

# STAT 88: Lecture 7 Continued

## Contents

### Section 3.5: Examples

#### Summary of Binomial versus Hypergeometric:

##### Binomial ( $n, p$ )

$n$  = # trials

$p$  = probability of success

sample with replacement

trial has two outcomes  
    < Success  
       Failure

$n$  independent trials

$X$  = # successes in sample

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

##### Binomial formula

Ex. pick 5 cards from a deck  
with replacement. What is chance  
you get 2 ace cards?

##### HG ( $N, G, n$ )

$N$  = population size

$G$  = # good elements in pop

$n$  = sample size

if w/ replacement,  $p = \frac{G}{N}$

sample without replacement

trial has two outcomes  
    < good  
       bad

$n$  dependent trials

$X$  = # good in sample

$$P(X=g) = \frac{\binom{G}{g} \binom{N-G}{n-g}}{\binom{N}{n}}$$

##### HG formula

Ex. pick 5 cards from a deck  
without replacement. What is chance  
you get 2 ace cards?

### 3.5. Examples

Problem solving techniques:

- Breaking the problem down into smaller pieces
- Examining the assumptions and hence deciding which distributions can be used
- Organizing the information to identify the parameters of the distributions
- Partitioning events into component pieces
- Using the addition and multiplication rules carefully

**Advisor meetings** An advisor at a university provides guidance to 10 students. Each student has to meet with her once a month during the school year which consists of nine months.

So each month the advisor schedules one day of meetings. Each student has to sign up for one meeting that day. Students have the choice of meeting her in the morning or in the afternoon.

Assume that every month each student, independently of other students and other months, chooses to meet in the afternoon with probability 0.75.

**Question.** What is the chance that she has both morning and afternoon meetings in all of the months except one?

8 months.

$X = \#$  of months have both morning and afternoon meetings.

$\uparrow$  Binomial( $n, p$ )

$n = 9$

$p = P(\text{Both morning and afternoon meeting})$

Complement rule  $\rightarrow 1 - P(\text{Both M and A})^c$

$= 1 - P(\text{all morning or all afternoon})$

Addition rule  $\rightarrow 1 - P(\text{all morning}) - P(\text{all afternoon})$

(Events of all M and all A ~ Mutually exclusive)  
 $= 1 - (0.25)^9 - (0.75)^9 \approx 0.94$

9 months  
 $\uparrow$  10 students, each of them sign up for one meeting.

$$P(X=8) = \binom{9}{8} (0.94)^8 (0.06)^1$$

**Randomized Controlled Experiments (RCE)** In a RCE, a simple random sample of half the participants will be assigned to the treatment group (T) and the other half to control (C). = sample (w/o replacement) 50 participants and assign them  $\leadsto$  H & to the Treatment group.

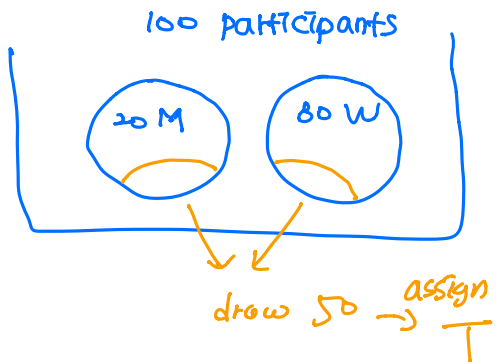
Experiment 1 has 100 participants of whom 20 are men.

**Question 1.** What is the chance that the treatment and control groups in Experiment 1 contain the same number of men?

$X = \# \text{ men in our sample}$   
 $\underbrace{\hspace{1.5cm}}$   
 treatment group

$N = 100$   
 $G = 20$   
 $n = 50$

$\rightarrow X \sim \text{HG}(100, 20, 50)$

$$P(X=10) = \frac{\binom{20}{10} \binom{80}{40}}{\binom{100}{50}}$$


100 participants

20 M 80 W

draw 50  $\rightarrow$  assign T

Experiment 1 has 100 participants of whom 20 are men.

Experiment 2 has 90 participants of whom 30 are men.

**Question 2.** What is the chance that the treatment groups in the two experiments have the same number of men?

$X_1$  = # men in the treatment group for Exp 1

$X_2$  = # men in the treatment group for Exp 2.

$$X_1 \sim HG(100, 20, 50)$$

$$X_2 \sim HG(90, 30, 50)$$

$$P(X_1 = X_2) = \underbrace{P(X_1 = 0, X_2 = 0)}_{\substack{\text{partition events} \\ \text{into component pieces}}} + P(X_1 = 1, X_2 = 1) + \dots + P(X_1 = 20, X_2 = 20)$$

"and"

$$= \sum_{g=0}^{20} P(X_1 = g \text{ and } X_2 = g)$$

$X_1$  and  $X_2$  indep.  $\nearrow$

$$= \sum_{g=0}^{20} P(X_1 = g) P(X_2 = g)$$

$$= \sum_{g=0}^{20} \frac{\binom{20}{g} \binom{80}{50-g}}{\binom{100}{50}} \cdot \frac{\binom{30}{g} \binom{60}{50-g}}{\binom{90}{50}}$$

$$\underline{\underline{\hspace{10cm}}}$$