AP Physics C: Electricity & Magnetism Notes

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1 Electric Charges, Fields and Gauss's Law

Brief Calculus Review

The derivative of a function at some point characterizes the rate of change of the function at that point; The rate of change of the function is basically the slope at that point.

Because the derivative is a slope, the notation can be written as

$$f'(x) = \frac{\mathrm{d}x}{\mathrm{d}t}$$

There are some derivative rules to know.

- $\frac{\mathrm{d}}{\mathrm{d}x} = 0$
- $\frac{\mathrm{d}}{\mathrm{d}x}(x) = C$
- $\frac{\mathrm{d}}{\mathrm{d}x}x^n = nx^{n-1}$
- $\frac{\mathrm{d}}{\mathrm{d}x}(f(x) \pm g(x)) = \frac{\mathrm{d}}{\mathrm{d}x}f(x) \pm \frac{\mathrm{d}}{\mathrm{d}x}g(x)$

An integral is simply finding the area under a curve.

The integral notation is:

$$f(x) = \int f'(x)\mathrm{d}x$$

In physics, we use the definite integral, where the area is found over an interval $[a, b]$.

The notation for this is:

$$A = \int_b^a f'(x)\mathrm{d}x = f(b) - f(a)$$

There are some integration rules to know as well.

- $\int \mathrm{d}x = x + C$
- $\int x^n \mathrm{d}x = \frac{1}{n+1}x^{n+1}, n \neq -1$
- $\int f(x) \pm g(x)\mathrm{d}x = \int f(x)\mathrm{d}x \pm \int g(x)\mathrm{d}x$

A differential equation is an equation involving one or more derivatives of an unknown function. The order of a differential equation is defined to be the order of the highest derivative it contains.

All differential equations are considered to be separable and can be solved by integration. This process is called separation of parts.

Much like derivatives there are set of integrals that don't follow the basic power rule of integration. There are some special integral rules.

- $\int e^{ax}\mathrm{d}x = \frac{1}{a}e^{ax} + C$
- $\int \frac{\mathrm{d}x}{x+n} = \ln|x + a| + C$
- $\int \cos(ax)\mathrm{d}x = \frac{1}{a}\sin(ax)$
- $\int \sin(ax)\mathrm{d}x = \frac{-1}{a}\cos(ax)$

Integration by substitution is a way of undoing the derivative's chain rule. You need the integral to look like this:

$$\int f(g(x))g'(x)\mathrm{d}x = \int f(u)\mathrm{d}u$$

Here are some special derivatives:

- $\frac{\mathrm{d}}{\mathrm{d}x}e^{ax} = ae^{ax}$
- $\frac{\mathrm{d}}{\mathrm{d}x}\ln ax = \frac{1}{x}$
- $\frac{\mathrm{d}}{\mathrm{d}x}\sin ax = a\cos ax$
- $\frac{\mathrm{d}}{\mathrm{d}x}\cos ax = -a\sin ax$

All derivatives on the formula sheet are written using the chain rule.

The chain rule follows this general rule:

$$f(g(x)) = f'(g(x)) \cdot g'(x)$$

A vector is a quantity that has both magnitude and direction. The length of the line shows its magnitude and the arrowhead points in the direction. To add vectors, place the tip of the first vector to the tail of the second vector. The resultant is the arrow drawn from the tail of the first vector to the tip of the second vector.

The goal of subtracting vectors is to turn it into addition by finding the inverse of the second vector. Basically:

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

When adding or subtracting vectors algebraically, the first thing you need to do is to resolve the vectors into components.

- $A_x = A\cos\theta$
- $A_y = A\sin\theta$

Once all the vectors are broken down, you can add the horizontal and vertical components. This will give you the horizontal and vertical components of the resultant.

To find the magnitude of the resultant, you can find the hypotenuse:

$$R = \sqrt{R_x^2 + R_y^2}$$

.

The direction can be found from:

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

There are times when you need to "scale" up or down a vector. To do so, you multiply the magnitude of a vector, but not the direction, by a scalar.

A unit vector has a magnitude of 1 and a direction that goes along one of the axes.

The dot product is the process of multiplying two vectors and getting a scalar answer in return. There are two ways to find this:

- $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$
- $\vec{a} \cdot \vec{b} = |a||b|\cos\theta$

The second method also helps determine if the vectors are orthogonal, or perpendicular to each other.

The cross product is the process of multiplying two vectors and getting a vector in return. The answer is a vector that is at a right angle to the two original vectors. The magnitude of the cross product equals the area of the parallelogram with the two original vectors as sides.

The cross product is zero in length when the original vectors point in the same or opposite directions. It reaches maximum length when the original vectors are at right angles to each other.

There are two ways to calculate the cross product.

The first is:

$$\vec{a} \text{x} \vec{b} = [|a||b|\sin\theta]\hat{n}$$

This method does not give you the direction of the vector.

The second way is to use a set of formulas to find the components:

- $C_x = a_y b_2 - a_2 b_y$
- $C_y = a_2 b_x - a_x b_2$
- $C_z = a_x b_y - a_y b_x$

The direction is determined by the right hand rule. Your index finger points in the direction of vector a, your middle points in the direction of b, and your thumb points in the direction of the answer.

1.1 Electric Charge and Electric Force

Electric charge is a fundamental property of all matter.

Charge is scalar value, which means it has no direction, and is described as either positive or negative.

The magnitude of charge on a single electron is the elementary charge which is $e = 1.6 \times 10^{-19}$ C (coulomb). The coulomb is the unit of charge.

Coulomb's Law describes the electrostatic force between two charges objects. The equation for this is:

$$F_{\mathsf{E}} = \frac{kq_1q_2}{r^2}$$

This equation is similar to the universal gravitation formula. Note that r can be written sometimes as d, it is the distance between the centers. k is the electrostatic constant and is equal to 9×10^9N·m^2/C^2. k is sometimes written as $\frac{1}{4\pi\epsilon_0}$.

The direction of the electrostatic force depends on the signs. Opposite charges attract and like charges repel. Electrostatic force can also cause other forces like tension, friction, and normal force.

Electrostatic force can be attractive (different signs) or repulsive (same signs), while gravitational force, which is similar, can only be attractive.

The electrostatic force has a much larger magnitude than gravitational force, but gravitational force acts on a larger scale in that the electrostatic force works at a microscropic scale, while gravitational force will be on a planetary scale.

Free space (a region where there is no electromagnetic or gravitational fields) has a constant value of electric (or vacuum) permittivity which is equal to $\epsilon_0 = 8.85 \times 10^{-12}$C^2/(N·m^2).

Example

Point charges Q$_1$ = 2.0μC and Q$_2$ = -4.0μC are located at $\vec{r_1} = (4.0\hat{i} - 2.0\hat{j} + 5.0\hat{k})$m and $\vec{r_2} = (8.0\hat{i} + 5.0\hat{j} - 9.0\hat{k})$m. What is the force of Q$_2$ on Q$_1$?

We have the equation

$$F_{\mathsf{E}} = \frac{kq_1q_2}{r^2}$$

We first have to find the distance between the charges. We can use the distance formula for this:

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z^1)^2}$$
$$r = \sqrt{(8-4)^2 + (5+2)^2 + (-9-5)^2}$$
$$r = 16.2\text{m}$$

Now we can plug this into the equation.

$$F_\mathsf{E} = \frac{kq_1q_2}{r^2}$$
$$F_\mathsf{E} = \frac{(9 \times 10^9)(2 \times 10^{-6})(-4 \times 10^{-6})}{(16.2)^2}$$
$$F_\mathsf{E} = -2.74 \times 10^{-4}\text{N}$$

Note that since both point charges have opposite signs, they will try and attract each other, which means the resulting force calculated will be negative.

1.2 Conservation of Electric Charge and the Process of Charging

The net charge or charge distribution of a system can change in response to the prescence of, or changes in, the net charge or charge distribution of other systems. For example, the net charge can change due to friction or contact between systems.

Induced charge separation occurs when electrostatic force between two systems alters the distribution or charges within the systems, resulting in the polarization of one or both systems. Induced charge separation can only occur in neutral systems.

Any change to a system's net charge is due to a transfer of charge between the system and its surroundings. Most of the time, this is the result of a transfer of electrons.

An application of this is grounding, which involves electricially connecting a charged object to a much larger and approximately neutral system (such as the Earth).

Example

There are two identical metal spheres on insulating stands. Sphere 1 has a charge of -1.02×10^{-16}C. Sphere 2 has a deficit of 841 electrons. The two spheres are brought together so that they touch each other. They are then separated again so that they are no longer touching. What charge does each sphere have after they have touched? Consider the new charge on Sphere 2. Does this correspond to a deficit or an excess of electrons? How many electrons is the deficit/excess?

First, we must find the charge on Sphere 2. Because the problem states that there is a deficit of electrons, the charge is positive.

The charge on Sphere 2 is:

$$841 \times (1.6 \times 10^{-19}) = 1.35 \times 10^{-16}\text{C}$$

Now we find the net charge on the system by adding it to the charge on Sphere 1:

$$1.35 \times 10^{-16} + (-1.02 \times 10^{-16}) = 3.3 \times 10^{-17}\text{C}$$

Now we simply divide this number by two since the charge is divded evenly between the two spheres:

$$\frac{3.3 \times 10^{-17}\text{C}}{2} = 1.6 \times 10^{-17}\text{C}$$

This answers the first part of the question.

The new charge is positive, which means there is an deficit of electrons. To find the amount of electrons in this deficit we must convert to electrons:

$$1.65 \times 10^{-17}\text{C} \times \frac{1e}{1.6 \times 10^{-19}\text{C}} = 103 \text{ electrons}$$

1.3 Electric Fields

Electric fields may originate from charged particles.

The electric field at a given point is the ratio of the electric force exerted on a test charge at the point to the charge and the charge on the test charge itself.

Mathematically this is:

$$E = \frac{F_E}{q}[N/C]$$

Another way of writing this is:

$$F_E = qE$$

The E-field points away from an isolated positive charge towards an isolated negative charge. Therefore, if the test charge is negative, the electric force points opposite the direction of the electric field.

Example

(a) Find the direction and magnitude of an electric field that exerts a 4.80×10^{-17} N westward force on an electron.

(b) What magnitude and direction force does this field exert on a proton?

For part (a), we use the formula:

$$E = \frac{F_E}{q} = \frac{4.80 \times 10^{-17} \text{ N}}{1.6 \times 10^{-19} \text{ C}} = 300 \text{ N/C East}$$

For part (b), the proton and electron have the same charge, so they experience the same electric field. The force and the E-field because of the positive charge results in the force pointing in the same direction, so 300 N/C West.

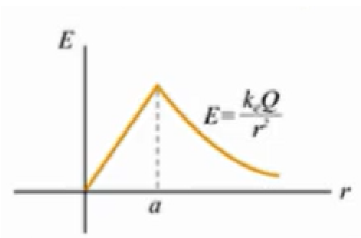
1.4 Electrostatic Equilibrium

Many problems on the FRQ section will involve conductors and insulators.

A conductor is an object or type of material that allows the flow or charge (electric current) in one or more directions. An insulator is a material in which electric current does not flow freely.

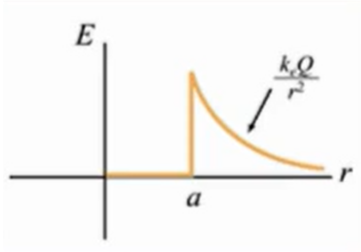
Electrostatic equilibrium occurs when there is no net motion of charge in an insulator or conductor.

When an insulator is in equilibrium, the excess charge of an insulator is distributed throughout the interior of the insulator as well as the surface. The electric field within the insulator may have a nonzero value.



In this, the peak is the surface of the insulator.

When in equilibrium, the excess charge on a conductor lies on the surface of the conductor, making the electric field equal to zero inside. The electric field is perpendicular to the surface of the conductor.



Example

A particle of charge 2.0×10^{-8}C experiences an upward force of magnitude 4.0×10^{-6}N when it is placed in a particular point in an electric field.

(a) What is the electric field at that point?

(b) If a charge $q = -1.0 \times 10^{-8}$C is placed there, what is the force on it?

For part A:

$$E = \frac{F_E}{q} = \frac{4.0 \times 10^{-6}\text{N}}{2.0 \times 10^{-8}\text{C}} = 200\text{N/C upward}$$

For part B:

$$F_E = gE = (1.0 \times 10^{-8}\text{C})(200\text{N/C}) = 2.0 \times 10^{-6}\text{N downwards}$$

1.5 Electric Fields of Charge Distributions

The expressions for the electric field of specified charge distributions can be found using integration and the principle of superposition.

Mathematically:

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$$

Symmetry considerations of certain charge distributions can simplify analysis of the electric field resulting from those charge distributions.

Some common distributions are: Infinite Wire, Finite Wire, Semicircle (Arc), Ring of Charge

There are three densities - linear, area, and volume.

Linear:

$$\lambda = \frac{Q}{L} \longrightarrow \lambda = \frac{dQ}{dL} \longrightarrow dQ = \lambda dL$$

Area:

$$\sigma = \frac{Q}{A} \longrightarrow \sigma = \frac{dQ}{dA} \longrightarrow dQ = \sigma dA$$

Volume:

$$\rho = \frac{Q}{V} \longrightarrow \rho = \frac{dQ}{dV} \longrightarrow DQ = \rho dV$$

At a point charge:

$$E_{PC} = \frac{kQ}{r^2} \therefore E = \frac{1}{4\pi\epsilon_0} \int \frac{dQ}{r^2} \hat{r}$$

Example

Find the electric field a distance z above the midpoint of a straight line segment of length L that carries a uniform line charge density λ.

There will be two electric fields, therefore

$$\vec{E_{\text{net}}} = \vec{E_1} + \vec{E_2}$$

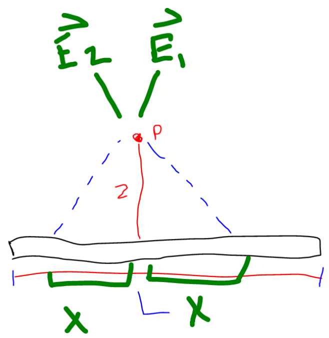
.

Therefore:

$$E_{1x}\hat{i} + E_{1z}\hat{k} + E_{2x}\hat{(} - \hat{i}) + E_{2z}\hat{k}$$

and the x-components cancel in this case and gives us:

$$E_{1z}\hat{k} + E_{2z}\hat{k}$$



Now we use calculus:

$$\vec{E}_{\text{net}} = \int \frac{k\cos\theta dQ}{r^2} + \int \frac{k\cos\theta dQ}{r^2} = 2\int \frac{k\cos\theta dQ}{r^2}$$

Now we have:

$$2\int \frac{k\cos\theta\lambda dL}{r^2}$$

Because we know

$$\cos\theta = \frac{z}{\sqrt{z^2+x^2}}, \text{we can plug that in for} \cos\theta$$

$$2\int \frac{k\cos\theta\lambda dL}{r^2}$$
$$= 2\int k\lambda\left[\frac{z}{\sqrt{x^2+z^2}}\right]\frac{1}{(\sqrt{x^2+z^2})^2}dx$$
$$= 2k\lambda z\int_0^{L/2}\frac{1}{(x^2+z^2)^{3/2}}dx$$

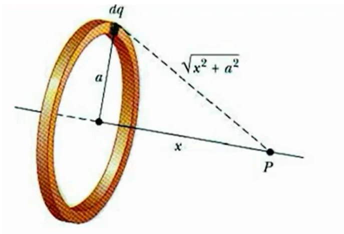
When we integrate this we get:

$$\begin{aligned} E &= 2k\lambda z \left[\frac{x}{z^2\sqrt{x^2+z^2}} \right]_0^{L/2} \\ &= 2k\lambda z \left[\frac{L/2}{z^2\sqrt{(L/2)^2+z^2} - 0} \right] \\ &= \frac{2k\lambda z}{z^2} \left[\frac{L}{2\sqrt{\frac{L^2}{4}+z^2}} \right] \\ &= \frac{kQ}{Lz} \left[\frac{L}{\sqrt{L^2/4+z^2}} \right] \\ &= \frac{kQ}{z\sqrt{L^2/4+z^2}} \end{aligned}$$

and we are done.

Example

Derive an expression for the electric field at the center of a ring of charge with radius a a distance P away from the center.



Note that $\sqrt{x^2+a^2}$, a, and x are constant.

Looking at point P, there will be an infinite amount of electric fields caused by dq at point P.

All the y-components will cancel, so we will integrate through x.

$$\begin{aligned} E_{net} &= dE_x = dE\cos\theta \\ E_x &= \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2} \end{aligned}$$

We want to substitute $\frac{x}{\sqrt{x^2+a^2}}$ for $\cos\theta$.

$$\begin{aligned} E_x &= \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2}\cos\theta \\ &= \frac{1}{4\pi\epsilon_0}\int \frac{dq}{\sqrt{x^2+a^2}} \cdot \frac{x}{\sqrt{x^2+a^2}} \\ &= \frac{x}{4\pi\epsilon_0(x^2+a^2)^{3/2}}\int dQ \\ &= \frac{qx}{4\pi\epsilon_0(x^2+a^2)^{3/2}} \end{aligned}$$

This is the expression for the ring of charge.

1.6 Electric Flux

Flux describes the amount of a given quantity that passes through a given area.

For an electric field that is constant across an area, the electric flux through the area is defined as:

$$\phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$$

The direction of the area vector is defined as perpendicular to the plane of the surface and outward from closed surface.

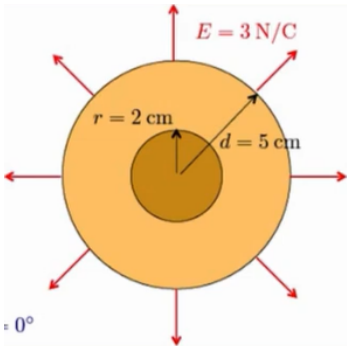
The sign of the flux is given by the dot product of the electric field vector and the area vector.

The total electric flux passing through a surface is defined by the surface integral of the electric field over the surface:

$$\phi_E = \int \vec{E} \cdot \mathrm{d}\vec{A} = EA$$

Example

A sphere of radius $r = 2$ cm creates an electric field $E = 3$N/C at a distance $d = 5$ cm from the center of the sphere. What is the electric flux through the surface of the sphere drawn at a distance $d = 5$cm?



The surface area of a sphere is $4\pi r^2$, so

$$\phi_E = \int \vec{E} \cdot \mathrm{d}\vec{A} = EA = E(4\pi r^2) = (3)(4\pi(0.05)^2) = 0.094\text{N} \cdot \text{m}^2/\text{C}$$

1.7 Gauss's Law

Gauss's law relates electric flux to a Gaussian surface to the charge enclosed by that surface:

$$\phi_E = \frac{q_{\text{enc}}}{\epsilon_0} = \oint \vec{E} \cdot \mathrm{d}\vec{A} = EA$$

A gaussian surface is a three-dimensional, closed surface.

The total electric flux through the surface is independent of the size of the Gaussian surface if the amount of enclosed charge remains constant.

Surfaces are constructed such that the electric field generated by the enclosed charge is either perpendicular or parallel to different regions of the Gaussian surface.

If a function of charge density is given for a charge distribution, the total charge can be determined by integrating the charge density of the length (1D), area (2D), or volume (3D).

Maxwell's equations are the collection of equations that fully describe electromagnetism. The first of these is Gauss's Law.

Example

A spherical cloud of charge radius R contains a total charge $+Q$ with a nonuniform charge density that varies according to the equation:

$$\rho(r) = \rho_0 \left(1 - \frac{r}{R}\right) \text{ for } r \leq R \text{ and}$$
$$\rho = 0 \text{ for } r > R,$$

where r is the distance from the center of the cloud. Express all algebraic answers in terms of Q, R, and fundamental constants. Determine the magnitude E of the electric field when $r > R$.

$$\frac{q_{\text{enc}}}{\epsilon_0} = \int \vec{E} \cdot \mathrm{d}\vec{A} = EA = E(4\pi r^2)$$
$$\implies E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2}$$

2 Electric Potential

2.1 Electric Potential Energy

The electric potential energy of a system of two point charges equals the amount of work required for an external force to bring point charges to their current positions from infinitely far away.

The general form of the electric potential energy between two charged objects is given by the equation:

$$U = \frac{kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1q_2}{r}$$

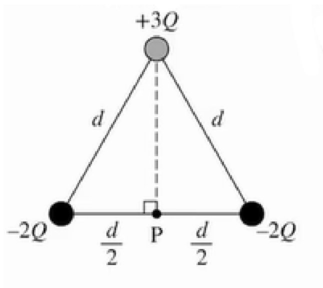
The total electric potential energy of a system can be determined by finding the sum of the electric potential energies of the individual interactions between each pair of charged objects in the system.

When there are opposite signs, the U value will decrease when close together.

When there are same signs, the U value will increase when close together.

Example

Derive an expression for the work required to assemble the charges in the configuration shown.



Remember that

$$W = \Delta U = \sum U$$

So we have:

$$\frac{kq_1q_2}{d} + \frac{kq_1q_3}{d} + \frac{kq_2q_3}{d} = k\left[\frac{(-2Q)(3Q)}{d} + \frac{(-2Q)(-2Q)}{d} + \frac{(-2Q)(3Q)}{d}\right] = 8Q$$

2.2 Electric Potential

Electric potential describes the electric potential energy per unit of charge at a point in space.

Expressions for the electric potential of charge distributions can be found by using integration and the principle of superposition:

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\mathrm{d}q}{r}$$

If there are multiple point charges, we just add up all the point charges.

The electric potential difference between two points is the change in the electric potential energy per unit charge when a test charge is moved between two points:

$$\Delta V = \frac{\Delta U_E}{q}$$

The value of the electric field component in any direction at a given point is equal to the negative of the rate of change in electric potential at that location:

$$E_x = -\mathrm{d}V/\mathrm{d}x$$

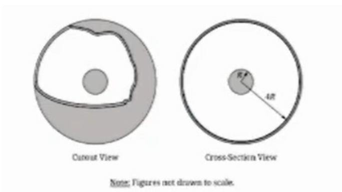
The change in electric potential between two points can be determined by integrating the dot product of the electric field and the displacement along the path connecting the points:

$$\Delta V = V_b - V_a = -\int \vec{E} \cdot \mathrm{d}\vec{r}$$

Equipotential lines represent lines of equal potential energy. These lines are perpendicular to the electric field vectors. Electric field vectors point in the direction of decreasing potential. There is no component of an electric field along an equipotential line.

Example

Derive an expression for the absolute value of the potential difference between the outer surface of the sphere and the inner surface of the shell. Express your answer in terms of Q, R, and physical constants, as appropriate.



Previously we would have found the electric field is:

$$E = -\frac{Q}{4\pi\epsilon_0 r^2}$$

So we must integrate:

$$\Delta V = -\int_R^{4R} \vec{E} \cdot \mathrm{d}\vec{R}$$

$$\Delta V = +\int_R^{4R} +\frac{Q}{4\pi\epsilon_0 r^2}\mathrm{d}r$$

$$\Delta V = \frac{Q}{4\pi\epsilon_0}\int_R^{4R} \frac{\mathrm{d}r}{r^2}$$

$$\Delta V = \frac{Q}{4\pi\epsilon_0}\left[+\frac{1}{r}\right]$$

Applying the limits of integration we get:

$$\Delta V = \frac{3Q}{4\pi\epsilon_0 R}$$

2.3 Conservation of Electric Energy

When a charged object moves between two locations with different electric potentials, the resulting change in the electric potential energy of the object-field system is given by:

$$\Delta U_E = q\Delta V$$

The movement of a charged object between two points with different electric potential results in a change in kinetic energy of the object consistent with the conservation of energy.

Example

A proton (mass $= 1.67 \times 10^{-27}$ kg) is accelerated through a potential difference of 4.5×10^6 V. (a) How much kinetic energy has the proton acquired? (b) If the proton started at rest, how fast is it moving.

For part A, we have

$$\begin{aligned} \Delta U = q\Delta V = \Delta K = K - K_0 \\ = (1.6 \times 10^{-19})(4.5 \times 10^6 \text{V}) \\ = 7.2 \times 10^{-13} \text{J} \end{aligned}$$

For part B:

$$\begin{aligned} K = \frac{1}{2}mv^2 \\ V = \sqrt{\frac{2K}{m}} \\ = \sqrt{\frac{2(7.2 \times 10^{-13})}{1.67 \times 10^{-27}}} = 2.94 \times 10^7 \text{m/s} \end{aligned}$$

3 Conductors and Capacitors

3.1 Electrostatics with Conductors

An ideal conductor is a material in which electrons are able to move freely.

When a conductor is in electrostatic equilibrium, mutual repulsion of excess charge carriers results in those charge carriers residing entirely on the surface of the conductor.

- In a conductor with a negative net charge, excess electrons reside on the surface of the conductor.
- In a conductor with a positive net charge, the surface becomes deficient in electrons and can be modeled as if positive charge carriers reside on the surface of the conductor.

Excess charges will move to the surface of a conductor to create a state of electrostatic equilibrium within the conductor.

- At electrostatic equilibrium, the electroc potential of the surface is the same everywhere and the conductor becomes an equipotential surface.

Recall there is no electric field inside of a conductor.

The charge density on the surface is greater where there are points or edges compared to planar areas.

Electrostatic shielding is the process of an area with a closed conducting shell to create a region inside the conductor that is far from external electric fields.

Example

A solid, uncharged conducting sphere of radius 3a contains a hollowed spherical region of radius a. A point charge +Q is placed at the common center of the spheres. Taking V = 0 as r approaches infinity, the potential at position r = 3a from the center of the sphere is:

We start with (treat as a point charge)

$$V = \frac{kq}{r}$$

and end up getting

$$V = \frac{k[Q]}{3a}$$

which is equal to

$$V = \frac{kQ}{3a}$$

3.2 Redistribution of Charge between Conductors

When conductors are in electrical contact, charges will be redistributed such that the surfaces of each conductor are at the same electric potential.

Ground is an idealized reference point that has zero electric potential and can absorb an infinite amount of charge without changing its electric potential. Charge can be induced by a conductor by grounding the conductor is the presence of an external electric field.

Example

Charge is placed on two conducting spheres that are very far apart and connected by a long thin wire. The radius of the smaller sphere is 5 cm and that of the larger sphere is 12 cm. The electric field at the surface of the larger sphere is 358 kV/m. Find the surface charge density on each sphere.

We start with electric field:

$$E = \frac{kQ}{R^2}$$

$$Q = \frac{ER^2}{k} = \frac{(358 \times 10^3 \text{V/m})}{(0.12\text{m})^2} 9 \times 10^9 = 5.78 \times 10^{-8}\text{C}$$

We know that $V_1 = V_2$, so

$$\frac{kQ_1}{R_1} = \frac{kQ_2}{R_2}$$

Rearranging for Q_2:

$$Q_2 = \frac{R_2}{R_1} Q_1 = \frac{5}{12}(5.78 \times 10^{-7}) = 2.4 \times 10^{-7}\text{C}$$

Now we can find the surface charge on each sphere.

$$\sigma_1 = \frac{Q_1}{A_1} \quad \sigma_2 = \frac{Q_2}{A_2}$$

So

$$\sigma_1 = 3.2 \times 10^{-6} C/m^2$$

and

$$\sigma_2 = 7.6 \times 10^{-6} C/m^2$$

3.3 Capacitors

A parallel-plate capacitor consists of two separated parallel conducting surfaces that can hold equal amounts of charge with opposite signs.

Capacitance relates the magnitude of the charge stored on each plate to the electric potential difference created by the separation of those charges.

$$C = Q/\Delta V$$

Unit = Farads

$$C = \frac{\kappa \epsilon_0 A}{d}$$

The electric field between the two charged plates with uniformly distributed electric charge is constant in both magnitude and direction, except nera the edges of the plates.

$$E = \frac{Q}{\epsilon_0 A} = \sigma/\epsilon_0$$

The electric potential stored in a capacitor is equal to the work done by an external force to separate that amount of charge on the capacitor.

$$U_c \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2 = \frac{1}{2}\frac{Q^2}{C}$$

Example

A capacitor with circular paralell plates of radius R that are separated by a distance d has a capacitance of C. What would the capacitance be if the plates has radius 2R and were separated by a distance d/2?

$$C_0 = \frac{\epsilon_0 A}{d}$$
$$C_0 = \frac{\epsilon_0 \pi r^2}{d} = \frac{\epsilon_0 \pi R^2}{d}$$

$$C = \frac{\epsilon_0 A}{d}$$
$$C = \frac{\epsilon_0 \pi r^2}{d}$$
$$C = \frac{\epsilon_0 \pi (2R)^2}{d/2} = \frac{8\epsilon_0 \pi R^2}{d}$$

3.4 Dielectrics

In a dielectric material, electric charges are not as free to move as they are in a conductor.

The material becomes polarized in the presence of an external electric field.

The dielectric constant of a material relates the electric permittivity of that material to the permittivity of free space.

$$\kappa = \epsilon/\epsilon_0$$

For a dielectric,

$$\int \vec{E} \cdot \mathrm{d}\vec{A} = \frac{Q}{\kappa\epsilon_0} = EA$$

The electric field created by a polarized dielectric is opposite in direction to the external field.

The electric field between the plates of an isolated parallel-plate capacitor decreases when a dielectric is placed between the plates.

$$\kappa = E_0/E$$

The insertion of a dielectric into the capacitor may change the capacitance of the capacitor.

$$C = \kappa C_0 = \frac{\kappa\epsilon_0 A}{d}$$

Example

A capacitor consists of two conducting, coaxial, cylindrical shells of radius a and b, respectively, and length L $>>$ b. The space between the cylinder is filled with oil that has a dielectric constant κ. Initially both cylinders are uncharged, but then a battery is used to charge the capacitor, leaving charge $+$Q on the inner cylinder and -Q on the outer cylinder. Let r be the radial distance from the axis of the capacitor. Determine.

- The electric field from a to b.
- The electric potential from a to b.
- The capacitance of the capacitor.

a.

$$\int \vec{E}\mathrm{d}\vec{A} = \frac{Q}{\epsilon}$$
$$EA = \frac{Q}{\kappa\epsilon_0}$$
$$E = \frac{Q}{2\kappa\pi\epsilon_0 Lr}$$

b.

$$V = -\int_a^b \vec{E}\cdot\mathrm{d}\vec{r}$$
$$V = \int_b^a \left(\frac{Q}{2\kappa\epsilon_0\pi L}\frac{1}{r}\right)\cdot\mathrm{d}r$$
$$V = \frac{Q}{2\kappa\epsilon_0\pi L}\int_b^a \frac{1}{r}\mathrm{d}r$$
$$V = \frac{Q}{2\kappa\epsilon_0\pi L}[\ln r]_b^a$$
$$V = \frac{Q}{2\kappa\epsilon_0\pi L}ln\left[\frac{b}{a}\right]$$

c.

$$C = \frac{Q}{V}$$
$$C = \frac{2\kappa\epsilon_0\pi L}{\ln b/a}$$

4 Electric Circuits

4.1 Electric Current

Current is the rate at which charge passes through a cross-sectional area of a wire.

$$I = \mathrm{d}q/\mathrm{d}t \implies q = \int I \mathrm{d}t$$

Current within a conductor consists charge carriers traveling through the conductor with an average drift velocity.

$$I = nqv_D A$$

Electric charge moves in a circuit in response to an electron potential difference, sometimes referred to as electromotive force, or emf (ϵ)

Current density if the flow of charge per unit area.

$$I = \int \vec{J} \cdot \mathrm{d}\vec{A} \implies I = JA$$

Current density is related to the motion of the charge carriers within a conductor and is a vector quantity.

$$J = nqv_D$$

A potential difference across a conductor creates an electric field within the conductor that is proportional to the resistivity and the current density.

$$\vec{E} = \rho \vec{J}$$

If a function of current density is given, the total current can be determined by integrating the density over the area.

$$I = \int J(\vec{r}) \cdot \mathrm{d}\vec{A}$$

gg6 Although current is a scalr quantity, it does have direction.

- The direction of conventional current is chosen to be the direction in which positive charge would move.
- In common circuits, the current is actually due to the movement of electrons.

Example

A long conducting cylinder has radius R, and carries a current to the left. The current density varies with distance r from the cylinder's central axis according to the equation $J = kr^2$, where $r \leq R$ and k is a positive constant. Derive an expression for the total current in the cylinder.

$$I = \int \vec{J}(r) \cdot \mathrm{d}\vec{A}$$

$$= \int (kr^2)(2\pi r \mathrm{d}r) = \frac{\pi k}{2} R^4$$

4.2 Simple Circuits

A circuit is composed of electrical loops, which can include wires, batteries, resistors, lightbulbs, capacitors, inductors, switches, ammeters, and voltmeters.

A closed electrical loop is a closed path through which charges may flow.

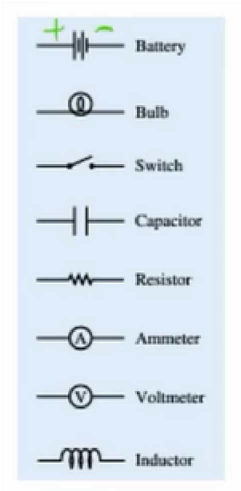
- A closed circuit is one in which charges would be able to flow.
- An open circuit is one in which charges would not be able to flow.
- A short circuit is one which charges would be able to flow with no change in potential difference.

Circuit schematics are representations used to describe and analyze electric circuits.

- The properties of an electric current are dependent on the physical arrangement of its constituent elements.

Circuit elements have common symbols that are used to create schematic diagrams.

- If an element is variable, then the element is indicated by a diagonal strikethrough arrow across the symbol.



4.3 Ohm's Law and Electrical Power

- Resistance is a measure of the degree to which an object opposes the movement of electrical charge.
- It is proportional to its resistivity and length and is inversely proportional to its cross-sectional area.

$$R = \frac{\rho l}{A}$$

- This assumes the resistivity to be uniform.
- If the resistivity is not uniform, meaning it varies along the length, use:

$$R = \int \frac{\rho(l)}{A} \mathrm{d}l$$

- Ohm's Law relates current, resistance, and potential difference across a conductive element of current.

$$I = \frac{\Delta V}{R} \implies V = IR$$

- The resistivity of an ohmic material is constant regardless of temperature.
- The resistance of an ohmic circuit element can be determined from the slope of a graph of the current in the element as a function of the potential difference across the element.
- The rate at which energy is transferred, converted or dissipated by a circuit element depends on the current in the element and the electrical potential difference across it.
- The brightness of a lightbulb increases with power, so power can be used to qualitatively predict the brightness of lightbulbs in a circuit.

4.4 Compound Direct Current Circuits

Circuit elements may be connected in series and/or parallel.

A series connection is one in which any charge passing through one circuit element must proceed through all elements in that connection and has no other path available.

The current in each series circuit element is the same.

A parallel connection is one in which chargesm ay pass through one of two or more paths.

Across each path, the potential difference is the same.

Ideal batteries and wires have negligible internal resistance.

If the battery is not ideal, then it has an internal resistance.

$$\Delta V_{\text{Terminal}} = \epsilon - IR$$

Ammeters are used to measure current at a specific point in a circuit. It must be connected in series in the circuit.

Voltmeters are used to measure the electric potential difference between two points in a circuit. They must be connected in parallel.

4.5 Kirchoff's Rules

- Kirchoff's Rules quantify how current flows through a circuit and how voltage varies around a loop in a circuit.
- Kirchoff's Loop Rule is a consequence of the conservation of energy.
  - This is sometimes called Kirchoff's Voltage Law.
  - It states that the sum of the potential differences across all circuit elements in a single closed loop must equal zero.

$$\sum V + 0$$

- Kirchoff's Junction Rule is a consequence of the conservation of charge.
  - This is sometimes called Kirchoff's Current Law.
  - It states the total amount of charge entering a junction per unit time must equal the total amount of charge exiting the junction per unit time.

$$\sum I = 0$$

4.6 Resistor-Capacitor (RC) Circuits

A collection of capacitors in a circuit may be analyzed as though it was a single capacitor with an equivalent capacitance C_{ep}.

As a result of conservation of charge, each of the capacitors in series must have the same magnitude of charge on each plate.

The charge on a capacitor or the current in a resistor in a RC circuit can be described by a fundamental differential equation derived from Kirchoff's loop rule.

$$\epsilon = \frac{\mathrm{d}q}{\mathrm{d}t} R + \frac{q}{C}$$

The time constant (τ) is a significant feature of an RC circuit.

The time constant of an RC circuit is a measure of how quickly the capacitor will charge or discharge and is defined as

$$\tau = RC$$

5 Magnetic Fields and Electromagnetism

5.1 Magnetic Fields

A magnetic field is a vector field that can be used to determine the magnetic force exerted on moving electric charges, electric currents or magnetic materials.

- Produced by magnetic dipoles or combinations of dipoles, but never by monopoles.
- Magnetic dipoles have north and south polarity.

A magnetic field can be represented by using vector field maps.

Magnetic field lines must form closed loops, as described by Gauss's law of magnetism.

- Gauss's law of magnetism is Maxwell's second equation.
- $\oint \vec{B} \cdot \mathrm{d}\vec{A} = 0$

Magnetic dipoles results from the circular of rotational motion of electric charges.

A magnetic dipole when placed in a magnetic field will align with the magnetic field.

A material's composition influences its magnetic behavior in the presence of an external magnetic field.

Magnetic permeability is a measurement of magnetization in a material in response to an external magnetic field.

Free space has a constant value of magnetic permeability, known as the vacuum permeability, μ_0, that appears in equations representing physical relationships.

$$\mu_0 = 4\pi \times 10^{-7}\text{T}\cdot \text{ m/A}$$

5.2 Magnetism and Moving Charges

A single moving charged object produces a magnetic field.

- It is dependent on the object's velocity and the distance between the point and the object.
- The direction of the magnetic field is perpendicular to both the velocity and the position vector from the object.
  - Determined by using a right-hand rule.

A magnetic field will exert a force on a charged object within that field, with magnitude and direction that depend on the cross-product of the charge's velocity and the magnetic field.

$$F_B = q(\vec{V} \times \vec{B}) = qvB\sin\theta$$

In a region containing both a magnetic field and an electric field, a moving charged particle will experience independent forces from each field.

The Hall effect describes the potential difference created in a conductor by an external magnetic field that has a component perpendicular to the direction of charges moving in the conductor.

5.3 Biot-Savart Law

The Biot-Savart law defines the magnitude and direction of a magnetic field created by an electric current.

$$\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi}\frac{I(\mathrm{d}l \times \hat{r})}{r^2}$$

The magnetic field vectors around a small segment of a current-carrying wire are tangent to concentric circles centered on that wire.

The Biot-Savart Law can be used to derive the magnitudes and directions of magnetic fields around segments of current-carrying wires.

$$B_{loop} = \frac{\mu_0 I}{2R}$$

A magnetic field will exert a force on a current-carrying wire.

$$F_B = \int I(\mathrm{d}\vec{l} \times \vec{B}) = IlB\sin\theta$$

5.4 Ampere's Law

Ampere's law relates the magnitude of the magnetic field to the current enclosed by an imaginary path called an Amperian Loop.

$$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I_{enc}$$

Ampere's law can be used to determine the magnetic field near a long, straight current carrying wire.

All solenoids are assumed to very long, with uniform magnetic fields inside the solenoids and negligible magnetic fields outside the solenoids.

Ampere's law can be used to determine the magnetic field inside of a long solenoid.

$$B_{sol} = \mu_0 nI$$

$n = \frac{\text{Turns}}{\text{Lenght}} = N/L$

An Amperian loop is a closed path arround a current-carrying conductor.

Ampere's law is the third of Maxwell's equations.

$$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{\mathrm{d}\varphi_E}{\mathrm{d}t}$$

6 Electromagnetic Induction

6.1 Magnetic Flux

For a magnetic field that is constant across an area, the magnetic flux through the area is defined as

$$\varphi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$$

The area vector is defined as perpendicular to the plane of the surface area and outward from a closed surface.

The sign of the flux is given by the dot product of the magnetic field vector and the area vector.

The total magnetic flux passing through a surface is defined by the surface integral of the magnetic field over the surface area.

$$\varphi_B = \oint \vec{B} \cdot \mathrm{d}\vec{A}$$

which is usually equal to BA.

6.2 Electromagnetic Induction

Faraday's law describes the relationship between changing magnetic flux and the resulting induced emf in a system.

$$\epsilon = \frac{-\mathrm{d}\varphi_B}{\mathrm{d}t} = \frac{-\mathrm{d}(\vec{B} - \vec{A})}{\mathrm{d}t} = \frac{-\mathrm{d}BA\cos\theta}{\mathrm{d}t}$$

Lenz's law is used to determine the direction of an induced emf resulting from a changing magnetic flux.

- An induced emf generates a current that creates a magnetic field that opposes the change in magnetic flux.

Faraday's law of induction is the third of Maxwell's equations.

$$\epsilon = \oint \vec{E} \cdot \mathrm{d}\vec{l} = \frac{\mathrm{d}\varphi_B}{\mathrm{d}t}$$

6.3 Induced Currents and Magnetic Forces

When an induced current is created in a conductive loop, the already present magnetic field will exert a magnetic force on the moving charge carriers in the loop.

$$F_B = \int I(\mathrm{d}\vec{l} \times \vec{B}) = ILB\cos\theta$$

When current is induced in a conducting loop, magnetic forces are only exerted on the segments of the loop that are within the external magnetic field.

The force on a conducting is proportional to the induced current in the loop, which depends on the rate of change of magnetic flux, the resistance of the loop and the velocity of the loop.

Newton's second law can be applied to a conducting loop moving in a magnetic field as it experiences an induced emf.

6.4 Inductance

Inductance is the tendency of a conductor to oppose a change in electrical current.

- Depends on the physical properties of the conductor.
- An inductor, such as solenoid, is a circuit element that has significant inductance.
- The inductance of a solenoid is

$$L_{sol} = \frac{\mu N^2 A}{l}$$

Inductors store energy in the magnetic field that is generated by current in the inductor.

$$U_L = \frac{1}{2} L I^2$$

The energy stored in the magnetic field generated by an inductor in which current is flowing can be dissipated through a resistor or used to charge a capacitor.

The transfer of energy generated in an inductor to other forms of energy obeys conservation laws.

By applying Faraday's law to an inductor and using the definition of inductance, induced emf can be related to inductance and the rate of change of current.

$$\epsilon_i = -L \frac{\mathrm{d}I}{\mathrm{d}t}$$

6.5 LR Circuits

A resistor will dissipate energy that was stored in an inductor as the current charges.

Kirchhoff's loop rule can be applied to a series LR circuit with a bettery emf ϵ, resulting in a differential equation that describes the current in the loop.

$$\epsilon = IR + L \frac{\mathrm{d}I}{\mathrm{d}t}$$

The time constant is a significant feature of the behavior of an LR circuit.

The time constant of a circuit is a measure of how quickly an LR circuit will reach a steady state and is described with the equation

$$\tau = \frac{L}{R}$$

The electric properties of inductors change during the time interval in which the current in the inductor changes, but will exhibit steady state behavior after a long time interval.

6.6 LC Circuits

In circuits containing only a charged capacitor and an inductor (LC Circuits), the maximum current in the inductor can be determined using conservation of energy within the circuit.

In LC circuits, the time dependence of the charge stroed in the capacitor can be modeled as simple harmonic motion:

$$\frac{\mathrm{d}^2 q}{\mathrm{d}t^2} = -\frac{I}{LC} q$$

The angular frequency of an oscillating LC circuit can be derived from the differnetial equation that describes an LC circuit.

$$\omega = \frac{1}{\sqrt{LC}} \implies \omega = \frac{2\pi}{T} \implies T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$$