# Algebra 2 - Functions

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*"Mathematics is the music of reason." - James Joseph Sylvester*

Note: It is expected that you try the examples to the best of your understanding, and complete the problem sets by the test date and ask for help where needed.

## 1 Domain and Range

There are two types of notation. One is set notation and the other is interval notation. You are likely familiar with interval notation already, so this will just go over set notation.

Set Notation uses the format $\{x|x \in \mathbb{R}\}$. In mathematics, the term $|$ means "such that", the term $\in$ means "element of", and $\mathbb{R}$ represents all real numbers.

Domain represents the x-values and goes left to right. Range represents the y-values and goes bottom to top.

For this course, there are three types of functions - a continuous function, a discontinuous function, and discrete data.

A continuous function is a graph that is connected.
A discontinuous function is a graph which has one ore more breaks or holes.
A discrete set of data is a function which only has separate points.

Example: For the function $x^3$, write the domain and range both in set and interval notation.

Solution:
Domain in Set Notation: $\{x|x \in \mathbb{R}\}$
Domain in Interval Notation $(-\infty, \infty)$

Range in Set Notation: $\{x|x \in \mathbb{R}\}$
Range in Interval Notation $(-\infty, \infty)$.

## 2 Transformations

There are general rules for transformations, which will be listed below.

Vertical Shifts
$f(x) + k$ represents a shift up $k$ units.
$f(x) - k$ represents a shift down $k$ units.

Horizontal Shifts
$f(x + h)$ represents a shift left $k$ units.
$f(x - h)$ represents a shift right $k$ units.

Reflections
$-f(x)$ represents a reflection over the x-axis.
$f(-x)$ represents a reflection over the y-axis.

Vertical Stretch/Compression
$a \cdot f(x)$ where $a > 1$ represents a vertical stretch by a factor of $a$.
$a \cdot f(x)$ where $0 < a < 1$ represents a vertical compression by a factor of $a$.

Example: Write the transformations in the function $-2\sqrt{x+1} - 4$.

Solution: Reflection over the x-axis, vertical stretch by a factor of 2, transformation left 1 unit, and down 4 units.

## 3 Inverse Functions

The relationship between inverse functions is that the x and y values will switch, the graph will reflect over the line $y = x$, the domain and range will switch, and they can "undo" each other.

To find the inverse of a function follow the steps:
1. Change $f(x)$ to y.
2. Switch x and y.
3. Solve for y.
4. Rewrite y as $f^{-1}(x)$.

Note that when taking a square root, half of the function will disappear. Make sure to use $\pm\sqrt{}$.
Also noted above, the domain and range switch for inverses. For square root functions, the range of the original function will become a restriction on the domain of the inverse function.

Example: Find the inverse of $f(x) = \sqrt{x+3}$.

Solution: $f^{-1}(x) = x^2 - 3$ with a domain of $[0,\infty]$.