Honors Pre Calculus Notes

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Contents

1 Functions, Rates, and Patterns

1.1 What is a Function?

1.2 Functions and Types of Functions

1.3 A Qualitative Look at Rates

1.4 Sequences and Triangular Differences

1.5 Functions Defined by Patterns

Problems

1. Reflect on your informal work in trying to describe what a function is and consider the various group definitions of function presented. Now revise the definition you originally created for describing a function in order to develop a more refined definition. Explain your reasons for refining your definition.
2. Why is it or is it not important to have a precise definition of the term function? Also, which of the supplied function definitions do you like best, and why?
3. In general, is it true that $f(g(x)) = g(f(x))$? State your conclusion as a property of the composition of functions.
4. See if you can identify the component functions $f(x)$ and $g(x)$ for $f[g(x)] = \log(2x - 5)$.
5. One property of invertible functions is that if f is invertible, then $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$. Can you verify this property?
6. Report on where, in your mathematical careers so far, you have encountered and work specifically with sequences, and in what capacity.
7. Try applying triangular differences for the sequence generated by n^2, n, and $n^2 - 3n$. Do you notice any patterns? What about $2n^2 + 4n$ or $5n^2 + 2n - 5$?

2 Algebra and Geometry

2.1 Algebra and Geometry

2.2 Complex Geometry and Roots

2.3 Conic Sections

2.4 Using Matrices to Find Models

2.5 Using Statistical Regression to Fit a Function to Bivariate Data

Problems

1. For $f(x) = ax^2 + bx + c$, find $d + ei$ that will generate real values.
2. Using the general formula $ax^2 + bx + c = 0$, derive the quadratic formula.
3. Given integers a, b, and c such that $0 < a < b$, and given

   $$P(x) = x(x-a)(x-b) - 13$$

   where $P(x)$ is divisible by $(x-c)$.

   Find a, b, and c. Is your solution unique? Justify your answer.
4. Can you provide an analytic definition of a circle?
5. Given: Two fixed points F, G, and a fixed positive number k. The ellipse consists of all points P such that $\overline{FP} + \overline{GP} = k$.

   The fixed points F and G have coordinates $(-c, 0)$ and $(c, 0)$ respectively. The points A and B are points where the ellipse intersects the positive x-axis and positive y-axis, respectively.

   Verify that $k = 2a$ and $c^2 = a^2 - b^2$, knowing that A and B are points on the ellipse.
6. Consider $0 = Ax^2 + Cy^2 + Dx + Ey + F$. This is the general form of the equation for any of the conic sections.

   How would one know which conic is represented by a given equation in this form?

   Put each of the conics represented in general form into the standard form of the specific conic.
7. Find the inverse of matrix A where: $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$ Based on this example, can you provide some general informal justification for why this process works?

3 Exponentials and Logarithms

3.1 Exponent Properties

3.2 A Special Number

3.3 The Natural Logarithm Function as the Inverse of e^x

3.4 Growth and Decay

3.5 Using Functions Defined by Patterns in Application

Problems

1. Can you prove that the logarithm properties follow directly from the laws of exponents?

4 Trigonometry

4.1 Working with Identities

4.2 Trigonometric Foundations

4.3 The Trigonometric Functions Off the Unit Circle

4.4 Angular and Linear Speed

4.5 Back to Identities

4.6 The Roller Coaster

Problems

1. Prove the identity $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$.
2. The domains of the $\csc(x)$, $\sec(x)$, and $\cot(x)$ trigonometric functions can also be restricted such that they are invertible. Do some research to find information about the inverses of these functions and their properties.
3. Find the formulas for $\sin(\varphi + \theta)$ and $\sin(\varphi - \theta)$.
4. Find a formula for $\sin(2\varphi)$, then determine two more formulas for $\cos(2\varphi)$.
5. Use the results from the above two problems to find an algebraic expression for $\cos\left(\frac{8\pi}{3}\right)$. Evaluate this expression by using one of the double-angle formulas derived previously. Verify the value you just calculated with the value of the cosine of the co-terminal angle on the Unit Circle.
6. Use the Law of Cosines and the distance formula to derive the trigonometric expansion identity for $\cos(\alpha - \beta)$.

5 Limits and Rates of Change of Functions

5.1 First, Some Background - Rational Functions

5.2 Limits

5.3 Approximating Rates of Change

5.4 The Derivative

Problems

1. Perform the division of the ration function $f(x) = \frac{x^2-x-2}{x-1}$ and sketch a graph. Make a conjecture about the equations for the asymptotes. Confirm your conjecture by analyzing the results of the function's division.

2. A formal definition of the limit of a function is:

$$L = \lim_{x \to c} f(x) \text{iff for any} \epsilon > 0 \text{there exists a number} \delta > 0$$
$$\text{such that if} 0 <\mid x - c \mid< \delta \text{then} \mid f(x) - L \mid< \epsilon$$

   On a pair of axes, precisely draw and label a piecture that illustrates the meaning of the given formal definition of limit.

6 Other Coordinate Systems

6.1 A Nonstandard Exploration of the Rate of Change of Functions

6.2 More Information Needed

6.3 Applications of Parametric Equations

6.4 Vectors

6.5 The Golf Shot

6.6 The Polar Coordinate System

6.7 Classic Polar Relations

6.8 Complex Numbers

Problems

1. Given a parabola $y = x^2$, construct graphs for $x(s)$ and $y(s)$.
2. At 1:00 pm, a ship is 10 miles due east of port. At 2:00 pm, it has sailed to a point that is 20 miles east and 50 miles north of the position at 1:00 pm. Assume that the ship continues to sail in this manner as it did from 1:00 pm to 2:00 pm.

   Write a function that will give the ship's position at any time.
3. Use the fact that $W = ||\vec{F}||||\vec{d}|| \cos\theta$, to show that a satellie orbiting earth does zero work as it travels around the earth.
4. Use Euler's Number $e^{i\theta} = \cos\theta + i \sin\theta$ to derive some common trigonometric identities.