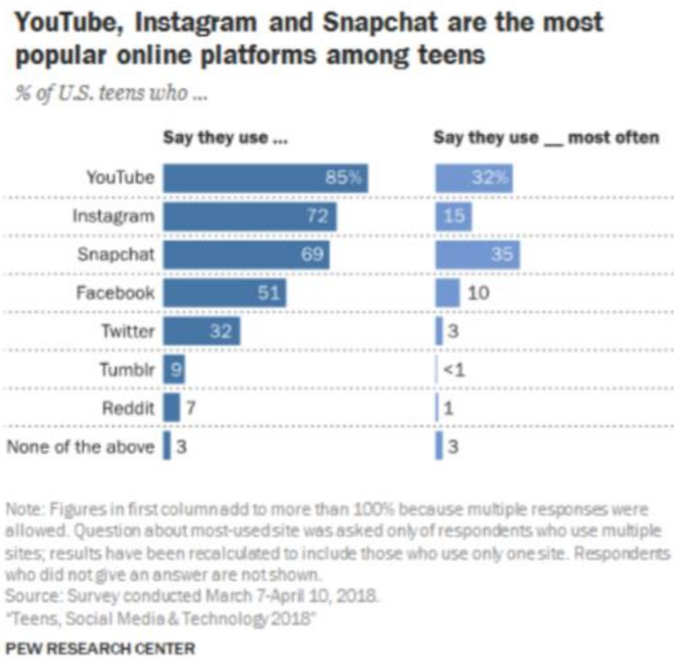
Probability and Mathematical Statistics in Data Science

Lecture 02 – Section 1.2: Exact Calculations and Bounds

Section 1.3: Fundamental Rules

Exact Calculations, or Bound?



Recall #3 about FB or Twitter. What was the answer?

What can you say about the chance that a randomly selected teen used FB or Twitter (not necessarily most often)?

Example with bounds

- Let A be the event that you catch the bus to class instead of walking, P(A) = 70%
- Let B be the event that it rains, P(B) = 50%
- Let C be the event that you are on time to class, P(C) = 10%

Q. What is the chance of at least one of these three events happening?

Q. What is the chance of all three of them happening?

Defining Events

Example: Flip a coin twice. Four possible outcomes, S={HH, HT, TH, TT}.

- Let A be the event that we obtain at least one H in the two flips. A={HH, HT, TH}.
- What is the P(A)?
- Let B be the event that we obtain two H's in the two flips. B={HH}.
- What is the P(B)?

Events and Sets

In a more abstract way, we can think about Sample Space, Outcomes and Events in terms of the Set Theory.

- Outcomes are the objects (elements)
- Events are sets (collections of elements)
- Sample Space is the whole set (collection of all elements)

We will treat events and sets synonymously.

Set Operations

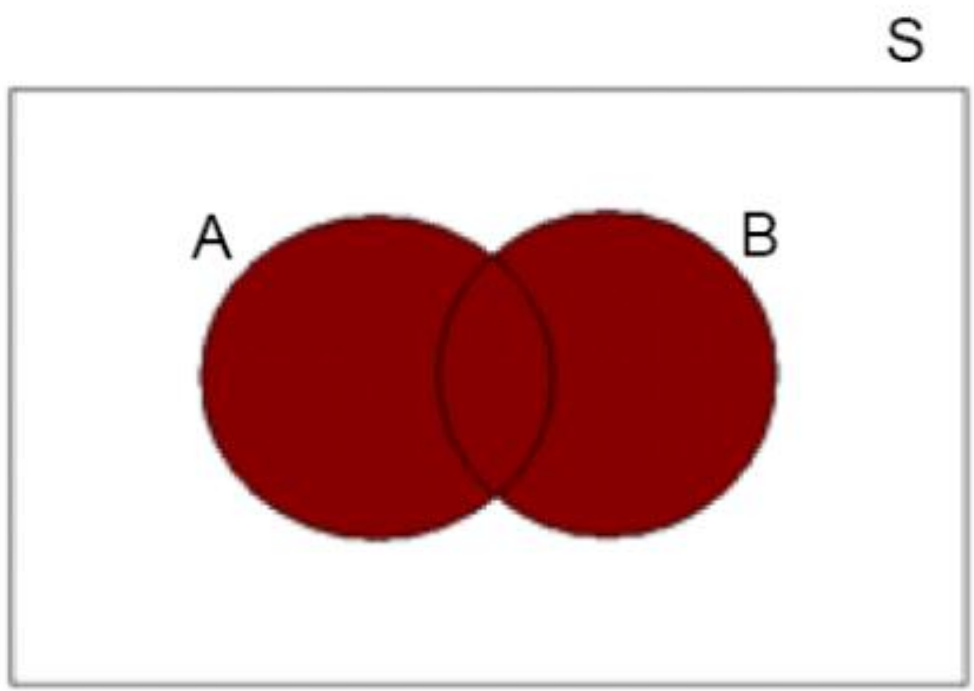
Given any two events (or sets) A and B, we have the following elementary set operations:

- The union
- The intersection
- The complement

Venn diagrams are often used to illustrate relationships between sets.

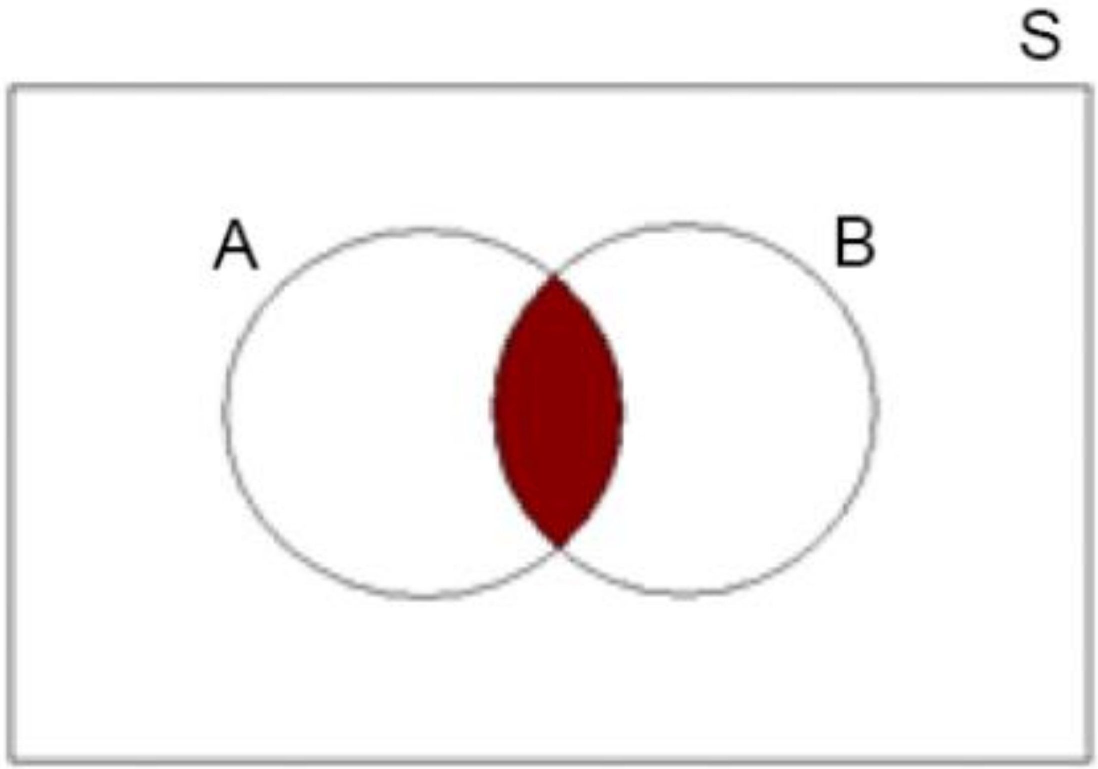
Union

The union of A and B, written as A U B and read "A or B", is the set of outcomes that belong to either A or B or both.



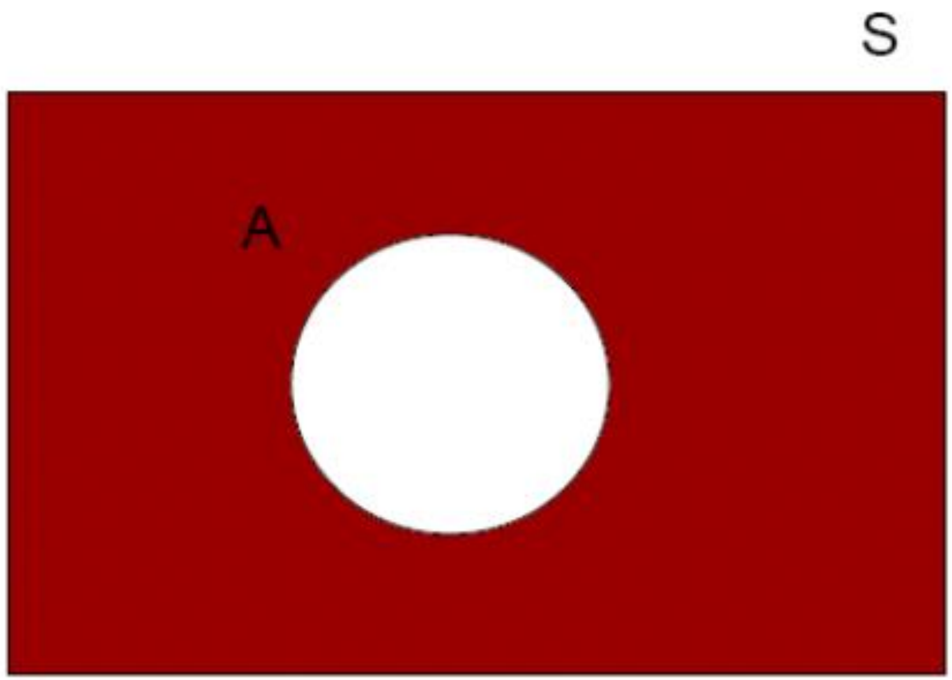
Intersection

- The intersection of A and B, written as A ∩ B, read "A and B", is the set of outcomes that belong to both A and B.



Complement

- The complement of A, written as A' or A^C, is the set of all outcomes in S that are not in A.



Example

Select a card at random from a standard deck of cards, and note its suit:

Clubs (Cl), Diamonds (D), Hearts (H) or Spades (Sp).

The sample space is S={Cl, D, H, Sp}.

Let: A={Cl, D}, B={D, H, Sp} and C={H}.

$A \cup B = \{Cl, D, H, Sp\} = S$

$A \cap B = \{D\}$

$A^C = \{H, Sp\}$

$A \cap C = \emptyset$ (null event – event consisting of no outcomes)

Disjoint events

- If $A \cap B = \emptyset$ then A and B are said to be mutually exclusive or disjoint events.
- Any event and its complement are disjoint!

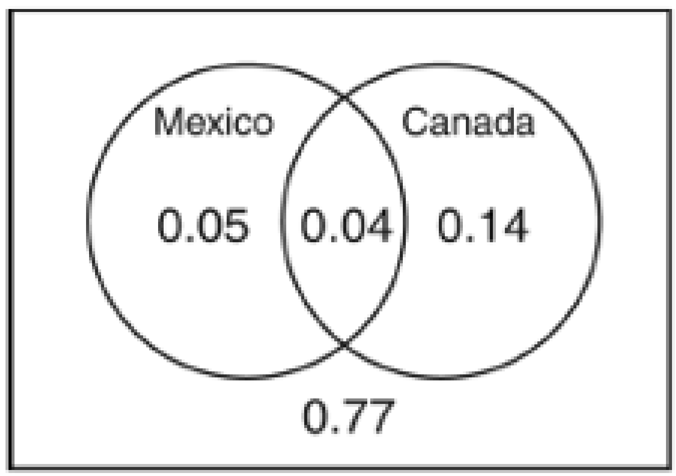
Question

Travel Suppose the probability that a U.S. resident has traveled to Canada is 0.18, to Mexico is 0.09, and to both countries is 0.04. What's the probability that an American chosen at random has

a) traveled to Canada but not Mexico?

b) traveled to either Canada or Mexico?

c) traveled to neither Canada or Mexico?



Probability models

- A probability model consists of a sample space (S) and the assignment of probabilities to each possible outcome.
- Probability that event A occurs is written as P(A), which will give a precise measure of the chance that A will occur.

Axioms of Probability

- To ensure the probability assignments will be consistent with our intuitive notions of probability, all assignments should satisfy the following axioms (basic properties) of probability.

1. For any event A, $P(A) \geq 0$.
2. $P(S)=1$.
3. If $A_1, A_2, A_3, \ldots$ is an infinite (finite) collection of disjoint events, then

$$P(A_1 \cup A_2 \cup A_3 \cup \ldots) = \sum P(A_i)$$

Propositions

- For any event A, $0 \le P(A) \le 1$.

- $P(A) + P(A^C) = 1$.

- If event A is contained in event B, in the sense that every outcome in A is also in B, then $P(A) \le P(B)$

- $P(\emptyset) = 0$

Thought Question

1. If you **flip a coin** and do it fairly, what is the ***probability that it will land heads up***?

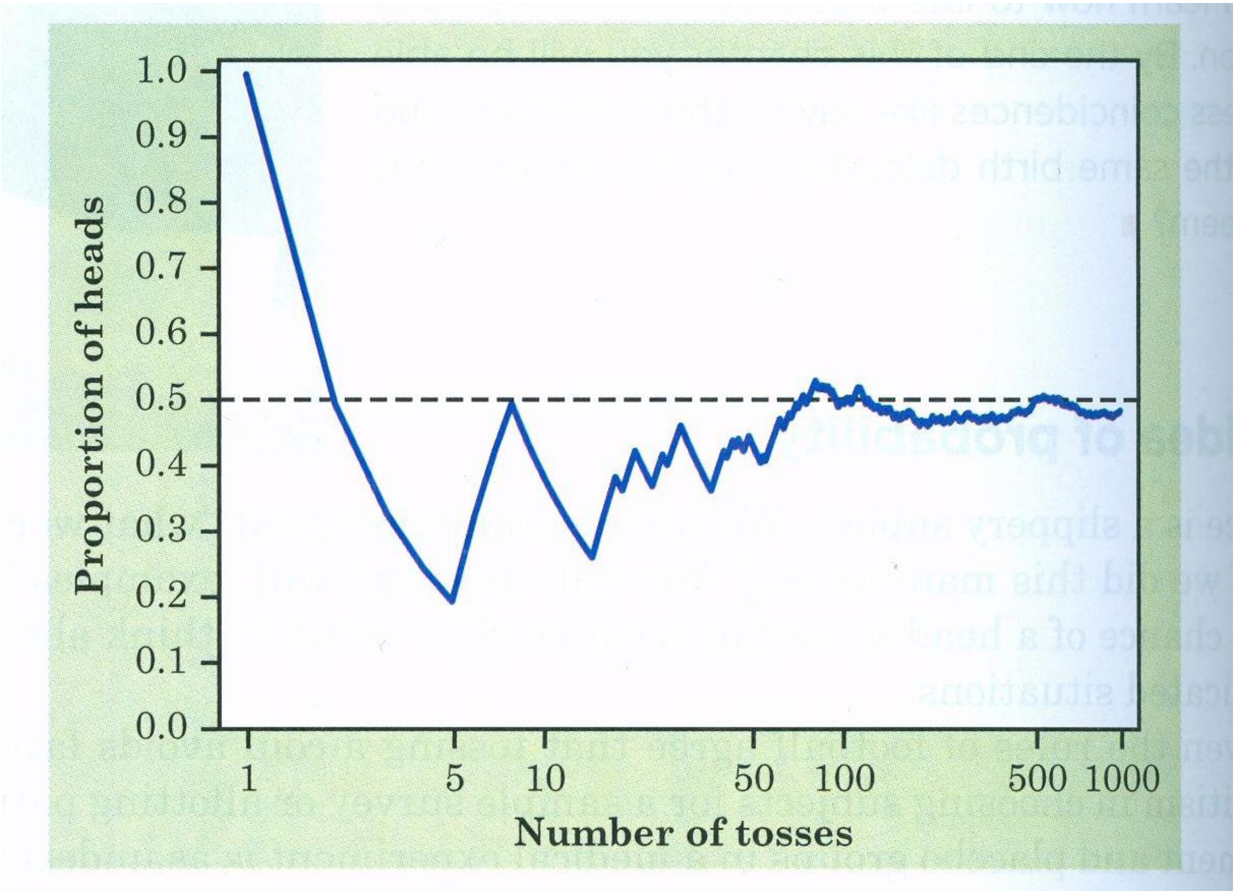
Interpreting Probability

- What does it mean when we say we have 50% chance of having a head when flipping a coin? Or what does it mean when we put $P(H)=0.5$?

- Probability is often treated as the long-term relative frequency or the limiting relative frequency

Interpreting Probability: Coin-Toss Example

- assume coins made such that they are equally likely to land with heads or tails up when flipped - **probability of a flipped coin showing heads up is ½.**



The Law of Large Numbers

First a definition . . .

- When thinking about what happens with combinations of outcomes, things are simplified if the individual trials are **independent**.
  - Roughly speaking, this means that the outcome of one trial doesn't influence or change the outcome of another.
  - For example, coin flips are independent.

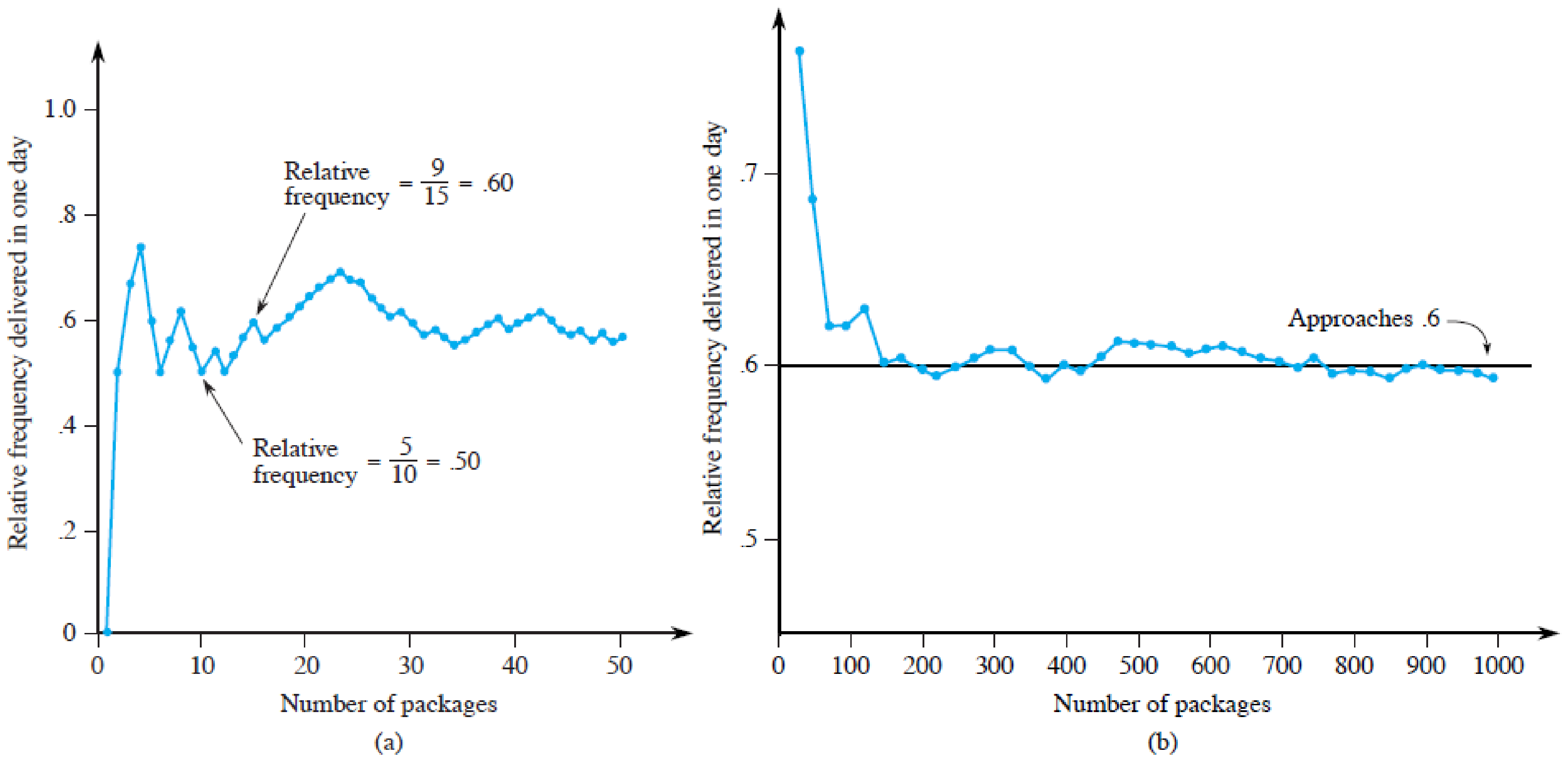
Law of Large Numbers

The law of large numbers says that the long-run relative frequency of repeated independent events gets closer and closer to the true relative frequency as the number of trials increases.

$$\frac{\text{Number of Occurences of Event A}}{\text{Number of Trials}} \rightarrow P(A)$$

as number of trials $n \rightarrow \infty$

Law of Large Numbers



Example

Roll a fair die. S={1,2,3,4,5,6}. Our sample space consists of 6 points, each of which is equally likely to occur.

- P(roll a 1) =
- Let A = roll a 4 or less = {1,2,3,4}. P(A) =
- Let B = roll an even number = {2,4,6}. P(B) =

Example

Roll two fair dice.

- There are 36 possible outcomes: {(1,1),(1,2),(1,3),…,(6,5),(6,6)}.

- Let A = sum of two rolls is 7; B = sum of two rolls is 11 or more. What are P(A) and P(B)?