

A "Basic" Mathematics Guide for High Schoolers

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2024

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Chapter 1

Proof

1.1 Proof

Introduction to Proof

In this section we will working with these topics:

- Consequence and Equivalence
- Proof by Exhaustion
- Proof by Deduction
- Disproof by Counter-Example
- Proof by Contradiction

When we look at consequence, we essentially say that “ a implies b ”, or:

$$a \rightarrow b$$

If the arrow points the other way, we say that “ b implies a ”, or:

$$a \leftarrow b$$

Let’s say that statement a states that p is a prime number > 2 .

Let’s say that statement b states that p is an odd number.

For these statements, we see that a does imply b , so we can write that

$$a \rightarrow b$$

The other way however does not work, since because p is an odd number, it does not imply that p is a prime number.

However, if this was true, we can write that a implies b and b implies a , or:

$$a \leftrightarrow b$$

which is sometimes written as “ a if and only b ” or “ a iff b ”.

Let’s show a logical equivalence. Let a be the statement n^2 is odd and b be the statement n is odd.

We know that when n^2 is odd, that n is odd when we list out the odd squared numbers. We can see the converse is true as well in this statement since every time a number n is squared, we are given an odd number, therefore:

$$a \leftrightarrow b$$

Proof by Exhaustion

Proof by Deduction

Disprove by Counter-Example

Proof by Contradiction

Chapter 2

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2.1 Indices

Subsets of Real Numbers

The Laws of Indices

2.2 Surds

Simplifying Surds

Rationalising the Denominator

Problem Solving

2.3 Quadratics

The Difference of Two Squares

Factorising Quadratics

Sketching Quadratics from Factorised Form

Completing the Square

Sketching Quadratics from Completed Square Form

Solving Quadratics

Using the Discriminant

Using the Quadratic Formula

Sketching Quadratics Using the Quadratic Formula

Sketching Quadratic Using a Calculator

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The Substitution Method

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2.5 Inequalities

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3.1 Coordinate Geometry

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Finding the Distance between Two Points

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Sketching Linear Graphs

Perpendicular Bisectors

Intersections of Lines

An Application of Linear Graphs

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SOHCAHTOA

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Introducing e

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Sketching $y = \log_b(x + a) + c$

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Sketching $y = \ln(x + a)$

Sketching $y = \ln(x + a) + b$

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The Laws of Logarithms

The Natural Logarithm

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Logging Both Sides

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