1 Electric Potential

1.1 Electric Potential Energy

The electric potential energy of a system of two point charges equals the amount of work required for an external force to bring point charges to their current positions from infinitely far away.

The general form of the electric potential energy between two charged objects is given by the equation:

$$U = \frac{kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1q_2}{r}$$

The total electric potential energy of a system can be determined by finding the sum of the electric potential energies of the individual interactions between each pair of charged objects in the system.

When there are opposite signs, the U value will decrease when close together.

When there are same signs, the U value will increase when close together.

Example

Derive an expression for the work required to assemble the charges in the configuration shown.
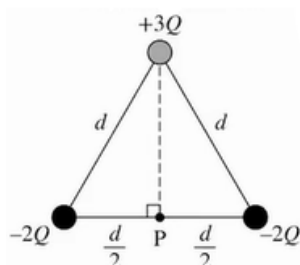
Remember that

$$W = \Delta U = \sum U$$

So we have:

$$\frac{kq_1q_2}{d} + \frac{kq_1q_3}{d} + \frac{kq_2q_3}{d} = k\left[\frac{(-2Q)(3Q)}{d} + \frac{(-2Q)(-2Q)}{d} + \frac{(-2Q)(3Q)}{d}\right] = 8Q$$

Exercise Two point charges, one with positive charge $+2Q$ and one with negative charge $-Q$, are fixed in a distance D apart. The electric potential energy of the two-point charge system is U_0. If a third, positive, point charge is added to the system, where should the point charge be located so that the electric potential energy of the three-point charge system is still U_0? What evidence or reasoning supports the claim?

Exercise Consider four different systems consisting of a pair of point charges. The point charges in the systems have the following charge values and separation distances.

- System 1: point charges of $+1$ nC and $+1$ nC, separated by 1 cm
- System 2: point charges of $+1$ nC and $+4$ nC, separated by 4 cm

- System 3: point charges of -1 nC and +6 nC, separated by 1 cm

- System 4: point charges of +2 nC and +2 nC, separated by 2 cm

Rank the electric potential energies U_1, U_2, U_3. and U_4 of the four systems. Take the potential energy of a system to be zero when the point charges are infinitely apart.

Exercise A system is comprised of two positive point charges, which have charge values of 1 nC and 2 nC. How does the amount of work done on the system by an external force or forces for the following three scenarios?

- Scenario 1: W_1 is the work done by an external force to move the 1 nC point charge from far away to a distance of 1 cm from the 2 nC point charge, which is fixed in place.

- Scenario 2: W_2 is the work done by an external force to move the 2 nC point charge from far away to a distance of 1 cm from the 1 nC point charge, which is fixed in place.

- Scenario 3: W_3 is the total work done by external forces to move both point charges simultaneously from far away to locations that are a distance of 1 cm from each other.

In each scenario, the initial positions of any point charges that get moved can be considered to be infinitely far away.

1.2 Electric Potential

Electric potential describes the electric potential energy per unit of charge at a point in space.

Expressions for the electric potential of charge distributions can be found by using integration and the principle of superposition:

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{\mathrm{d}q}{r}$$

If there are multiple point charges, we just add up all the point charges.

The electric potential difference between two points is the change in the electric potential energy per unit charge when a test charge is moved between two points:

$$\Delta V = \frac{\Delta U_E}{q}$$

The value of the electric field component in any direction at a given point is equal to the negative of the rate of change in electric potential at that location:

$$E_x = -\mathrm{d}V/\mathrm{d}x$$

The change in electric potential between two points can be determined by integrating the dot product of the electric field and the displacement along the path connecting the points:

$$\Delta V = V_b - V_a = -\int \vec{E} \cdot \mathrm{d}\vec{r}$$

Equipotential lines represent lines of equal potential energy. These lines are perpendicular to the electric field vectors. Electric field vectors point in the direction of decreasing potential. There is no component of an electric field along an equipotential line.

> **Example**
>
> Derive an expression for the absolute value of the potential difference between the outer surface of the sphere and the inner surface of the shell. Express your answer in terms of Q, R, and physical constants, as appropriate.

Cutout View   Cross-Section View

Note: Figures not drawn to scale.
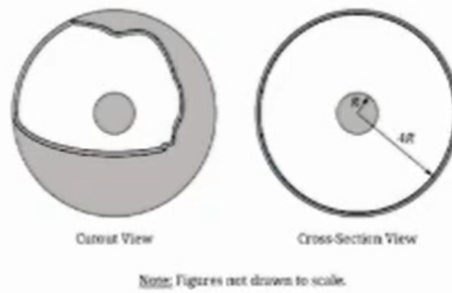
Previously we would have found the electric field is:

$$E = -\frac{Q}{4\pi\epsilon_0 r^2}$$

So we must integrate:

$$\Delta V = -\int_{R}^{4R} \vec{E} \cdot \mathrm{d}\vec{R}$$

$$\Delta V = +\int_{R}^{4R} +\frac{Q}{4\pi\epsilon_0 r^2}\mathrm{d}r$$

$$\Delta V = \frac{Q}{4\pi\epsilon_0}\int_{R}^{4R}\frac{\mathrm{d}r}{r^2}$$

$$\Delta V = \frac{Q}{4\pi\epsilon_0}\left[+\frac{1}{r}\right]$$

Applying the limits of integration we get:

$$\Delta V = \frac{3Q}{4\pi\epsilon_0 R}$$

Exercise Two point charges are located on the x-axis. A -4.0 nC point charge is at $x = -0.20$ m, and a $+5.0$ nC point charge is at $x = +0.10$ m. What is the electric potential on the x-axis at $x = 0.00$ m? Take the potential to be zero at an infinite distance from the point charges.

1.3 Conservation of Electric Energy

When a charged object moves between two locations with different electric potentials, the resulting change in the electric potential energy of the object-field system is given by:

$$\Delta U_E = q\Delta V$$

The movement of a charged object between two points with different electric potential results in a change in kinetic energy of the object consistent with the conservation of energy.

> **Example**
>
> A proton (mass = 1.67×10^{-27} kg) is accelerated through a potential difference of 4.5×10^6 V. (a) How much kinetic energy has the proton acquired? (b) If the proton started at rest, how fast is it moving?
>
> For part A, we have
>
> $$\Delta U = q\Delta V = \Delta K = K - K_0$$
> $$= (1.6 \times 10^{-19})(4.5 \times 10^6 \text{V})$$
> $$= 7.2 \times 10^{-13}\,\text{J}$$

For part B:

$$K = \frac{1}{2}mv^2$$

$$V = \sqrt{\frac{2K}{m}}$$

$$= \sqrt{\frac{2(7.2 \times 10^{-13})}{1.67 \times 10^{-27}}} = 2.94 \times 10^7 \text{m/s}$$

Exercise A proton in an electric field is initially located at point P_1 on the x-axis. The proton is given an initial velocity in the positive x-direction, moves in a straight line along the axis, and continually slows down and comes instantaneously to rest at point P_2. What can be concluded about the electric potential at points P_1 and P_2, and what evidence or reasoning supports the claim?

Exercise An electron is accelerated from rest through a potential difference of ΔV_0, reaching a speed of v_1. A different electron is accelerated from rest through a potential difference of $4\Delta V_0$, reaching a speed of v_2. The ratio of $\frac{v_2}{v_1}$ is?