

Section 11/9

1 Question 11.2

Let $\theta_1 < \theta_2$ and suppose X_1, X_2, \dots, X_n are i.i.d. uniform on the interval (θ_1, θ_2) . Let $\theta = \theta_2 - \theta_1$ be the length of the interval.

a) Let $M_1 = \min(X_1, X_2, \dots, X_n)$ be the sample minimum and $M_2 = \max(X_1, X_2, \dots, X_n)$ be the sample max. The statistic $T_1 = M_2 - M_1$ is called the range of the sample and is a natural estimator of θ . Without calculation, explain why T_1 is biased, and say whether it underestimates or overestimates θ .

b) Find the bias of T_1 and confirm that its sign is consistent with your answer to Part a. For large n , is the size of the bias large or small?

c) Use T_1 to construct an unbiased estimator of θ . Call the new estimator T_2 .

d) Compare $SD(T_1)$ and $SD(T_2)$. Which one is bigger? For large n , is it a lot bigger or just a bit bigger?