Algebra 2 - Rationals

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"Time is a created thing. To say 'I don't have time,' is like saying, 'I don't want to.'" - Lao Tzu

Note: It is expected that you try the examples to the best of your understanding, and complete the problem sets by the test date and ask for help where needed.

1 Factoring Review

You likely already know how to factor.

Example: Factor $2x^2 + 5x + 3$.

Solution: $(x + 1)(2x + 3)$

2 Graphing Rationals

A reciprocal function is a function belonging to $f(x) = \frac{1}{x}$, where $x \neq 0$.

A vertical asymptote is a vertical line that a graph approaches as the value of y becomes extremely small or large, but the function can never cross.

A horizontal asymptote is a horizontal line that a graph approaches as the value of x becomes extremely small or large, and the function can cross this asymptote.

The steps for graphing a rational function is the following:
1. Factor the top and bottom. If a factor cancels, set them equal to zero and there is a hole in the graph.

2. Set the numerator equal to zero. These are the x-intercepts. These will be the only time the function will cross the x-axis.

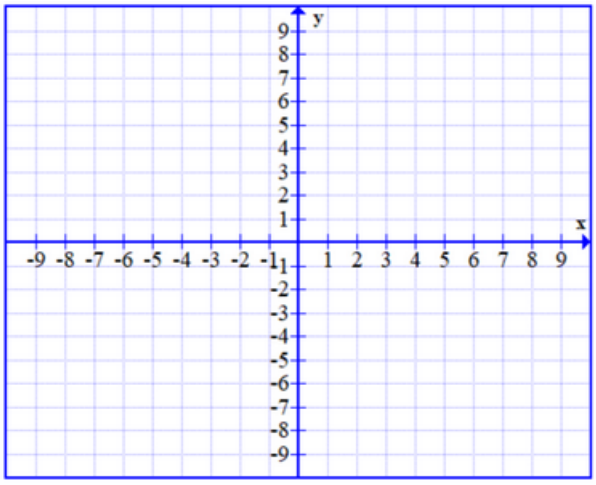
3. Set the denominator equal to zero. These are the domain restrictions. These are the vertical asymptotes, which a function cannot cross.

4. In order to find tangency or togetherness, you have to see if there are factors with a multiplicity of two. If there are twins in the numerator, there is tangency, which means the graph will touch the x-axis then turn around. If the twins are in the denominator, the graph will come together at the vertical asymptote.

5. The horizontal asymptote has 3 possibilities. You will have to compare the degrees in the numerator and denominator. If it is a higher degree in the numerator, there may be no asymptote or a slant asymptote. If the degrees are equal, the asymptote are the coefficients. If the denominator has a higher degree, then the asymptote is 0.

6. To find the y-intercept, plug in $x = 0$.

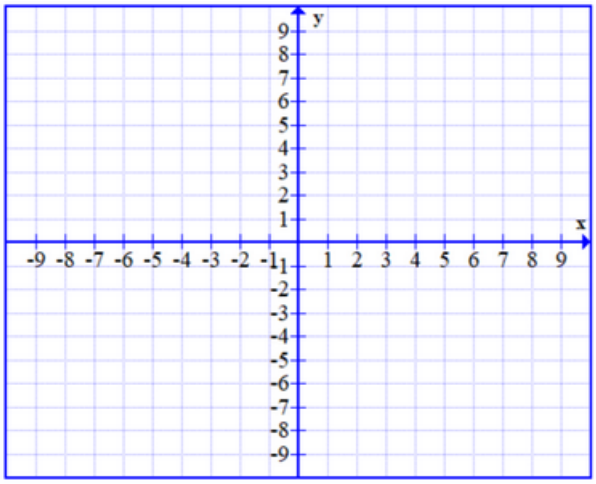
Example: Graph the function $f(x) = \frac{x-2}{x+4}$.



Solution: Use Desmos

3 Tangency and Togetherness

Example: Graph the function $f(x) = \frac{x(x^2+4x+4)}{(x-2)(x^2+2x-8)}$.

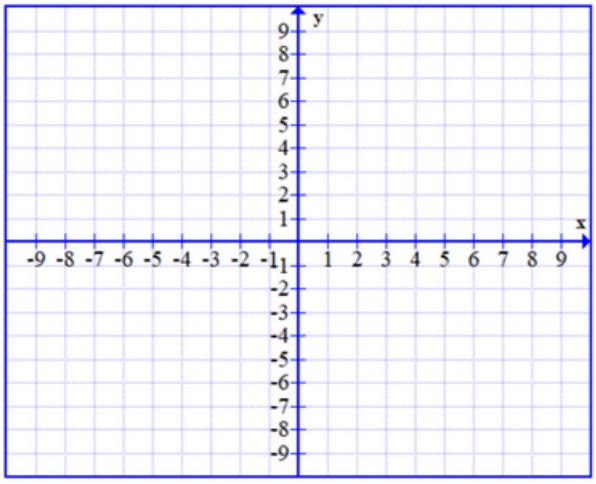


Solution: Use Desmos

4 Slant Asymptotes

To find the slant asymptote, you use polynomial long division of the original function.

Example: Graph the function $f(x) = \frac{x^2-4}{x}$.



Solution: Use Desmos

5 Multiplying and Dividing Rationals

Steps to multiplying or dividing rational functions:
1. Factor both the numerator and denominator.
2. Identify the undefined values.
3. Cancel common factors that are being divided.

Example: Simplify $\frac{x^2-2x-3}{x^2+6x+5}$.

Solution: $\frac{x-3}{x+5}$

For dividing rationals, recall that you keep the first fraction, change the division sign to multiplication, and flip the second fraction.

Example: Simplify $\frac{x+2}{8} \div \frac{x+2}{64}$.

Solution: 8

6 Adding and Subtracting Rationals

The steps to add or subtract rationals are as follows:
1. Factor the denominator.
2. Identify undefined values.
3. Find the least common denominator.
4. Add or subtract the numerators.
5. Factor the numerator.
6. Simplify.

Example: Simplify $\frac{m+1}{m^2+8m+12} + \frac{4m}{m+2}$.

Solution: $\frac{4m^2+25m+1}{(m+6)(m+2)}$

7 Complex Rationals

A complex fraction is a fraction where the numerator or denominator or both are fractions.

Example: Simplify $\frac{\frac{3}{x-4}}{1-\frac{2}{x-4}}$.

Solution: $\frac{3}{x-6}$

8 Applications of Rationals

The formula for the time for two people to do something is $\frac{1}{a} + \frac{1}{b} = \frac{1}{t}$.

Example: Yasmin can paint a fence in 8 hours. Jannet can do it in 4 hours. How long will it take for them to do the job if they work together?

Solution: 2 hours 40 minutes.

The formula for distance problems is $d = r \cdot t$.

Example: Jesus can jog 5 miles downhill in the same time it takes him to jog 3 miles uphill. He jogs downhill 4 mph faster than he jogs uphill. Find his jogging rate each way.

Solution: 6 mph uphill and 10 mph downhill.