

# Linear Algebra Notes

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- 1.3 Vector Equations**
- 1.4 The Matrix Equation  $Ax = b$**
- 1.5 Solution Sets of Linear Systems**
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