Stat 88 Fall 2019 **MIDTERM ANSWERS** **A. Adhikari**

1. [See e.g. Ch 3 Ex 2a]

$\sum_{k=3}^{20} \binom{20}{k} 0.1^k 0.9^{20-k}$, or $1 - \sum_{k=0}^{2} \binom{20}{k} 0.1^k 0.9^{20-k}$

2. [See e.g. Ch 2 Ex 6, HW 2 Ex 3b(ii)]

$\frac{\frac{45}{60} \times 1}{\frac{45}{60} \times 1 \ + \ \frac{15}{60} \times 0.5}$

3. [See e.g. HW 6 Ex 3b]

$(\frac{7}{12} \times 10) \ + \ (\frac{3}{12} \times 4) \ + \ (\frac{2}{12} \times 1)$

4. [See e.g. HW 2 Ex 5]

They won't contradict each other if any random birth is far more likely to be to a younger mother than to an older one, which agrees with the reality that "younger" is by definition a much larger set of ages of mothers than "older". This is all that is required as a midterm answer.

Formally, if for a random birth D denotes Down's Syndrome, M_y denotes a younger mother, and M_o denotes and older mother, then the first condition is $P(D \mid M_o) > P(D \mid M_y)$ and the second condition is $P(M_y \mid D) > P(M_o \mid D)$. By Bayes' rule, $P(M_y \mid D) = P(M_y)P(D \mid M_y)/P(D)$ and $P(M_o \mid D) = P(M_o)P(D \mid M_o)/P(D)$.

If $P(M_y)$ is much larger than $P(M_o)$ then we can have $P(M_y \mid D) > P(M_o \mid D)$ even if $P(D \mid M_y) < P(D \mid M_o)$.

5. [See e.g. HW 1 Ex 2b and 4b]

Can't find the exact chance since we don't have a measure of the dependence of the events on each other.
Lower bound: $P(\text{all overbooked}) \le 0.99$ since the intersection is smaller than each individual event.
Upper bound: $P(\text{all overbooked}) = 1 - P(\text{at least one is not overbooked}) \ge 1 - 10(0.01) = 0.9$.

6. [See HW 3 Ex 3a, HW 5 Ex 4, Ch 5 Ex 8]

Let X be the number of groups we are counting. Then $X = I_1 + I_2 + I_3$ where I_j is the indicator of the event that Group j contains more than 60 smokers.

For each j, $P(I_j = 1) = \sum_{k=61}^{100} \frac{\binom{180}{k}\binom{120}{100-k}}{\binom{300}{100}}$ by the symmetry of simple random sampling.

So $E(X) = 3 \times \sum_{k=61}^{100} \frac{\binom{180}{k}\binom{120}{100-k}}{\binom{300}{100}}$