Stat 88 lec 3

warm up 2:00–2:10

(ex 1.4.5)

If a student applies to ten colleges with a 20% chance of being accepted to each, what are the chances that he will be accepted by at least one college?

Be clear about any assumptions you are making.

Making no assumptions:

C_i = event get into i^{th} college

$P(A_1 \cup \cdots A_{10}) \geq P(A_1)$

Bernoulli shows

$P(C_1 \cup \cdots \cup C_{10}) \leq \sum_{i=1}^{10} P(C_i)$

$\Rightarrow \boxed{.2 \leq P(C_1 \cup \cdots \cup C_{10}) \leq 1}$

exact

$$P(C_1 \cup \cdots \cup C_{10}) = 1 - P((C_1 \cup \cdots C_{10})^c)$$
$$= 1 - P(C_1^c \cdots C_{10}^c)$$
$$= 1 - \underbrace{P(C_1^c)}_{= .8} \cdots \underbrace{P(C_{10}^c)}_{= .8} \quad \text{assuming indep.}$$
$$= 1 - \underbrace{(.8)^{10}}_{= .11} = .89$$

Last time

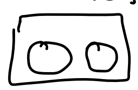
Sec 1.3 Fundamental rules and bounds

Addition rule (OR rule)

$P(A \cup B) = P(A) + P(B)$ (OR: $\cup$) if A and B are mutually exclusive

without assumptions





$\max(P(A), P(B)) \le P(A \cup B) \le P(A) + P(B)$

$\uparrow$ Bonferroni / Boole ineq

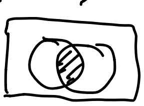
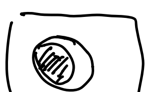
Notation: AB means $A \cap B$

De Morgan rule: $(A^c \cup B^c)^c = AB$

so $P(AB) = P((A^c \cup B^c)^c) = 1 - P(A^c \cup B^c)$ $\le P(A^c) + P(B^c)$

Note $P(A^c \cup B^c) \le P(A^c) + P(B^c) \Rightarrow -P(A^c \cup B^c) \ge -P(A^c) - P(B^c)$

$\Rightarrow \quad P(AB) \ge 1 - P(A^c) - P(B^c)$

Also $\quad P(AB) \le \min(P(A), P(B))$

So $\quad 1 - P(A) - P(B) \le P(AB) \le \min(P(A), P(B))$

Today ① Sec 2.1 The chance of an intersection

② Sec 2.2 Symmetry in sampling

① Sec 2.1 The chance of an intersection

3 card deck $\{R, B, G\}$

Pick 2 cards w/o replacement

Find $P(1^{st}$ card B and 2^{nd} card R$)$

$$\Omega = \{RB, RG, BG, BR, GR, GB\}$$

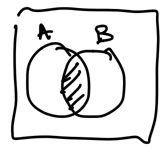
all equally likely

answ $\boxed{1/6}$

Alternatively, use the multiplication rule

$$P(BR) = \underbrace{P(2^{nd} R \mid 1^{st} B)}_{1/2} \cdot \underbrace{P(1^{st} B)}_{1/3}$$

mult rule (and)



$$P(AB) = P(B \mid A)\,P(A)$$

Note $AB = BA$ since $A \cap B = B \cap A$.

$$\Rightarrow P(AB) = P(BA) = P(A \mid B)\,P(B)$$

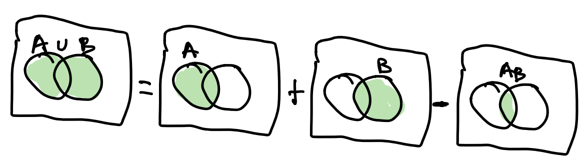
ex What is chance 1^{st} card in a 52 card deck is queen and the last is queen?

$$P(1^{st}\,Q \text{ and last } Q) = \underbrace{P(1^{st}\,Q)}_{\frac{4}{52}}\,\underbrace{P(\text{last } Q \mid 1^{st}\,Q)}_{\frac{3}{51}}$$

$$\boxed{\frac{4}{52} \cdot \frac{3}{51}}$$

Inclusion-Exclusion (or)

$$P(A \cup B) = P(A) + P(B) - P(AB)$$



ex What is chance 1^{st} card in a 52 card deck is queen or the last is queen?

$$P(A) + P(B) - P(AB)$$

$$\frac{4}{52} + \frac{4}{52} - \frac{4}{52} \cdot \frac{3}{51}$$

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1. A deck of cards is shuffled. What is the chance that the top card is the **king** of spades **or** the bottom card is the **king** of spades

a $\frac{1}{52} + \frac{1}{52} - \frac{1}{52} \times \frac{1}{52}$
b $\frac{1}{52} + \frac{1}{51}$
c $\frac{1}{52} + \frac{1}{52} - \frac{1}{52} \times \frac{1}{51}$
d none of the above

Assume you deal cards w/o replacement unless told otherwise.

$$P(KS_1 \text{ or } KS_{52}) = \underset{\frac{1}{52}}{\underset{\shortparallel}{P(KS_1)}} + \underset{\frac{1}{52}}{\underset{\shortparallel}{P(KS_{52})}} - \underset{0}{\underset{\shortparallel}{P(KS_1 \cdot KS_{52})}} = \boxed{\frac{1}{52} + \frac{1}{52}}$$

ex

Consider a poker hand (5 cards randomly drawn w/o replacement)

What is chance all 5 cards are hearts?

$$\frac{13}{52}\frac{12}{51}\frac{11}{50}\frac{10}{49}\frac{9}{48}$$

$$P(H_1)P(H_2|H_1)P(H_3|H_1H_2)\cdots$$

What is the chance all 5 cards are of the same suit?
(i.e have a flush?)

4 × answ. part 1

$$\frac{52}{52}\frac{12}{51}\frac{11}{50}\frac{10}{49}\frac{9}{48}$$

② Sec 2.2 <u>Symmetry in Simple Random Sampling</u>

Recall a deck of 52 cards has 4 suits

How many possible pairs of cards are there?

$$\boxed{52 \cdot 51}$$

If you deal 2 cards, what is the chance the 2^{nd} card is red?

method 1 (easiest)

Imagine deck of cards before it is dealt.
nothing special about 1^{st} card.

$$P(R_2) = P(R_1) = \frac{26}{52}$$

method 2 (partition event according to what first card is)

$$P(R_2) = P(B_1 R_2) + P(R_1 R_2)$$

$$= \frac{26}{52}\cdot\frac{26}{51} + \frac{26}{52}\cdot\frac{25}{51} = \frac{26}{52}\left(\frac{26+25}{51}\right) = \frac{26}{52} \checkmark$$

$51/51$