STAT 88: Lecture 7 Continued

Contents

Summary of Binomial versus Hypergeometric:

Binomial(n,p)	$HG(N,G,n)$
n = # trials	N = population size
p = probability of success	G = # good elements in pop
	n = sample size
sample with replacement	if w/ replacement, $p=\frac{G}{N}$ sample without replacement
trial has two outcomes: success, failure	trial has two outcomes: good, bad
n independent trials	n dependent trials
X = # successes in sample	X = # good in sample
$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$	$P(X=g) = \dfrac{\binom{G}{g}\binom{N-G}{n-g}}{\binom{N}{n}}$
Binomial formula	HG formula
Ex. pick 5 cards from a deck with replacement. What is chance you get 2 ace cards?	Ex. pick 5 cards from a deck without replacement. What is chance you get 2 ace cards?

3.5. Examples

Problem solving techniques:

- Breaking the problem down into smaller pieces
- Examining the assumptions and hence deciding which distributions can be used
- Organizing the information to identify the parameters of the distributions
- Partitioning events into component pieces
- Using the addition and multiplication rules carefully

Advisor meetings An advisor at a university provides guidance to 10 students. Each student has to meet with her once a month during the school year which consists of nine months.

So each month the advisor schedules one day of meetings. Each student has to sign up for one meeting that day. Students have the choice of meeting her in the morning or in the afternoon.

Assume that every month each student, independently of other students and other months, chooses to meet in the afternoon with probability 0.75.

Question. What is the chance that she has both morning and afternoon meetings in all of the months except one?

8 Months.

9 months

10 students, each of them sign up for one meeting.

X = # of months have both morning and afternoon meetings.

Binomial(n, p)

$n = 9$

$p = P(\text{Both morning and afternoon meeting})$

Complement rule

$= 1 - P((\text{Both M and A})^c)$

$= 1 - P(\text{all morning or all afternoon})$

Addition rule (Events of all M and all A ~ Mutually exclusive)

$= 1 - P(\text{all morning}) - P(\text{all afternoon})$

$= 1 - (0.25)^{10} - (0.75)^{10} \approx 0.94$

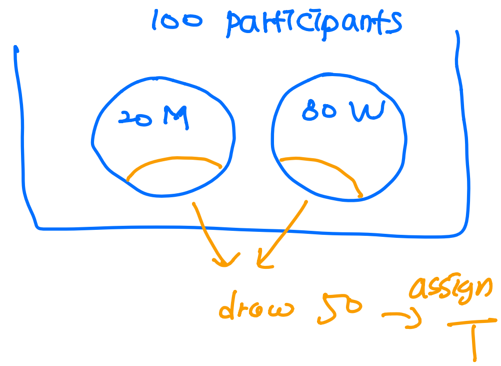
$$P(X = 8) = \binom{9}{8}(0.94)^8(0.06)^1$$

Randomized Controlled Experiments (RCE) In a RCE, a simple random sample of half the participants will be assigned to the treatment group (T) and the other half to control (C). = sample (w/o replacement) 50 participants and assign them to the Treatment group. ~> HG

Experiment 1 has 100 participants of whom 20 are men.

Question 1. What is the chance that the treatment and control groups in Experiment 1 contain the same number of men?

X = # men in $\underbrace{\text{our sample}}_{\text{" treatment group}}$



$N = 100$
$G = 20$
$n = 50$

$\rightarrow X \sim HG(100, 20, 50)$

$$P(X=10) = \frac{\binom{20}{10}\binom{80}{40}}{\binom{100}{50}}$$

Experiment 1 has 100 participants of whom 20 are men.

Experiment 2 has 90 participants of whom 30 are men.

Question 2. What is the chance that the treatment groups in the two experiments have the same number of men?

X_1 = # men in the treatment group for Exp 1

X_2 = # men in the treatment group for Exp 2.

$X_1 \sim HG(100, 20, 50)$

$X_2 \sim HG(90, 30, 45)$

Note Fixed mistake from lecture

$$P(X_1 = X_2) = P(X_1 = 0, X_2 = 0) + P(X_1 = 1, X_2 = 1) + \cdots + P(X_1 = 20, X_2 = 20)$$

partition events into component pieces

"and"

$$= \sum_{g=0}^{20} P(X_1 = g \text{ and } X_2 = g)$$

X_1 and X_2 indep.

$$= \sum_{g=0}^{20} P(X_1 = g)\, P(X_2 = g)$$

$$= \sum_{g=0}^{20} \frac{\binom{20}{g}\binom{80}{50-g}}{\binom{100}{50}} \cdot \frac{\binom{30}{g}\binom{60}{45-g}}{\binom{90}{45}}$$

Note

Fixed mistake from lecture