

A Basic Guide to High School Mathematics

Stasya (Discord: stasssiee)

2024

Contents

1	Proof	9
2	Algebra & Functions	11
2.1	Indices	12
2.2	Surds	12
2.3	Quadratics	12
2.4	Simultaneous Equations	12
2.5	Inequalities	12
2.6	Polynomials & Rational Expressions	12
2.7	Graphs & Proportion	12
2.8	Functions	12
2.9	Graph Transformations	12
2.10	Algebraic Fractions	12
2.11	Modelling	12
3	Coordinate Geometry	13
3.1	Coordinate Geometry	13
3.2	Circles	13
3.3	Parametric Equations	13
3.4	Parametric Equation Modelling	13
4	Sequences & Series	15
4.1	Binomial Expansion	15
4.2	Sequences	15
4.3	Sigma Notation	15
4.4	Arithmetic Sequences	15
4.5	Geometric Sequences	15
4.6	Modelling with Sequences	15
5	Trigonometry	17
5.1	Trigonometry	18
5.2	Small Angle Approximation	18
5.3	Trig Graphs	18
5.4	Further Trigonometry	18
5.5	Trigonometric Identities	18
5.6	Compound Angles & Equivalent Forms	18
5.7	Trig Equations	18
5.8	Proving Trigonometric Identities	18
5.9	Trigonometry in Context	18
6	Exponentials & Logarithms	19
6.1	Exponentials	19
6.2	Exponential Models	19
6.3	Logarithms	19
6.4	Laws of Logarithms	19
6.5	Exponential & Logarithmic Equations	19
6.6	Reduction to Linear Form	19
6.7	Exponential Growth & Decay	19
7	Differentiation	21
7.1	Differentiation from First Principles	22

7.2	Differentiation	22
7.3	Gradients	22
7.4	Further Differentiation	22
7.5	Implicit Differentiation & Parametric Differentiation	22
7.6	Forming Differential Equations	22
8	Integration	23
8.1	Fundamental Theorem of Calculus	23
8.2	Indefinite Integrals	23
8.3	Definite Integrals & Parametric Integration	23
8.4	Integration as the Limit of a Sum	23
8.5	Further Integration	23
8.6	Integration with Partial Fractions	23
8.7	Differential Equations	23
8.8	Differential Equations in Context	23
9	Numerical Methods	25
9.1	The Change of Sign Method	25
9.2	The $x=g(x)$ Method & The Newton-Raphson Method	25
9.3	Numerical Integration	25
9.4	Numerical Methods in Context	25
10	Vectors	27
10.1	Introducing Vectors	27
10.2	Magnitude & Direction of a Vector	27
10.3	Resultant & Parallel Vectors	27
10.4	Position Vectors	27
10.5	Vector Problems	27
11	Statistical Sampling	29
12	Data Presentation & Interpretation	31
12.1	Box Plots, Cumulative Frequency, & Histograms	31
12.2	Scatter Graphs	31
12.3	Central Tendency & Variation	31
12.4	Outliers & Cleaning Data	31
13	Probability	33
13.1	Venn Diagrams, Tree Diagrams, & Two-Way Tables	33
13.2	Conditional Probability	33
13.3	Modelling with Probability	33
14	Statistical Distributions	35
14.1	Discrete Random Variables & The Binomial Distribution	35
14.2	The Normal Distribution	35
14.3	Appropriate Distributions	35
15	Hypothesis Testing	37
15.1	Introducing Hypothesis Testing	37
15.2	Binomial Hypothesis Testing	37
15.3	Sample Means Hypothesis Testing	37
16	Quantities & Units in Mechanics	39
17	Kinematics	41
17.1	Displacement, Velocity, & Acceleration	41

17.2	Graphs of Motion	41
17.3	SUVAT	41
17.4	Calculus in Kinematics	41
17.5	Projectiles	41
18	Forces & Newton's Laws	43
18.1	Introducing Forces & Newton's First Law	43
18.2	Newton's Second Law	43
18.3	Weight & Tension	43
18.4	Newton's Third Law and Pulleys	43
18.5	$F=ma$ & Differential Equations	43
18.6	The Coefficient of Friction	43
19	Moments	45
20	Proof	47
21	Complex Numbers	49
21.1	Introducing Complex Numbers	50
21.2	Working with Complex Numbers	50
21.3	Complex Conjugates	50
21.4	Introducing the Argand Diagram	50
21.5	Introducing Modulus-Argument Form	50
21.6	Multiply and Divide in Modulus-Argument Form	50
21.7	Loci with Argand Diagrams	50
21.8	De Moivre's Theorem	50
21.9	$z = re^{i\theta}$	50
21.10	nth Roots of Unity	50
21.11	Geometrical Problems	50
22	Matrices	51
22.1	Introducing Matrices	52
22.2	The Zero & Identity Matrices	52
22.3	Matrix Transformations	52
22.4	Invariance	52
22.5	Determinants	52
22.6	Inverse Matrices	52
22.7	Simultaneous Equations	52
22.8	Geometrical Interpretation	52
22.9	Factorising Determinants	52
22.10	Eigenvalues and Eigenvectors	52
22.11	Diagonalisation	52
22.12	Cayley-Hamilton Theorem	52
23	Further Algebra & Functions	53
23.1	Roots of Polynomials	54
23.2	Forming New Equations	54
23.3	Summations	54
23.4	Method of Differences	54
23.5	Introducing Maclaurin Series	54
23.6	Standard Maclaurin Series	54
23.7	Limits and l'Hospital's Rule	54
23.8	Polynomial Inequalities	54
23.9	Rational Function Inequalities	54
23.10	Modulus of Functions	54

23.11 Reciprocal Graphs	54
23.12 Linear Rational Graphs	54
23.13 Quadratic Rational Functions	54
23.14 Discriminants	54
23.15 Conic Sections	54
23.16 Transformations	54
24 Further Calculus	55
24.1 Improper Integrals	55
24.2 Volumes of Revolution	55
24.3 Mean Value	55
24.4 Partial Fractions	55
24.5 Differentiating Inverse Trig	55
24.6 Integrals of the Form $\sqrt{a^2 - x^2}$ and $1/(a^2 + x^2)$	55
24.7 Arc Length and Sector Area	55
24.8 Reduction Formulae	55
24.9 Limits	55
25 Further Vectors	57
25.1 Equations of Lines	57
25.2 Equations of Planes	57
25.3 The Scalar Product	57
25.4 Perpendicular Vectors	57
25.5 Intersections	57
25.6 The Vector Product	57
26 Polar Coordinates	59
26.1 Polar Coordinates	59
26.2 Polar Curves	59
26.3 Polar Integration	59
27 Hyperbolic Functions	61
27.1 Hyperbolic Functions	61
27.2 Hyperbolic Calculus	61
27.3 Hyperbolic Inverse	61
27.4 Hyperbolic Inverse	61
27.5 Hyperbolic Integration	61
27.6 Hyperbolic Identities	61
27.7 Hyperbolic Identities	61
28 Differential Equations	63
28.1 1st Order Differential Equations - Integrating Factors	63
28.2 1st Order Differential Equations - Particular Solutions	63
28.3 Modelling	63
28.4 2nd Order Homogeneous Differential Equations	63
28.5 2nd Order Non-Homogeneous Differential Equations	63
28.6 2nd Order Non-Homogeneous Differential Equations	63
28.7 Simple Harmonic Motion	63
28.8 Damped Oscillations	63
28.9 Systems of Differential Equations	63
28.10 Hooke's Law	63
28.11 Damping Force	63
29 Numerical Methods	65
29.1 Mid-Ordinate Rule & Simpson's Rule	65

29.2 Euler's Step by Step Method	65
29.3 Euler's Improved Step by Step Method	65
30 Tracing an Algorithm	67
30.1 Tracing an Algorithm	67
30.2 Complexity	67
31 Bin Packing	69
31.1 Bin Packing	69
31.2 Complexity	69
32 Sorting Algorithms	71
32.1 Introduction	71
32.2 Quick Sort	71
32.3 Bubble Sort	71
33 Graph Theory	73
34 Minimum Spanning Trees	75
34.1 Introduction	75
34.2 Kruskal's Algorithm	75
34.3 Prim's Algorithm	75
34.4 Prim's Algorithm with a Matrix	75
35 Dijkstra's Algorithm	77
36 Critical Path Analysis	79
36.1 Critical Path Analysis (CPA)	79
36.2 Precedence Tables	79
36.3 Activity Networks	79
36.4 Dummy Activities	79
37 Network Flows	81
37.1 Network Flows	81
37.2 Cuts	81
37.3 Supersinks & Supersources	81
38 Linear Programming	83
38.1 Drawing Inequalities & The Objective Function	83
38.2 Formulating an LP Problem	83
38.3 3-Variable to 2-Variable	83
39 Simplex Algorithm	85
40 LP Solvers	87
40.1 Indicator Variables	87
40.2 Shortest Path (Dijkstra's)	87
40.3 Longest Path (CPA)	87
40.4 Network Flows	87
40.5 Critical Path Analysis (Alternative)	87
40.6 Matching	87
40.7 Allocation	87
40.8 Transportation	87
40.9 LINDO	87
41 PMCC	89

41.1 Bivariate Data	89
41.2 Correlation & Association	89
41.3 The PMCC	89
42 Linear Regression	91
42.1 Introduction	91
42.2 Calculating Regression Lines	91
42.3 Interpreting	91
43 PMCC Hypothesis Testing	93
43.1 PMCC Hypothesis Testing	93
43.2 Effect Sizes	93
44 Spearman's Rank	95
44.1 Spearman's Rank Correlation Coefficient	95
44.2 Hypothesis Testing	95
45 Chi-Squared Contingency Table Tests	97
45.1 The Chi-Squared Statistic	97
45.2 Hypothesis Testing	97
46 Discrete Random Variables	99
46.1 Discrete Random Variables	99
46.2 The Expected Value $E(X)$	99
46.3 The Variance $\text{Var}(X)$	99
46.4 $E(aX+b)=aE(X)+b$	99
46.5 $\text{Var}(aX+b)=a^2 \text{Var}(X)$	99
46.6 $E(X+Y) = E(X) + E(Y)$ and $\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y)$	99
47 Discrete Uniform Distributions	101
48 Geometric Distributions	103
49 Binomial Distributions	105
50 Poisson Distribution	107
51 Goodness of Fit Tests	109
51.1 Goodness of Fit Tests	109
51.2 The Uniform Distribution	109
51.3 The Poisson Distribution	109
51.4 The Binomial Distribution	109
51.5 The Left Hand Tail	109
52 Energy	111
52.1 Introduction to Energy	111
52.2 Conservation of Mechanical Energy	111
52.3 The Work-Energy Principle	111
53 Power	113
53.1 Introduction to Power	113
53.2 Horsepower	113
53.3 Maximum Speed	113
53.4 Work, Energy, & Power	113
54 Friction	115

54.1 Introduction to Friction	115
54.2 Block Sliding Down a Slope	115
54.3 Friction Examples	115
54.4 Exam-Style Question	115
55 Momentum & Impulse	117
55.1 Momentum	117
55.2 Impulse	117
56 Collisions	119
56.1 Conservation of Linear Momentum	119
56.2 The Coefficient of Restitution	119
56.3 Hitting the Ground/Hitting the Wall	119
57 Moments	121
57.1 Moments - The Basics	121
57.2 Couples	121
57.3 Ladders	121
57.4 Pivots/Hinges	121
57.5 Sliding & Toppling	121
58 Centre of Mass	123
58.1 Introducing CoM	123
58.2 Laminas	123
58.3 Suspending a Lamina	123
58.4 Triangles	123
58.5 Other Shapes	123
59 Dimensional Analysis	125
59.1 Introducing Dimensional Analysis	125
59.2 Dimensional Consistency	125
59.3 Finding Formulae	125
59.4 Triangles	125
59.5 Other Shapes	125

1 Proof

Introduction to Proof

In this section we will working with these topics:

- Consequence and Equivalence
- Proof by Exhaustion
- Proof by Deduction
- Disproof by Counter-Example
- Proof by Contradiction

When we look at consequence, we essentially say that “ a implies b ”, or:

$$a \rightarrow b$$

If the arrow points the other way, we say that “ b implies a ”, or:

$$a \leftarrow b$$

Let's say that statement a states that p is a prime number > 2 .

Let's say that statement b states that p is an odd number.

For these statements, we see that a does imply b , so we can write that

$$a \rightarrow b$$

The other way however does not work, since because p is an odd number, it does not imply that p is a prime number.

However, if this was true, we can write that a implies b and b implies a , or:

$$a \leftrightarrow b$$

which is sometimes written as “ a if and only b ” or “ a iff b ”.

Let's show a logical equivalence. Let a be the statement n^2 is odd and b be the statement n is odd.

We know that when n^2 is odd, that n is odd when we list out the odd squared numbers. We can see the converse is true as well in this statement since every time a number n is squared, we are given an odd number, therefore:

$$a \leftrightarrow b$$

Proof by Exhaustion

Proof by Deduction

Disprove by Counter-Example

Proof by Contradiction

2 Algebra & Functions

2.1 Indices

Subsets of Real Numbers

The Laws of Indices

2.2 Surds

Simplifying Surds

Rationalising the Denominator

Problem Solving

2.3 Quadratics

The Difference of Two Squares

Factorising Quadratics

Sketching Quadratics from Factorised Form

Completing the Square

Sketching Quadratics from Completed Square Form

Solving Quadratics

Using the Discriminant

Using the Quadratic Formula

Sketching Quadratics Using the Quadratic Formula

Sketching Quadratic Using a Calculator

Using Quadratic Methods for Solving

2.4 Simultaneous Equations

The Elimination Method

The Substitution Method

Further Simultaneous Equations

2.5 Inequalities

Introducing Inequalities, Set Notation and Interval Notation

Linear Inequalities

Quadratic Inequalities

Discriminant Inequalities

More Inequalities

3 Coordinate Geometry

3.1 Coordinate Geometry

Introduction to Coordinate Geometry

Finding the Midpoint

Finding the Distance between Two Points

Finding the Gradient

The Equation of a Line

Parallel and Perpendicular Lines

Sketching Linear Graphs

Perpendicular Bisectors

Intersections of Lines

An Application of Linear Graphs

3.2 Circles

The Equation of a Circle

Sketching Circles

Circles: Completing the Square

Intersections with Circles

Circle Theorems

Circles: Perpendicular Bisectors

Tangents and Normals

3.3 Parametric Equations

Introducing Parametric Equations

Cartesian to Parametric

Graphing Parametric Curves

Parametric to Cartesian

Ellipses

3.4 Parametric Equation Modelling

4 Sequences & Series

4.1 Binomial Expansion

The Factorial Function

Pascal's Triangle

Algebra Problems with nCr

Binomial Expansion

Finding a Coefficient

Approximating using Binomial Expansion

Further Binomial Expansion

The Range of Validity

4.2 Sequences

GCSE Sequences Revision

Inductive Definitions and Recurrence Relations

Describing Sequences

4.3 Sigma Notation

4.4 Arithmetic Sequences

Introducing Arithmetic Sequences

Arithmetic Series

Simultaneous Equation Problems

4.5 Geometric Sequences

Introducing Geometry Sequences

Geometric Series

Sum to Infinity

Simultaneous Equation Problems

4.6 Modelling with Sequences

5 Trigonometry

5.1 Trigonometry

SOHCAHTOA

The Sine Rule

The Cosine Rule

The Area of a Triangle

Radians

Arc Length

Area of a Sector

5.2 Small Angle Approximation

5.3 Trig Graphs

Sketching $\sin(x)$, $\cos(x)$, and $\tan(x)$

Radians

5.4 Further Trigonometry

Cosec(x), Sec(x), Cot(x)

Sketching cosec(x), sec(x), and cot(x)

Inverse Trigonometric Functions

5.5 Trigonometric Identities

Trigonometric Identities

Further Trigonometric Identities

5.6 Compound Angles & Equivalent Forms

Compound Angle Formulae

Double Angle Formulae

Equivalent Forms

5.7 Trig Equations

Basic Trigonometric Equations

Quadratic Trigonometric Equations

Using $\tan(x) = \sin(x)/\cos(x)$

Trigonometric Equations with Transformations

More Quadratic Trigonometric Equations

6 Exponentials & Logarithms

6.1 Exponentials

Introducing a^x

Introducing e

6.2 Exponential Models

6.3 Logarithms

Introducing Logarithms

Introducing Logarithmic Graphs

Sketching $y = \log_b(x + a)$

Sketching $y = \log_b(x + a) + c$

Introducing the Natural Logarithm

Sketching $y = \ln(x + a)$

SKetching $y = \ln(x + a) + b$

6.4 Laws of Logarithms

The Laws of Logarithms

The Natural Logarithm

6.5 Exponential & Logarithmic Equations

Solving $a^x = b$

Logging Both Sides

Inequalities

Hidden Quadratics

Solving $e^x = k$

Logarithmic Equations

Solving $\ln(x)=k$

6.6 Reduction to Linear Form

6.7 Exponential Growth & Decay

7 Differentiation

7.1 Differentiation from First Principles

Gradient of a Straight Line

Differentiating Polynomials

Gradients of Gradient Functions

Second Derivatives

Differentiation from First Principles

Convex and Concave

7.2 Differentiation

Differentiating x^n

Differentiating Standard Functions

7.3 Gradients

Gradients of Functions

Tangents and Normals

Stationary Points

Increasing and Decreasing

The Second Derivative Test

Types of Stationary Point

Convex and Concave

Points of Inflection

Points of Inflection of the Normal Distribution

Optimisation

7.4 Further Differentiation

The Chain Rule

Connected Rates of Change

The Product Rule

The Quotient Rule

Choosing Between Rules

Differentiating an Inverse Function

7.5 Implicit Differentiation & Parametric Differentiation

8 Integration

8.1 Fundamental Theorem of Calculus

8.2 Indefinite Integrals

Integrating ax^n

Finding the Constant of Integration

Integrating Standard Functions

8.3 Definite Integrals & Parametric Integration

Finding Areas

Definite Integrals

Areas Between a Curve and a Line

Areas between Two Curves

Parametric Integration

8.4 Integration as the Limit of a Sum

8.5 Further Integration

Reversing the Chain Rule

Integrating by Substitution

Integration by Parts

Integrating $\ln(x)$

Integration by Parts Twice

The Tabular Method for Integration by Parts

Further Integration

8.6 Integration with Partial Fractions

8.7 Differential Equations

8.8 Differential Equations in Context

9 Numerical Methods

9.1 The Change of Sign Method

The Need for Numerical Methods

The Change of Sign Method

9.2 The $x=g(x)$ Method & The Newton-Raphson Method

The $x=g(x)$ Method

The Newton-Raphson Method

9.3 Numerical Integration

Estimating Areas with Rectangles

The Trapezium Rule

9.4 Numerical Methods in Context

10 Vectors

10.1 Introducing Vectors

What is a Vector?

Finding the Vector between Two Points

Vectors in 3D

10.2 Magnitude & Direction of a Vector

The Magnitude & Direction of a 2D Vector

Finding the Angle Between two Vectors

The Magnitude of a 3D Vector

The Angle between two 3D Vectors

10.3 Resultant & Parallel Vectors

Resultant Vectors

Parallel Vectors

Collinear Points

10.4 Position Vectors

10.5 Vector Problems

11 Statistical Sampling

The Large Data Set

Types of Sample and Sampling Methods

12 Data Presentation & Interpretation

12.1 Box Plots, Cumulative Frequency, & Histograms

Introducing Data Representation

Box Plots/Box and Whisker Diagrams

Cumulative Frequency Curves

Histograms

12.2 Scatter Graphs

Bivariate Data

The Product Moment Correlation Coefficient

Regression Lines

Interpolation vs Extrapolation

12.3 Central Tendency & Variation

Ungrouped Data: Mean, Mode, Median & Quartiles

Grouped Data: Mean, Mode, Median & Quartiles

The Interquartile Range

The Midrange

Comparing Data Sets

Variance and Standard Deviation

Linear Coding

12.4 Outliers & Cleaning Data

13 Probability

13.1 Venn Diagrams, Tree Diagrams, & Two-Way Tables

Basic Probability Concepts

Venn Diagrams

Independent Events / Mutually Exclusive Events

Tree Diagrams

Two-Way Tables

Probability with a Histogram

13.2 Conditional Probability

13.3 Modelling with Probability

14 Statistical Distributions

14.1 Discrete Random Variables & The Binomial Distribution

Introducing Discrete Random Variables

Discrete Probability Distributions as Algebraic Functions

Discrete Uniform Distributions

Cumulative Distribution Functions

The Binomial Distribution

14.2 The Normal Distribution

Introducing the Normal Distribution

Finding Probabilities

The Inverse Normal

Normal to Binomial Problem

Normal to Histogram

Approximating the Binomial Distribution

14.3 Appropriate Distributions

15 Hypothesis Testing

15.1 Introducing Hypothesis Testing

Introducing Hypothesis Testing

Product Moment Correlation Coefficient Hypothesis Testing

Rank Correlation Coefficient Hypothesis Testing

15.2 Binomial Hypothesis Testing

Binomial Hypothesis Testing

Finding the Critical Region

The Critical Region Method

15.3 Sample Means Hypothesis Testing

Introducing Sample Means Hypothesis Testing

Example 1

Example 2

Example 3

16 Quantities & Units in Mechanics

17 Kinematics

17.1 Displacement, Velocity, & Acceleration

Position vs Displacement vs Distance

Velocity vs Speed

Acceleration and Deceleration

17.2 Graphs of Motion

Displacement / Time Graphs

Velocity / Time Graphs

Acceleration / Time Graphs

Graphs of Motion

17.3 SUVAT

Deriving the SUVAT Formulae

Using the SUVAT Formulae

Gravity

More Complicated SUVAT Problems

SUVAT in 2D

17.4 Calculus in Kinematics

General Motion in 1D

General Motion in 2D

17.5 Projectiles

Introducing Projectiles

Projectiles from the Ground

Projectiles from a Height

18 Forces & Newton's Laws

18.1 Introducing Forces & Newton's First Law

Introducing Forces

Force Diagrams

Resultant Forces

Newton's First Law

18.2 Newton's Second Law

Newton's Second Law

Working with the SUVAT Equations

18.3 Weight & Tension

18.4 Newton's Third Law and Pulleys

Newton's Third Law

Pulleys

Lifts and Scale Pans

18.5 $F=ma$ & Differential Equations

$F=ma$ in Two Dimensions

$F=ma$ as Differential Equations

18.6 The Coefficient of Friction

19 Moments

Introducing Moments

Centre of Mass

Equilibrium of a Rigid Body

Tilting

Non-Parallel Forces with Pivots and Ladders

20 Proof

Introducing Proof by Induction

Sums of Series

Divisibility

Sequences

Matrices

Inequalities

Extras

21 Complex Numbers

21.1 Introducing Complex Numbers

Introducing Complex Numbers

Solving Polynomial Equations with Real Coefficients

21.2 Working with Complex Numbers

Real and Imaginary Parts

Working with Complex Numbers

21.3 Complex Conjugates

The Complex Conjugate

Complex Conjugate Pairs

21.4 Introducing the Argand Diagram

21.5 Introducing Modulus-Argument Form

Introducing the Modulus and Argument

Modulus-Argument Form

21.6 Multiply and Divide in Modulus-Argument Form

21.7 Loci with Argand Diagrams

Circles

Perpendicular Bisectors

Loci Problems with Circles & Perpendicular Bisectors

Half-Lines

Loci Problems with Circles, Perpendicular Bisectors and Half-Lines

21.8 De Moivre's Theorem

Introducing De Moivre's Theorem

Expansions of $\cos(n\theta)$ and $\sin(n\theta)$

21.9 $z = re^{i\theta}$

Introducing $z = re^{i\theta}$

Summing Series

21.10 n th Roots of Unity

22 Matrices

22.1 Introducing Matrices

Introducing Matrices

Multiplying Matrices

22.2 The Zero & Identity Matrices

The Zero Matrix

The Identity Matrix

22.3 Matrix Transformations

2D Transformations

3D Transformations

22.4 Invariance

22.5 Determinants

Introducing Determinants

2x2 Matrix Determinants

Negative Determinants and Orientation

3x3 Matrix Determinants

Determinant Problems

22.6 Inverse Matrices

Notation

2x2 Inverse Matrices

Singular Matrices

3x3 Inverse Matrices

22.7 Simultaneous Equations

Two-Variable Simultaneous Equations

Three-Variable Simultaneous Equations

22.8 Geometrical Interpretation

Two Dimensions

Three Dimensions

22.9 Factorising Determinants

23 Further Algebra & Functions

23.1 Roots of Polynomials

23.2 Forming New Equations

Quadratics

Cubics

Quartics

The Substitution Method

23.3 Summations

Introduction

Examples

23.4 Method of Differences

Method of Differences

Method of Differences with Partial Fractions

23.5 Introducing Maclaurin Series

23.6 Standard Maclaurin Series

23.7 Limits and l'Hospital's Rule

Finding a Limit using Maclaurin Series

l'Hopital's Rule

23.8 Polynomial Inequalities

Cubic Inequalities

Quartic Inequalities

23.9 Rational Function Inequalities

Introducing Rational Function Inequalities

Rational Function Inequality Examples

23.10 Modulus of Functions

Modulus of Functions

Solving Equations

Solving Inequalities

24 Further Calculus

24.1 Improper Integrals

Introducing Improper Integrals

Integration Techniques Part 1

Integration Techniques Part 2

24.2 Volumes of Revolution

Introducing Volumes of Revolution

Revolution about the x-axis

Parametric Equations

24.3 Mean Value

Introducing the Mean Value

Examples

24.4 Partial Fractions

Re-Introducing Partial Fractions

Quadratic Factors in the Denominator

24.5 Differentiating Inverse Trig

24.6 Integrals of the Form $\sqrt{a^2 - x^2}$ and $1/(a^2 + x^2)$

24.7 Arc Length and Sector Area

Arc Length

Surface Area

24.8 Reduction Formulae

24.9 Limits

25 Further Vectors

25.1 Equations of Lines

The Vector Equation of a Line

The Cartesian Equation of a Line

25.2 Equations of Planes

25.3 The Scalar Product

25.4 Perpendicular Vectors

25.5 Intersections

Two Lines Intersecting

Intersection of a Line and a Plane

Distance between Two Lines

Distance from a Point to a Line

Distance from a Point to a Plane

25.6 The Vector Product

Introducing the Vector Product

Using the Vector Product

Distances

26 Polar Coordinates

26.1 Polar Coordinates

Introducing Polar Coordinates

Converting between Polar and Cartesian Coordinates

26.2 Polar Curves

Polar Curves

Limacons

Rhodonea / Rose Curves

Further Polar Curves

26.3 Polar Integration

The Area enclosed by a Polar Curve

Polar Tangents

27 Hyperbolic Functions

27.1 Hyperbolic Functions

Introducing Hyperbolic Functions

Hyperbolic Identities & Equations

27.2 Hyperbolic Calculus

Differentiation & Integration

Differentiation

Integration

27.3 Hyperbolic Inverse

27.4 Hyperbolic Inverse

Logarithmic Forms

Differentiation

27.5 Hyperbolic Integration

Differentiating Standard Forms

Integration

27.6 Hyperbolic Identities

Proving "Double Angle" formulae

Using Identities

27.7 Hyperbolic Identities

28 Differential Equations

28.1 1st Order Differential Equations - Integrating Factors

Introduction

Integrating Factors

28.2 1st Order Differential Equations - Particular Solutions

28.3 Modelling

28.4 2nd Order Homogeneous Differential Equations

Introduction

The Auxiliary Equation

28.5 2nd Order Non-Homogeneous Differential Equations

28.6 2nd Order Non-Homogeneous Differential Equations

28.7 Simple Harmonic Motion

28.8 Damped Oscillations

28.9 Systems of Differential Equations

28.10 Hooke's Law

28.11 Damping Force

29 Numerical Methods

29.1 Mid-Ordinate Rule & Simpson's Rule

Mid-Ordinate Rule

Simpson's Rule

29.2 Euler's Step by Step Method

29.3 Euler's Improved Step by Step Method

30 Tracing an Algorithm

30.1 Tracing an Algorithm

30.2 Complexity

31 Bin Packing

31.1 Bin Packing

31.2 Complexity

32 Sorting Algorithms

32.1 Introduction

32.2 Quick Sort

32.3 Bubble Sort

33 Graph Theory

34 Minimum Spanning Trees

34.1 Introduction

34.2 Kruskal's Algorithm

34.3 Prim's Algorithm

34.4 Prim's Algorithm with a Matrix

35 Dijkstra's Algorithm

36 Critical Path Analysis

36.1 Critical Path Analysis (CPA)

36.2 Precedence Tables

36.3 Activity Networks

36.4 Dummy Activities

37 Network Flows

37.1 Network Flows

37.2 Cuts

37.3 Supersinks & Supersources

38 Linear Programming

38.1 Drawing Inequalities & The Objective Function

38.2 Formulating an LP Problem

38.3 3-Variable to 2-Variable

39 Simplex Algorithm

40 LP Solvers

- 40.1 Indicator Variables**
- 40.2 Shortest Path (Dijkstra's)**
- 40.3 Longest Path (CPA)**
- 40.4 Network Flows**
- 40.5 Critical Path Analysis (Alternative)**
- 40.6 Matching**
- 40.7 Allocation**
- 40.8 Transportation**
- 40.9 LINDO**

41 PMCC

41.1 Bivariate Data

41.2 Correlation & Association

41.3 The PMCC

42 Linear Regression

42.1 Introduction

42.2 Calculating Regression Lines

42.3 Interpreting

43 PMCC Hypothesis Testing

43.1 PMCC Hypothesis Testing

43.2 Effect Sizes

44 Spearman's Rank

44.1 Spearman's Rank Correlation Coefficient

44.2 Hypothesis Testing

45 Chi-Squared Contingency Table Tests

45.1 The Chi-Squared Statistic

45.2 Hypothesis Testing

46 Discrete Random Variables

46.1 Discrete Random Variables

46.2 The Expected Value $E(X)$

46.3 The Variance $\text{Var}(X)$

46.4 $E(aX+b)=aE(X)+b$

46.5 $\text{Var}(aX+b)= a^2 \text{Var}(X)$

46.6 $E(X+Y) = E(X) + E(Y)$ and $\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y)$

47 Discrete Uniform Distributions

48 Geometric Distributions

49 Binomial Distributions

50 Poisson Distribution

51 Goodness of Fit Tests

51.1 Goodness of Fit Tests

51.2 The Uniform Distribution

51.3 The Poisson Distribution

51.4 The Binomial Distribution

51.5 The Left Hand Tail

52 Energy

52.1 Introduction to Energy

52.2 Conservation of Mechanical Energy

52.3 The Work-Energy Principle

53 Power

53.1 Introduction to Power

53.2 Horsepower

53.3 Maximum Speed

53.4 Work, Energy, & Power

54 Friction

54.1 Introduction to Friction

54.2 Block Sliding Down a Slope

54.3 Friction Examples

54.4 Exam-Style Question

55 Momentum & Impulse

55.1 Momentum

55.2 Impulse

56 Collisions

56.1 Conservation of Linear Momentum

56.2 The Coefficient of Restitution

56.3 Hitting the Ground/Hitting the Wall

57 Moments

57.1 Moments - The Basics

57.2 Couples

57.3 Ladders

57.4 Pivots/Hinges

57.5 Sliding & Toppling

58 Centre of Mass

58.1 Introducing CoM

58.2 Laminas

58.3 Suspending a Lamina

58.4 Triangles

58.5 Other Shapes

59 Dimensional Analysis

59.1 Introducing Dimensional Analysis

59.2 Dimensional Consistency

59.3 Finding Formulae

59.4 Triangles

59.5 Other Shapes