Algebra 2 - Quadratics

Stasya

"Mathematics is a place where you can do things which you can't do in the real world." - Marcus du Sautoy

Note: It is expected that you try the examples to the best of your understanding, and complete the problem sets by the test date and ask for help where needed.

1 Solving Quadratics

The term "solution" can also mean x-intercept, roots, or zeroes.
The zero-product property states that if $a \cdot b = 0$, then $a = 0$ and/or $b = 0$.

Example: Solve the quadratic equation $2x^2 - 2x = 12$ by factoring.

Solution: $x = 3$ and $x = -2$

To solve a quadratic system, you are looking for the points of intersection.
1. Solve each equations for y.
2. Set the equations equal to teach other and move all terms to one side.
3. Solve by factoring.

Example: Solve the following system of equations.

$$\begin{aligned} y &= -x^2 + 2x + 4 \\ x + y &= 4 \end{aligned}$$

Solution: $(0, 4)$ and $(3, 1)$

2 Completing the Square

To convert from standard form ($y = ax^2 + bx + c$) to vertex form ($y = a(x - h)^2 + k$), you have to complete the square.
In order to complete the square you have to follow these steps:
1. Move the x terms to one side.
2. Factor out a if $a \neq 1$.

3. Add $\left(\frac{b}{2}\right)^2$ to each side.
4. Factor out the trinomial as $\left(x+\frac{b}{2}\right)^2$.
5. Solve for y.

Example: Find the vertex form of $y = x^2 + 4x - 7$, then find the vertex, axis of symmetry, and the range.

Solution: $(x+2)^2 - 11$. Vertex - $(-2, -11)$, Axis of symmetry - $x = -2$, Range $[-11, \infty]$

Example: Solve $18x + 3x^2 = 45$ by completing the square.

Solution: $x = -3 \pm \sqrt{24}$

3 Quadratic Inequalities

To solve a quadratic inequality follow the steps:
1. Move all terms to one side of the inequality sign and factor.
2. Solve to find the critical points (roots).
3. Draw a number line with the critical points on it.
4. If plugging in $x = 0$ into the original inequality is true, if $x = 0$ is located in between your two critical points, the solution is (a, b) otherwise it is $(-\infty, a) \cup (b, \infty)$.

If plugging in $x = 0$ into the original inequality is false, if $x = 0$ is located in between your two critical points, the solution is $(-\infty, a) \cup (b, \infty)$ otherwise it is (a, b).

Note: whether or not you include or exclude your critical points will depend on the context of the problem.

Example: Solve the inequality $2x^2 - 4 \geq 7x$.

Solution: $(-\infty, -\frac{1}{2}] \cup [4, \infty)$

4 Quadratic Formula

The quadratic formula is $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$b^2 - 4ac$ is defined as the discriminant. When the discriminant is greater than zero, there are two real solutions, otherwise when it is equal to zero, there is one real solution, and when it is less than zero, there are two imaginary solutions.

Example: Find the discriminant and roots of $f(x) = x^2 + 10x + 2$.

Solution: Discriminant: 92, Roots: $x = -5 \pm \sqrt{23}$

5 Complex Solutions

An imaginary number is defined as a number that is expressed in terms of a square root of a negative number.

$i = \sqrt{-1}$ and $i^2 = -1$

Example: Simplify $3i(2 - 7i)$

Solution: $21 + 6i$

6 Applications of Quadratics

Example: You are placing a mat around a 25 in (length) by 21 in (width) picture as shown. You want the mat to be twice as wide to the left as it is to the top and bottom. You have 705 square inches of mat that you can use. How wide should the mat be to the left and right of the picture?

Solution: 2.5 inches