Algebra 2 - Exponentials and Logs

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"If I were again beginning my studies, I would follow the advice of Plato and start with mathematics." - Galileo Galilei

Note: It is expected that you try the examples to the best of your understanding, and complete the problem sets by the test date and ask for help where needed.

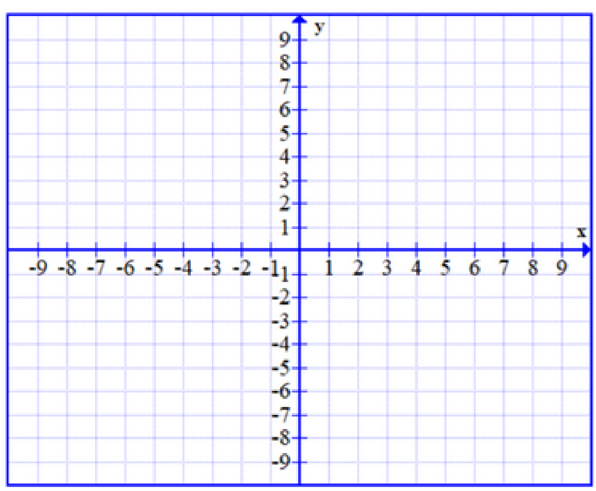
1 Graphing Exponential and Logs

Exponential functions are given in the form $f(x) = a(b)^{x-h} + k$. The two key points are $(0, 1)$ and $(1, b)$ (after shifting of course). The exponential graph has a horizontal asymptote.

Logarithmic functions are given in the form $f(x) = a \cdot \log_b(x - h) + k$. The two key points are $(1, 0)$ and $(b, 1)$. The logarithmic graph has a vertical asymptote.

Note that the exponential and logarithmic function are inverses of each other.

Example: Graph $f(x) = -\log_2(x + 1) + 5$.



Solution: Use Desmos

2 Properties of Logs

To convert from a logarithm to an exponential equation: $\log_b y = x$ is equal to $b^x = y$.

A log without a base is assumed to be base-10.

There are three properties of logs:
1. Product $\log_b(xy) = \log_b(x) + \log_b(y)$
2. Quotient $\log_b(\frac{x}{y}) = \log_b(x) - \log_b(y)$
3. Power $\log_b(x)^y = \log_b(x^y) = y\log_b(x)$

Example: Simplify $\log_2(4(x-1))$.

Solution: $4 + 2\log_2(x-1)$

3 Solving Log Equations

Example: Solve $\log_2(x+3) + \log_2(x-4) = \log_2(8)$ for x.

Solution: $x = 5$

4 Solving Exponential Equations

Example: Solve $5^{x-1} = 3^x$ for x.

Solution: $x = -\frac{1}{\log_5(3)-1}$

5 Natural Log and Base e

The natural logarithm is $\log_e(x) = \ln(x)$.

Properties of natural logs:

$\ln(e) = 1$
$\ln(1) = 0$

Example: Simplify $3\ln(3) - 4\ln(2)$.

Solution: $\ln\left(\frac{27}{16}\right)$

The formula for compounding continuously is $A = Pe^{rt}$, where A is the final amount, P is the initial amount, r is the rate, and t is time.

Example: What is the amount of a \$1,000 investment after 10 years if 5% interest is compounded continuously?

Solution: \$1648.72