Stat 88 Midterm Practice Questions

Spring 2021 Course Staff

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Cupcakes

There are 10 chocolate, 10 strawberry, and 10 vanilla cupcakes on a plate. Adalie, Nancy, and Emily take turns and each choose 5 cupcakes randomly from the plate without putting them back.

- Find the chance that Nancy takes at least 3 chocolate cupcakes.
- Find the expected number of chocolate cupcakes that Nancy takes.
- Find the chance that Emily takes 2 vanilla cupcakes given that Adalie takes 3 vanilla cupcakes and 2 chocolate cupcakes.

Geckos

Samarth, an avid reptile collector, visits the East Bay Vivarium in Berkeley to purchase some geckos. He buys n geckos, selecting at random without replacement from a population of size N with 20% green geckos, 30% blue geckos, 40% red geckos, and 10% white geckos. In the questions that follow, you may leave your answer in terms of N and n.

- What is the distribution of the number of green or white geckos in Samarth's sample?
- Assume n is an even number. What is the probability Samarth selects fewer than $\frac{n}{2}$ red geckos?

Now, you may assume $N >> n$ (that is, the population is much larger than the sample size) and each of Samarth's selections has no discernible effect on future choices.

- What is the expected number of colors that don't appear in Samarth's sample?
- Does the quantity in (c) increase or decrease as n increases? What about as N increases? Does this make sense?

GSIs and a Game of Chance

Shashank is playing a game of chance. In this game the player rolls a die 10 times. The player wins if there are 4 or more sixes in the 10 rolls.

- Find the chance that Shashank wins.

Other GSIs like this game and want to try their luck. Danny, David, Jamaracus, Adalie, Annie, Anton, Chris, Emily, Meghana, Nancy, Shashank and Shide – all 12 GSIs – try the game independently of each other, in that order.

- Find the expected number of GSIs who win.
- Find the chance that Nancy is the 4th GSI to win.

Statistical Stocks

3 friends Ophelia (O), Xinyi (X), and Aditya (A) are interested in investing in STAT88 stocks. On any given day, the probability that O, X, and A invest is $0.4, 0.2$, and 0.5 respectively. For all parts, you must show your work using mathematical calculations and/or Venn diagrams.

- Find or bound the probability that none of them invest.
- On Fridays, if O decides to invest, both X and A will not invest. In other words, if O invests on a Friday, they will be the only person out of the three who invests. With this information, find or bound the probability that X and A invest on a Friday.
- On Mondays, if A decides to invest, the probability that X invests as well increases to 0.4. With this information, find or bound the probability that all three friends invest on a Monday.

Tough Enemy

Shide is trying to kill a tough enemy in his video game. Being a senior player, Shide has many powerful weapons at his command. This enemy has two phases of attack; once it loses half its health, the second phase begins. The enemy can only be killed in the second phase.

Shide wields the master sword first. With clear focus and his mastery of combat skill, he hits the enemy 7 times and triggers the second phase. Shide changes his weapon to an ancient dagger whenever he reaches the second phase. Now,

- There's a 5% chance that Shide will deal critical damage and kill the enemy in one hit.
- There's a 15% chance that the enemy will dodge and use its magical spell: the enemy will recover to full health, forcing Shide to switch back to his master sword and hit 7 more times to trigger the second phase attack again.
- There's an 80% chance that Shide will be able to attack continuously without interruption, with an expected number of 8 hits until the enemy dies.

What is the expected number of hits for Shide to finish his enemy, including both phases? Round to the nearest integer.

Monopoly

Imagine that you are playing a game of Monopoly with 5 friends. Assume each of you has an equal chance of winning the game.

- Assume that you will stop playing once you win one game. What is the probability you will have to play at least ten games?
- You decide you will continue to play until you win three games. What is the probability you stop playing at exactly the 12th game?

Witchcraft: The Meeting

Suppose for the card game *Witchcraft: The Meeting* there is a 1/10 chance of finding a rare card in a booster pack, independent of all other packs (max one rare per pack). Each rare card could be a blue dragon or a red dragon card with probabilities 1/5 and 1/10 of appearing respectively.

Assume that Chris opens 100 packs of Witchcraft cards (a large number of packs indeed!). Find the following:

- Approximate the probability that Chris finds at least 2 blue dragon cards.
- Approximate the probability that Chris finds exactly 4 of any type of dragon card (blue or red).

Now assume that dragon cards (of either variety) sell for \$30 apiece, and that each booster costs exactly \$1.

- Approximate the probability that Chris (still buying 100 booster packs) gains money overall.

Zoning Zones

Chris has been hired by the city of Berkeley to assess housing costs. His goal is to decide if Berkeley should change its policies and allow developers to build more units in various neighborhoods. He takes a sample of 100 residents' incomes $X_1, X_2, \ldots, X_{100}$. Assume each resident is independent and identically distributed with $E(X_i) = \mu$.

- Always practical, Anton proposes that Chris' sampling of 100 households is a waste of money. He says that we only need one household for an unbiased estimator of the average income. Is he correct? Why or why not?
- Always clever, Chris responds that if we only need one estimator as Anton proposed, then why not take a sample of 100 and take the maximum? Will this statement disprove Anton? In other words, why is the maximum of 100 incomes not usually an unbiased estimator?
- Anton says, but of course it will be unbiased, under a certain condition. Under which condition will the previous statement be true? Is this a realistic/practical assumption?
- Suppose the average Berkeley income is \$50,000. Chris realizes that incomes in Berkeley are very diverse and creates a new strategy: he will sample a people from Northside, b from Southside, c from Downtown, and d from the Berkeley Hills, with respective neighborhood population average incomes of \$75,000, \$50,000, \$30,000, and \$100,000. Each neighborhood's incomes are i.i.d. within the neighborhood. Let a, b, c, d be any non-negative integers that sum to 100. How many people should Chris sample from each neighborhood to make an unbiased estimator?

Drawing Faces

Jamarcus and Danny are playing a game with 3 coins and a deck of cards. Jamarcus flips 3 coins; he counts the number of coins that show heads, then removes that many face cards from the deck. (If he sees 0 heads, then he leaves the deck as is. If he sees 1 head, then he removes 1 face card from the deck, and so on). Then, Danny draws 13 cards without replacement from the deck. He counts the number of face cards in his hand.

- Find the expected number of face cards in Danny's hand if Jamarcus removed 1 face card from the deck.
- Find the expected number of face cards in Danny's hand.

Mailman

There are n mailboxes and $2n$ letters to distribute. An irresponsible mailman randomly puts each letter in one of the mailboxes. Assume for each letter, the chance of ending up in each mailbox is the same and independent of the other letters.

- Find the chance that the first mailbox is empty.
- Assuming n is large, find an exponential approximation for the probability in part (1).
- Let X be the number of letters **not** in the first mailbox. What is the distribution of X?
- Again assume n is large. Using the Law of Small Numbers, find $P(X = n)$ and check how it relates to the probability you found in part (2)

Library Policies

In the year 2022, you, a successful graduate of Stat 88, are hired by Main Stacks (a library on campus) to conduct a survey regarding a new library policy. This policy will give priority to more senior students on campus. To get a fair representation of students' opinion, you ask undergraduate students around campus whether they are in favor of this new policy. Suppose there is an **equal** number of freshmen, sophomores, juniors, and seniors on campus. For a surveyee, the probability of he/she being in favor of this new policy is summarized below, where θ is a fixed, unknown population parameter. Assume all probabilities below are between 0 and 1. Your goal is to estimate θ.

Year	Probability of supporting the new policy
Freshman	$0.5 - 2\theta$
Sophomore	$0.5 - 1\theta$
Junior	0.5
Senior	$0.5 + 3\theta$

- Let I be the opinion of a random student you encounter on campus, where $I = 1$ if they support the new policy and $I = 0$ if they don't. Find $E[I]$.
- Suppose you decide to survey 100 freshmen, 100 sophomores, and 100 juniors because apparently many senior students don't go to campus any more. Let $\bar{X}$ be the proportion of students who are in favor of this policy from all 300 students in the sample. Use $\bar{X}$ to construct an unbiased estimator of θ.
- Suppose you are also a GBO leader and decide to ask freshmen in your group about their opinions regarding this new policy. You decide to ask N freshmen where N follows a $Uniform\{51, 52, ..., 99, 100\}$ distribution. Suppose Y of the N students support the policy. Use an expression of $\frac{Y}{N}$ to construct an unbiased estimator of θ.

Biased Coins

- A box has three coins. One has heads on both sides, another has tails on both sides, and the last is a fair coin. A coin is chosen at random and tossed. What is the probability that the chosen coin has heads on both sides given it landed heads?
- You have two fair coins and one coin with heads on both sides. You pick a coin at random and toss it twice. If it reads heads both times, what is the probability it also reads heads after a third toss?
- A bag contains one fair coin, two coins with heads on both sides, and three coins with tails on both sides. Each of the six coins is flipped, but the outcomes of five randomly-selected coins are hidden from you. If the outcome you see is heads, what is the probability that the fair coin (which may or may not be the coin that you see) landed heads?