Stat 88 Midterm Practice Problems

Note: These exercises are taken from *Probability* by Jim Pitman. Do not distribute these beyond the course.

Question 1 A coin which lands heads with probability p is tossed repeatedly. Assuming independence of the tosses, find formulae for

a. P(Exactly 5 heads appear in the first 9 tosses)

b. P(The first head appears on the 7th toss)

c. P(The fifth head appears on the 12th toss)

d. P(The same number of heads appear in the first 8 tosses as in the next 5 tosses)

Question 2 Radioactive substances emit α-particles. The number of such particles reaching a counter over a given time period follows the Poisson distribution. Suppose two substances emit α-particles independently of each other. The first substance gives out α-particles which reach the counter according to the Poisson (3.87) distribution, while the second substance emits α-particles which reach the counter according to the Poisson (5.41) distribution. Find the chance that the counter is hit by at most 4 particles.

Question 3 Suppose you roll a fair six-sided die repeatedly until the first time you roll a number that you have rolled before.

a. For each $r = 1, 2, ...$ calculate the probability p_r that you roll exactly r times.

b. Without calculation, write down the value of $p_1 + p_2 + ... + p_{10}$. Explain.

c. Check that your calculated values of p_r have this value for their sum.

Question 4 Let X be the number of spades in 7 cards dealt from a well-shuffled deck of 52 cards containing 13 spades. Find $E(X)$.

Question 5 A typical slot machine in a Nevada casino has three wheels, each marked with twenty symbols at equal spacings around the wheel. The machine is engineered so that on each play the three wheels spin independently, and each wheel is equally likely to show anyone of its twenty symbols when it stops spinning. On the central wheel, nine out of the twenty symbols are bells, while there is only one bell on the left wheel and one bell on the right wheel. The machine pays out the jackpot only if the wheels come to rest with each wheel showing a bell.

a. Calculate the probability of hitting the jackpot.

b. Calculate the probability of getting two bells but not the jackpot.

c. Suppose that instead there were three bells on the left, one in the middle, and three on the right. How would this affect the probabilities in a) and b)? Explain why the casino might find the 1 - 9 - 1 machine more profitable than a 3 - 1 - 3 machine.

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Question 6 A manufacturing process produces integrated circuit chips. Over the long run the fraction of bad chips produced by the process is around 20%. Thoroughly testing a chip to determine whether it is good or bad is rather expensive, so a cheap test is tried. All good chips will pass the cheap test, but so will 10%, of the bad chips.

a. Given a chip passes the cheap lest, what is the probability that it is a good chip?

b. If a company using this manufacturing process sells all chips which pass the cheap test, over the long run what percentage of chips sold will be bad?

Question 7 A deck of cards is shuffled and dealt to four players, with each receiving 13 cards. Find:

a. the probability that the first player holds all the aces;

b. the probability that the first player holds all the aces given that she holds the ace of hearts;

c. the probability that the first player holds all the aces given that she holds at least one;

d. the probability that the second player holds all the aces given that he holds all the hearts.

Question 8 A hat contains a number of cards, with 30% white on both sides; 50% black on one side and white on the other; 20% black on both sides. The cards are mixed up, then a single card is drawn at random and placed on the table. If the top side is black, what is the chance that the other side is white?

Question 9 Suppose a word is picked at random from this sentence.

a. What is the distribution of the length of the word picked?

b. What is the distribution of the number of vowels in the word?

Question 10 A box contains 8 tickets. Two are marked 1, two marked 2, two marked 3, and two marked 4. Tickets are drawn at random from the box without replacement until a number appears that has appeared before. Let X be the number of draws that are made. Make a table to display the probability distribution of X.

Question 11 A book has 200 pages. The number of mistakes on each page is a Poisson random variable with mean 0.01, and is independent of the number of mistakes on all other pages.

a. What is the expected number of pages with no mistakes?

b. A person proofreading the book finds a given mistake with probability 0.9. What is the expected number of pages where this person will find a mistake?

c. What, approximately, is the probability that the book has two or more pages with mistakes?

Question 12 In a circuit containing n switches, the ith switch is closed with probability $p_i, i = 1, ..., n$. Let X be the total number of switches that are closed. What is $E(X)$? Or is it impossible to say without further assumptions?

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Question 13 Suppose electric power is supplied from two independent sources which work with probabilities 0.4, 0.5, respectively. If both sources are providing power enough power will be available with probability 1. If exactly one of them works there will be enough power with probability 0.6. Of course, if none of them works the probability that there will be sufficient supply is O.

a. What are the probabilities that exactly k sources work for $k = 0, 1, 2$?

b. Compute the probability that enough power will be available.

Question 14 A die is rolled 8 times. Given that there were 3 sixes in the 8 rolls, what is the probability that there were 2 sixes in the first five rolls?

Question 15 Suppose an airline accepted 12 reservations for a commuter plane with 10 seats. They know that 7 reservations went to regular commuters who will show up for sure. The other 5 passengers will show up with a 50% chance, independently of each other.

a. Find the probability that the flight will be overbooked, i.e., more passengers will show up than seats are available.

b. Find the probability that there will be empty seats.

c. Let X be the number of passengers turned away. Find $E(X)$.