Algebra 2 - Systems of Equations

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"Nature is an infinite sphere of which the center is everywhere and the circumference nowhere." - Blaise Pascal

Note: It is expected that you try the examples to the best of your understanding, and complete the problem sets by the test date and ask for help where needed.

1 Review

In Algebra 1, you likely learnt how to find the solutions for a system of equations.

If you are given two equations with a different slope, there will be one solution. If you are given two equations with the same slope, but different y-intercepts, there will be no solutions, since the lines are parallel. If you are given two equations that are the same, there are infinitely many solutions.

Example: Determine the number of solutions for the system below.

$$y = \frac{3}{7}x - \frac{9}{7}$$
$$y = -\frac{1}{2}x - \frac{1}{4}$$

Solution: One Solution

To use substitution follow these steps:
1: Isolate one variable
2: Substitute into the other equation
3: Solve for one variable
4: Solve for the other variable by substitution

In order to use elimination, you have to make one variable cancel out, and you need coefficients with opposite signs.

Example: Determine the solutions for the system below by using substitution.

$$\frac{2}{3}x + \frac{1}{3}y = 2$$
$$y - 9x = 1$$

Solution: $(\frac{5}{11}, \frac{56}{11})$

In order to solve systems of inequalities follow these steps:
1: Draw the lines
2: A solid line means a $\leq$ or $\geq$, and a dotted line means $<$ or $>$.
3: Shade above the line if the y has a $>$ or $\geq$ and below the line if y has a $<$ or $\leq$.
4: The solutions for a system of inequalities are any ordered pairs that satisfy both inequalities.

2 Systems involving 3 variables

In order to solve systems with 3 variables, you will use the same technique as you did with systems with 2 variables in the review.

Example: The sum of three numbers is -4. The second number decreased by the third is equal to the first. The sum of the first and second number is -5. Find the product of the three numbers.

Solution: 6.

3 Matrices and Row Operations

A matrix is defined as a rectangular array of values in rows and columns. It has dimensions m x n. There are three legal operations:
1: Row Switching - You can switch rows around to put matrix in a nicer row arrangement.
2: Row Multiplication - You can multiply by a non-zero number to create a new row.
3: Row Addition - You can add one row to another row to create a new row.

Example: Use the systems of equations to do the matrix row operation $2R_1 + R_2 = R_1$.

$$\begin{aligned} x + y - 4z &= 12 \\ -2x + 3y - z &= -5 \\ 3x - 2y + z &= 0 \end{aligned}$$

Solution: $\begin{bmatrix} 0 & 5 & -9 & 19 \\ -2 & 3 & -1 & -5 \\ 3 & -2 & 1 & 0 \end{bmatrix}$

4 Gaussian Elimination

Gaussian elimination is a way of row operations to solve systems of equations. This is done by getting the bottom left corner of the matrix and the number to the right and above it to equal zero.

Example: Solve the system of equations using Gaussian Elimination.

$$\begin{aligned} x + 2y + 3z &= 9 \\ y - z &= -4 \\ x + 4y - z &= -5 \end{aligned}$$

Solution: $(2, -1, 3)$

5 Using a Calculator for Gaussian Elimination

Follow these steps to solve a matrix:
1: Create an augmented matrix by going to 2nd $\rightarrow x^{-1} \rightarrow$ Edit[A].
2: Enter Rows and columns
3: Enter the coefficients and then click 2nd $\rightarrow$ Mode
4: 2nd $\rightarrow x^{-1} \rightarrow$ MATH B:rref(
5: 2nd $\rightarrow x^{-1} \rightarrow$ Names[A], and click enter and the calculator will solve the matrix for you.

Example: Lauren bought 3 types of candy for Halloween: Nerds, Skittles, and AirHeads. She brought home 14 bags of candy and paid \$33.75. She remembers seeing a sign that said Skittles were twice the price of Nerds and AirHeads were 75 cents less than Skittles. She has 3 bags of Nerds, 6 bags of Skittles, and 5 bags of Airheads. Which candy was the most expensive?

Solution: Skittles was at \$3.00.