On Course website, there are two past MTs.
Also a MT prepare sheet (later today).

Tell me what you want to hear on Wednesday
on Piazza / by email / leave a msg.

Midterm preparation

1. Textbook - do some exercises
2. HW - some questions in MT might be similar.
3. Quizzes - look at what you got wrong!

100-minute-long, open book, open notes
5 Question

During the exam.

- Do not overthink, and do sanity check
  Q. expected number of H's in 200 coin tosses
  A: 1682.0173
- Read the questions carefully
  eg. the 4^{th} H is on the 20^{th} toss
  the 4^{th} H is after the 20^{th} toss
  4 Hs in 20 tosses

  3H / 19 — H
  ≤3H / 20
  4H / 20
- Re-state the questions verbally.
  e.g. Draw cards until you see 3 Aces.
  $X :=$ # of cards drawn
  Find $P(X > x)$
  $= \mathbb{P}$(You need $> x$ draws to see 3 Aces)
  $= \mathbb{P}$(Sees < 3 Aces in the first x draws)

- Visualize your thoughts
  Venn diagram
  Tree diagram
  distr. table / joint distr. table
  more.
- What you should leave as your answer.
  No simplification needed.
  What we wish to see is your understanding
  Give some verbal reasoning if you are doing a mult-step computation
- About questions with multiple parts
  Usually, you need the result from previous part(s) to proceed.
  (a), (b) ⇒ (c)
  If you cannot solve one part, you may still proceed as if you've solved that part.
  Sometimes, there are parallel questions where the conditions are altered / removed / added
  If there are many parts, each should be answered concisely.

- Math requirement.
  Should not be hard.

- Counting / equally likely outcomes.
- Bounds (of intersection / union)
- Conditional expectation, Bayes rule.
- Multiplication rule, independence.
- Distributions. ① Range ② PMF ③ Expectation ④ Note
  Ben (p)
  Unif $\{1,2,\dots,n\}$
  Binom (n,p)
  Geom (p) ☆
  Hypergeom (N,G,n) SRS w/o. repl. sample size $= n$
  Poisson (λ)
  NegBinom (r,p)

- Expectations
  Defn
  conditioning.
  additivity.
  method of indicators
  - Define the indicators
  - Find expectation of these indicators as prob.
  - Add them ?

- Variance / SD
  (two ways)

Example (Bayes rule)
Computer virus
30% Mac — 65%
50% Win — 82%
20% Lin — 50%

Q: $\mathbb{P}(\text{Win} \mid V) = \dfrac{\mathbb{P}(V \mid \text{Win})\,\mathbb{P}(\text{Win})}{\mathbb{P}(V \mid \text{Mac})\,\mathbb{P}(\text{Mac}) + \mathbb{P}(V \mid \text{Win})\,\mathbb{P}(W) + \mathbb{P}(V \mid \text{Lin})\,\mathbb{P}(\text{Lin})}$

82% × 50%

"Prob. by conditioning"

Example (Poisson)
Writing python codes. Each line he writes has no bug with prob 99.9%, indpt. of others
What is the chance that he makes less than 3 mistakes after writing a program containing 1500 lines of codes, approximately?
Binom(1500, 0.001) $\approx$ Poisson(1.5)

- explicitly (Sees Poisson(λ) in question)
- implicitly (usually law of small numbers)

Example (Method of indicators).
A fair die is rolled 14 times.
$X :=$ $\boxed{\text{\# of}}$ faces that appear exactly twice.
→ method of indicators?
Find $\mathbb{E}X$.

Range(X) $= \{0, 1, 2, \dots, 6\}$

$I_j = \begin{cases} 1 & \text{if face } j \text{ appears exactly twice} \\ 0 & \text{o/w.} \end{cases}$

$X = \sum_{j=1}^{6} I_j$. $\qquad \mathbb{E} I_j = \boxed{\mathbb{P}(I_j = 1)}$ Mini-question

$\mathbb{E} X = \mathbb{E} \sum I_j = \sum \mathbb{E} I_j$

Example.
A particle is moving along the real line.
It starts at the origin, and jumps to one of the neighbor integer each second in the following manner.
w/. prob. p $+1$ (right)
w/. prob. $1-p$ -1 (left).
independent of other moves.
"biased Random Walk"
$X :=$ position after a minute. (60 seconds)

$\mathbb{E}X$

$I_j = \begin{cases} 1 & \text{moving right on the } j^{th} \text{ second} \\ -1 & \text{moving left — — — .} \end{cases}$

$X := I_1 + I_2 + \dots + I_{60}$

$\mathbb{E}X = \sum_{j=1}^{60} \mathbb{E} I_j = 60\,\mathbb{E} I_1 = \dots$

Example (Distribution).
5 B 7 G W/o. repl. until I collect both colors
$X :=$ # of cards drawn
What is the distr. of X?
Range(X) $= \{2, 3, 4, \dots, 8\}$
"Worst case scenario" $g^7 b$

⇒ Need to find the pmf of X at $2, 3, \dots, 8$ $\qquad$ 5B 7G

$\mathbb{P}(X=2) = \mathbb{P}(bg) + \mathbb{P}(gb)$
$\mathbb{P}(X=3) = \mathbb{P}(bbg) + \mathbb{P}(ggb)$ → $\mathbb{P}(bbg) = \mathbb{P}(bb)\,\mathbb{P}(g \mid bb)$
$= \text{HG} \cdot \frac{7}{10}$
$\vdots$
$\mathbb{P}(X=8) = \underbrace{\mathbb{P}(b^7 g)}_{=0} + \mathbb{P}(g^7 b)$

Quiz 3 Q5
$E \sim \text{Poisson}(6)$
$C \sim \text{Poisson}(4)$ } $M \sim \text{Poisson}(4+3)$
$L \sim \text{Poisson}(3)$
<\$20 ⟺ < 10 milk sold
$\mathbb{P}$(Sells less than 10 milk)
$= \mathbb{P}(M < 10) = \sum_{k=0}^{9} e^{-7} \cdot \frac{7^k}{k!}$