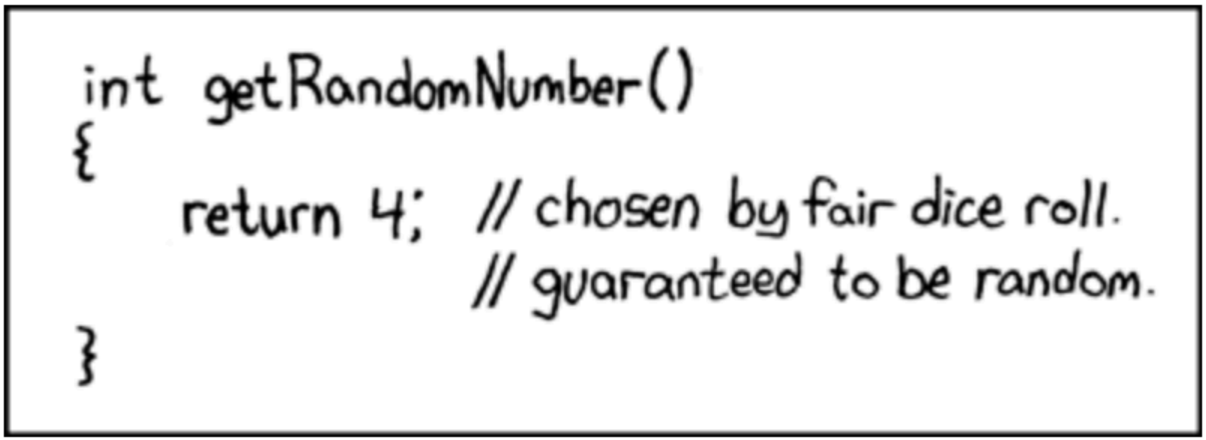
Stat 88: Probability and Mathematical Statistics in Data Science



https://xkcd.com/221/

Lecture 2: 1/22/2021

Axioms of Probability

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Agenda

- The Basics:
  - Review of 1.1 + FB question, and conditional probability
  - Extra problems
  - Section 1.2: Exact Calculation or Bound
  - Section 1.3: Fundamental Rules

So far:

- If all the possible outcomes are *equally likely*, then each outcome has probability $1/n$, where $n = \#(\Omega)$

$\Omega = \{1, 2, 3, 4, 5, 6\}$

A = collection of outcomes

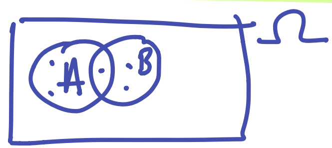
- Let $A \subseteq \Omega$, $P(A) = \frac{\#(A)}{\#(\Omega)}$

$A = \{2, 3, 5\}$ $\quad P(A) = \frac{3}{6} = \frac{1}{2}$

- Probabilities as proportions
- Sum of the probabilities of all the distinct outcomes should add to 1
- $0 \leq P(A) \leq 1, \mathrm{A} \subseteq \Omega$

$P(\cancel{1}) \times \quad P(\{1\})$

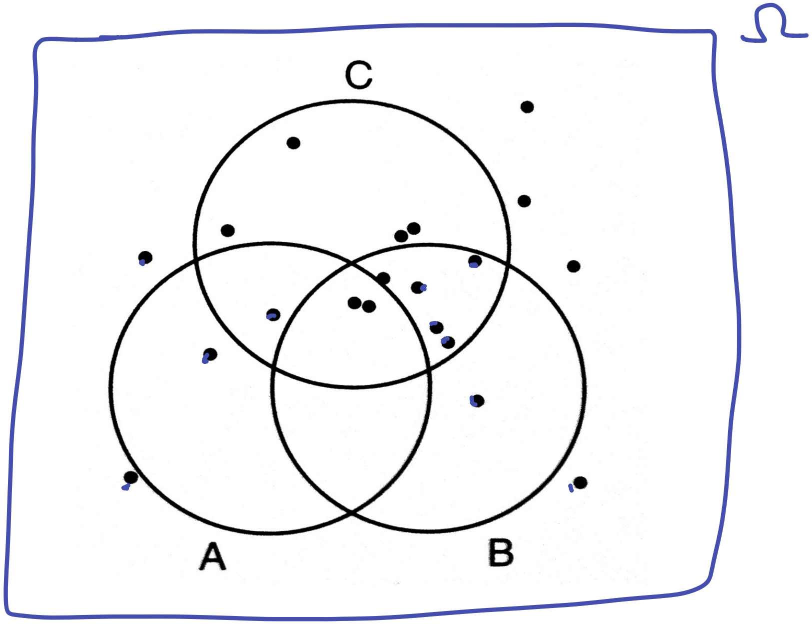
- A *distribution* of the outcomes over different categories is when each outcome appears in one and only one category.
- Venn diagrams



$A = \{2, 3, 5\}$
$B = \{2, 4, 6\}$

- When we get some information about the outcome or event whose probability we want to figure out, our outcome space reduces, incorporating that information.

Extra problem 1



$\#(\Omega) = 20$

Consider the Venn diagram above. (The sample space consists of all the dots.) What is the probability of A? What about A or B? A or B or C?

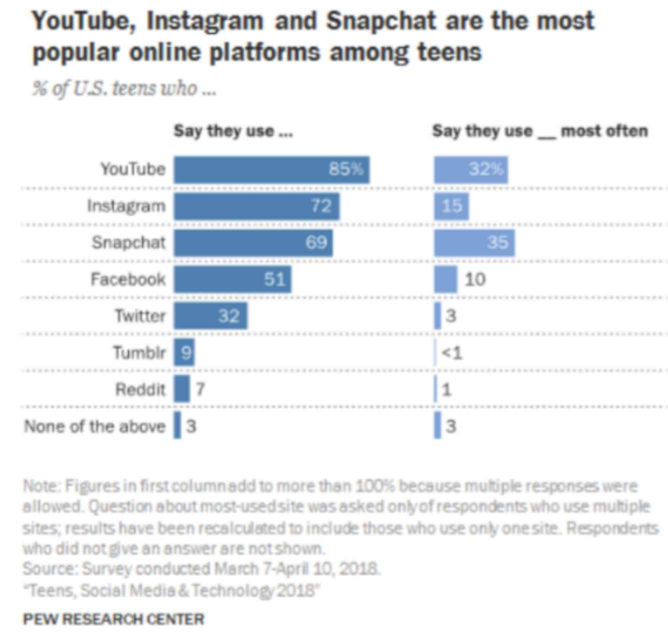
$P(A) = \frac{4}{20}$ $\quad P(A \text{ or } B) = \frac{10}{20}$

$\rightarrow P(A \cup B) =$

A union B

$P(A \text{ or } B \text{ or } C) = P(A \cup B \cup C) = \frac{14}{20}$

Not equally likely outcomes



1. What is the chance that a randomly picked teen uses FB most often?

~10%

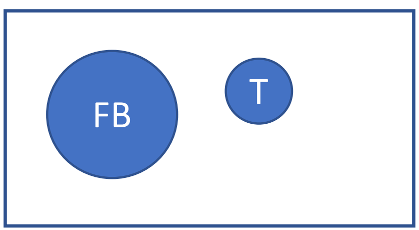
2. What is the chance that a randomly picked teen did *not* use FB most often?

~90%

3. What is the chance that FB *or* Twitter was their favorite?

10% + 3% = 13%

3. Venn diagram:



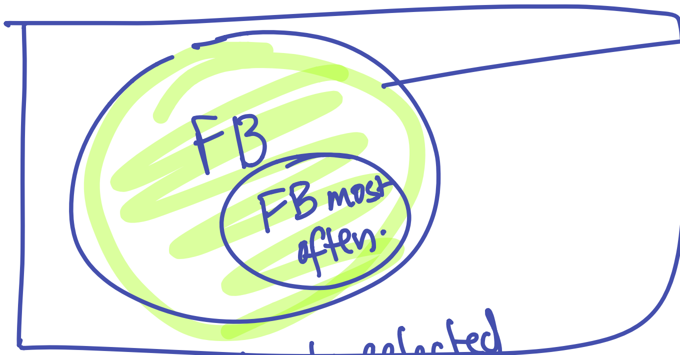
4. What is the chance that the teen used FB, just not most often?

51%-10% = 41% of all teens used FB but not most often

5. ***Given*** that the teen used FB, what is the chance that they used it most often?

Conditional probability

- In the last question, we used the information that the teen used FB. We were told the teen used FB, and *then* asked to compute the chance that FB was their favorite.
- This is called the *conditional probability that the teen used Facebook most often, given that they used Facebook* and denoted by:



everyone using FB

a randomly selected teen

Prob of using FB most often, given the info that they use FB $= \dfrac{\#(A)}{\#(\Omega_{new})} = \dfrac{10}{51} \approx 0.2$

P(a randomly selected teenager uses FB most often | they use FB)

("GIVEN")

Conditional probability

- This probability we computed is called a ***conditional probability***. It puts a condition on the teen, and *changes* (restricts) the universe (the sample space) of the next outcome, a teen who likes FB best.
- To compute a conditional probability:
  - First restrict the set of all outcomes as well as the event to *only* the outcomes that *satisfy* the given **condition**
  - Then calculate proportions accordingly
- How do the probabilities in #1 and #5 compare?

$$P(A) = \frac{\#(A)}{\#(\Omega)}$$, given information B.

$P(A \mid B) \leftarrow$ read as prob of A given B

conditional prob of A given B

$$\frac{\#(A \cap B) / \#(\Omega)}{\#(B) / \#(\Omega)} = \frac{P(A \cap B)}{P(B)} = P(A \mid B)$$

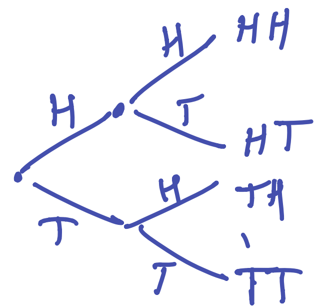
Extra problem 2

result of 1 roll

$\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

- A ten-sided fair die is rolled twice:
  - If the first roll lands on 1, what is the chance that the second roll lands on a number bigger than 1?
  - Find the probability that the second number is greater than the **twice** the first number.

$\#(\Omega) = 100$ (2 rolls of die, 100 possible outcomes)



But GIVEN first roll is 1, so universe reduces to 10 outcomes

new $\Omega = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (1,9), (1,10)\}$

$P(2^{nd}\text{roll} > 1 \mid 1^{st} \text{ roll} = 1) = \frac{9}{10} = 0.9$

A is event where 2^{nd} roll is greater than twice the 1^{st}

$\#(3, >6) = 4$, $\#(4, >8) = 2$

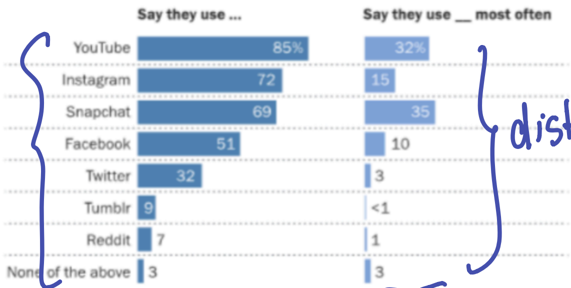
$\#((1,>2)) = 8$, $\#((2,>4)) = 6$, $\#((3,>6))$

$\#(A) = 8+6+4+2 = 20$ $\quad P(A) = \frac{20}{100}$

Section 1.2: Exact Calculations, or Bound?

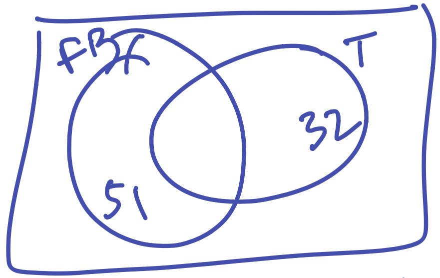
YouTube, Instagram and Snapchat are the most popular online platforms among teens

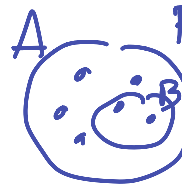
% of U.S. teens who ...



Note: Figures in first column add to more than 100% because multiple responses were allowed. Question about most-used site was asked only of respondents who use multiple sites; results have been recalculated to include those who use only one site. Respondents who did not give an answer are not shown.
Source: Survey conducted March 7-April 10, 2018.
"Teens, Social Media & Technology 2018"
PEW RESEARCH CENTER

Recall #3 about FB or Twitter. What was the answer? What can you say about the chance that a randomly selected teen used FB or Twitter (not necessarily most often)?

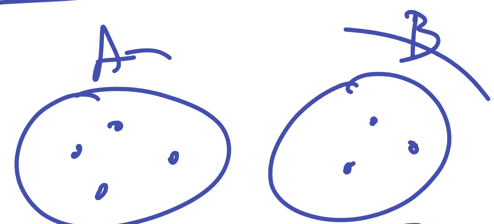




$P(A \text{ or } B) = P(A)$ smallest

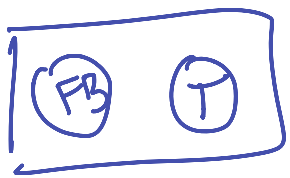
Most overlap = 32%

This is when $P(FB \text{ or } T)$ ⇒ smallest = 51%



$P(A \cup B) = 7$

$P(FB \text{ or } T)$



$\leq (51+32)\% = 83\%$

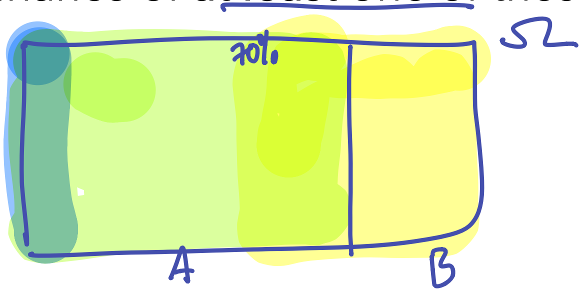
$P(A \cup B)$

$51\% \leq P(FB \cup T) \leq 83\%$

= biggest

Example with bounds

- Let A be the event that you catch the bus to class instead of walking, P(A) = 70%
- Let B be the event that it rains, P(B) = 50%
- Let C be the event that you are on time to class, P(C) = 10%
- What is the chance of **at least** one of these three events happening?

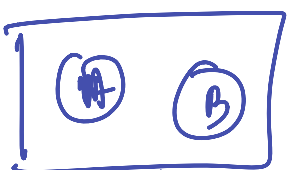


$P(A \cup B \cup C)$

$\leq \underbrace{P(A) + P(B) + P(C)}_{130\%}$

- What is the chance of **all three** of them happening?

$P(A \cup B \cup C) \leq 100\% =)$



$P(A \cap B) = 0$

"mutually exclusive."

$0 \leq P(A \cap B \cap C) \leq 10\%$

Section 1.3: Fundamental Rules

- Also called "Axioms of probability", first laid out by Kolmogorov
- Recall Ω, the outcome space. Note that Ω can be finite or infinite.

- <u>First, some notation:</u>
- Events are denoted (usually) by $\mathrm{A}, \mathrm{B}, \mathrm{C} \ldots$

- Note that Ω is itself an event (called the ***certain*** event) and so is the empty set (denoted $\emptyset$, and called the ***impossible*** event or the *empty set*)

- The *complement* of an event A is everything ***else*** in the outcome space (all the outcomes that are *not* in A). It is called "not A", or the complement of A, and denoted by A^c

Intersections and Unions

- When two events A and B ***both*** happen, we call this the *intersection* of A and B and write it as

$$A \text{ and } B = A \cap B$$

- When either A *or* B happens, we call this the *union* of A and B and write it as

$$A \text{ or } B = A \cup B$$

- If two events A and B ***cannot both occur*** at the same time, we say that they are *mutually exclusive* or *disjoint*.

$$A \cap B = \emptyset$$

empty set ϕ