A Basic Guide to High School Mathematics

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1 Proof

Introduction to Proof

In this section we will working with these topics:

- Consequence and Equivalence
- Proof by Exhaustion
- Proof by Deduction
- Disproof by Counter-Example
- Proof by Contradiction

When we look at consequence, we essentially say that "a implies b", or:

$$a \to b$$

If the arrow points the other way, we say that "b implies a", or:

$$a \leftarrow b$$

Let's say that statement a states that p is a prime number > 2.

Let's say that statement b states that p is an odd number.

For these statements, we see that a does imply b, so we can write that

$$a \to b$$

The other way however does not work, since because p is an odd number, it does not imply that p is a prime number.

However, if this was true, we can write that a implies b and b implies a, or:

$$a \leftrightarrow b$$

which is sometimes written as "a if and only b" or "a iff b".

Let's show a logical equivalence. Let a be the statement n^2 is odd and b be the statement n is odd.

We know that when n^2 is odd, that n is odd when we list out the odd squared numbers. We can see the converse is true as well in this statement since every time a number n is squared, we are given an odd number, therefore:

$$a \leftrightarrow b$$

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