AP Physics C: Mechanics Notes

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1 Kinematics

1.1 Scalars and Vectors

Scalars are quantities described by magnitude only, vectors are quantities described by both magnitude and direction.

Vectors can be visually modeled as arrows with appropriate direction and lengths proportional to their magnitudes.

Vectors can be expressed in unit vector notation or as a magnitude and a direction.

- Unit vector notation can be used to represent vectors as the sum of their constituent components in the x, y, and z directions, denoted by $\hat{i}, \hat{j}$ and $\hat{k}$.

$$\vec{r} = A\hat{i} + B\hat{j} + C\hat{k}$$

- The position vector of a point is given by $\vec{r}$ and the unit vector in the direction of the position vector is denoted $\hat{r}$.
- A resultant vector is the vector sum of the addend vectors' components.

$$\vec{C} = \vec{A} + \vec{B} = (A_x + B_y)\hat{i} + (A_y + B_y)\hat{j}$$

In a given one-dimensional coordinate system, opposite directions are denoted by opposite signs.

1.2 Displacement, Velocity, and Acceleration

When using the object model, the size, shape and internal configuration are ignored.

- The object may be treated as a single point with extensive properties such as mass and charge.

Displacement is the change in an object's position: $\Delta x = x - x_0$

Averages of velocity and acceleration are calculated considering the initial and final states of an object over an interval of time.

Average velocity is the displacement of an object divided by the interval of time in which that displacement occurs:

$$\vec{v}_{avg} = \frac{\Delta\vec{x}}{\Delta t}$$

Average acceleration is the change in velocity divided by the interval of time in which that change in velocity occurs.

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$$

As the time interval used to calculate the average value of a quantity approaches zero, the average value of that quantity approaches the value of the quantity that is instant, called the instantaneous value.

- $\vec{v} = \frac{\mathrm{d}x}{\mathrm{d}t}$
- $\vec{a} = \frac{\mathrm{d}v}{\mathrm{d}t}$

Time dependent functions and instantaneous values of position, velocity and acceleration can be determined using differentiation and integration.

1.3 Representing Motion

Motion can be represented by motion diagrams, figures, graphs, equations and narrative descriptions.

For constant acceleration, three kinematics equations can used to describe the instantaneous linear motion in one dimension:

- $v = v_0 + at$
- $x = x_0 + v_0 t + \frac{1}{2}at^2$
- $v^2 = v_0^2 + 2a\Delta x$

Near the surface of the Earth, the vertical acceleration caused by the force of gravity is downward, constant and has a measured value of $g = 9.8$ m/s^2 or $g = 10$ m/s^2.

Graphs of position, velocity and acceleration as functions of time can be used to find the relationships between those quantities.

1.4 Reference Frames and Relative Motion

The choice of reference frame will determine the direction and magnitude of quantities measured by an observer in that reference frame.

Measurements from a given reference frame may be converted to measurements from another reference frame.

The observed velocity of an object results from the combination of the object's velocity and the velocity of the observer's reference frame.

- Combining the motion of an object and the motion of an observer in a given reference frame involves the addition or subtraction of vectors.
- The acceleration of any object is the same as measured from all inertial reference frames.

1.5 Motion in Two or Three Dimensions

Motion in two or three dimensions can be analyzed using one-dimensional kinematic relationships if the motion is separated into componenets.

Velocity and acceleration may be different in each dimension and be nonuniform.

Motion in one dimension may be changed without causing a change in the perpendicular dimension.

Projectile motion is a special case of two-dimensional motion that has zero acceleration in one dimension and constant, nonzero acceleration in the second dimension.

2 Force and Translational Dynamics

2.1 Systems and Center of Mass

- System properties are determined by the interactions between objects within the system.
- If the properties or interactions of the constituent objects within a system are not important in modeling the behavior of a macroscopic system, the system can itself be treated as a single object.
- Systems may allow interactions between constituent parts of the system and the environment, which may result in the transfer of energy or mass.
- For objects with symmetrical mass distributions, the center of mass is located on lines of symmetry.
- The location of a system's center of mass along a given axis can be calculated using the equation:

$$x_{cm} = \frac{\sum m_i x_i}{M}$$

- For a nonuniform solid that can be considered as a collection of differential masses, $\mathrm{d}m$, the solid's center of mass can be calculated using

$$x_{cm} = \int x \mathrm{d}m / M$$

- A system can be modeled as a singular object that is located at the system's center of mass.

2.2 Forces and Free-Body Diagrams

- Forces are vector quantitites that describe interactions between objects or systems.
- Contact forces describe the interaction of an object or system touching another object or system.
- Free-body diagrams (FBDs) are useful tools for visualizing forces exerted on a single object or system and for determining the equations that represent a physical situation.
- The FBD of an object or system shows each of the forces exerted on the object or system by the environment.
- Forces exerted on an object or system are represented as vector originating from the center of mass, such as a dot.
- Choose a coordinate system such that one axis is parallel to the acceleration of the object or system.

2.3 Newton's Third Law

Newton's third law describes the interaction of two objects or systems in terms of the paired forces that exerts on the other.

$$\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$$

Interactions between objects within a system do not influence the motion of a system's center of mass.

Tension is the macroscopic net results of forces that infinitesimal segments of a string, cable, chain or similar systme exert on each other in response to an external force.

- An ideal string has negligible mass and does not stretch when under tension.
- The tension in an ideal string is the same at all points within the string.

- In a string with nonneglibible mass, tension may not be the same at all points within the string.
- An ideal pulley that has negligible mass and rotates about an axle through its center of mass with negligible friction.

2.4 Newton's First Law

The net force on a system is the vector sum of all forces exerted on the system.

Translational equilibrium is the configuration of forces that the net force exerted on a system is zero.

$$\sum F = 0$$

Newton's first law states that if the net force exerted on a system is zero, the velocity of that system will remain constant.

Forces may be balanced in one dimension but unbalanced in another.

2.5 Newton's Second Law

Unbalanced forces are a configuration of forces such that the net force exerted on a system is not equal to zero.

Newton's second law of motion states that the acceleration of a system's center of mass has a magnitude proportional to the magnitude of the net force exerted on the system and is in the same direction of the force.

$$\sum F = ma = 0$$

The velocity of a system's center of mass will only change if a nonzero net external force is exerted on that system/

2.6 Gravitational Force

Newton's law of universal gravitation describes the gravitational force between two objects as directly proportional to each of their masses and inversely proportional to the square of the distance between their centers.

$$F_G = \frac{Gm_1m_2}{d^2}$$

A field models the effects of a noncontact force exerted on an object at various positions in space.

The magnitude of the gravitational field created by a system of mass M at a point in space is equal to the ratio of the gravitational force exerted by the system on a test object of mass m to the mass of the test object.

$$\vec{g} = \frac{\vec{F}_g}{m}$$

If a system is accelerating, the apparent weight of the system is not equal to the magnitude of the gravitational force exerted on the system.

Newton's shell law theorem describes the net gravitational force exerted on an object by a uniform spherical shell of mass.

2.7 Kinetic and Static Friction

Kinetic friction occurs when two surfaces in contact move relative to each other.

- It opposes the direction of motion.
- The surface area of contact is not a factor.

The magnitude of the kinetic friction force exerted on an object is the product of the normal force the surface exerts on the object and the coefficient of kinetic friction.

$$f_k = \mu_k F_N$$

Static friction may occur between the contacting surfaces of two objects that are not moving relative to each other.

Static friction adopts the value and direction required to prevent an object from slipping or sliding on a surface.

$$f_s \leq \mu_s F_N$$

The coefficient of static friction is typically greater than the coefficient of kinetic friction for a given pair of surfaces.

2.8 Spring Forces

An ideal spring has negligible mass and exerts a force that is proportional to the change in its length as measured from its relaxed length.

The magnitude of the force exerted by an ideal spring on an object is given by Hooke's Law:

$$F_{sp} = -k\Delta x$$

The force exerted on an object by a spring is always directed toward the equilibrium position of the object-spring system.

A collection of springs that exert forces on an object may behave as though they were a single spring with an equivalent spring constant.

- Springs in series: $\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots$
- Springs in parallel: $k_{eff} = k_1 + k_2 + \ldots$

2.9 Resistive Forces

A resistive force is defined as a velocity-dependent force in the opposite direction of an object's velocity.

$$F_R = -kv[F_R = -bv^2]$$

Applying Newton's second law to an object upon which a resistive force is exerted results in a differential equation for velocity.

- The differential portion of a=the equation comes from substituting in $a = \frac{\mathrm{d}v}{\mathrm{d}t}$

Terminal velocity is defined as the maximum speed achieved by an object moving under the influence of a constant force and a resistive force that are exerted on the object in opposite directions.

- For a falling object, this occurs when the air resistance equals the weight of the object.

2.10 Circular Motion

Centripetal acceleration is the component of an object's acceleration directed toward the center of the object's circular path.

- The magnitude of the acceleration for an object moving in a circular path is the ratio of the object's tangential speed squared to the radius of the circular path.

$$a_c = v^2/r$$

Centripetal acceleration can result from a single force, more than one force, or components of forces that are exerted on an object in circular motion.

Tangential acceleration is the rate at which an object's speed changes and is directed tangent to the object's circular path.

$$a = \sqrt{a_c^2 + a_T^2}$$

The net acceleration of an object moving in a circle is the vector sum of the centripetal acceleration and tangential acceleration.

The revolution of an object traveling in a circular path at a constant speed (UCM) can be described using period and frequency.

$$v = \frac{2\pi r}{T} = 2\pi r f \qquad T = \frac{1}{f}$$

3 Work, Energy and Power

3.1 Translational Kinetic Energy

An object's translational kinetic energy is given by the equation

$$K = \frac{1}{2}mv^2$$

Translational kinetic energy is a scalar quantity.

Different observers may measure different values of the translational kinetic energy of an object, depending on the observer's frame of reference.

3.2 Work

Work is the amount of energy transferred into or out of a system by a force exerted on that system over a distance.

- The work done by a conservative force is path-independent and only depends on the initial and final configurations of that system.
- The work done by a nonconservative force is path-dependent.

Work is scalar quantity that may be positive, negative or zero.

The work done on an object by a variable force is calculated as

$$W = \int_a^b \vec{F}(r) \cdot \mathrm{d}\vec{r} \qquad [W = Fd\cos\theta]$$

The work-energy theorem states that the change in an object's kinetic energy is equal to the sum of the work being done by all forces exerted on the object.

$$\delta K = W$$

Work is equal to the area under the curve of a graph of F as a function of displacement.

3.3 Potential Energy

A system composed of two or more objects has potential energy if the objects within that system only interact with each other through conservative forces.

Potential energy is a scalar quantity associated with the position of objects within a system.

The definition of zero potential energy for a given system is a decision made by the observer considering the situation to simplify or otherwise assist in analysis.

The relationship between conservative forces exerted on a system and the system's potential energy is

$$\delta U = -\int \vec{F}(r) \cdot \mathrm{d}\vec{r}$$

The conservative forces exerted on a single dimension can be determined using the slope of a system's potential energy with respect to position in that dimension, these forces point in the direction of decreasing potential energy.

$$F_x = -\mathrm{d}u(x)/\mathrm{d}x$$

The potential energy of common physical systems can be described using the physical properties of that system.

3.4 Conservation of Energy

A system that contains objects that interact via conservative forces or that can change its shape reversibly may have both kinetic and potential energies.

Mechanical energy is the sum of a system's kinetic and potential energy.

A system may be selected so that the total energy of the system is constant.

If the total energy of a system changes, that change will be equivalent to the energy transferred into or out of the system.

Energy is conserved in all interactions.

If the work done on a selected system is zero and there are no nonconservative interactions within the system, the total mechanical energy of the system is constant.

If the work done on a selected system is nonzero, the energy is transferred between the system and the environment.

3.5 Power

Power is the rate at which energy changes with respect to time, either by transfer into or out of a system or by converstion from one type to another within the system.

Average power is the amount of energy being transferred or converted, divided by the time it took for that transfer to happen.

The instantaneous power delivered to an object by a force is given by the equation:

$$p_{ins} = \frac{\mathrm{d}E}{\mathrm{d}t}$$

The instantaneous power delivered to an object by the component of a constant force parallel to the object's velocity can be described with the derived equation:

$$p_{ins} = Fv\cos\theta$$

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