STATISTICS 88 **Shobhana M. Stoyanov**
Spring 2021

Midterm practice

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1. A pair of dice are thrown.
   (a) Find the probability that both dice show the same number of spots.
   (b) Show that the event that the sum of the spots on the dice is 7 is independent of the number of spots on the first die.

2. A, B and C are mutually independent events that occur with probabilities $P(A) = 0.3, P(B) = .2, P(C) = 0.5$.
   (a) Find the probability that at least one of the events occurs.
   (b) Find the probability that exactly 2 of the events occur.

3. My aunt loves to try her luck at the slot machines in the local casino. Suppose the chance of her winning on any one round is about one in a thousand. What is the chance that she will win at least once in n tries? Derive an exponential approximation to this probability.

4. A player throws darts at a target. On each try, his probability of hitting the bullseye is 0.05, independently of the other tries. How many times should he throw so that his probability of hitting the bullseye at least once is 0.5?

5. A and B are **independent** events. The chance of A is **0.3**, and the chance of B is **0.5**. Fill in the blanks, choosing from the options given below. Explain your answer.
   (a) The chance of **both** A **and** B happening is ______________.
   (i)0.15 (ii) 0 (iii)0.8 (iv) 0.3 (v) 0.5 (vi) Need more information.
   (b) The chance of A happening, **given** that B happened is ______________.
   (i)0.15 (ii) 0 (iii)0.8 (iv) 0.3 (v) 0.5 (vi) Need more information.
   (c) The chance of **either** A **or** B happening is ______________.
   (i)0.15 (ii) 0 (iii)0.8 (iv) 0.3 (v) 0.5 (vi) 0.65 (vii)Need more information.
   (d) The chance of **neither** A **nor** B happening is ______________.
   (i)0.35 (ii) 0 (iii)0.2 (iv) 0.65 (v) 0.5 (vi) Need more information.

6. A box contains 6 tickets, numbered 1 through 6. Three tickets are drawn at random, without replacement, from the box. Find the chance that the three tickets left in the box are 4, 5, and 6.

7. About 8% of males are colorblind. A researcher needs some colorblind subjects for an experiment, and begins checking potential subjects. You may assume that being colorblind is independent from person to person.
   (a) On average, how many men should she expect to check before finding one who is colorblind?
   (b) What is the probability that she won't find any colorblind men among the first 4 men that she checks?
   (c) What is the probability that the first colorblind man is the sixth person that she checks?
   (d) What is the probability that the **fourth** colorblind male that she finds is the twentieth person that she checks?

8. So the Cubs won the World Series, coming back from being down 1-3. Who would have thunk it?! I read an article from USA today that put their chances of winning at that point at 1 in 8 (12.5%). Let's pretend to go back in time, before the World Series began and let's suppose that for each game, the Cubs have a 0.51 chance of winning, independent of the other games. What is the probability that they would win the World Series? Note that in order to win the World Series, you have to **win 4 games** (best of seven games), so a team can win by winning the first 4 games (and the series is over - no more games will be played), or in 5 games etc. *You do not have to calculate your answer out, just show us the computation with the numbers for the chance that the Cubs will win the World Series. Note that you can do this problem in 2 ways, try to do it both ways and make sure you get the same answer.*

9. You and a friend are rolling a fair die and betting money on the outcomes. You keep rolling the die over and over. If you roll a six, your friend gives you a dollar. If you don't, you give your friend a dollar. You stop if your **net gain** is two dollars.

(a) What is the probability that you will stop after exactly **two** rolls?

(b) What is the probability that you will stop after exactly **three** rolls?

(c) What is the probability that you will stop after exactly **four** rolls?

10. If X and Y are jointly distributed discrete random variables with $P(X = i, Y = j) = \frac{ij}{c}$ for $i = 1, 2$ and $j = 1, 2, 3$, find

    (a) c. Then using the c you found, compute:

    (b) $P(X + Y > 3)$.

    (c) Find $E(X + Y)$

    (d) Are X and Y independent?

11. 100 people need to be tested for a certain disease. The chance that a person has the disease is 0.1 for everyone, independent of each other. Instead of testing each person separately, the 100 people are grouped into groups of 10. The blood samples of the 10 people in each group are pooled and tested. If the test is negative, this one test suffices. If the test is positive (that is, at least one person in group has the disease), then each person in the group is tested individually, and so 11 tests in all will be made on this group. Write down the expression for the expected number of tests that will be made on each group.

12. A fair four-sided die is rolled, and then a fair coin is tossed as many times as there are spots that show on the die. Let X be the number of spots rolled, and let Y be the total number of heads we get once we are done tossing the coin.

    (a) Write down the joint distribution table for X and Y.

    (b) Write down the marginal distribution of Y and find its expectation.

    (c) Given $Y = 1$, find the conditional distribution of X.

13. Say we take two independent samples from a population, and the sample means are respectively $\overline{X}_1$ and $\overline{X}_2$. We put them together to estimate the population mean, μ taking a weighted sum to define a new estimator $W = \alpha\overline{X}_1 + \beta\overline{X}_2$. What conditions must we put on α and β to ensure that W is unbiased?

14. If X and Y are jointly distributed discrete random variables with $P(X = i, Y = j) = \frac{i+j}{15}$ for $i = 1, 2, 3$ and $j = 0, 1$, find

    (a) the marginal distributions of X and Y.

    (b) Find $E(X + Y)$

    (c) Find $P(X + Y \geq 2)$

    (d) Are X and Y independent?

    (e) Find $E(XY)$. Is $E(XY) = E(X)E(Y)$?

15. A population consists of 1000 people. A sample of n people is drawn at random with replacement from the population. For $n \leq 1000$, write a formula for the chance that the sample consists of n different people.

16. In a class of 20 students, assuming every student has a birthday in one of the twelve months at random, independently of one other, what is the expected number of months in which **none** of the students has a birthday?

17. A man has five coins, two of which are double-headed, one double-tailed, and two that are normal (fair) coins.

    (a) He shuts his eyes, picks a coin at random, and tosses it. What is the probability that the **lower** face of the coin is a head?

    (b) He opens his eyes, and sees that the coin has landed heads. What is the probability that the **lower** face of the coin is a head?

    (c) He tosses the coin again, and sees that it lands heads again. What is the probability that the coin is double-headed?

The following problems are taken from *Probability* by Jim Pitman. Do not distribute these problems beyond our course.

18. A coin which lands heads with probability p is tossed repeatedly. Assuming independence of the tosses, find formulae for:

   (a) P(Exactly 5 heads appear in the first 9 tosses)

   (b) P(The first head appears on the 7th toss)

   (c) P(The fifth head appears on the 12th toss)

   (d) P(The same number of heads appear in the first 8 tosses as in the next 5 tosses)

19. Radioactive substances emit $\alpha-$particles. The number of such particles reaching a counter over a given time period follows the Poisson distribution. Suppose two substances emit $\alpha-$particles independently of each other. The first substance gives out $\alpha-$particles which reach the counter according to the Poisson (3.87) distribution, while the second substance emits $\alpha-$particles which reach the counter according to the Poisson (5.41) distribution. Find the chance that the counter is hit by at most 4 particles.

20. Suppose you roll a fair six-sided die repeatedly until the first time you roll a number that you have rolled before.

   (a) For each $r = 1, 2, \ldots$, calculate the probability p_r that you roll exactly r times.

   (b) Without calculation, write down the value of $p_1 + p_2 + \ldots + p_10$. Explain.

   (c) Check that your calculated values of p_r have this value for their sum.

21. A typical slot machine in a Nevada casino has three wheels, each marked with twenty symbols at equal spacings around the wheel. The machine is engineered so that on each play the three wheels spin independently, and each wheel is equally likely to show anyone of its twenty symbols when it stops spinning. On the central wheel, nine out of the twenty symbols are bells, while there is only one bell on the left wheel and one bell on the right wheel. The machine pays out the jackpot only if the wheels come to rest with each wheel showing a bell.

   (a) Calculate the probability of hitting the jackpot.

   (b) Calculate the probability of getting two bells but not the jackpot.

   (c) Suppose that instead there were three bells on the left, one in the middle, and three on the right. How would this affect the probabilities in (a) and (b)? Explain why the casino might find the 1 - 9 - 1 machine more profitable than a 3 - 1 - 3 machine.

22. A manufacturing process produces integrated circuit chips. Over the long run the fraction of bad chips produced by the process is around 20%. Thoroughly testing a chip to determine whether it is good or bad is rather expensive, so a cheap test is tried. All good chips will pass the cheap test, but so will 10%, of the bad chips.

   (a) Given a chip passes the cheap lest, what is the probability that it is a good chip?

   (b) If a company using this manufacturing process sells all chips which pass the cheap test, over the long run what percentage of chips sold will be bad?

23. A deck of cards is shuffled and dealt to four players, with each receiving 13 cards. Find:

   (a) the probability that the first player holds all the aces;

   (b) the probability that the first player holds all the aces given that she holds the ace of hearts;

   (c) the probability that the first player holds all the aces given that she holds at least one;

   (d) the probability that the second player holds all the aces given that he holds all the hearts.

24. A hat contains a number of cards, with 30% white on both sides; 50% black on one side and white on the other; 20% black on both sides. The cards are mixed up, then a single card is drawn at random and placed on the table. If the top side is black, what is the chance that the other side is white?

25. Suppose a word is picked at random from this sentence.

   (a) What is the distribution of the length of the word picked?

   (b) What is the distribution of the number of vowels in the word?

26. A box contains 8 tickets. Two are marked 1, two marked 2, two marked 3, and two marked 4. Tickets are drawn at random from the box without replacement until a number appears that has appeared before. Let X be the number of draws that are made. Make a table to display the probability distribution of X.

27. A book has 200 pages. The number of mistakes on each page is a Poisson random variable with mean 0.01, and is independent of the number of mistakes on all other pages.

    (a) What is the expected number of pages with no mistakes?

    (b) A person proofreading the book finds a given mistake with probability 0.9. What is the expected number of pages where this person will find a mistake?

    (c) What, approximately, is the probability that the book has two or more pages with mistakes?

28. In a circuit containing n switches, the ith switch is closed with probability $p_i, i = 1, \ldots, n$. Let X be the total number of switches that are closed. What is $E(X)$? Or is it impossible to say without further assumptions?

29. Suppose electric power is supplied from two independent sources which work with probabilities 0.4, 0.5, respectively. If both sources are providing power enough power will be available with probability 1. If exactly one of them works there will be enough power with probability 0.6. Of course, if none of them works the probability that there will be sufficient supply is 0.

    (a) What are the probabilities that exactly k sources work for $k = 0, 1, 2$?

    (b) Compute the probability that enough power will be available.

30. A die is rolled 8 times. Given that there were 3 sixes in the 8 rolls, what is the probability that there were 2 sixes in the first five rolls?

31. Suppose an airline accepted 12 reservations for a commuter plane with 10 seats. They know that 7 reservations went to regular commuters who will show up for sure. The other 5 passengers will show up with a 50% chance, independently of each other.

    (a) Find the probability that the flight will be overbooked, i.e., more passengers will show up than seats are available.

    (b) Find the probability that there will be empty seats.

    (c) Let X be the number of passengers turned away. Find E(X).