1 Extra Topics

Problems

1. Which equality axiom of addition is demonstrated by $(ax + by) + c = ax + (by + c)$?

2. Which of the following numbers is considered to be an "abundant" number?

    (A) 26 **(B)** 28 **(C)** 30 **(D)** 32 **(E)** 34

3. $ABC1_{16} + ABC1_{15} = $ _____$_{14}$

4. Let $P = \{2, 3, 5\}, Q = \{2, 4, 6\}$, and $R = \{3, 5, 7\}$. How many elements are in $(P \cap R) \cup (R \cap Q)$?

5. Which of the following sets is closed under addition and subtraction?

    (A) Positive Even Numbers   **(B)** Integers   **(C)** Positive Odd Numbers   **(D)** Primes   **(E)** Wholes

6. Find the harmonic mean of 4 and 9.

7. Which of the following numbers is considered to be an "deficient" number?

    (A) 24 **(B)** 56 **(C)** 66 **(D)** 92 **(E)** 112

8. In the decimal number $2x3y4z$, the letters x, y, and z represent digits where all six digits are distinct. If the number is divisible by 30 then $x + y + z$ could be:

9. $888_9 + 555_6 + 222_3 = $ _____$_3$

10. Use the Fibonacci characteristic sequence $\cdots - 1.5, p, q, 3, r, \ldots$ to Find $p + q + r$.

11. One of Eratosthenes of Cyrene's main contributions to mathematics involved a method for finding _____.

12. I'm an unhappy deficient number but a number that is lucky to be prime. Which of the following numbers am I?

13. Which equality axiom of multiplication is demonstrated by $(a)(a)^{-1} = 1$?

14. How many subsets containing 4 members can be made from the set $\{2, 1, 3, 4, 7, 11\}$?

15. Which of the following was the first Nigerian woman to be awarded a doctorate in mathematics?

16. Find the harmonic mean of the roots of $x^3 - 7x^2 + 14x - 8 = 0$.

17. If R, S, and T are distinct digits then $RST_2 - ST_3 - R_4$ has a numeric value in base 10 of:

18. The set $\{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$ is closed under which of the following operations:

    I. addition   II. subtraction   III. multiplication   IV. division

19. $F_0 = 0$ and F_1 are the first two Fibonacci numbers. Find F_{10}.

20. Let $R = \{1, 3, 5\}, S = \{0, 2, 4\}$, and $T = \{1, 2, 3\}$. How many elements are in $(R \cup T) \cap (S \cup T)$?

21. $(p - q) \times r = pr - qr$ is an example of which property of equality?

22. The 8th Fibonacci number is 13. The 10th Fibonacci number is 34. Find the 9th Lucas number.

23. Find L_9 if $L_0 = 2, L_1 = 1$, and $L_n = L_{(n-1)} + L_{(n-2)}$, where $n \geq 2$.

24. The universal set $U = \{2, 3, 5, 7, 11, 13, 15, 17, 19\}$. Subset $L = \{5, 7, 15, 17\}$, subset $M = \{3, 13\}$. How many elements are in the complement set of $L \cup M$?
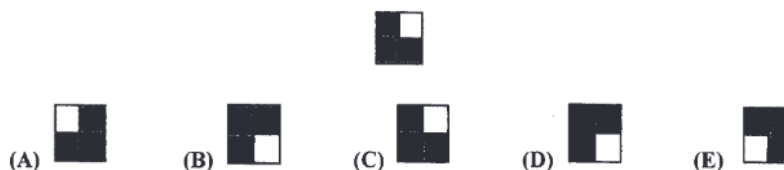
25. Which of the following mathematicians was most remembered as the inventor of logarithms?

26. In the binomial expansion of $(3x - 1)^5$, the coefficient of the fourth term is:

27. What are the odds that a factor of 2010 is a prime number?

28. The formula $e^{ix} = \cos x + i \sin x$, where e is the base of the natural logarithm and i is the imaginary unit, is named after:

29. The odd numbers from 1 to 17 are to be placed in this magic square in which the rows and columns have the same sum. Find the value of x.

	1	
5		13
x		

30. $P = \{p, l, u, s\}, Q = \{m, i, n, u, s\}$, and $R = \{t, i, m, e, s\}$. How many elements are in $(P \cup Q) \cap (P \cup R)$?

31. The number 12010 in base 3 is equivalent to the number $wxyz$ in base 5, where w, x, y, and z are digits. Find $w + x + y + z$.

32. $3(x + 4) = 5$ and $3(4 + x) = 5$ is an example of the _____property.

33. If $ax + b = c$ and $c = dx + e$, then $ax + b = dx + e$ is an example of the _____property.

34. Which of the following is true about the relation graphed below?
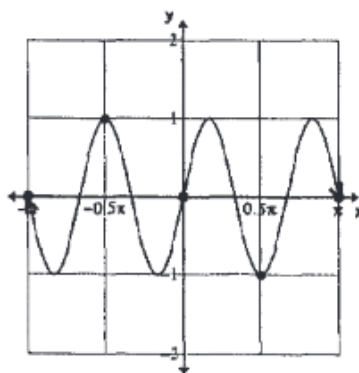
35. Integers x & y exist such that $x = 2y$ and the arithmetic mean of x & y is 1 more than the harmonic mean of x & y. Find the geometric mean of x & y.

36. The figure shown is reflected over a negative diagonal. Which of the following figures is the result of that single transformation?

37. A recent visit to the planet Strangebase discovered that the equation, $3S^2 - 25S + 66 = 0$, has two solutions, 4 and 9. What base was being used for the number system on planet Strangebase?

38. Evaluate: $\prod\limits_{n=2}^{6}\left(1 + \frac{1}{n}\right)$

39. Which of the following mathematicians created an abacus for calculating products and quotients and extracting square roots that was based on Arab mathematics and lattice multiplication.

40. Polly Euler folds the net shown into a cube. What letter will be on the opposite side of side S?
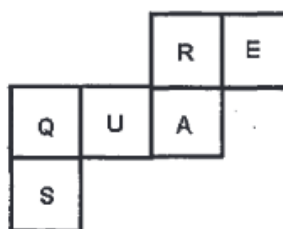
41. If $\sqrt{x\sqrt[3]{x\sqrt[4]{x}}} = \sqrt[n]{x^k}$, where k and n are relatively prime, then $k =$?

42. The universal set $U = \{1, 2, 3, 5, 8, 13, 21, 34\}$. Subset $A = \{1, 3, 8, 21, 34\}$ and subset $B = \{2, 3, 5, 13, 21\}$. How many elements are in the complement set of $A \cap B$?
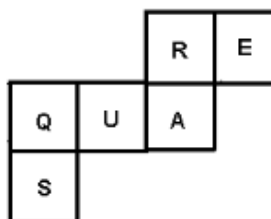
43. Which of the following numbers is an unhappy number and evil number?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

44. $2 \times 4 \times 8 = 8 \times 8 = 64$ and $2 \times 4 \times 8 = 2 \times 32 = 64$ are examples of the ? property of equality.

45. If $2(3 + 5) = 16$ and $16 = 4^2$ then $2(3 + 5) = 4^2$. Which of the following properties does this example illustrate?

46. Pauline Gone folds the net shown into a cube. What letter will be on the opposite side of face E?

47. Find $a + b + c + d$ given the Fibonacci characteristic sequence: $3, a, b, 17, c, d, 71, \ldots$.

48. Which of the following mathematicians is known for developing a "machine" that uses a system of rules, states, and transitions used to decide a language or to solve mathematical functions? It is a powerful tool used in computer science and code breaking?

49. Mr. White's 'bath tub mat' pattern table consists of 19 columns and 12 rows. Only 7 rows are shown. Determine the sum of the numbers in the 8th row.

50. The harmonic mean of the real roots of $3x^3 + 2x^2 + 5x + 4 = 0$ is _____.

51. Which of the following pairs of numbers are considered to be 'fangs' of a 'vampire' number?

I. (35, 41)   II. (21, 87)   III. (72, 27)   IV. (51, 63)

52. Let $4022_b - k_b = 1665_b$, where k_b is a four digit number. Find k_b in base 10.

53. 7,158,AB3 ÷ 9 has a remainder of 7. Find $A + B$.

54. A square-free semiprime is a composite number that is the product of two different primes. How many composite numbers less than 20 are considered square-free semiprimes?

55. Let U (universal set) $= \{u, i, l, m, a, t, h, b\}$, $B = \{b, u, i, l, t\}$, and $T = \{t, h, u, m, b\}$. Let $I = (B \cap T)^C$. Set I contains how many distinct elements?

56. Which of the following sets of numbers is closed under multiplication and addition?

I. Primes   II. Integers   III. Wholes   IV. Rationals

57. For how many different positive integers n is each of n, $n + 2$, and $n + 4$ a prime number?

58. What's the only positive integer whose two largest divisors have a sum of 111?

59. For how many different pairs of positive integers (a, b), with greatest common factor 1, and with $a > b$, does $ab = 30!$? (Note: 30! is the product of the first 30 positive integers.)

60. Leonardo Pisano Bigollo was an Italian mathematician who referenced and made known which of the following special sequences of numbers to Western mathematics?

61. Which of the following numbers is an abundant, happy, and lucky number?

(A) 28 **(B)** 31 **(C)** 44 **(D)** all of these   **(E)** none of these

62. Find $a + b + c + d$ given the Fibonacci characteristic sequence: $a, 2, b, c, 20, d, 51, \dots$.

63. The set of Lucas numbers is $1, 3, 5, 7, 11, \dots$, where $L_1 = 1$. The set of Fibonacci numbers is $\{1, 1, 2, 3, 5, \dots\}$, where $F_1 = 1 = F_2$. If $L_{10} = F_x + F_y$, where $y > x$, then y is _____.

64. If the following pattern continues, determine which of the following numbers will be in row 10.

```
            1           row 0
          1   1         row 1
        1   2   1       row 2
        1   3   3       row 3
      1   4   6   4   1     row 4
    1   5   10   10   5   1     row 5
```

65. An "emirp" number is a prime number that becomes a new prime number when the digits are reversed. Single digit primes and palindromic primes cannot be emirp numbers. How many prime numbers less than 20 are considered to be emirp numbers?

66. Let $(131_b) \times 3_b = k_b$, where k_b is a 3-digit number. Find b if $k_b = 1323_4$.

67. If P, Q, and R are different digits, then the largest possible three-digit sum for $PPP + QP + P = ?$ has which of the following forms?

68. Let $A = \{a, c, u, t, e\}, O = \{o, b, t, u, s, e\}$, and $R = \{r, i, g, h, t\}$. The number of elements in $(A \cup R) \cap O$ is:

69. $111A09201B \div 9$ has a remainder of 5. Find the least value of $A + B$.

70. Which of the following mathematicians is noted for his work with sets, probability, and logic?

71. $236_7 + 453_7 - 165_7 =$ _____$_7$.

72. Find $a + b + c + d$ given the Fibonacci characteristic sequence: $a, -3, b, -1, c, 0, d, 1, \ldots$

73. Evaluate: $\displaystyle\prod_{n=2}^{6} (-1)^{n-2} - n$

74. Find the sum of the first three Mersenne Primes such that the sum is a prime but is not a Mersenne Prime.

75. Eratosthenes shifted out all of the prime numbers less than 100. How many of these primes contained the digit 9?

76. Let $S = \{7, 11, 18\}$. The arithmetic mean of S is A. The geometric mean of S is G. And the harmonic mean of S is H. Which of the following is the correct order of the means A, G, and H from least to greatest?

77. Expand $10^B \div (10^{(2B)} - 10^B - 1)$ for $B = 2$. What is the 20th digit after the decimal place?

78. How many even four-digit counting numbers are less than 7000?

79. Which mathematician created the Method of Exhaustion?

80. Which of the following numbers is a counting number?

(A) $3i$ **(B)** 3 **(C)** -1 **(D)** $\sqrt{5}$

81. Let $U = \{0, 1, 2, 3, 5, 6, 9\}$ be the universal set, $J = \{1, 2, 5\}$, and $F = \{1, 3, 6\}$. What is the number of elements in $\{J' \cap F'\}$, where J' and F' denote the complement sets?

82. Find the 5th term of row 10.

```
            1       row 0
          1   1       row 1
        1   2   1       row 2
      1   3   3   1       row 3
      1   4   6   4   1       row 4
                ...
```

83. Which mathematician is noted for work on conic sections and the construction of astrolabes used for navigation?

84. 4 bushels + 2 pecks + 1 gallon + 3 quarts = _____pints.

85. $(613_8 - 316_8) \times 4_8 =$ _____$_8$

86. Find the greatest common divisor of $270, 504,$ and 882.

87. $2153A16B19 \div 11$ has a remainder of 6. Find $A - B$.

88. Andy Foundette knows of an odd integer between 600 and 800 that is divisible by both 7 and 9. He added the digits of this number. What sum did Andy get?

89. The number 215 is a member of which of the following sets of special types of numbers:

(E)vil   (H)appy   (O)dious   (U)nhappy

90. Given that the set of natural numbers continue in the triangular pattern shown below, find the sum of the 7th number in row 8 and the 8th number in row 9.

```
                1       (row 1)
              2   3   4       (row 2)
            5   6   7   8   9       (row 3)
        10   11   12   13   14   15   16       (row 4)
                    ...       (...)
```

91. Let $\begin{bmatrix} a & 2 \\ 1 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 6 & b \end{bmatrix} = \begin{bmatrix} 15 & 19 \\ 33 & 46 \end{bmatrix}$. Find $a + b$.

92. Three integers, p, q, and r exist such that they form an arithmetic progression and their product is a prime number. Find the absolute value difference of the smallest and largest of the three integers.

93. If $P = \{2, 3, 5, 7, 11\}$, $T = \{1, 3, 6, 10, 15\}$, $F = \{1, 2, 3, 5, 8\}$, and $L = \{1, 2, 3, 4, 7\}$, then $(P \cup T) \cap (F \cup L)$ contains how many elements?

Solutions

1. Associative

2. C

3. 21411

4. 2

5. B

6. $5\frac{7}{13}$

7. D

8. 12

9. 689

10. 6.75

11. prime numbers

12. 37

13. Commutative

14. 15

15. Grace Alele Williams

16. $1\frac{5}{7}$

17. $3R - S$

18. I, II & III

19. 55

20. 3

21. distributive

22. 47

23. 76

24. 3

25. John Napier

26. -90

27. $\frac{1}{3}$

28. Leonard Euler

29. 7

30. 6

31. 6

32. commutative

33. distributive

34. It is a one-to-one function.

35. $3\sqrt{2}$

36. D

37. base 5

38. 11.39

39. Sophie Germain

40. A

41. 12

42. 6

43. C

44. associative

45. transitive

46. U

47. 88

48. Alan Turing

49. 898

50. -2.4

51. I & II

52. 1,117

53. 10

54. 4

55. 5

56. II, III, & IV

57. 1

58. 74

59. 512

60. $1, 1, 2, 3, 5, 8, \ldots$

61. E

62. 58

63. 11

64. 252

65. 2

66. 5

67. QQR

68. 3

69. 8

70. John Venn

71. 524

72. 9

73. -360

74. 41

75. 6

76. H, G, A

77. 5

78. 3000

79. Archimedes

80. B

81. 2

82. 210

83. Hypatia

84. 302

85. 1364

86. 18

87. 5

88. 18

89. E & U

90. 128

91. 10

92. 4

93. 5