1 Electric Potential

1.1 Electric Potential Energy

The electric potential energy of a system of two point charges equals the amount of work required for an external force to bring point charges to their current positions from infinitely far away.

The general form of the electric potential energy between two charged objects is given by the equation:

$$U = \frac{kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1q_2}{r}$$

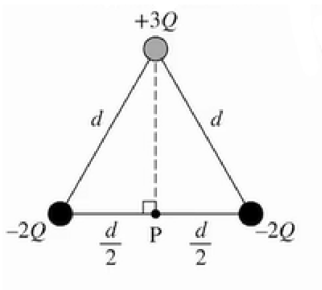
The total electric potential energy of a system can be determined by finding the sum of the electric potential energies of the individual interactions between each pair of charged objects in the system.

When there are opposite signs, the U value will decrease when close together.

When there are same signs, the U value will increase when close together.

Example

Derive an expression for the work required to assemble the charges in the configuration shown.



Remember that

$$W = \Delta U = \sum U$$

So we have:

$$\frac{kq_1q_2}{d} + \frac{kq_1q_3}{d} + \frac{kq_2q_3}{d} = k\left[\frac{(-2Q)(3Q)}{d} + \frac{(-2Q)(-2Q)}{d} + \frac{(-2Q)(3Q)}{d}\right] = 8Q$$

1.2 Electric Potential

Electric potential describes the electric potential energy per unit of charge at a point in space.

Expressions for the electric potential of charge distributions can be found by using integration and the principle of superposition:

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\mathrm{d}q}{r}$$

If there are multiple point charges, we just add up all the point charges.

The electric potential difference between two points is the change in the electric potential energy per unit charge when a test charge is moved between two points:

$$\Delta V = \frac{\Delta U_E}{q}$$

The value of the electric field component in any direction at a given point is equal to the negative of the rate of change in electric potential at that location:

$$E_x = -\mathrm{d}V/\mathrm{d}x$$

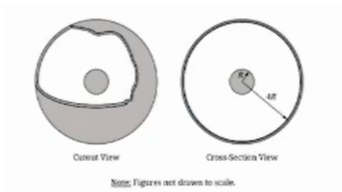
The change in electric potential between two points can be determined by integrating the dot product of the electric field and the displacement along the path connecting the points:

$$\Delta V = V_b - V_a = -\int \vec{E} \cdot \mathrm{d}\vec{r}$$

Equipotential lines represent lines of equal potential energy. These lines are perpendicular to the electric field vectors. Electric field vectors point in the direction of decreasing potential. There is no component of an electric field along an equipotential line.

Example

Derive an expression for the absolute value of the potential difference between the outer surface of the sphere and the inner surface of the shell. Express your answer in terms of Q, R, and physical constants, as appropriate.



Previously we would have found the electric field is:

$$E = -\frac{Q}{4\pi\epsilon_0 r^2}$$

So we must integrate:

$$\Delta V = -\int_R^{4R} \vec{E} \cdot \mathrm{d}\vec{R}$$
$$\Delta V = +\int_R^{4R} +\frac{Q}{4\pi\epsilon_0 r^2}\mathrm{d}r$$
$$\Delta V = \frac{Q}{4\pi\epsilon_0}\int_R^{4R} \frac{\mathrm{d}r}{r^2}$$
$$\Delta V = \frac{Q}{4\pi\epsilon_0}\left[+\frac{1}{r}\right]$$

Applying the limits of integration we get:

$$\Delta V = \frac{3Q}{4\pi\epsilon_0 R}$$

1.3 Conservation of Electric Energy

When a charged object moves between two locations with different electric potentials, the resulting change in the electric potential energy of the object-field system is given by:

$$\Delta U_E = q\Delta V$$

The movement of a charged object between two points with different electric potential results in a change in kinetic energy of the object consistent with the conservation of energy.

Example

A proton (mass $= 1.67 \times 10^{-27}$ kg) is accelerated through a potential difference of 4.5×10^6 V. (a) How much kinetic energy has the proton acquired? (b) If the proton started at rest, how fast is it moving.

For part A, we have

$$\begin{aligned} \Delta U = q\Delta V = \Delta K = K - K_0 \\ = (1.6 \times 10^{-19})(4.5 \times 10^6 \text{V}) \\ = 7.2 \times 10^{-13} \text{J} \end{aligned}$$

For part B:

$$\begin{aligned} K = \frac{1}{2}mv^2 \\ V = \sqrt{\frac{2K}{m}} \\ = \sqrt{\frac{2(7.2 \times 10^{-13})}{1.67 \times 10^{-27}}} = 2.94 \times 10^7 \text{m/s} \end{aligned}$$