# Stat 88 Review Section 1/29

Bounds, Independence, Probability

January 2021

## 1 Problem 1

We are playing roulette (a traditional roulette wheel contains 18 red pockets, 18 black pockets, and 2 green pockets). I bet on red at roulette and you bet on black. Both bets are placed on the same wheel. What is the probability that
a) We both lose?
b) At least one of us wins?
c) At least one of us loses?

## 2 Problem 2

2 dice are rolled. Find the probabilities of the following events:
a) The maximum of the 2 numbers rolled is less than or equal to 2
b) The maximum of the 2 numbers rolled is less than or equal to 3
c) The maximum of the 2 numbers rolled is exactly equal to 3

## 3 Problem 3

Suppose there exists a 6-sided die with faces as shown in the following diagram. The faces showing 1 and 6 are square faces with side of length one unit, but the thickness of the shape is a length less than or equal to 1 1. So, each of the faces 2, 3, 4, and 5 is a rectangle instead of a square. This type of die can either land flat (1 or 6) or on its side (2, 3, 4, 5). Suppose that the thickness is such that the chance of landing flat is 1/2.

To see an image, go to tinyurl.com/stat88-dice

What is the probability of:
a) Rolling a number 3 or greater
b) Rolling a number 3 or greater given the decide rolled on its side.

# 4 Problem 4

Say we are creating sequences of 7 letters using the Hindi alphabet, which has 46 letters and does not distinguish between upper case and lower case. If the letters can be repeated, what is the chance of a sequence such that the first four letters are all different, and the last three are all the same as one of the first four letters.

# 5 Problem 5

Say we have 2 events, A and B, which are independent with P(A)=0.4, P(B)=0.5. Find the probability that exactly one of the events occurs.

Challenge: Using the fact that since A and B are independent, then P(AB) = P(A) P(B), try to show that A and $B^c$ are independent (aka P(A $B^c$) = $P(A)P(B^c)$)

# 6 Problem 6

Suppose that A, B and C are 3 events with probabilities 0.6, 0.53, and 0.25 respectively.
a) What is the smallest that P(A $\cap$ B) can be?
b) What is the smallest that P(A $\cap$ B $\cap$ C) can be?

# 7 Problem 7

Economists are making predictions about the financial health of three countries. For i = 1, 2, 3, let $G_i$ be the event that Country i has no recession next year. The intersection G = G1 $\cap$ G2 $\cap$ G3 is the event that all three countries have no recession next year. Another way to say this: G is the event that none of the countries has a recession next year.

a) Fill in the blank with one of the symbols $\leq$, =, and $\geq$ so that the resulting statement is always true, and explain your answer: P(G) (blank) P(G1).

b) Suppose the economists' predictions are all rosy: P(G1) = 0.9, P(G2) = 0.95, and P(G3) = 0.99. Fill in the blank with the best lower bound you can find, and justify your answer: Without making any further assumptions, we can say that P(G) $\geq$ .

Hint: Use complements. Think about what must happen if G does not occur. Can you find a bound on that?

The method you have discovered in this exercise is commonly used to find bounds on chances of intersections, for example the chance that several estimates based on a random sample all have acceptable accuracy.