(-2Q)(3Q)}{d}\right] = 8Q$$

Exercise Two point charges, one with positive charge $+2Q$ and one with negative charge $-Q$, are fixed in a distance D apart. The electric potential energy of the two-point charge system is U_0. If a third, positive, point charge is added to the system, where should the point charge be located so that the electric potential energy of the three-point charge system is still U_0? What evidence or reasoning supports the claim?

Exercise Consider four different systems consisting of a pair of point charges. The point charges in the systems have the following charge values and separation distances.

- System 1: point charges of $+1$ nC and $+1$ nC, separated by 1 cm
- System 2: point charges of $+1$ nC and $+4$ nC, separated by 4 cm

- System 3: point charges of -1 nC and +6 nC, separated by 1 cm
- System 4: point charges of +2 nC and +2 nC, separated by 2 cm

Rank the electric potential energies U_1, U_2, U_3. and U_4 of the four systems. Take the potential energy of a system to be zero when the point charges are infinitely apart.

Exercise A system is comprised of two positive point charges, which have charge values of 1 nC and 2 nC. How does the amount of work done on the system by an external force or forces for the following three scenarios?

- Scenario 1: W_1 is the work done by an external force to move the 1 nC point charge from far away to a distance of 1 cm from the 2 nC point charge, which is fixed in place.
- Scenario 2: W_2 is the work done by an external force to move the 2 nC point charge from far away to a distance of 1 cm from the 1 nC point charge, which is fixed in place.
- Scenario 3: W_3 is the total work done by external forces to move both point charges simultaneously from far away to locations that are a distance of 1 cm from each other.

In each scenario, the initial positions of any point charges that get moved can be considered to be infinitely far away.

2.2 Electric Potential

Electric potential describes the electric potential energy per unit of charge at a point in space.

Expressions for the electric potential of charge distributions can be found by using integration and the principle of superposition:

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{\mathrm{d}q}{r}$$

If there are multiple point charges, we just add up all the point charges.

The electric potential difference between two points is the change in the electric potential energy per unit charge when a test charge is moved between two points:

$$\Delta V = \frac{\Delta U_E}{q}$$

The value of the electric field component in any direction at a given point is equal to the negative of the rate of change in electric potential at that location:

$$E_x = -\mathrm{d}V/\mathrm{d}x$$

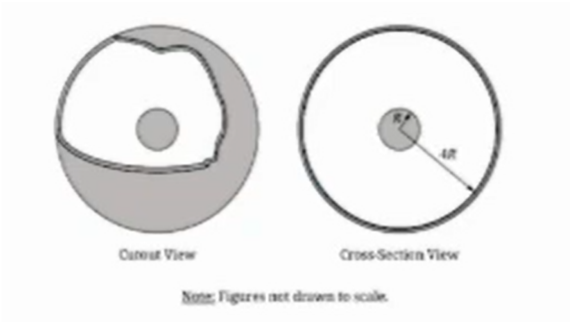
The change in electric potential between two points can be determined by integrating the dot product of the electric field and the displacement along the path connecting the points:

$$\Delta V = V_b - V_a = -\int \vec{E} \cdot \mathrm{d}\vec{r}$$

Equipotential lines represent lines of equal potential energy. These lines are perpendicular to the electric field vectors. Electric field vectors point in the direction of decreasing potential. There is no component of an electric field along an equipotential line.

Example

Derive an expression for the absolute value of the potential difference between the outer surface of the sphere and the inner surface of the shell. Express your answer in terms of Q, R, and physical constants, as appropriate.



Note: Figures not drawn to scale.

Previously we would have found the electric field is:

$$E = -\frac{Q}{4\pi\epsilon_0 r^2}$$

So we must integrate:

$$\Delta V = -\int_R^{4R} \vec{E} \cdot \mathrm{d}\vec{R}$$
$$\Delta V = +\int_R^{4R} +\frac{Q}{4\pi\epsilon_0 r^2}\mathrm{d}r$$
$$\Delta V = \frac{Q}{4\pi\epsilon_0}\int_R^{4R} \frac{\mathrm{d}r}{r^2}$$
$$\Delta V = \frac{Q}{4\pi\epsilon_0}\left[+\frac{1}{r}\right]$$

Applying the limits of integration we get:

$$\Delta V = \frac{3Q}{4\pi\epsilon_0 R}$$

Exercise Two point charges are located on the x-axis. A -4.0 nC point charge is at $x = -0.20$ m, and a $+5.0$ nC point charge is at $x = +0.10$ m. What is the electric potential on the x-axis at $x = 0.00$ m? Take the potential to be zero at an infinite distance from the point charges.

2.3 Conservation of Electric Energy

When a charged object moves between two locations with different electric potentials, the resulting change in the electric potential energy of the object-field system is given by:

$$\Delta U_E = q\Delta V$$

The movement of a charged object between two points with different electric potential results in a change in kinetic energy of the object consistent with the conservation of energy.

Example

A proton (mass $= 1.67 \times 10^{-27}$ kg) is accelerated through a potential difference of 4.5×10^6 V. (a) How much kinetic energy has the proton acquired? (b) If the proton started at rest, how fast is it moving?

For part A, we have

$$\Delta U = q\Delta V = \Delta K = K - K_0$$
$$= (1.6 \times 10^{-19})(4.5 \times 10^6 \text{V})$$
$$= 7.2 \times 10^{-13}\,\text{J}$$

For part B:

$$K = \frac{1}{2}mv^2$$
$$V = \sqrt{\frac{2K}{m}}$$
$$= \sqrt{\frac{2(7.2 \times 10^{-13})}{1.67 \times 10^{-27}}} = 2.94 \times 10^7 \text{m/s}$$

Exercise A proton in an electric field is initially located at point P_1 on the x-axis. The proton is given an initial velocity in the positive x-direction, moves in a straight line along the axis, and continually slows down and comes instantaneously to rest at point P_2. What can be concluded about the electric potential at points P_1 and P_2, and what evidence or reasoning supports the claim?

Exercise An electron is accelerated from rest through a potential difference of ΔV_0, reaching a speed of v_1. A different electron is accelerated from rest through a potential difference of $4\Delta V_0$, reaching a speed of v_2. The ratio of $\frac{v_2}{v_1}$ is?

3 Conductors and Capacitors

3.1 Electrostatics with Conductors

An ideal conductor is a material in which electrons are able to move freely.

When a conductor is in electrostatic equilibrium, mutual repulsion of excess charge carriers results in those charge carriers residing entirely on the surface of the conductor.

- In a conductor with a negative net charge, excess electrons reside on the surface of the conductor.
- In a conductor with a positive net charge, the surface becomes deficient in electrons and can be modeled as if positive charge carriers reside on the surface of the conductor.

Excess charges will move to the surface of a conductor to create a state of electrostatic equilibrium within the conductor.

- At electrostatic equilibrium, the electroc potential of the surface is the same everywhere and the conductor becomes an equipotential surface.

Recall there is no electric field inside of a conductor.

The charge density on the surface is greater where there are points or edges compared to planar areas.

Electrostatic shielding is the process of an area with a closed conducting shell to create a region inside the conductor that is far from external electric fields.

Example

A solid, uncharged conducting sphere of radius $3a$ contains a hollowed spherical region of radius a. A point charge $+Q$ is placed at the common center of the spheres. Taking $V = 0$ as r approaches infinity, the potential at position $r = 3a$ from the center of the sphere is:

We start with (treat as a point charge)

$$V = \frac{kq}{r}$$

and end up getting

$$V = \frac{k[Q]}{3a}$$

which is equal to

$$V = \frac{kQ}{3a}$$

Exercise A spherical conductor of radius r is given a charge $+Q$. What is the electric potential inside the spherical conductor at half of its radius and why?

3.2 Redistribution of Charge between Conductors

When conductors are in electrical contact, charges will be redistributed such that the surfaces of each conductor are at the same electric potential.

Ground is an idealized reference point that has zero electric potential and can absorb an infinite amount of charge without changing its electric potential. Charge can be induced by a conductor by grounding the conductor is the presence of an external electric field.

Example

Charge is placed on two conducting spheres that are very far apart and connected by a long thin wire. The radius of the smaller sphere is 5 cm and that of the larger sphere is 12 cm. The electric field at the surface of the larger sphere is 358 kV/m. Find the surface charge density on each sphere.

We start with electric field:

$$E = \frac{kQ}{R^2}$$

$$Q = \frac{ER^2}{k} = \frac{(358 \times 10^3 \text{V/m})}{(0.12\text{m})^2} 9 \times 10^9 = 5.78 \times 10^{-8}\text{C}$$

We know that $V_1 = V_2$, so

$$\frac{kQ_1}{R_1} = \frac{kQ_2}{R_2}$$

Rearranging for Q_2:

$$Q_2 = \frac{R_2}{R_1} Q_1 = \frac{5}{12}(5.78 \times 10^{-7}) = 2.4 \times 10^{-7}\text{C}$$

Now we can find the surface charge on each sphere.

$$\sigma_1 = \frac{Q_1}{A_1} \quad \sigma_2 = \frac{Q_2}{A_2}$$

So

$$\sigma_1 = 3.2 \times 10^{-6} C/m^2$$

and

$$\sigma_2 = 7.6 \times 10^{-6} C/m^2$$

Exercise A conducting sphere with net charge Q_0 and radius R is connected by a wire to an initially uncharged conducting sphere of radius $2R$. After electrostatic equilibrium has been reached, describe the electric potential at the surface of the smaller sphere.

Exercise Two conducting spheres are initially isolated. Sphere 1 has radius r_1 and is initially uncharged. Sphere 2 has radius r_2, such that $r_1 > r_2$ and has a charge of $+Q$. The two spheres are then connected by a conducting wire and allowed to reach equilibrium, resulting in a charge of Sphere 1. Represent the charge q on Sphere 1 after the two spheres have reached equilibrium.

3.3 Capacitors

A parallel-plate capacitor consists of two separated parallel conducting surfaces that can hold equal amounts of charge with opposite signs.

Capacitance relates the magnitude of the charge stored on each plate to the electric potential difference created by the separation of those charges.

$$C = Q/\Delta V$$

Unit = Farads

$$C = \frac{\kappa \epsilon_0 A}{d}$$

The electric field between the two charged plates with uniformly distributed electric charge is constant in both magnitude and direction, except nera the edges of the plates.

$$E = \frac{Q}{\epsilon_0 A} = \sigma/\epsilon_0$$

The electric potential stored in a capacitor is equal to the work done by an external force to separate that amount of charge on the capacitor.

$$U_c \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2} \frac{Q^2}{C}$$

Example

A capacitor with circular parallel plates of radius R that are separated by a distance d has a capacitance of C. What would the capacitance be if the plates has radius $2R$ and were separated by a distance $d/2$?

$$C_0 = \frac{\epsilon_0 A}{d}$$
$$C_0 = \frac{\epsilon_0 \pi r^2}{d} = \frac{\epsilon_0 \pi R^2}{d}$$

$$C = \frac{\epsilon_0 A}{d}$$
$$C = \frac{\epsilon_0 \pi r^2}{d}$$
$$C = \frac{\epsilon_0 \pi (2R)^2}{d/2} = \frac{8\epsilon_0 \pi R^2}{d}$$

Exercise A parallel-plate capacitor with plates of area A and plate separation d is attached to a bettery and given a charge Q. The potential energy stored in the capacitor is U_1. The capacitor is then detached from the battery and then the plates are pulled apart to a distance of $2d$. The potential energy stored in the capacitor is now U_2. Describe the ratio of the potential energies $\frac{U_1}{U_2}$.

Exercise An isolated, charged parallel-plate capacitor has charge Q and the absolute value of the potential difference across the plates is $|\Delta V|$. A slab of conductive material is inserted between the plates such that it fills half of the distance between the plates. Describe the change, if any, in Q and $|\Delta v|$ as the conductive material is inserted between the plates.

3.4 Dielectrics

In a dielectric material, electric charges are not as free to move as they are in a conductor.

The material becomes polarized in the presence of an external electric field.

The dielectric constant of a material relates the electric permittivity of that material to the permittivity of free space.

$$\kappa = \epsilon/\epsilon_0$$

For a dielectric,

$$\int \vec{E} \cdot \mathrm{d}\vec{A} = \frac{Q}{\kappa \epsilon_0} = EA$$

The electric field created by a polarized dielectric is opposite in direction to the external field.

The electric field between the plates of an isolated parallel-plate capacitor decreases when a dielectric is placed between the plates.

$$\kappa = E_0/E$$

The insertion of a dielectric into the capacitor may change the capacitance of the capacitor.

$$C = \kappa C_0 = \frac{\kappa \epsilon_0 A}{d}$$

Example

A capacitor consists of two conducting, coaxial, cylindrical shells of radius a and b, respectively, and length $L >> b$. The space between the cylinder is filled with oil that has a dielectric constant κ. Initially both cylinders are uncharged, but then a battery is used to charge the capacitor, leaving charge $+Q$ on the inner cylinder and $-Q$ on the outer cylinder. Let r be the radial distance from the axis of the capacitor. Determine:

- The electric field from a to b.
- The electric potential from a to b.
- The capacitance of the capacitor.

a.

$$\int \vec{E}\mathrm{d}\vec{A} = \frac{Q}{\epsilon}$$
$$EA = \frac{Q}{\kappa\epsilon_0}$$
$$E = \frac{Q}{2\kappa\pi\epsilon_0 Lr}$$

b.

$$V = -\int_a^b \vec{E}\cdot\mathrm{d}\vec{r}$$
$$V = \int_b^a \left(\frac{Q}{2\kappa\epsilon_0\pi L}\frac{1}{r}\right)\cdot\mathrm{d}r$$
$$V = \frac{Q}{2\kappa\epsilon_0\pi L}\int_b^a \frac{1}{r}\mathrm{d}r$$
$$V = \frac{Q}{2\kappa\epsilon_0\pi L}[\ln r]_b^a$$
$$V = \frac{Q}{2\kappa\epsilon_0\pi L}ln\left[\frac{b}{a}\right]$$

c.

$$C = \frac{Q}{V}$$
$$C = \frac{2\kappa\epsilon_0\pi L}{\ln b/a}$$

Exercise An air-filled parallel plate capacitor has a capacitance of $C = 12$ pF. The space between the plates is filled with a dielectric, and the new capacitance of the capacitor is $C = 48$ pF. What is the dielectric constant for the dielectric?

Exercise A capacitor initially has a capacitance of C_1. If a dielectric with a dielectric constant of 3 is added between the plates of the capacitor the new capacitance is C_2. Expresse the ratio of the new capacitance C_2 to C_1.

4 Electric Circuits

4.1 Electric Current

Current is the rate at which charge passes through a cross-sectional area of a wire.

$$I = \mathrm{d}q/\mathrm{d}t \implies q = \int I \mathrm{d}t$$

Current within a conductor consists charge carriers traveling through the conductor with an average drift velocity.

$$I = nqv_D A$$

Electric charge moves in a circuit in response to an electron potential difference, sometimes referred to as electromotive force, or emf (ϵ)

Current density if the flow of charge per unit area.

$$I = \int \vec{J} \cdot \mathrm{d}\vec{A} \implies I = JA$$

Current density is related to the motion of the charge carriers within a conductor and is a vector quantity.

$$J = nqv_D$$

A potential difference across a conductor creates an electric field within the conductor that is proportional to the resistivity and the current density.

$$\vec{E} = \rho \vec{J}$$

If a function of current density is given, the total current can be determined by integrating the density over the area.

$$I = \int J(\vec{r}) \cdot \mathrm{d}\vec{A}$$

Although current is a scalr quantity, it does have direction.

- The direction of conventional current is chosen to be the direction in which positive charge would move.
- In common circuits, the current is actually due to the movement of electrons.

Example

A long conducting cylinder has radius R, and carries a current to the left. The current density varies with distance r from the cylinder's central axis according to the equation $J = kr^2$, where $r \leq R$ and k is a positive constant. Derive an expression for the total current in the cylinder.

$$I = \int \vec{J}(r) \cdot \mathrm{d}\vec{A}$$

$$= \int (kr^2)(2\pi r \mathrm{d}r) = \frac{\pi k}{2} R^4$$

Exercise Two different wires are both carrying uniform currents. Wire 1 has a cross-sectional area A_1, resistivity ρ_1 and a current I_1. Wire 2 has a cross-sectional area $2A_1$, resistivity $3\rho_1$, and a current $2I_1$. What is the ratio $E_2 : E_1$ of the electric magnitude in wire 2 to the electric field magnitude in wire 1?

Exercise Two conducting wires, Wire X and Wire Y, have the same length and cross-sectional area but are made of different materials. The charge carriers in two wires are electrons, and have densities in wires X and Y of 4.22×10^{22} cm^{-3} and 8.0×10^{22} cm^{-3} respectively. Wire Y has three times the current of Wire X. What is the ratio $\frac{v_y}{v_x}$ of the average drift velocity v_Y in wire Y to the average drift velocity v_X in wire X?

4.2 Simple Circuits

A circuit is composed of electrical loops, which can include wires, batteries, resistors, lightbulbs, capacitors, inductors, switches, ammeters, and voltmeters.

A closed electrical loop is a closed path through which charges may flow.

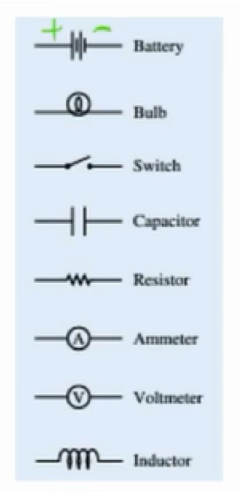
- A closed circuit is one in which charges would be able to flow.
- An open circuit is one in which charges would not be able to flow.
- A short circuit is one which charges would be able to flow with no change in potential difference.

Circuit schematics are representations used to describe and analyze electric circuits.

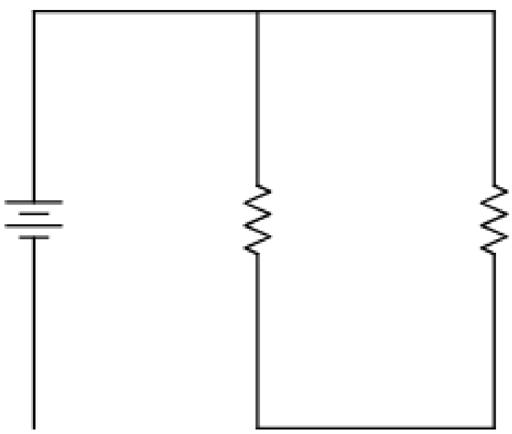
- The properties of an electric current are dependent on the physical arrangement of its constituent elements.

Circuit elements have common symbols that are used to create schematic diagrams.

- If an element is variable, then the element is indicated by a diagonal strikethrough arrow across the symbol.



Exercise



The figure shows a battery connected to two resistors. Is there current in the resistors? What evidence or reasoning supports the claim?

4.3 Ohm's Law and Electrical Power

- Resistance is a measure of the degree to which an object opposes the movement of electrical charge.

- It is proportional to its resistivity and length and is inversely proportional to its cross-sectional area.

$$R = \frac{\rho l}{A}$$

- This assumes the resistivity to be uniform.
- If the resistivity is not uniform, meaning it varies along the length, use:

$$R = \int \frac{\rho(l)}{A} \mathrm{d}l$$

- Ohm's Law relates current, resistance, and potential difference across a conductive element of current.

$$I = \frac{\Delta V}{R} \implies V = IR$$

- The resistivity of an ohmic material is constant regardless of temperature.
- The resistance of an ohmic circuit element can be determined from the slope of a graph of the current in the element as a function of the potential difference across the element.
- The rate at which energy is transferred, converted or dissipated by a circuit element depends on the current in the element and the electrical potential difference across it.
- The brightness of a lightbulb increases with power, so power can be used to qualitatively predict the brightness of lightbulbs in a circuit.

Example

Long cables can sometimes act like antennas, picking up electronic noise, which are signals from other equipment and applicances. Coaxial cables are used for many applications that require this noise to be eliminated. For example, they can be found in the home in cable TV connections or other audiovisual connections. Coaxial cables consist of an inner conductor of radius r_i surrounded by a second, outer concentric conductor with radius r_0. The space between the two is normally filled with an insulator such as polyethylene plastic. A small amount of radial leakage current occurs between the two conductors. Determine the resistance of a coaxial cable of length L.

$$R = \frac{\rho L}{A} \implies \mathrm{d}R = \frac{\rho}{A}\mathrm{d}r \implies R = \frac{P}{2\pi L}\int_{r_i}^{r_o} \frac{1}{r}\mathrm{d}r$$

Solving this results in $\frac{P}{2\pi L} \ln\left(\frac{r_0}{r_i}\right)$

Exercise A wire has a length of 0.5 m, a circular cross section of radius 2.0×10^{-4} m, and a resistance of 2.5Ω. What is the resistivity of the material used to make the wire?

Exercise A 10Ω resistor has a current that varies with time t according to the equation $I(t) = I_0 e^{-\alpha t}$, where $I_0 = 4$A and $\alpha = 2$ s^{-1}. What is the total energy dissipated in the resistor between $t = 0$ and a long time later?

4.4 Compound Direct Current Circuits

Circuit elements may be connected in series and/or parallel.

A series connection is one in which any charge passing through one circuit element must proceed through all elements in that connection and has no other path available.

The current in each series circuit element is the same.

A parallel connection is one in which chargesm ay pass through one of two or more paths.

Across each path, the potential difference is the same.

Ideal batteries and wires have negligible internal resistance.

If the battery is not ideal, then it has an internal resistance.

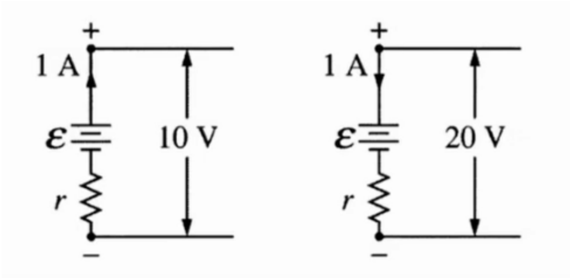
$$\Delta V_{\text{Terminal}} = \epsilon - IR$$

Ammeters are used to measure current at a specific point in a circuit. It must be connected in series in the circuit.

Voltmeters are used to measure the electric potential difference between two points in a circuit. They must be connected in parallel.

Example

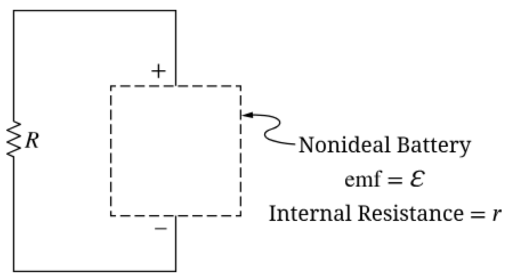
The figures below show parts of two circuits, each containing a battery of emf ϵ and internal resistance r. The current in each battery is 1 A, but the direction in one battery is opposite to that in the other. If the potential differences across the batteries' terminals are 10 V and 20 V as shown, what are the value of ϵ and r?



From the Kirchoff we have both $10 = \epsilon - (1)r$ and $-20 = -\epsilon + (1)r$.

Solving this system of equations gives $\epsilon = 15$ V and $r = 5\Omega$.

Exercise



The figure shows a circuit with resistor of resistance R connected to a nonideal battery. The battery has emf ϵ and an internal resistance r. What is the current generated by the battery?

Exercise Two batteries have the same emf, but one battery is ideal while the other battery is nonideal. The batteries are connected to identical resistors. Does the resistor connected to the nonideal battery have the same or less current compared to the resistor connected to the ideal battery? What evidence or reasoning helps support this claim?

4.5 Kirchoff's Rules

- Kirchoff's Rules quantify how current flows through a circuit and how voltage varies around a loop in a circuit.
- Kirchoff's Loop Rule is a consequence of the conservation of energy.
  - This is sometimes called Kirchoff's Voltage Law.

- It states that the sum of the potential differences across all circuit elements in a single closed loop must equal zero.

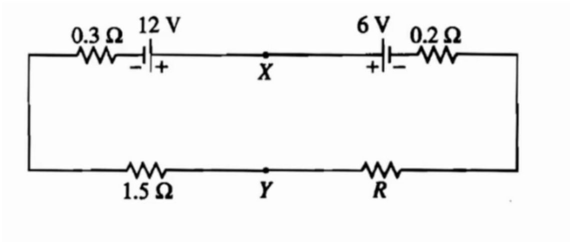
$$\sum V = 0$$

- Kirchoff's Junction Rule is a consequence of the conservation of charge.
  - This is sometimes called Kirchoff's Current Law.
  - It states the total amount of charge entering a junction per unit time must equal the total amount of charge exiting the junction per unit time.

$$\sum I = 0$$

Example

In the circuit below, the emf's and the resistances have the values shown. The current I in the circuit is 2 amperes. The potential difference between points X and Y are?

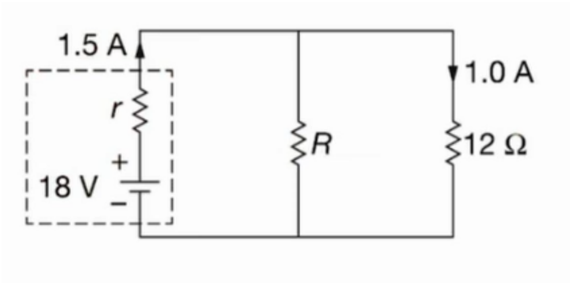


The voltage equation for this circuit is 12V - $6V$ - $(0.2)(2)$ - $2R$ - $(1.5)(2)$ - $0.3(2) = 0$. Solving this gives $R = 1$.

Plugging this back in we get that $6 + (0.2)(2) + (1)(2) = 8.4$ V.

Example

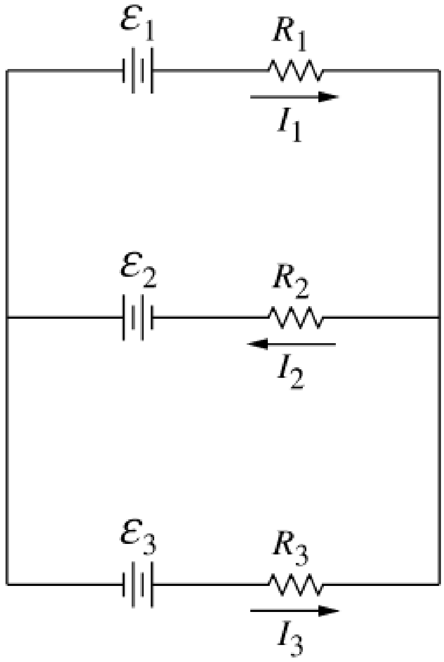
Two resistors of resistances R and 12Ω are connected to a battery of emf 18 V, as shown in the figure below. The battery has an internal resistance of r. The current in the battery is 1.5 A, and the current in the 12Ω resistor is 1.0 A. What is the resistance R?



We have two equations, the first being $18V - 1.5R - (1.0)(12) = 0$ and $-18V + 1.5R + 0.5R = 0$.

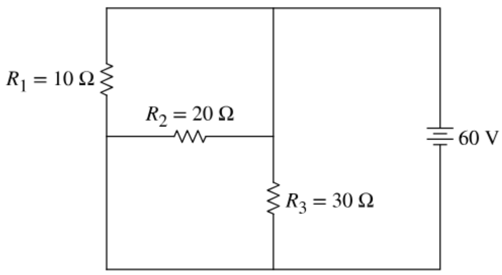
Solving for R we get that this is equal to 24Ω.

Exercise



The figure shows a circuit with three batteries and three resistors with the emf of each battery and the resistance of each resistor as indicated. Write an equation that expresses a Kirchoff's loop rule for a portion of the circuit.

Exercise



The figure shows three resistors connected to a battery, with the resistances and emf indicated. Rank the potential differences ΔV_1, ΔV_2, and ΔV_3 across resistors R_1, R_2, and R_3, respectively?

4.6 Resistor-Capacitor (RC) Circuits

A collection of capacitors in a circuit may be analyzed as though it was a single capacitor with an equivalent capacitance C_{ep}.

As a result of conservation of charge, each of the capacitors in series must have the same magnitude of charge on each plate.

The charge on a capacitor or the current in a resistor in a RC circuit can be described by a fundamental differential equation derived from Kirchoff's loop rule.

$$\epsilon = \frac{\mathrm{d}q}{\mathrm{d}t}R + \frac{q}{C}$$

The time constant (τ) is a significant feature of an RC circuit.

The time constant of an RC circuit is a measure of how quickly the capacitor will charge or discharge and is defined as

$$\tau = RC$$

Example

A relaxation oscillator is used to control a pair of windshield wipers. The relaxation oscillator consists of a 10.00 mF capacitor and a 10.00 kΩ variable resistor known as a rheostat. A knob connected to the variable resistor allows the resistance to be adjusted from 0.00Ω to $10.00\mathrm{k}\Omega$.

The output of the capacitor is used to control a voltage-controlled switch. The switch is normally open, but when the output reaches 10.00 V, the switch closes, energizing an electric motor and discharging the capacitor. The motor causes the windshield wipers to sweep once across the windshield and the capacitor begins to charge again. To what resistance should the rheostat be adjusted for the period of the wiper blades to be 10.00 seconds?

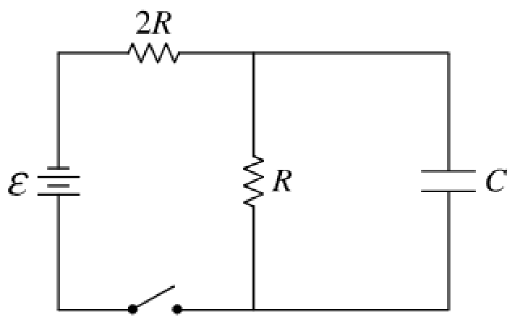
We start with $\epsilon = [\frac{\mathrm{d}q}{\mathrm{d}t} + \frac{q}{c}]/R$.

From this we have that $\mathrm{d}t = \frac{\mathrm{d}q}{\frac{\epsilon}{R} - \frac{q}{RC}}$.

Solving for R gives $R = \frac{-t}{C[\ln[1-\frac{V_D}{V}]]} = 558.11\Omega$.

Exercise N identical capacitors are placed in series with each other. What is the ratio of the equivalent capacitance of the series set to the capacitance of a single capactior?

Exercise



In the circuit shown in the figure, the switch has initially been opened for a long time. The switch is then closed. Immediately after the switch is closed, the current in resistor 2R is I_i. A long time after the switch is closed, the current in the resistor 2R is I_f. What is the approximate value of the ratio $\frac{I_f}{I_I}$?

5 Magnetic Fields and Electromagnetism

5.1 Magnetic Fields

A magnetic field is a vector field that can be used to determine the magnetic force exerted on moving electric charges, electric currents or magnetic materials.

- Produced by magnetic dipoles or combinations of dipoles, but never by monopoles.
- Magnetic dipoles have north and south polarity.

A magnetic field can be represented by using vector field maps.

Magnetic field lines must form closed loops, as described by Gauss's law of magnetism.

- Gauss's law of magnetism is Maxwell's second equation.
- $\oint \vec{B} \cdot \mathrm{d}\vec{A} = 0$

Magnetic dipoles results from the circular of rotational motion of electric charges.

A magnetic dipole when placed in a magnetic field will align with the magnetic field.

A material's composition influences its magnetic behavior in the presence of an external magnetic field.

Magnetic permeability is a measurement of magnetization in a material in response to an external magnetic field.

Free space has a constant value of magnetic permeability, known as the vacuum permeability, μ_0, that appears in equations representing physical relationships.

$$\mu_0 = 4\pi \times 10^{-7}\text{T}\cdot\text{ m/A}$$

Exercise Two dipole magnets are mounted such that they can rotate freekly around their respective centers of mass. Describe an orientation in which the two magnets are stable.

Exercise Two nails are placed in a strong, external magnetic field. One nail is made of iron and the other is made of aluminum. After a few minutes, the nails are removed from the external magnetic field and placed a large distance away from each other. How does the new magnetic field of the iron nail B_{Fe} compare to the magnetic field of the aluminum nail B_{Al}?

5.2 Magnetism and Moving Charges

A single moving charged object produces a magnetic field.

- It is dependent on the object's velocity and the distance between the point and the object.
- The direction of the magnetic field is perpendicular to both the velocity and the position vector from the object.
  - Determined by using a right-hand rule.

A magnetic field will exert a force on a charged object within that field, with magnitude and direction that depend on the cross-product of the charge's velocity and the magnetic field.

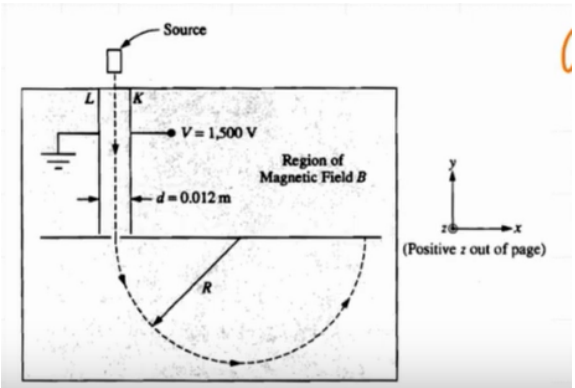
$$F_B = q(\vec{V} \times \vec{B}) = qvB\sin\theta$$

In a region containing both a magnetic field and an electric field, a moving charged particle will experience independent forces from each field.

The Hall effect describes the potential difference created in a conductor by an external magnetic field that has a component perpendicular to the direction of charges moving in the conductor.

Example

A mass spectrometer is used for determining the mass of singly ionized positively charged ions. There is a uniform magnetic field $B = 0.20$ T perpendicular to the page in the shaded region of the diagram. A potential difference $V = 1,500$ V is applied across the parallel plates L and K, which are separated by a distance $d = 0.012$ m and which act as a velocity selector. (a) Calculate the magnitude of the electric field between the plates. (b) Calculate the speed of a particle that can pass between the parallel plates without being deflected.

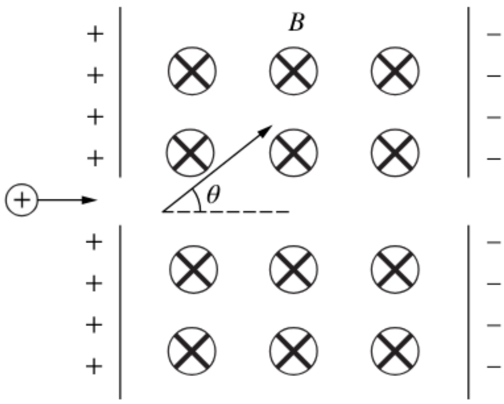


For part (a) use the formula $E = \frac{V}{D} = 1.3 \times 10^5$ N/C

For part (b) use $F_B = F_E = qvB = qE$. SOlving for v we get 6.3×10^5 m/s.

Exercise A particle with charge q and mass m is accelerated from rest through a potential difference into a region where both a magnetic field B and an electric field E are present. The electric and magnetic fields are such that the particle moves in a straight line at constant speed. What is the potential difference through which the particle was accelerated?

Exercise



A positively charged particle moves with speed v into the region between two parallel plates. The left plate is positively charged and the right plate is negatively charged to produce an electric field with magnitude E. A uniform magnetic field B directed into the page exists between the plates. Write a correct equation that could be used to determine the angle θ relative to the horizontal of the sum of forces on the particle immediately after is enters the region. Gravitational effects can be ignored.

5.3 Biot-Savart Law

The Biot-Savart law defines the magnitude and direction of a magnetic field created by an electric current.

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I(dl \times \hat{r})}{r^2}$$

The magnetic field vectors around a small segment of a current-carrying wire are tangent to concentric circles centered on that wire.

The Biot-Savart Law can be used to derive the magnitudes and directions of magnetic fields around segments of current-carrying wires.

$$B_{loop} = \frac{\mu_0 I}{2R}$$

A magnetic field will exert a force on a current-carrying wire.

$$F_B = \int I(\mathrm{d}\vec{l} \times \vec{B}) = IlB\sin\theta$$

Example

A wire carries a current I in a circular arc with radius R swept through an arbitrary angle θ. Calculate the magnetic field at the center of this arc at point P.

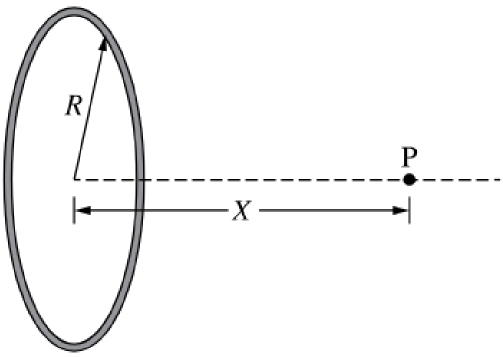
Let $\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi}\frac{t\mathrm{d}l \times \vec{r}}{r^2}$.

Integrating both sides gives us $B = \frac{I\mu_0}{4\pi}\int \frac{\mathrm{d}l}{r^2}$.

Substituting $l = r\theta \implies \mathrm{d}l = r\mathrm{d}\theta$ gives us $B = \frac{\mu_0 I}{4\pi}\int_0^\theta \frac{\mathrm{d}\theta}{r}$

Simplifying this (after taking out r as a constant) gives us $B = \frac{\mu_0 I\theta}{4\pi r}$.

Exercise



A circular loop of wire has a radius R and carries a current I. Point P is along the central axis of the loop a distance X away from the center of the loop, as shown. What is the magnitude of the magnetic field at point P?

Exercise

Three long, parallel wires are placed in a horizontal plane and the direction of the current in each wire is as shown. The wire on the left carries current into the page, and the wires in the middle and on the right both carry current out of the page. The middle wire is equidistant from the other two wires. The direction of the current in the middle wire is then reversed. How does this affect the direction of the magnetic force exerted on the middle wire?

5.4 Ampere's Law

Ampere's law relates the magnitude of the magnetic field to the current enclosed by an imaginary path called an Amperian Loop.

$$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I_{enc}$$

Ampere's law can be used to determine the magnetic field near a long, straight current carrying wire.

All solenoids are assumed to very long, with uniform magnetic fields inside the solenoids and negligible magnetic fields outside the solenoids.

Ampere's law can be used to determine the magnetic field inside of a long solenoid.

$$B_{sol} = \mu_0 n I$$

$n = \frac{\text{Turns}}{\text{Length}} = N/L$

An Amperian loop is a closed path arround a current-carrying conductor.

Ampere's law is the third of Maxwell's equations.

$$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{\mathrm{d}\phi_E}{\mathrm{d}t}$$

Example

Using Ampere's Law, derive the magnetic field for (a) inside and (b) outside of an infinite cylinder. Be sure to define the Amperian loop. (Assume all currents are distributed uniformly).

Let's define r as the distance from the center of the cylinder to the Amperian loop and R to be the distance from the center of the cylinder to the surface of the cylinder.

For part (a) we start with $JA_{loop} = I_{encl}$.

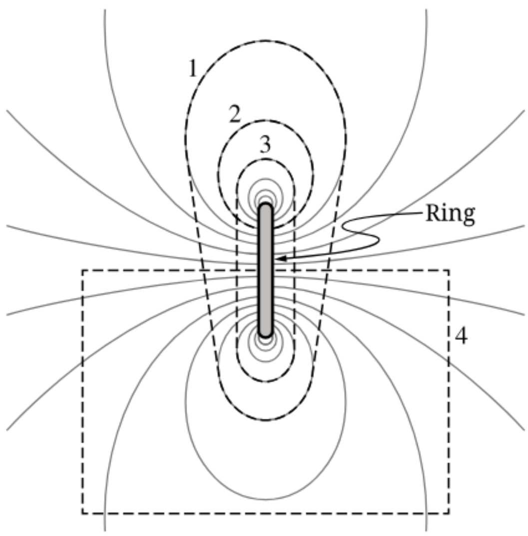
From this we get that $I_{encl} = \frac{I}{\pi R^2}(\pi r^2) = \frac{Ir^2}{R^2}$.

We know that $Bl = \mu_0 I_{encl}$, so plugging this in gives $B = \frac{\mu_0 I r}{2\pi R^2}$

For part (b) we have that $Bl = \mu_o I \implies B = \frac{\mu_0 I}{2\pi r}$.

Exercise Two long wires are parallel to each other and are 2.0 m apart. Each wire has a current of 3.0 A, but in opposite directions. What is the magnetic field at the midpoint between the wires?

Exercise



A circular ring carries and electric current. The magnetic field diagram shown is in a plane that passes through the center of the ring, and which is perpendicular to the plane of the ring. Four parths 1, 2, 3, and 4, represented by dashed lines, are in the plane of the magnetic field along which the integral $\oint \vec{B} \cdot \mathrm{d}\vec{l}$ may be evaluated. Which of the paths would yield a result of zero when evaluating this integral?

6 Electromagnetic Induction

6.1 Magnetic Flux

For a magnetic field that is constant across an area, the magnetic flux through the area is defined as

$$\phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$$

The area vector is defined as perpendicular to the plane of the surface area and outward from a closed surface.

The sign of the flux is given by the dot product of the magnetic field vector and the area vector.

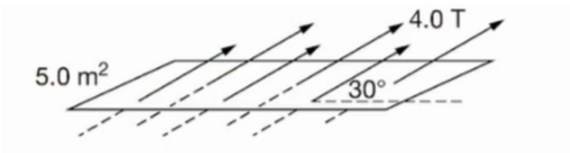
The total magnetic flux passing through a surface is defined by the surface integral of the magnetic field over the surface area.

$$\phi_B = \oint \vec{B} \cdot \mathrm{d}\vec{A}$$

which is usually equal to BA.

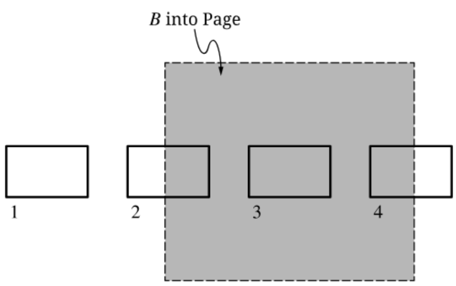
Example

A magnetic field of magnitude 4.0 T is directed at an angle of 30° to the plane of a rectangular loop of area 5.0 m^2, as shown below. What is the magnetic flux through the loop?

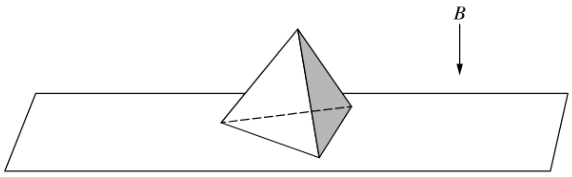


From $\phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta = 10$ Wb.

Exercise A rectangular loop of wire is moved into, through, and out of a region with a uniform magnetic field directed into the page, as shown in the figure. At Position 1, the loop has not yet entered the magnetic field. At Position 2, half of the loop has entered the field. At Position 3, the loop is entirely within the field. At Position 4, only half of the loop remains in the field. Compare the magnetic flux $\phi_1, \phi_2, \phi_3, \phi_4$ through the loop when the loop is at positions $1, 2, 3$, and 4 respectively?



Exercise A four-faced pyramid whose faces are comprised of equilateral triangles is at rest on a horizontal table. The pyramid is composed of a material that does not interact with magnetic fields. A uniform magnetic field of constant magnitude B is directed vertically downward throughout the entire region shown. If the area of each face of the pyramid is A, what is the net magnetic flux passing through the top three surfaces of the pyramid that are not in contact with the table?



6.2 Electromagnetic Induction

Faraday's law describes the relationship between changing magnetic flux and the resulting induced emf in a system.

$$\epsilon = \frac{-\mathrm{d}\phi_B}{\mathrm{d}t} = \frac{-\mathrm{d}(\vec{B} - \vec{A})}{\mathrm{d}t} = \frac{-\mathrm{d}BA\cos\theta}{\mathrm{d}t}$$

Lenz's law is used to determine the direction of an induced emf resulting from a changing magnetic flux.

- An induced emf generates a current that creates a magnetic field that opposes the change in magnetic flux.

Faraday's law of induction is the third of Maxwell's equations.

$$\epsilon = \oint \vec{E} \cdot \mathrm{d}\vec{l} = \frac{\mathrm{d}\phi_B}{\mathrm{d}t}$$

Example

A single loop of wire is at rest in a magnetic field that is directed into the page. The loop has a radius r and a resistance of R. The magnetic field has a magnitude B that changes as a function of time t according to the equation $B = \alpha t + \beta t^2, \alpha$ and β are both positive constants in units of T/s and T/s^2, respectively. Derive an expression for the current in the loop as a function of time.

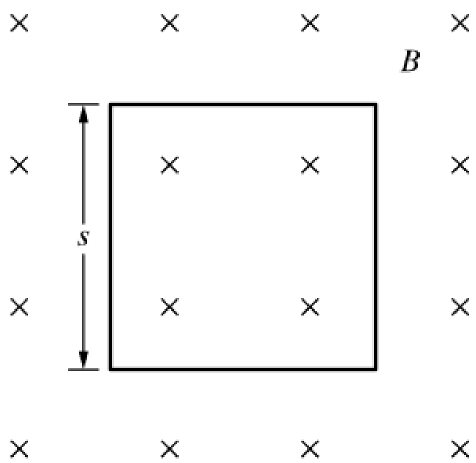
We know that $I = \frac{V}{R}$.

We know that $\epsilon = \frac{\mathrm{d}\phi_B}{\mathrm{d}t} = \frac{\mathrm{d}\vec{B}\cdot\vec{A}}{\mathrm{d}t} = \frac{\mathrm{d}BA}{\mathrm{d}t} = \frac{A\mathrm{d}B}{\mathrm{d}t}$.

From this we take the derivative and get $(\pi r^2)(\alpha + 2\beta t)$, so $I = \frac{\pi r^2(\alpha+2\beta t)}{R}$.

Exercise The magnitude of the magnetic field in a region of space as a function of time is $B(t) = 3t^2 - 2t + 1$. The magnetic field is directed out of the page. A circular loop in the plane of the page has a radius of R. Write an equation that represents the magnitude of the induced emf ϵ in the loop as a function of time.

Exercise



A single loop of wire with resistance R is placed in a magnetic field that is directed into the page, as shown. The loop is a square with each side having a length s. The magnitude of the magnetic field as a function of time is given by $B(t) = 4t^3 + 3$. What is the magnitude of the current induced in the wire due to the changing magnetic field as a function of time.

6.3 Induced Currents and Magnetic Forces

When an induced current is created in a conductive loop, the already present magnetic field will exert a magnetic force on the moving charge carriers in the loop.

$$F_B = \int I(\mathrm{d}\vec{l} \times \vec{B}) = ILB\cos\theta$$

When current is induced in a conducting loop, magnetic forces are only exerted on the segments of the loop that are within the external magnetic field.

The force on a conducting is proportional to the induced current in the loop, which depends on the rate of change of magnetic flux, the resistance of the loop and the velocity of the loop.

Newton's second law can be applied to a conducting loop moving in a magnetic field as it experiences an induced emf.

Example

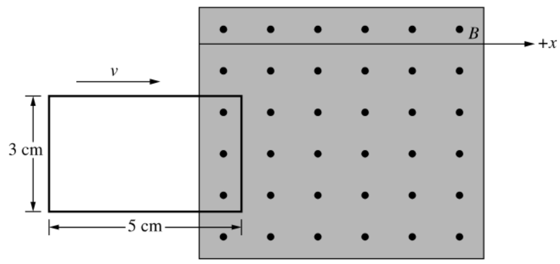
A rectangular loop of resistance R is partly in a region of uniform magnetic field. The loop's height and length are H and l, respectively. The magnetic field is directed out of the page. A constant force F_0 is exerted on the loop to the right such that the loop moves with constant speed v. Derive an expression for v.

We know that $F_B = \int I(\mathrm{d}l \times B) = IHB$.

From this we solve for $I = \frac{V}{R} = \frac{1}{R}\left[\frac{\mathrm{d}\phi_B}{\mathrm{d}t}\right] = \frac{BHv}{R}$.

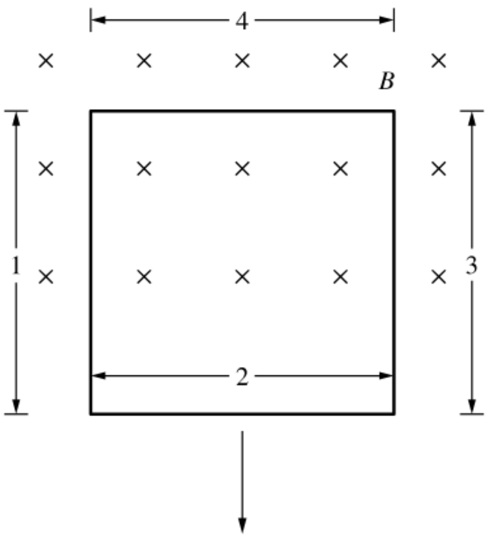
Equating the above two gives $v = \frac{RF_0}{B^2H^2}$

Exercise



A 3.0 cm wide and 5.0 cm long rectangular loop of conducting wire of resistance 0.2Ω, is moved at a constant speed of 3.0 m/s to the right. The loop enters, passes through, and exits a 0.5 T magnetic field directed out of the page, as shown. The loop is always moved at a constant speed. What is the work done by the magnetic field on the loop?

Exercise



A square loop os wire is moving at speed v as it exists a magnetic field directed into the page, as shown. Segments 1 and 3 of the wire are halfway in the magnetic field, Segment 2 has completely exited the magnetic field, and Segment 4 is completely in the magnetic field. Which segment(s) of the loop, if any, will experience a magnetic force from the external magnetic field?

6.4 Inductance

Inductance is the tendency of a conductor to oppose a change in electrical current.

- Depends on the physical properties of the conductor.
- An inductor, such as solenoid, is a circuit element that has significant inductance.
- The inductance of a solenoid is
$$L_{sol} = \frac{\mu N^2 A}{l}$$

Inductors store energy in the magnetic field that is generated by current in the inductor.

$$U_L = \frac{1}{2} L I^2$$

The energy stored in the magnetic field generated by an inductor in which current is flowing can be dissipated through a resistor or used to charge a capacitor.

The transfer of energy generated in an inductor to other forms of energy obeys conservation laws.

By applying Faraday's law to an inductor and using the definition of inductance, induced emf can be related to inductance and the rate of change of current.

$$\epsilon_i = -L \frac{\mathrm{d}I}{\mathrm{d}t}$$

Example

An inductor with inductance $L = 0.30$ H is connected in series with a resistor and both are connected to a power supply. The power supply generates a current that is given as a function of time t by the equation $I = I_0(1 - t/k)$, where $I_0 = 4.0$ A and $k = 2.0$ s. What is the magnitude of the potential difference across the inductor induced by the changing current?

We know that $\epsilon = L\frac{\mathrm{d}I}{\mathrm{d}t}$.

To find $\frac{\mathrm{d}I}{\mathrm{d}t}$ we know this is equal to $\frac{\mathrm{d}}{\mathrm{d}t}\left[I_0\left(1 - \frac{t}{k}\right)\right]$. This is equal to -2.

The answer is therefore 0.60 V.

Exercise What combination of physical changes to a solenoid would definitely cause an increase in its inductance?

Exercise The current as a function of time in a circuit that includes an inductor of inductance L is given by the equation $I = C\sqrt{t}$, where C is a positive constant with appropriate units. What is an equation for the emf ϵ across the inductor as a function of time?

6.5 LR Circuits

A resistor will dissipate energy that was stored in an inductor as the current charges.

Kirchhoff's loop rule can be applied to a series LR circuit with a bettery emf ϵ, resulting in a differential equation that describes the current in the loop.

$$\epsilon = IR + L\frac{\mathrm{d}I}{\mathrm{d}t}$$

The time constant is a significant feature of the behavior of an LR circuit.

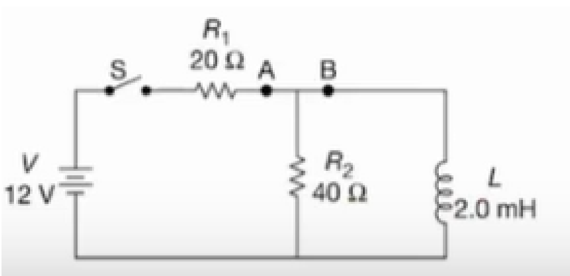
The time constant of a circuit is a measure of how quickly an LR circuit will reach a steady state and is described with the equation

$$\tau = \frac{L}{R}$$

The electric properties of inductors change during the time interval in which the current in the inductor changes, but will exhibit steady state behavior after a long time interval.

Example

After a long time of the switch being closed, it is now opened. Derive an equation to show that the equation or the current as a function of time is $I = I_0 e^{-t/\tau}$ where $\tau = L/R^2$.



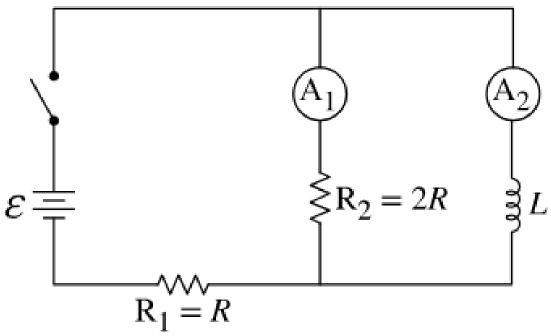
We start with $V_L - V_R = 0 \implies -L\frac{\mathrm{d}I}{\mathrm{d}t} - IR_2 = 0$.

We can get the differential equation $\int_{I_0}^{I(t)} \frac{\mathrm{d}I}{I} = \int_0^t \frac{-R_2}{L}\mathrm{d}t$.

Solving this equation gives $\ln I(t) - \ln I_0 = -\frac{R_2}{L}t$.

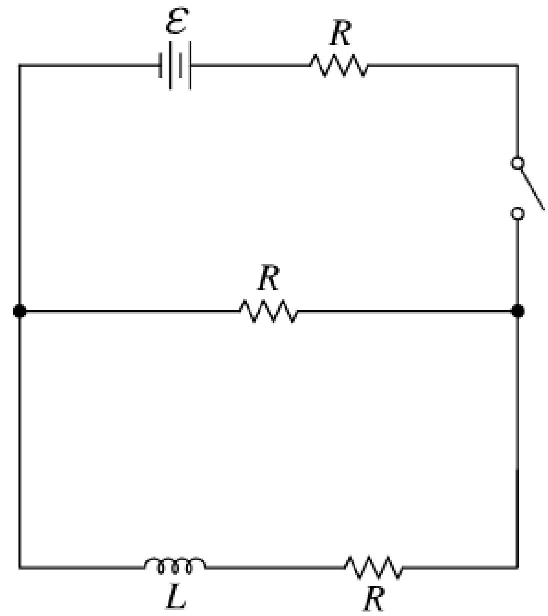
The simplification is left to the reader but it is equivalent to $I(t) = I_0 e^{-t/\tau}$.

Exercise



A circuit containing an inductor of inductance L, resistors R_1 and R_2 of resistance R and $2R$, respectively, a battery of emf ϵ, two ammeters, and a switch is constructed as shown in the diagram. After the switch is closed and the circuit reaches steady state, how does the current I_1 measured by Ammeter A_1 compare to the current I_2 measured by Ammeter A_2, and why?

Exercise



A circuit is constructed from a battery of emf ϵ, a switch, an inductor of inductance L, and three identical resistors of resistance R, connected as shown in the diagram. The switch is initially open. Write an expression that best indicates the energy stored in the inductor a long time after the switch has been closed.

6.6 LC Circuits

In circuits containing only a charged capacitor and an inductor (LC Circuits), the maximum current in the inductor can be determined using conservation of energy within the circuit.

In LC circuits, the time dependence of the charge stroed in the capacitor can be modeled as simple harmonic motion:

$$\frac{\mathrm{d}^2 q}{\mathrm{d}t^2} = -\frac{I}{LC} q$$

The angular frequency of an oscillating LC circuit can be derived from the differnetial equation that describes an LC circuit.

$$\omega = \frac{1}{\sqrt{LC}} \implies \omega = \frac{2\pi}{T} \implies T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$$

Example

A 50 μF capacitor is fully charged to 40 μC and then connected in series to a 20 mH inductor and an open switch. After the switch is closed, what is the maximum current through the inductor?

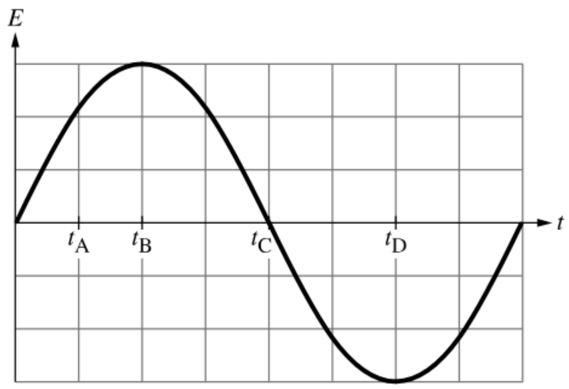
We know that $Q = CV$ and $U_C = \frac{1}{2}CV^2$.

From $U_C = U_L$, we get that $\frac{Q^2}{LC} = I^2$.

Solving for I and plugging in numbers gives us 0.04 A.

Exercise A capacitor is charged using a battery until the energy stored in the electric field between the plates is equal to U. The capacitor is removed from the battery and then connected to an inductor. The time it takes for the capacitor to discharge with the same initial polarity is T. At what time will the energy stored in the magnetic field of the inductor be equal to U?

Exercise



A circuit is constructed by connecting a capacitor and an inductor. The electric field E in the capacitor varies sinusoidally with time t. Times t_A, t_B, t_C, and t_D are indicated in the figure. Rank the magnitudes of the currents I_A, I_B, I_C and I_D in the inductor at times t_A, t_B, t_C, and t_D respectively.