

Differential Equations Notes

anastasia

Spring 2025

Contents

1	Introduction to Differential Equations	3
1.1	Background	3
1.2	Solutions and Initial Value Problems	3
1.3	Direction Fields	3
1.4	The Approximation Method of Euler	3
2	First-Order Differential Equations	4
2.1	Separable Equations	4
2.2	Linear Equations	4
2.3	Exact Equations	4
2.4	Special Integrating Factors	4
2.5	Substitutions and Transformations	4
3	Mathematical Models and Numerical Methods Involving First-Order Equations	5
3.1	Compartmental Analysis	5
3.2	Numerical Methods: A Closer Look At Euler's Algorithm	5
3.3	Higher-Order Numerical Methods: Taylor and Runge-Kutta	5
4	Linear Second-Order Equations	6
4.1	Introduction: The Mass-Spring Oscillator	6
4.2	Homogeneous Linear Equations: The General Solution	6
4.3	Auxiliary Equations with Complex Roots	6
4.4	Nonhomogeneous Equations: the Method of Undetermined Coefficients	6
4.5	The Superposition Principle and Undetermined Coefficients Revisited	6
4.6	Variation of Parameters	6
4.7	Variable-Coefficient Equations	6
5	Introduction to Systems	7
5.1	Differential Operators and the Elimination Method for Systems	7
5.2	Solving Systems and Higher-Order Equations Numerically	7
6	Theory of Higher-Order Linear Differential Equations	8
6.1	Basic Theory of Linear Differential Equations	8
6.2	Homogeneous Linear Equations with Constant Coefficients	8
6.3	Undetermined Coefficients and the Annihilator Method	8
6.4	Method Of Variation of Parameters	8
7	Laplace Transforms	9
7.1	Definition of the Laplace Transform	9
7.2	Properties of the Laplace Transform	9
7.3	Inverse Laplace Transform	9
7.4	Solving Initial Value Problems	9
7.5	Transforms of Discontinuous Functions	9
7.6	Transforms of Periodic and Power Functions	9
7.7	Convolution	9
7.8	Impulses and the Dirac Delta Function	9
7.9	Solving Linear Systems with Laplace Transforms	9
8	Series Solutions of Differential Equations	10
8.1	Introduction: The Taylor Polynomial Approximation	10
8.2	Power Series and Analytic Functions	10
8.3	Power Series Solutions to Linear Differential Equations	10
8.4	Equations with Analytic Coefficients	10
8.5	Method of Frobenius	10

9	Matrix Methods for Linear Systems	11
9.1	Introduction	11
9.2	Review 1: Linear Algebraic Equations	11
9.3	Review 2: Matrices and Vectors	11
9.4	Linear Systems in Normal Form	11
9.5	Homogeneous Linear Systems with Constant Coefficients	11
9.6	Complex Eigenvalues	11
9.7	Nonhomogeneous Linear Systems	11

1 Introduction to Differential Equations

1.1 Background

1.2 Solutions and Initial Value Problems

1.3 Direction Fields

1.4 The Approximation Method of Euler

2 First-Order Differential Equations

2.1 Separable Equations

2.2 Linear Equations

2.3 Exact Equations

2.4 Special Integrating Factors

2.5 Substitutions and Transformations

3 Mathematical Models and Numerical Methods Involving First-Order Equations

3.1 Compartmental Analysis

3.2 Numerical Methods: A Closer Look At Euler's Algorithm

3.3 Higher-Order Numerical Methods: Taylor and Runge-Kutta

4 Linear Second-Order Equations

- 4.1 Introduction: The Mass-Spring Oscillator**
- 4.2 Homogeneous Linear Equations: The General Solution**
- 4.3 Auxiliary Equations with Complex Roots**
- 4.4 Nonhomogeneous Equations: the Method of Undetermined Coefficients**
- 4.5 The Superposition Principle and Undetermined Coefficients Revisited**
- 4.6 Variation of Parameters**
- 4.7 Variable-Coefficient Equations**

5 Introduction to Systems

5.1 Differential Operators and the Elimination Method for Systems

5.2 Solving Systems and Higher-Order Equations Numerically

6 Theory of Higher-Order Linear Differential Equations

6.1 Basic Theory of Linear Differential Equations

6.2 Homogeneous Linear Equations with Constant Coefficients

6.3 Undetermined Coefficients and the Annihilator Method

6.4 Method Of Variation of Parameters

7 Laplace Transforms

7.1 Definition of the Laplace Transform

7.2 Properties of the Laplace Transform

7.3 Inverse Laplace Transform

7.4 Solving Initial Value Problems

7.5 Transforms of Discontinuous Functions

7.6 Transforms of Periodic and Power Functions

7.7 Convolution

7.8 Impulses and the Dirac Delta Function

7.9 Solving Linear Systems with Laplace Transforms

8 Series Solutions of Differential Equations

8.1 Introduction: The Taylor Polynomial Approximation

8.2 Power Series and Analytic Functions

8.3 Power Series Solutions to Linear Differential Equations

8.4 Equations with Analytic Coefficients

8.5 Method of Frobenius

9 Matrix Methods for Linear Systems

9.1 Introduction

9.2 Review 1: Linear Algebraic Equations

9.3 Review 2: Matrices and Vectors

9.4 Linear Systems in Normal Form

9.5 Homogeneous Linear Systems with Constant Coefficients

9.6 Complex Eigenvalues

9.7 Nonhomogeneous Linear Systems