Last time

Integer-valued R.V.s

- Bernoulli. Bern (p)
- Uniform Unif ({1, 2, ..., n})
- Binomial Binom (n, p)
- Geometric Geom (p)
- Hypergeometric Hypergeom (N, G, n)

This time:

- problem solving techniques
- Negative Binomial Distr.
- Cumulative Distribution Function
- Python

· problem solving techniques

1. Breaking the problem into pieces   distr. = distribution
2. Examining the assumptions and deciding the distr.
3. Organizing the information to identify the parameter(s)
4. Partitioning events into components parts.
5. Using the add./multi. rules carefully.

Example. (3.5.1)

Advisor meeting.

10 students

meet each student once a month.

9 months.

The students can choose to meet < morning (0.25), afternoon 0.75

Assume they choose indpt.

and indpt. among months.

Q: What is the chance that the advisor has both morning and afternoon meetings in all months except one?

1. Breaking the problem into pieces

1.1. P(She has morning & afternoon)

10 students, < morning $1-0.75$, afternoon 0.75 indpt.

= 1 − P(She only has morning or afternoon)   4. Partitioning events into components parts.

= 1 − (P(only morning) + P(only afternoon))

$= 1 - \left((1-0.75)^{10} + 0.75^{10} \right) := p_1$ denote / defined as.

5. Using the add./multi. rules carefully.

with prob. p_1

1.2 P("that thing" happens in exactly 8 out of 9 months)

2. Examining the assumptions and deciding the distr.

In 1.2, we should use Binomial distr.

3. Organizing the information to identify the parameter(s)

$X \sim \text{Binom}(n = 9,\ p = p_1)$

$P(X = 8) = \binom{9}{8} p_1^{8} (1-p_1)^{1}$.

Textbook Section 3.5 : Reading.

Negative Binomial. − generalization of Geometric. $(r=1)$

Waiting time until the r-th success

Setup: i.i.d. trials, with chance of success p,   identically distributed, indpt. = independent.

Let T_r denote the trials needed until the r-th success (including).

Find $P(T_r = k)$ <
- the last (k-th) trial has to be a success   A
- there are (r−1) successes in the first (k−1) trials.   B

What does T_r mean? both considered as constants

Are A & B indpt.? Yes! Since A − the k-th, B − the 1st − (k−1)-th

$\Rightarrow P(T_r = k) = P(A) \cdot P(B)$

$= p \cdot P(B)$ $= p \binom{k-1}{r-1} p^{r-1} (1-p)^{k-r}$ $k = r, r+1, \ldots$

Check assumptions to use Binomial distr. in B.

$X \sim \text{Binom}(k-1, p)$

$P(B) = P(X = r-1) = \binom{k-1}{r-1} p^{r-1} (1-p)^{(k-1)-(r-1)}$

Cumulative Distribution Function CDF c.d.f.

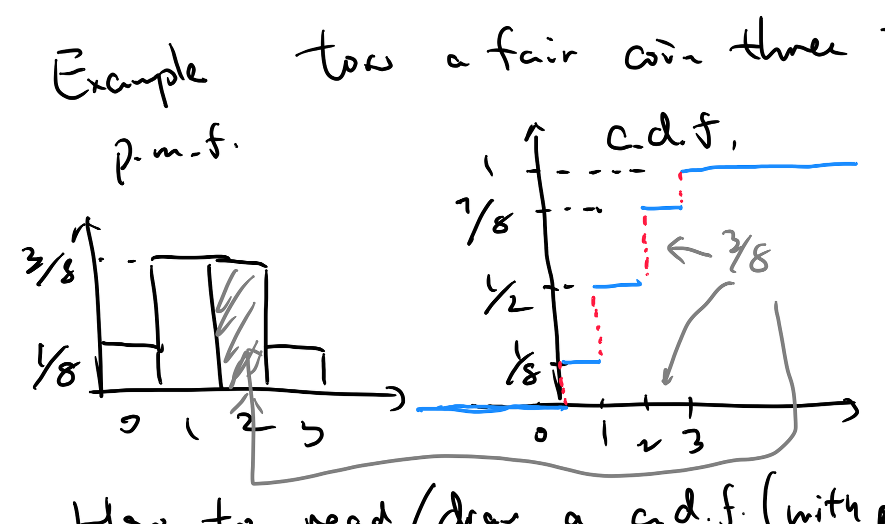
$P(X \le x)$ v.s. p.m.f. $P(X = x)$

summation ← / → difference

① Sometimes c.d.f. is easier to calculate < Geom, NB
(or its complement, known as 'tail prob.')
$P(X > x)$

② It can be used for R.V.s other than discrete ones.

Example toss a fair coin three times Binom(3, 1/2)



properties:
① within [0, 1]
② non-decreasing
③ $\lim_{x \to -\infty} P(X \le x) = 0$, $\lim_{x \to +\infty} P(X \le x) = 1$

How to read/draw a c.d.f. (with p.m.f.)

① flat parts (platforms)
Cannot take values inside a platform.
e.g. platform (a, b) ⇒ $P(X \le a) = P(X \le b)$
⇒ $P(a < X \le b) = 0$

② jump
a positive prob. that it could take / an 'atom'
e.g. a jump at a. means $P(X < a) < P(X \le a)$
let $p_a := P(X \le a) - P(X < a) = P(X = a)$