

1 Force and Translational Dynamics

1.1 Systems and Center of Mass

- System properties are determined by the interactions between objects within the system.
- If the properties or interactions of the constituent objects within a system are not important in modeling the behavior of a macroscopic system, the system can itself be treated as a single object.
- Systems may allow interactions between constituent parts of the system and the environment, which may result in the transfer of energy or mass.
- For objects with symmetrical mass distributions, the center of mass is located on lines of symmetry.
- The location of a system's center of mass along a given axis can be calculated using the equation:

$$x_{cm} = \frac{\sum m_i x_i}{M}$$

- For a nonuniform solid that can be considered as a collection of differential masses, dm , the solid's center of mass can be calculated using

$$x_{cm} = \int x dm / M$$

- A system can be modeled as a singular object that is located at the system's center of mass.

1.2 Forces and Free-Body Diagrams

- Forces are vector quantities that describe interactions between objects or systems.
- Contact forces describe the interaction of an object or system touching another object or system.
- Free-body diagrams (FBDs) are useful tools for visualizing forces exerted on a single object or system and for determining the equations that represent a physical situation.
- The FBD of an object or system shows each of the forces exerted on the object or system by the environment.
- Forces exerted on an object or system are represented as vector originating from the center of mass, such as a dot.
- Choose a coordinate system such that one axis is parallel to the acceleration of the object or system.

1.3 Newton's Third Law

Newton's third law describes the interaction of two objects or systems in terms of the paired forces that exerts on the other.

$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$$

Interactions between objects within a system do not influence the motion of a system's center of mass.

Tension is the macroscopic net results of forces that infinitesimal segments of a string, cable, chain or similar system exert on each other in response to an external force.

- An ideal string has negligible mass and does not stretch when under tension.
- The tension in an ideal string is the same at all points within the string.

- In a string with nonnegligible mass, tension may not be the same at all points within the string.
- An ideal pulley that has negligible mass and rotates about an axle through its center of mass with negligible friction.

1.4 Newton's First Law

The net force on a system is the vector sum of all forces exerted on the system.

Translational equilibrium is the configuration of forces that the net force exerted on a system is zero.

$$\sum F = 0$$

Newton's first law states that if the net force exerted on a system is zero, the velocity of that system will remain constant.

Forces may be balanced in one dimension but unbalanced in another.

1.5 Newton's Second Law

Unbalanced forces are a configuration of forces such that the net force exerted on a system is not equal to zero.

Newton's second law of motion states that the acceleration of a system's center of mass has a magnitude proportional to the magnitude of the net force exerted on the system and is in the same direction of the force.

$$\sum F = ma = 0$$

The velocity of a system's center of mass will only change if a nonzero net external force is exerted on that system/

1.6 Gravitational Force

Newton's law of universal gravitation describes the gravitational force between two objects as directly proportional to each of their masses and inversely proportional to the square of the distance between their centers.

$$F_G = \frac{Gm_1m_2}{d^2}$$

A field models the effects of a noncontact force exerted on an object at various positions in space.

The magnitude of the gravitational field created by a system of mass M at a point in space is equal to the ratio of the gravitational force exerted by the system on a test object of mass m to the mass of the test object.

$$\vec{g} = \frac{\vec{F}_g}{m}$$

If a system is accelerating, the apparent weight of the system is not equal to the magnitude of the gravitational force exerted on the system.

Newton's shell law theorem describes the net gravitational force exerted on an object by a uniform spherical shell of mass.

1.7 Kinetic and Static Friction

Kinetic friction occurs when two surfaces in contact move relative to each other.

- It opposes the direction of motion.
- The surface area of contact is not a factor.

The magnitude of the kinetic friction force exerted on an object is the product of the normal force the surface exerts on the object and the coefficient of kinetic friction.

$$f_k = \mu_k F_N$$

Static friction may occur between the contacting surfaces of two objects that are not moving relative to each other.

Static friction adopts the value and direction required to prevent an object from slipping or sliding on a surface.

$$f_s \leq \mu_s F_N$$

The coefficient of static friction is typically greater than the coefficient of kinetic friction for a given pair of surfaces.

1.8 Spring Forces

An ideal spring has negligible mass and exerts a force that is proportional to the change in its length as measured from its relaxed length.

The magnitude of the force exerted by an ideal spring on an object is given by Hooke's Law:

$$F_{sp} = -k\Delta x$$

The force exerted on an object by a spring is always directed toward the equilibrium position of the object-spring system.

A collection of springs that exert forces on an object may behave as though they were a single spring with an equivalent spring constant.

- Springs in series: $\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} + \dots$
- Springs in parallel: $k_{eff} = k_1 + k_2 + \dots$

1.9 Resistive Forces

A resistive force is defined as a velocity-dependent force in the opposite direction of an object's velocity.

$$F_R = -kv[F_R = -bv^2]$$

Applying Newton's second law to an object upon which a resistive force is exerted results in a differential equation for velocity.

- The differential portion of a = the equation comes from substituting in $a = \frac{dv}{dt}$

Terminal velocity is defined as the maximum speed achieved by an object moving under the influence of a constant force and a resistive force that are exerted on the object in opposite directions.

- For a falling object, this occurs when the air resistance equals the weight of the object.

1.10 Circular Motion

Centripetal acceleration is the component of an object's acceleration directed toward the center of the object's circular path.

- The magnitude of the acceleration for an object moving in a circular path is the ratio of the object's tangential speed squared to the radius of the circular path.

$$a_c = v^2/r$$

Centripetal acceleration can result from a single force, more than one force, or components of forces that are exerted on an object in circular motion.

Tangential acceleration is the rate at which an object's speed changes and is directed tangent to the object's circular path.

$$a = \sqrt{a_c^2 + a_T^2}$$

The net acceleration of an object moving in a circle is the vector sum of the centripetal acceleration and tangential acceleration.

The revolution of an object traveling in a circular path at a constant speed (UCM) can be described using period and frequency.

$$v = \frac{2\pi r}{T} = 2\pi r f \quad T = \frac{1}{f}$$