STAT 88: Lecture 28

Contents

Last time

Testing hypotheses:

Hypothesis testing has 5 steps:

(a) Null hypothesis H_0 — a specific distribution (null distribution).

$H_0: \mu = 0$

(b) Alternative hypothesis H_A — one or two sided depending on the context of the problem (alternative distribution). ( )

$H_A: \mu > 0$ $H_A: \mu \neq 0$

(c) Test statistic — a random variable that we can compute based on our samples and whose distribution under H_0 we know so we can compute a p-value.

(d) p-value — chance of being as or more extreme than the observed value of our test statistic in the direction of the alternative.

(e) Conclusion — accept null if p-value $\geq$ leve of test; reject otherwise.

5% level : Accept Null $\Leftrightarrow$ p-value > 0.05

Reject Null $\Leftrightarrow$ p-value ≤ 0.05

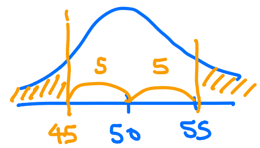
$\sigma = 20$

Warm up: A population distribution is known to have an SD of 20. Determine the p-value of a test of the hypothesis that the population mean is equal to 50, if the average of a sample of 64 observations is 55. $\mu = 50$

$x_1, \ldots, x_{64}$ $\qquad$ $\bar{x} = 55$

- $H_0: \mu = 50$
- $H_A: \mu \neq 50$
- Test statistic $\bar{X}$, obs-value $= 55$

Under H_0, $\bar{X} \sim N\left(50, \left(\frac{20}{\sqrt{64}}\right)^2\right) = N\left(50, \left(\frac{20}{8}\right)^2\right)$



- p-value $= P(\bar{X} \geq 55 \text{ or } \bar{X} \leq 45)$

$$= 2 \cdot P(\bar{X} \leq 45)$$

$$= 2 \cdot P\left(\frac{\bar{X} - 50}{20/8} \leq \frac{45 - 50}{20/8}\right)$$

$$= 2 \cdot P(Z \leq -2)$$

$$= 2 \cdot \Phi(-2)$$

$$= 0.0455$$

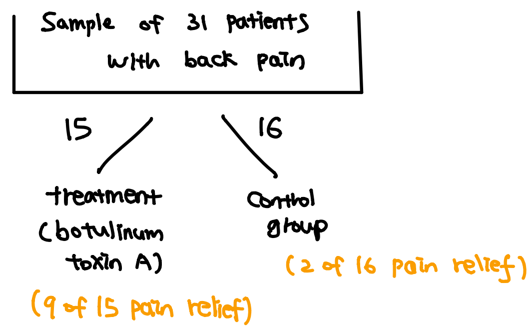
- P-val ≤ 0.05 $\rightsquigarrow$ Reject H_0 at 5% level

9.2. A/B Testing: Fisher's Exact Test

A/B testing is the shorthand for a simple controlled experiment (or just comparing two distributions).

$$\text{A} = \text{Control group}; \quad \text{B} = \text{Treatment group}.$$

Example: (Study for treatment of chronic back pain)



Is the treatment effective?

(a) State H_0. Treatment has no effect ; any difference between two groups is due to random assignments.

(b) State H_A. Treatment has an effect on back pain. (good or bad)

(c) Find test statistic. X = # treated patients w/ pain relief , obs-value = 9

Under H_0

31

11 | 20

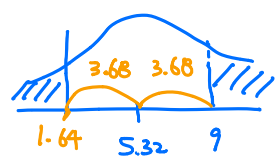
Pain relief

↓ 15

9 out 15 pain relief.

Under H_0, $X \sim HG(31, 11, 15)$; $11 = G$

$E(X) = 15 \cdot \frac{11}{31} = 5.32$



(d) Find p-value.

$$\begin{aligned} \text{p-val} &= P(X \geq 9 \text{ or } X \leq 1.64) \\ &= P(X \geq 9) + P(X \leq 1.64) = \sum_{g=9}^{11} \frac{\binom{11}{g}\binom{20}{15-g}}{\binom{31}{15}} + \sum_{g=0}^{1} \frac{\binom{11}{g}\binom{20}{15-g}}{\binom{31}{15}} = 0.0092. \end{aligned}$$

(e) Conclusion (5% level).

Reject H_0 ($\because$ p-val < 0.05)

This is called Fisher's Exact test

200

95 trt | 105 Control

75 of 95 test | 70 of 105 test

Example: (Exercise 9.5.9) A randomized controlled trial was conducted as part of a effort to encourage high school students from under-resourced communities to apply for college. The trial had 200 participants. A simple random sample of 95 participants received special coaching for the ACT. The remaining participants received no intervention.

"treatment"

At the end of the experiment, the participants got to decide whether or not they would take the ACT. Among the 95 students in the treatment group, 75 decided to take the test. Among the 105 students in the control group, 70 decided to take it.

Is the difference statistically significant? Answer this question by performing a test of whether or not the intervention had any effect.

(a) State H_0. Treatment (special coaching) did nothing.

(b) State H_A. Treatment did something good or bad

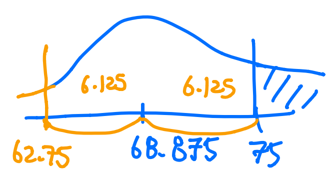
(c) Find test statistic. X = # test taken in treatment group. Obs value = 75

$75+70=145$ | 200 | 55 | $n=95$

Under H_0, $X \sim HG(200, 145, 95)$

75 out of 95 took the test.

$E(X) = 95 \cdot \frac{145}{200} = 68.875.$



(d) Find p-value.

$$p\text{-val} = P(X \geq 75) + P(X \leq 62.75)$$

$$= \sum_{g=75}^{95} \frac{\binom{145}{g}\binom{55}{95-g}}{\binom{200}{95}} + \sum_{g=0}^{62} \frac{\binom{145}{g}\binom{55}{95-g}}{\binom{200}{95}} = 0.0582$$

$\sum_{g=40}^{62}$

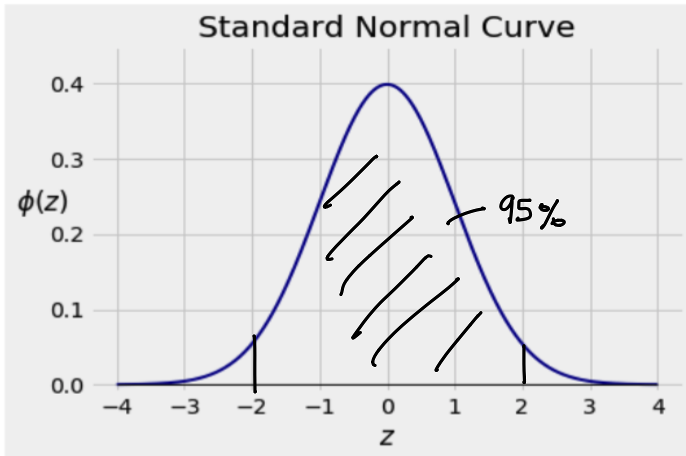
(e) Conclusion (5% level).

p-val > 5%

→ Do not reject H_0.

9.3. Confidence Intervals: Method

Preliminary The standard normal curve:



Confidence interval A confidence interval is an interval of estimates of a *fixed* but unknown parameter, based on data in a random sample.

Let $X_1, \ldots, X_n$ be i.i.d. with mean μ and SD σ. We know $\bar{X}$ is an unbised estimator of μ (i.e $E(\bar{X}) = \mu$), and $\mathrm{SD}(\bar{X}) = \sigma/\sqrt{n}$ is a measure of the average spread of $\bar{X}$.

If n is large, the Central Limit Theorem tells us that the distribution of $\bar{X}$ is roughly normal, so

$$P\left(-2 < \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < 2\right) \approx 0.95.$$

We rewrite this equation as follows:

$$P\left(\bar{X} - 2\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + 2\frac{\sigma}{\sqrt{n}}\right) \approx 0.95$$
$$\iff P\left(\mu \in \left(\bar{X} - 2\frac{\sigma}{\sqrt{n}}, \bar{X} + 2\frac{\sigma}{\sqrt{n}}\right)\right) \approx 0.95.$$

The *random* interval

$$\left(\bar{X} - 2\frac{\sigma}{\sqrt{n}}, \bar{X} + 2\frac{\sigma}{\sqrt{n}}\right)$$

is called an approximate 95% confidence interval for μ. It is a random interval because its endpoints depend on the sample mean $\bar{X}$ which is a random variable whose value varies across samples.

Interpretation: the chance that this *random interval* contains the *fixed parameter* is about 95%.

Example: (From warm up) A population distribution is known to have an SD of 20. You test if the population mean is equal to 50. The average of a sample of 64 observations is 55. What is your 95% confidence interval for the population mean?

Example: (Proportion of undecided voters) In a simple random sample of 400 voters in a state, 23% are undecided about which way they will vote. Find a 95% CI for the proportion of undecided voters in the state.

Confidence Level

In above problem, find 99.7% confidence interval.

To find 90% confidence interval,

So 90% CI is

$$\left(\bar{X} - \qquad \frac{\sigma}{\sqrt{n}}, \bar{X} + \qquad \frac{\sigma}{\sqrt{n}}\right).$$