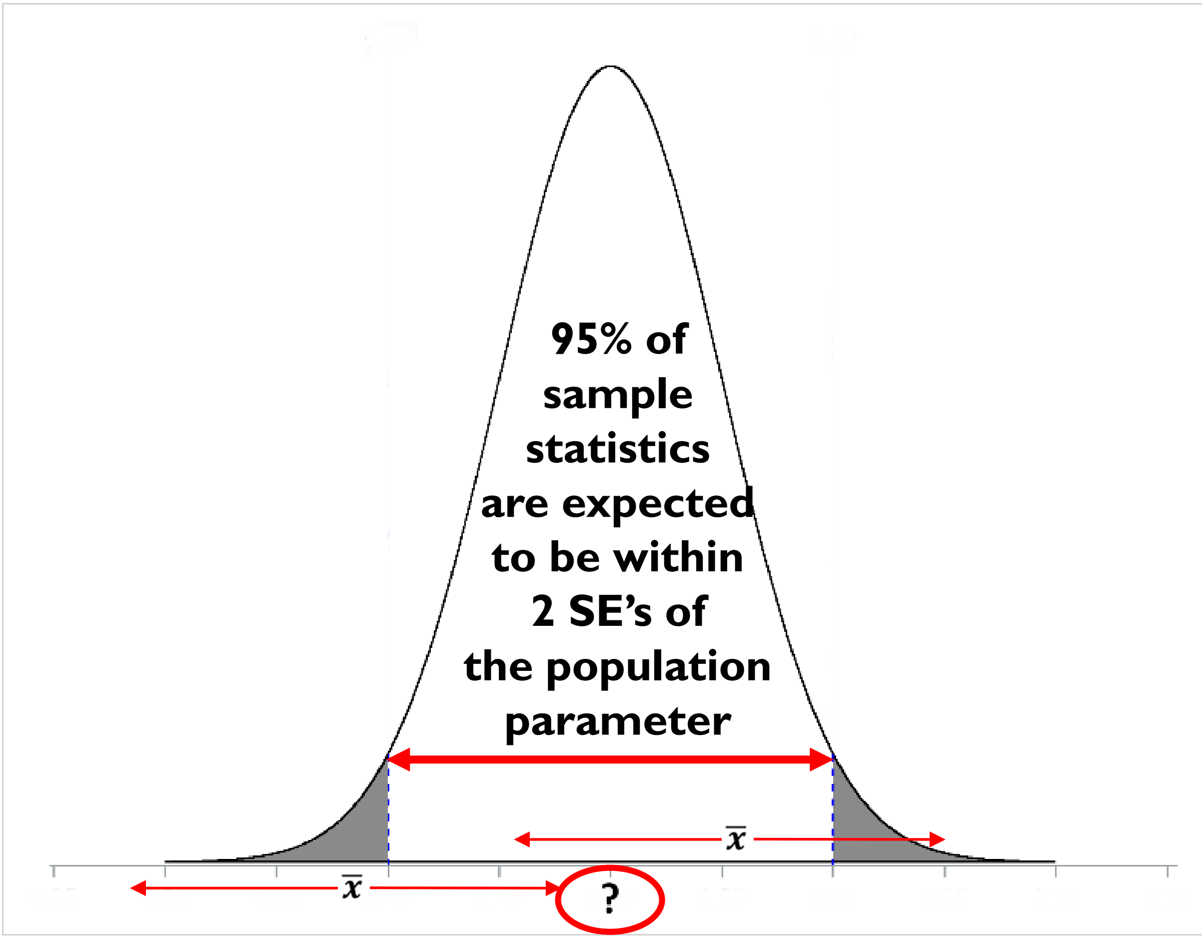
Probability and Mathematical Statistics in Data Science

Lecture 26: Section 9.4: Confidence Intervals: Interpretation

Confidence Intervals: Interpretation

The Sampling Distribution



95% Confidence Interval: sample statistic ± 2 x (standard error)

Interpretation

- Chance that sample mean is less than 2 SDs away from population mean is about 0.95
- Therefore the chance that population mean is less than 2 SDs away from sample mean is about 0.95
- Does it make sense to say “The probability that the number 2 is between 3 and 5 is 0.95” ?
- Does it make sense to say “The probability that the population mean is between 18 and 26 is 0.95”?

Interpretation

- Let's think about tossing coins. *Before* we toss a coin some number of times, we can say that the number of heads is random, since we *don't know* how many heads we will get.

- Suppose we have tossed the coin (say 100 times) and we see 53 heads, can we say 53 is a random number and the chance that 53 lies between 40 and 50 is 95%?

- 53 is our ***realization*** of the random "number of heads" in this *particular* instance of 100 tosses.

Confidence intervals: What is random?

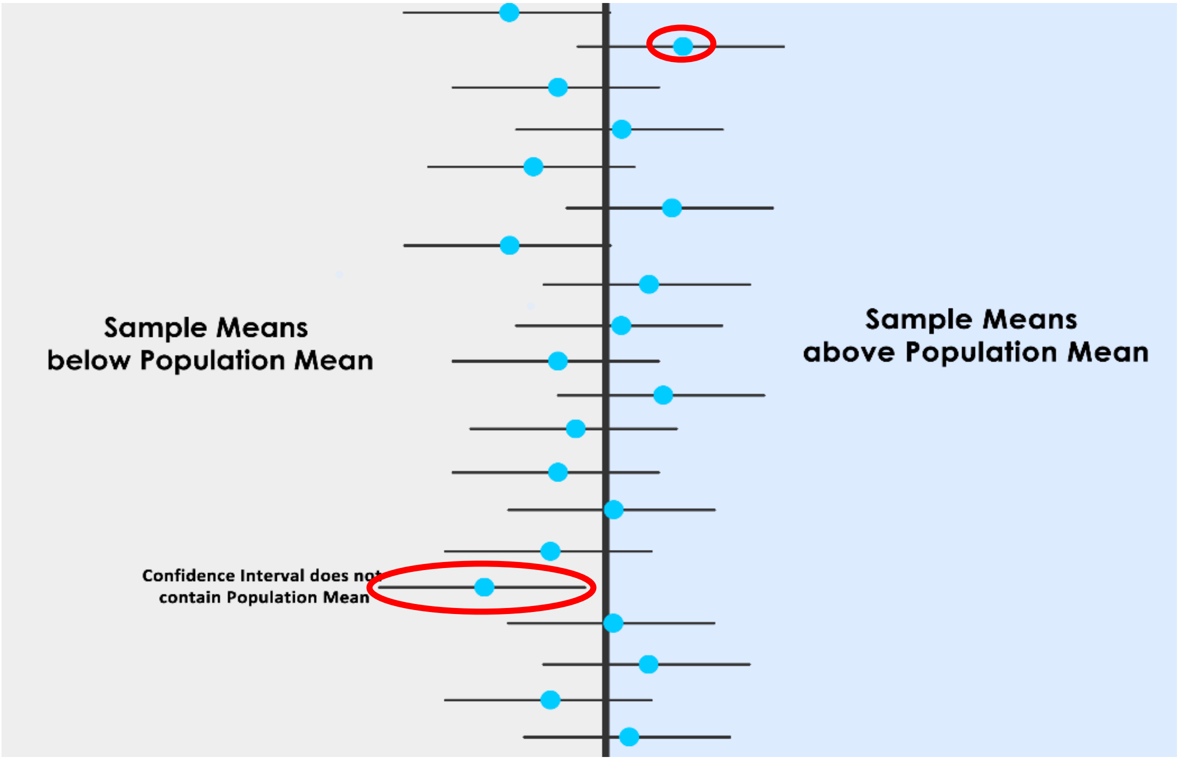
- Note that if we use the sample mean and extend one or two SDs in either direction, we *may* or *may not* cover the true population percentage.

- The *interval* is random, since we use a realization of the random variable $(\bar{X})$ to compute it.

- What fraction of such intervals (each interval computed from a random sample of data) will cover the true value μ?

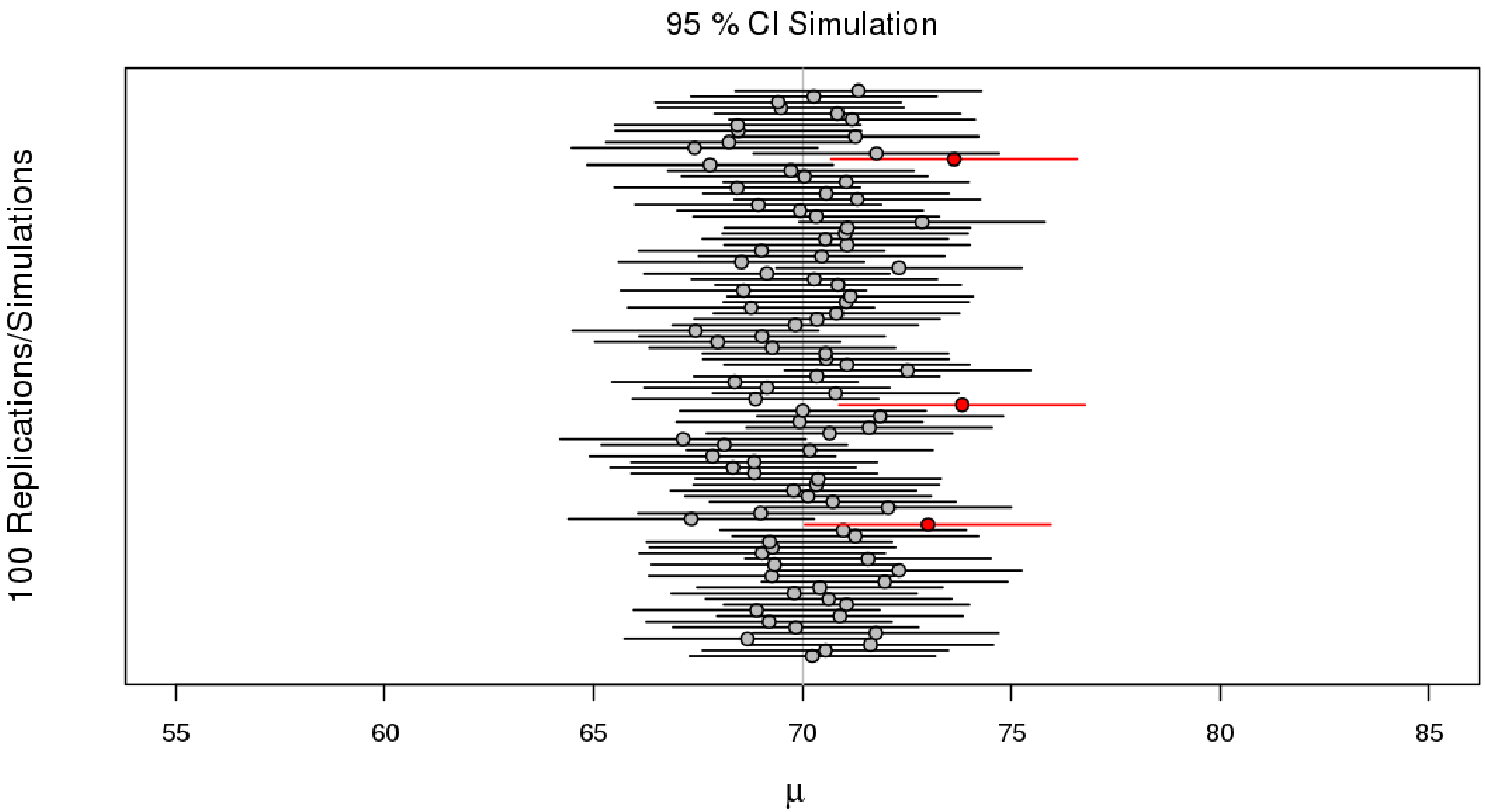
- This *coverage probability* **(before we actually collect the data)** is called the ***confidence level*** of the confidence interval.

Confidence Level: 95% Confidence Interval



- We expect19 out of every 20 "95% confidence intervals"
- ….to contain the population parameter

Confidence Interval for a Sample Mean: A simulation



source: https://shiny.rit.albany.edu/stat/confidence/

Confidence Intervals

1. Which would be wider : a 99% CI or a 95% CI?

2. What about a 90% CI? 68%?

3. The ________ the confidence level, the ________ the interval

Poll

IER Diet: 95% Confidence Interval : 4.8 kg to 7.9 kg means that:

A. "95% of women in U.K. will lose between 4.8 and 7.9 kg on the IER diet."

B. "We are 95% confident that a randomly selected women in U.K. will lose between 4.8 and 7.9 kg on the IER diet"

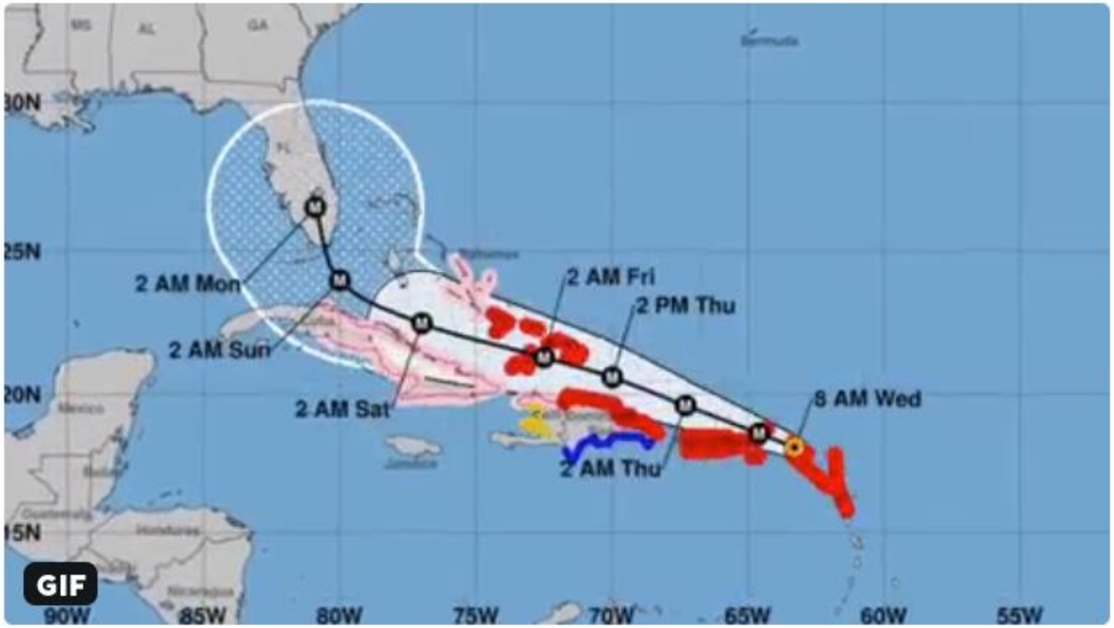
C. "We are 95% confident that the population mean weight loss for women in U.K. on the IER diet is between 4.8 kg and 7.9 kg"

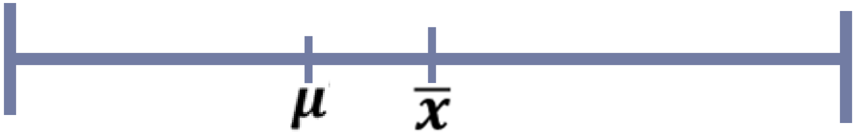
D. "The population mean weight loss is 6.4kg 95% of the time."

Irma Shifting Forecasts: It's All a Matter of Probability – New York Times

"Many people trying to use forecasts like those provided by the National Hurricane Center, however, do not fully understand the cone of probability and focus instead on the line that runs down the middle, taking it as an accurate prediction of the storm's path."

Irma Shifting Forecasts: It's All a Matter of Probability – New York Times





source: https://newyorktimes.com

The New York Times

Opinion

Those Hurricane Maps Don't Mean What You Think They Mean

We use hurricane forecasts to warn people. Why do we misinterpret them so often?

By Alberto Cairo
With Tala Schlossberg

- https://www.nytimes.com/interactive/2019/08/29/opinion/hurricane-dorian-forecast-

Margin of error

- We have a confidence interval. Now we want to keep the **same confidence level**, but want to improve our accuracy. For example, say our *margin of error* is 4 percentage points, and we want it to be 1 percentage point. What should we do?

A. Increase width of CI 4 times by increasing SD
B. Decrease width of CI by increasing *n* by 4 times
C. Decrease width of CI by increasing *n* by 16 times

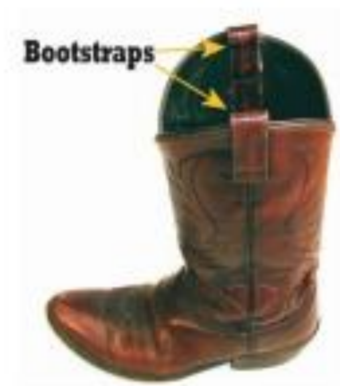
Alternative Method - Bootstrapping

- Quantitative research: testing and estimation require information about the population distribution.

- Bootstrap: Estimates the sampling distribution by using the information based on a number of resamples from the single sample you obtained.

Bootstrap

- Wikipedia: Boots may have a tab, loop or handle at the top known as a bootstrap, allowing one to use fingers or a tool to provide greater force in pulling the boots on.



Bootstrap Method

- Use the information of a number of resamples from the sample to estimate the sampling distribution

- Given a sample of size n:

Procedure

- • treat the sample as population
- • Draw B samples of size n with replacement from your sample (the bootstrap samples)
- • Compute for each bootstrap sample the statistic of interest (for example, the mean)
- • Estimate the sampling distribution of the statistic by the bootstrap sampling distribution

EX: Sample from Z Normal – Mean=0, SD=1

Case number	Original data	B_sample 1	B_sample 2	B_sample 3
1	1.22	-0.49	1.30	1.40
2	-0.49	-0.93	-1.20	-0.07
3	0.61	0.72	-0.26	-0.55
4	0.72	1.22	-0.08	0.13
5	0.58	-0.49	0.40	1.27
6	-0.08	-0.49	-0.15	-0.08
7	0.07	1.22	-0.55	-0.08
8	-0.26	-0.17	1.22	0.61
9	-0.07	0.40	0.11	-0.15
.			.	.
.			.	.
25	-0.17	1.27	0.52	0.15
26	-0.41	-0.52	1.40	0.72
27	1.40	-1.20	1.30	1.22
28	1.27	-0.15	-0.55	0.07
29	0.26	0.26	0.07	0.40
30	-0.55	-0.17	1.40	0.13
Mean	0.17	0.20	1.11	0.32
Sd	0.81	0.73	0.83	0.85

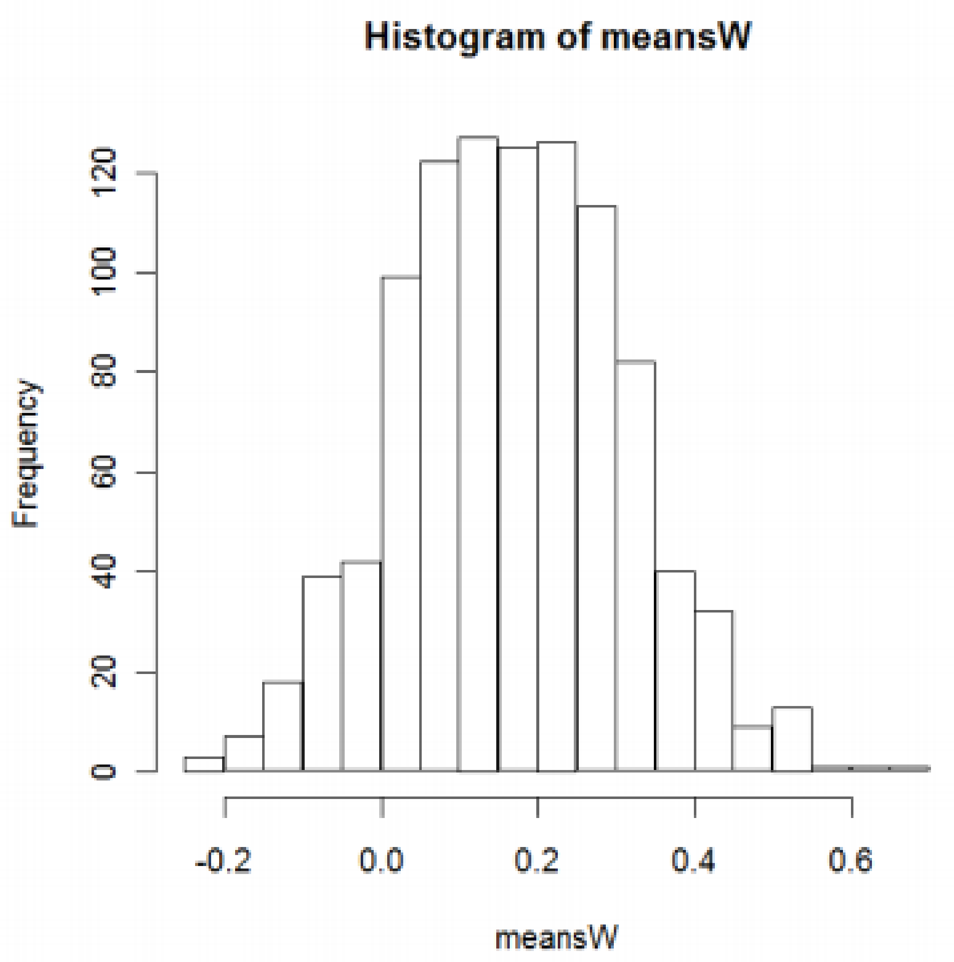
R-Script

```
set.seed(146035)  #start seed for simulation

sampleW <- rnorm(30, 0, 1) # draw sample of size 30
meansW <- numeric(1000)  #declaration of de bootstrap mean vector

# bootstrap procedure
for (i in 1:1000){
            meansW[i] <- mean(sample(sampleW, replace = T))}
```

R-Script Continued

```
# bootstrap results
hist(meansW,30)
summary(meansW)
quantile(meansW, c(0.025, 0.975))


# traditional approach
summary(sampleW)
mean(sampleW)
CI_up = mean(sampleW)+1.96*sd(sampleW)/sqrt(30)
CI_lo = mean(sampleW)-1.96*sd(sampleW)/sqrt(30)
```

Bootstrap Result



Example

- Population: standard normal distribution, mean=0, std. dev = 1

- Sample with sample size = 30.

- 1000 bootstrap samples.

- Results sample (traditional analysis)
- sample mean: 0.1698
- 95% Confidence interval: (-0.12,0.46)

- Bootstrap results:
- sample mean: 0.1689;
- 95% Confidence interval : (-0.10, 0.45)

Why does bootstrapping work?

- Basic idea: If the sample is a good approximation of the population, bootstrapping will provide a good approximation of the sampling distribution. Justification:

- 1. If the sample is representative of the population, the sampling distribution (empirical distribution) approaches the population (theoretical) distribution if n increases.

- 2. As the number of resamples (B) from the original sample increases, the bootstrap distribution approaches the sampling distribution