Stat 88 lec 20 (no lec 19 due to midterm)

warmup 2:00-2:10

b) Let the distribution of X be

$(x-\mu_x)^2$	$(1.1)^2$	$(.1)^2$	$(.9)^2$
x	1	2	3
$P(X=x)$	0.2	0.5	0.3

Calculate a) $\mu_x = E(X)$

b) Find $(x-\mu_x)^2$ in table

c) $E((X-\mu_x)^2)$

a) $E(X) = 1(.2) + 2(.5) + 3(.3) = \boxed{2.1}$

c) $E((X-\mu_x)^2) = (1.1)^2(.2) + (.1)^2(.5) + (.9)^2(.3)$

$= \boxed{.49}$

$\parallel$

$Var(X)$

Announcement

There was a lot of great learning in first half of the course. Keep it up!

Class will be course captured.

Today

sec 6.1 Variance and Standard Deviation

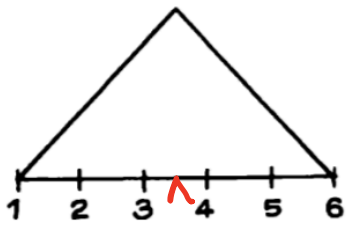
Sec 6.2 Simplifying the Calculation

sec 6.1 Variance and Standard Deviation

Expectation is the center of a distribution

Standard Deviation is the average spread of a distribution about the center.

. What is the SD of the following figure?



a) .5

b) 1 ←

c) 2

Variance

$D = X - \mu_X$ (deviation from expected value)

$Var(X) = E(D^2) = E((X-\mu_X)^2)$

We saw how to calculate this in the warm up.

Units are squared

Standard deviation

$SD(X) = \sqrt{Var(X)} = \sqrt{E((X-\mu_X)^2)}$

interpretation:

SD(X) is the "average" variation from the center.

ex

$D^2 = (y-\mu_Y)^2 \quad (-.8)^2 \quad (.2)^2 \quad (1.2)^2$

y	3	4	5
$P(Y=y)$	0.55	0.1	0.35

Calculate $E(Y)$ — $3(.55) + 4(.1) + 5(.35) = \boxed{3.8}$

$Var(Y)$ — $(-.8)^2(.55) + (.2)^2(.1) + (1.2)^2(.35) = \boxed{.86}$

$SD(Y) = \sqrt{.86} = \boxed{.93}$

In Python:

```
variance_table_Y
```

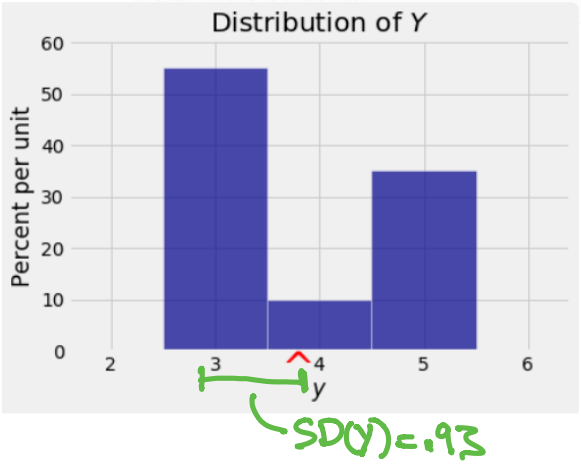
y	(y - E(Y))**2	P(Y = y)
3	0.64	0.55
4	0.04	0.1
5	1.44	0.35

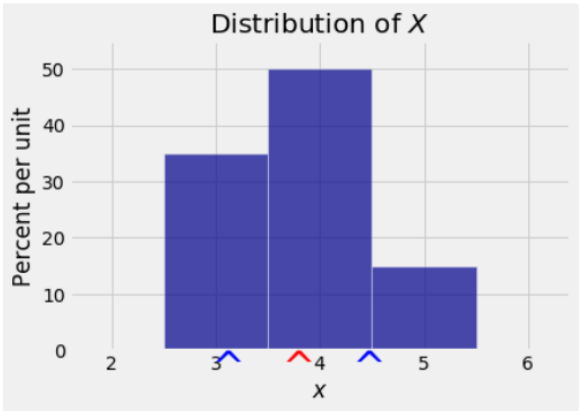
2

```
var_Y = sum(variance_table_Y.column(1) * variance_table_Y.column(2))
sd_Y = var_Y ** 0.5
sd_Y
```

```
0.9273618495495703
```

Picture





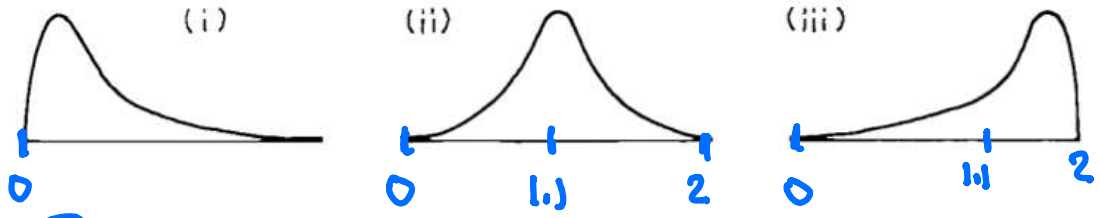
$E(X) = E(Y) = 3.8$

How does SD(x) and SD(Y) Compare?

Tinyurl.com/March10-pt1

avg = 1.1
SD = 1.5

One term, about 700 Statistics 2 students at the University of California, Berkeley, were asked how many college mathematics courses they had taken, other than Statistics 2. The average number of courses was about 1.1; the SD was about 1.5. Would the histogram for the data look like (i), (ii), or (iii)? Why?



2 **a** i

b ii

c iii

The right two pictures have all students take between 0 and 2 math classes. If the average is 1.1 that would mean that all students are within 1 SD of the mean which isn't possible since the SD is the avg dist from the mean.

The far left picture allows the possibility that there are some students who take many math classes.

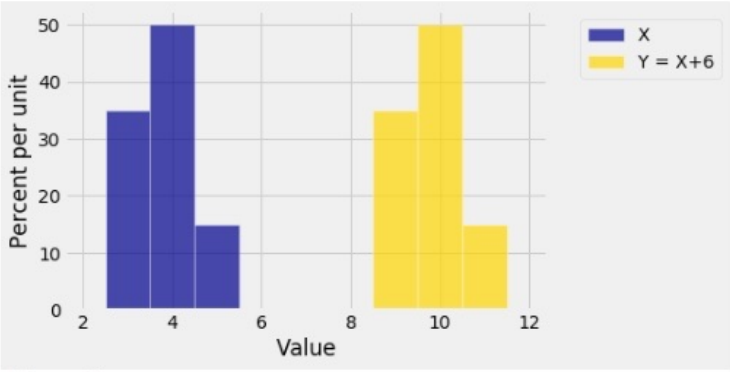
Sec 6.2 Simplifying the Calculation

Linear transformations

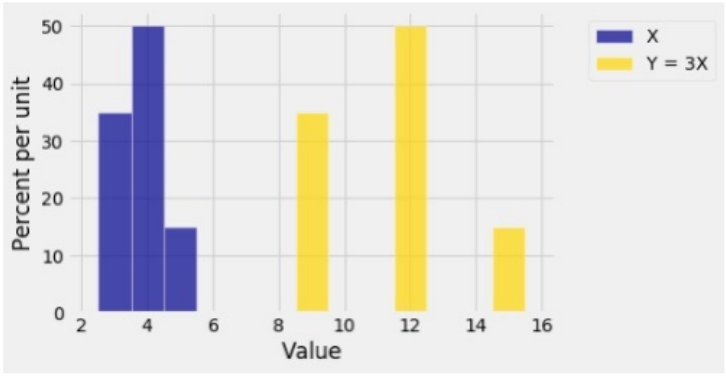
Celsius – Fahrenheit conversion

$$C = \frac{9}{5}F + 32$$

How does SD(C) compare to SD(F)?

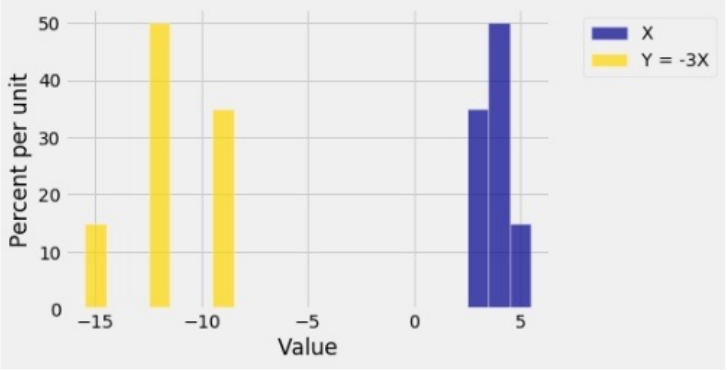


$$SD(X+b) = SD(X)$$



$a > 0$,

$$SD(aX) = a\,SD(X)$$



$a < 0$,

$$SD(aX) = |a|\,SD(X)$$

So $SD(aX+b) = |a|\,SD(X)$

$Var(aX+b) = a^2 Var(X)$

Hence

$$C = \tfrac{9}{5}F + 32$$

$$\Rightarrow SD(C) = \tfrac{9}{5} SD(F).$$

A different way to calculate variance

An algebraic simplification for calculating variance:

$$Var(X) = E\left((X-\mu_X)^2\right)$$

$$= E\left(X^2 - 2X\mu_X + \mu_X^2\right)$$

$$= E\left(X^2\right) - 2\mu_X E(X) + \mu_X^2$$

($E(X) = \mu_X$)

$$= E(X^2) - 2\mu_X^2 + \mu_X^2$$

$$= E(X^2) - \mu_X^2 = E(X^2) - (E(X))^2$$

($\mu_X = E(X)$)

Ex

y	3	4	5
$P(Y = y)$	0.55	0.1	0.35

$E(Y) = 3.8$

Calculate $\text{Var}(Y) = E(Y^2) - E(Y)$

$E(Y^2) = 3^2(.55) + 4^2(.1) + 5^2(.35) = 15.3$

$\text{Var}(Y) = 15.3 - (3.8)^2 = .86$

$SD(Y) = \sqrt{.86} \approx .93$

Ex 5. Let $p \in (0, 1)$ and let X be the number of spots showing on a flattened die that shows its six faces according to the following chances:

- $P(X = 1) = P(X = 6)$
- $P(X = 2) = P(X = 3) = P(X = 4) = P(X = 5)$
- $P(X = 1 \text{ or } 6) = p$

	1	2	3	4	5	6
	$\frac{p}{2}$	$\frac{1-p}{4}$	$\frac{1-p}{4}$	$\frac{1-p}{4}$	$\frac{1-p}{4}$	$\frac{p}{2}$

E(X)

Find $SD(X)$.

$E(X) = \frac{7}{2}$ by symmetry of the above table

$E(X^2) = \frac{p}{2}(1^2 + 6^2) + \frac{1-p}{4}(2^2 + 3^2 + 4^2 + 5^2)$

$= \frac{74p}{4} + \frac{54}{4} - \frac{54p}{4}$

$= \frac{20p}{4} - \frac{54}{4} = 5p - \frac{54}{4}$

$SD(X) = \sqrt{5p - \frac{54}{4} - \frac{49}{4}} = \sqrt{5p - \frac{5}{4}}$

. A study on college students found that the men had an average weight of about 66 kg and an SD of about 9 kg. The women had an average weight of about 55 kg and SD of 9 kg. If you took the men and women together, would the SD of their weights be:

a) smaller than 9kg

b) just about 9 kg

c) bigger than 9kg

d) you need more information