Stat 88 Chapter 1 Exercise Solutions

Question 1

a. NET is the percent of people in each broad group

b. 0.66

c. 78.6%

d. 18 - 29 should have a smaller percent

e. Consistent with (d)

Question 2

a. $\frac{1}{3}$

b. $\frac{1}{2}$

c. False, $\frac{2}{3}$

Question 3

a. Not possible

b. $\frac{1}{2}$

c. The chance that the student got an A given that the student did not get a B is larger

Question 4

a. Not correct, too big

b. Not correct, too big

Question 5

Between 20% and 100%

Stat 88 Chapter 2 Exercise Solutions

Question 1

ii)

Question 2

a. $(\frac{8}{8})(\frac{6}{7})(\frac{4}{6})(\frac{2}{5})$

b. $(\frac{800}{800})(\frac{600}{799})(\frac{400}{798})(\frac{200}{797})$

c. i) $(\frac{8}{8})(\frac{6}{8})(\frac{4}{8})(\frac{2}{8})$

ii) $(\frac{800}{800})(\frac{600}{800})(\frac{400}{800})(\frac{200}{800})$

Question 3

a. $26 * 25 * 24 * 23 * 22 * 21$

b. $\frac{1}{26*25*24*23*22*21}$

c. $\frac{6*5*4*3*2*1-1}{26*25*24*23*22*21}$

Question 4

a. True

b. True

c. False

Question 5

a. $1 - (\frac{364}{365})^n$

b. $1 - (\frac{365}{365})(\frac{364}{365})...(\frac{365-(n-1)}{365})$

Question 6

$\frac{\frac{1}{2}*\frac{1}{3}}{\frac{1}{6}+\frac{1}{3}}$

Question 7

Option (iv): $\frac{2}{3}$

Question 8

a. $\frac{5}{100}$

b. $\frac{5}{100}$

c. $2 \cdot \frac{5}{100} \cdot \frac{95}{99}$

Question 9

Stat 88 Chapter 2 Exercise Solutions

a. 0.95

b. $\frac{(0.95*0.8)}{(0.95*0.8+0.2*0.85)}$

Question 10

a. $1 - (\frac{N-1}{N})^n$

b. $\frac{n}{N}$

Question 11

$\frac{(0.8*0.15)}{(0.85*0.2+0.8*0.15)}$

Question 12

a. $P(\text{evidence} \mid \text{one of 10,000 other possible suspects}) = \frac{1}{1000}$, not $P(\text{innocent} \mid \text{evidence})$

b. $\frac{1}{11}$

Stat 88 Chapter 3 Exercise Solutions

Question 1

a.

2	3	4	5	6	7	8	9	10	11	12
1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

b. $\sum_{i=1}^{5} \frac{i}{36}$

c. $P(5 \leq S \leq 9) = \frac{4}{36} + \frac{5}{36} + \frac{6}{36} + \frac{5}{36} + \frac{4}{36}$

Question 2

a. $P(R > 16) = \sum_{k=17}^{20} \binom{20}{k} (\frac{1}{2})^k (\frac{1}{2})^{20-k}$

b. $(1 - (part\, a))^3$

Question 3

a. $\binom{10}{4} (\frac{18}{38})^4 (\frac{20}{38})^6$

b. $\sum_{k=0}^{4} \binom{10}{k} (\frac{18}{38})^k (\frac{20}{38})^{10-k}$

c. $\binom{9}{2} (\frac{18}{38})^2 (\frac{20}{38})^7 (\frac{18}{38})$

d. $\sum_{k=0}^{4} \binom{10}{k} (\frac{18}{38})^k (\frac{20}{38})^{10-k}$

Question 4

Let X be the amount of money he makes. $P(X > 0) = \sum_{k=6}^{90} \binom{90}{k} (\frac{2}{38})^k (\frac{36}{38})^{90-k}$

Question 5

a. $\frac{\binom{4}{2}\binom{48}{11}}{\binom{52}{13}}$

b. $\frac{\binom{4}{3}\binom{48}{10}}{\binom{52}{13}} + \frac{\binom{4}{4}\binom{48}{9}}{\binom{52}{13}}$

c. $\frac{\binom{12}{6}\binom{40}{7}}{\binom{52}{13}}$

Question 6

a. $\frac{\binom{70}{10}\binom{130}{30}}{\binom{200}{40}}$

b. $\sum_{i=11}^{40} \frac{\binom{70}{i}\binom{130}{40-i}}{\binom{200}{40}}$

c. $\sum_{i=11}^{40} \frac{\binom{70}{i}\binom{130}{40-i}}{\binom{200}{40}}$

Question 7

$n > -1,000,000 \cdot log(0.5)$

$n > 693,148$

Stat 88 Chapter 3 Exercise Solutions

Question 8

a. $1 - (\frac{1}{4})^{15}$

b. $(\frac{1}{2})^{15}$

Question 9

a. No

b. Yes

Question 10

a. No

b. Yes

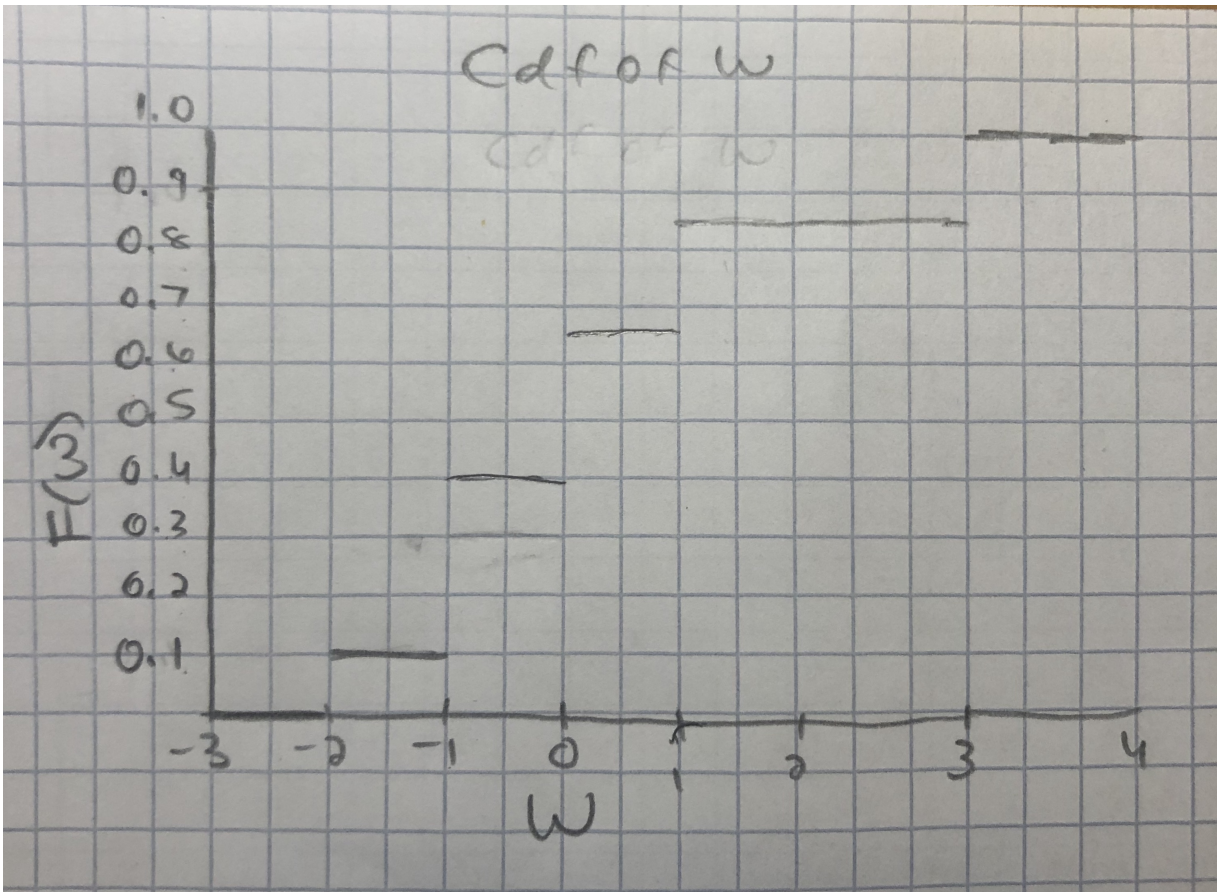
Stat 88 Chapter 4 Exercise Solutions

Question 1

a. $0.8^4 * 0.2$

b. 0.8^{16}

Question 2



Question 3

$0.7^9(0.3) + 0.3^9(0.7)$

Question 4

a. $0.8^4 + 0.2^4$

b. $\binom{5}{2}(0.8)^2(0.2)^3 + \binom{5}{3}(0.8)^3(0.2)^2$

c. $\binom{5}{3}(0.8)^3(0.2)^2(0.8)$

d. $\binom{3}{3}(0.8)^3(0.2)^0(0.8) + \binom{4}{3}(0.8)^3(0.2)(0.8) + \binom{5}{3}(0.8)^3(0.2)^2(0.8) + \binom{6}{3}(0.8)^3(0.2)^3(0.8)$

Question 5

a. $P(\text{3 aces dealt in 38 cards, 39th is an ace}) = \frac{\binom{4}{3}\binom{48}{35}}{\binom{52}{38}} * \frac{1}{14}$

Stat 88 Chapter 4 Exercise Solutions

b. $P(\text{at most 3 aces dealt in first 20 cards}) = \sum_{i=0}^{3} \frac{\binom{4}{i}\binom{48}{20-i}}{\binom{52}{20}}$

Question 6

$1 - e^{-1/2}$

Question 7

a. $1 - \sum_{i=0}^{2} e^{-2}\frac{2^k}{k!}$

b. $(e^{-2})^{20}$

c. $\binom{20}{2}\left(e^{-2}\frac{2^3}{3!}\right)^2\left(1 - e^{-2}\frac{2^3}{3!}\right)^{18}$

Question 8

a. $e^{-9}\frac{9^{12}}{12!}$

b. $1 - \sum_{k=0}^{12} e^{-9}\frac{9^k}{k!}$

c. $(1 - e^{-8}) * e^{-1}$

Question 9

F(x) = 0 if x < 6
F(x) = $1 - \sum_{k=0}^{5} \binom{x}{k}(\frac{1}{38})^k(\frac{37}{38})^{x-k}$ otherwise

Question 10

$\binom{24}{10} * (0.7)^{14} * 0.3^{10}$

Question 11

Let $p = 1 - \sum_{k=0}^{5} \frac{e^{-10}10^k}{k!}$
$1 - \sum_{j=0}^{90} \binom{100}{j}p^j(1-p)^{100-j}$

Question 12

a. $\frac{(\frac{e^{-4}*4^3}{3!})(\frac{e^{-4}*4^7}{7!})}{\frac{e^{-8}*8^{10}}{10!}}$

b. Binomial, 10, $\frac{1}{2}$

Stat 88 Chapter 5 Exercise Solutions

Question 1

a. -0.45

b. -1.45

c. $3(0.2) + 2(0.25) + 1(0.35) + 0(0.2) = 1.45$

d. $9(0.2) + 4(0.25) + 1(0.35) + 0(0.2) = 3.15$

Question 2

a. 17.5

b. $\sum_{k=11}^{15} \binom{15}{k} 0.5^k 0.5^{15-k}$

Question 3

a. Possible values: 1, 2, 3, 4, 5. Each of the probabilities is $\frac{1}{5}$

b. 3

c. No

d. $\frac{51}{2} + 1$

Question 4

a. $200\big(17(2/38) + (-1)(36/38)\big) \approx -10.53$ dollars

b. No

Question 5

a. No, both are Binomial(3, 0.5)

b. No

c.

	$X = 0$	$X = 1$	$X = 2$	$X = 3$
$Y = 0$	$\frac{1}{16}$	$\frac{1}{16}$	0	0
$Y = 1$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	0
$Y = 2$	0	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{1}{16}$
$Y = 3$	0	0	$\frac{1}{16}$	$\frac{1}{16}$

d. $\frac{8}{16}$

e. 0.5

f. Agrees

Question 6

a. 2

Stat 88 Chapter 5 Exercise Solutions

b. 6

c. $6 * (\frac{5}{6})^{12}$

d. 6− part c

Question 7

$70(0.4) - 30(0.4) = 16 = 2(70(0.4)) - 40$

Question 8

a. $\frac{1}{n}$

b. 1

c. B has the distribution $Poisson(1)$

d. Yes

Question 9

a. parameters: iii, v. statistics: i, ii, iv

b. unbiased estimators: i, ii, iii, v

Question 10

a. $2p - 1$

b. $2\frac{X}{100} - 1$

Question 11

$\frac{(\frac{(X_1+X_2+...+X_n)}{n}-2)}{-5}$

Question 12

a. Binomial$(3, 0.5)$, 1.5

b. $E(Y \mid X = 0) = 0.5$ $E(Y \mid X = 1) = 1.1667$ $E(Y \mid X = 2) = 1.8333$ $E(Y \mid X = 3) = 2.5$

c. 1.5

Question 13

1.2495

Question 14

a. 3

Stat 88 Chapter 5 Exercise Solutions

b. 30

c. $\frac{11}{5}$

Question 15

19.89 minutes

Stat 88 Chapter 6 Exercise Solutions

Question 1

$E(X) = 2.1, SD(X) = 0.7$

Question 2

$E(X) = 22, SD(X) = 2$

Question 3

a. $P(X = 10) = 1$

b. $P(X = -10) = 0.5, P(X = 30) = 0.5$

c. $P(X = 0) = \frac{4}{5}, P(X = 50) = \frac{1}{5}$

Question 4

a. Equal

b. $SD(X)$

Question 5

$SD(X) = \sqrt{5p + \frac{54}{4} - (3.5)^2}$

Question 6

a. $P(X \geq 80) \leq \frac{1}{2}$

b. $P(10 < X < 70) \geq 1 - (\frac{1}{2})^2$

c. $P(10 \leq X \leq 70) \geq 1 - (\frac{1}{2})^2$

Question 7

Best lower bound: 0, Best upper bound: 0.16

Question 8

$E(X) \geq 0.5m$

Question 9

$\frac{416}{1000}$

Question 10

a. Yes

Stat 88 Chapter 6 Exercise Solutions

b. Yes

c. Yes

Stat 88 Chapter 7 Exercise Solutions

Question 1

$E(V) = 75, Var(V) = 289, SD(V) = 17$

Question 2

$E(R) = 20, SD(R) = 3.65$

Question 3

a. $E(X) = -5.26, SD(X) = 40.19$

b. 43.13%

Question 4

a. $E(X_1) = 3.5, SD(X_1) = 1.71$

b. $SD(\bar{X}) = \frac{\sigma}{\sqrt{n}}$ where $\sigma = \sqrt{\frac{N^2-1}{12}}$

c. $\frac{2\sigma}{\sqrt{n}}$ where $\sigma = \sqrt{\frac{N^2-1}{12}}$

d. As n gets large, $SD(\hat{N})$ gets small, so the distribution of $\hat{N}$ gets more concentrated around its expectation

Question 5

$E(X) = 3.2, SD(X) = 1.78$

Question 6

a. $E(X+Y) = \mu + \lambda$, $SD(X+Y) = \sqrt{\mu + \lambda}$

b. Poisson$(\mu + \lambda)$

Question 7

$E(X) = 6, SD(X) = 2.45$

Question 8

a. $E(M_T) = 15, SD(M_T) = 1.75$

b. $E(M_C) = 15, SD(M_C) = 1.75$

c. False

Question 9

a. (ii)

Stat 88 Chapter 7 Exercise Solutions

b. (iii)

Question 10

No

Question 11

a. Binomial(1300, 0.95)

b. $E(X) = 1300 * 0.95, SD(X) = \sqrt{1300 * 0.95 * 0.05}$

c. $\sum_{k=1251}^{1300} \binom{1300}{k} * (0.95)^k (0.05)^{1300-k}$

Question 12

a. $E(U) < E(W) < E(C)$

b. $SD(C) = SD(U) < SD(W)$

Stat 88 Chapter 8 Exercise Solutions

Question 1

a. 8000, 424.26

b. Normal(8000, 424.26)

c. 40, 2.12

d. Normal(40, 2.12)

Question 2

0.106

Question 3

a. Right skewed

b. Resembles a

c. Same as a

d. Approximately normal with mean $70,000$ and SD $1,500$

Question 4

a. $60,000$

b. 1725.52

Question 5

68798.55

Question 6

a. Binomial(100, 0.5)

b. Normal, mean = 50, SD = 5

c. 68%

d. 0.729

e. $45, 55, 44.5, 55.5$

f. 0.729

Stat 88 Chapter 9 Exercise Solutions

Question 1

a. H_0: The unknown mean is 98.6. The temperatures $X_1, X_2, \ldots, X_{100}$ are i.i.d. with $\mu = E(X_1) = 98.6$

b. H_A: $\mu < 98.6$

c. $\bar{X} = \frac{1}{100}\sum_{i=1}^{100} X_i$. Small values support the alternative

d. By the CLT, the distribution of $\bar{X}$ is approximately normal; the mean depends on the hypothesis. $E_{H_0}(\bar{X}) = 98.6$. Under both hypotheses, $SD(\bar{X}) = \frac{\sigma}{\sqrt{100}} \approx \frac{1.5}{\sqrt{100}} = 0.15$.

The p-value is $P_{H_0}(\bar{X} < 98.2) \approx \Phi((98.2 - 98.6)/0.15)) \approx 0.004$.

e. $p < 5\%$, so the data support the alternative more than they support the null.

Question 2

a. H_0: The null hypothesis is Mendel's model. The plants are 1064 i.i.d. Bernoulli (0.25) random variables.

b. H_A: Mendel's model is not good.

c. Let X be the number of short plants in the sample. Under H_0, the distribution of X is binomial $(1064, 0.25)$ so $E_{H_0}(X) = 1064 \times 0.25 = 266$. So use $T = |X - 266|$ as the statistic. Large values of T favor the alternative. You could also use $|Y - 0.25|$ as the statistic where Y is the proportion of short plants in the sample. But you would have to convert to counts in the next part.

d. The observed value of T is $|277 - 266| = 11$. So the p-value is

$P_{H_0}(T \geq 11) = P_{H_0}(|X - 266| \geq 11) = P_{H_0}(X \leq 266 - 11) + P_{H_0}(X \geq 266 + 11) \approx 0.46$.

e. The data support Mendel's model. The p-value is substantial, and much bigger than 5%.

Question 3

a. Yes

b. No

c. Alternative

Question 4

a. False

b. False

Question 5

$X < 180$ or $X > 220$

Question 6

$[16.1828, 16.4572]$

Question 7

Stat 88 Chapter 9 Exercise Solutions

a. $[2.6455, 2.9545]$

b. False

c. $[7.793, 12.207]$

Question 8

a. Not possible

b. 20

c. $n = 625$

d. 17.37% to 22.63%

Question 9

H_0: The treatment did nothing.
H_A: The treatment did something, good or bad.
Let X be the number of test takers in the treatment group. Under H_0, X is hypergeometric $(200, 145, 95)$ and $E_{H_0}(X) = 95(145/200) = 68.875$.
Test statistic: $T = |X - 68.875|$. Large values favor the alternative.
Observed statistic: $|75 - 68.875| = 6.125$.
p-value: $P_{H_0}(X \leq 68.875 - 6.125) + P_{H_0}(X \geq 68.875 + 6.125)$
$= P_{H_0}(X \leq 62) + P_{H_0}(X \geq 75) = \sum_{g=0}^{62} \frac{\binom{145}{g}\binom{55}{95-g}}{\binom{200}{95}} + \sum_{g=75}^{95} \frac{\binom{145}{g}\binom{55}{95-g}}{\binom{200}{95}} \approx 28.5\%$
Not statistically significant. The data are consistent with H_0.

Question 10

a. 0.0143

b. Yes

Question 11

Let I_i be the indicator of the event that Robot i is faster after modification. Then $I_1, I_2, \ldots, I_{12}$ are i.i.d. Bernoulli (p) for some p.
H_0: $p = 0.5$
H_A: $p > 0.5$
Use the number of negative signs $X = \sum_{i=1}^{12} I_i$ as the statistic; large values favor the alternative. The data are consistent with the null.

Question 12

a. city

b. (iii) and (iv)

Stat 88 Chapter 9 Exercise Solutions

c. False

d. the average income of adults in the city, $40,000$

e. the average income of adults in the city, $2,000$

Stat 88 Chapter 10 Exercise Solutions

Question 1

a. $3 * 50^3$

b. $F(x) = 1 - \frac{50^3}{x^3}$, $x > 50$. $F(x) = 0$, $x \leq 50$.

c. 75

d. 1875

e. 87.6%

Question 2

a. 2.5 minutes after 3:07

b. $(\frac{3}{5})^7$

c. $F_X(x) = 0$ for $x \leq$ 3:07 and $F_X(x) = 1$ for $x \geq$ 3:12. For x between 3:07 and 3:12 write $x =$ 3:07 $+ t$. $F_X(x) = P(X \leq x)$ is the chance that all seven students arrive before time x. That's $(t/5)^7$.

Question 3

a. e^{-3}

b. e^{-2}

Question 4

$\frac{-log(\frac{1}{3})}{\frac{log(2)}{28.8}}$

Question 5

$w = 1.645 * \sqrt{20^2 * 5} + 750$

Question 6

159.15

Question 7

a. $((4 - 1.5) - 1.96 * \sqrt{(\frac{1.5^2}{300} + \frac{2^2}{200})}, (4 - 1.5) + 1.96 * \sqrt{(\frac{1.5^2}{300} + \frac{2^2}{200})})$

b. $((0.5 - 0.2) - 1.96 * \sqrt{(\frac{0.5*0.5}{300} + \frac{0.2*0.8}{200})}, (0.5 - 0.2) + 1.96 * \sqrt{(\frac{0.5*0.5}{300} + \frac{0.2*0.8}{200})})$

Question 8

Sample A is i.i.d. Bernoulli (p_A) and sample B is i.i.d. Bernoulli (p_B). The sample proportion X_A is approximately normal with mean p_A and variance $\frac{p_A q_A}{500}$. The sample proportion X_B is approximately normal with mean p_B and variance $\frac{p_B q_B}{700}$. X_A and X_B are independent.

Stat 88 Chapter 10 Exercise Solutions

H_0: $p_A = p_B = p$ in the above.
H_A: $p_A > p_B$.
Test statistic: $T = X_A - X_B$. Large values favor the alternative.
Under H_0 we have $p_A = p_B = p$ which is estimated from the data as $\hat{p} = 0.2 \cdot \frac{5}{12} + 0.16 \cdot \frac{7}{12} = 0.1767$.
$E_{H_0}(T) = 0$ and $Var_{H_0}(T) = \frac{\hat{p}(1-\hat{p})}{500} + \frac{\hat{p}(1-\hat{p})}{700}$ so $SD_{H_0}(T) \approx 2.3\%$.
z-score: $\frac{(0.04-0)}{0.023} = 1.739$
p-value: 0.041
The data support the alternative.

Question 9

0.346

Question 10

a. 0 to ∞

b. $F(v) = 0$ for $v \leq 0$, $1 - e^{-5v}$ for $v > 0$

c. Exponential(5)

Question 11

False

Question 12

The GPAs from Sample A $X_1, X_2, \ldots, X_{100}$ are i.i.d. with $E(X_1) = \mu_X$
The GPAs from Sample B $Y_1, Y_2, \ldots, Y_{150}$ are i.i.d. with $E(Y_1) = \mu_Y$
The X_is and Y_is are independent.
H_0: $\mu_X = \mu_Y = \mu$ in the above.
H_A: μ_X is not equal to μ_Y.
Test statistic: $T = \mid \bar{X} - \bar{Y} \mid$. Large values favor the alternative.
$E_{H_0}(T) = 0$ and $Var_{H_0}(T) \approx \frac{0.5^2}{100} + \frac{0.3^2}{150}$ so $SD_{H_0}(T) \approx 0.0557$.
z-score: $\frac{(0.2-0)}{0.0557} = 3.592$
p-value: 0.00033
The data support the alternative.

Stat 88 Chapter 11 Exercise Solutions

Question 1

The two are not the same. The bias of the estimator in the report is $\frac{-1}{n}$. The variances are the same.

Question 2

a. Underestimates θ

b. $\frac{-2}{n+1}\theta$. For large n the size of the bias is small.

c. $\frac{n+1}{n-1}T_1$

d. $SD(T_2) = \frac{n+1}{n-1}SD(T_1)$. $SD(T_2)$ is bigger by a factor of $\frac{n+1}{n-1} = 1 + \frac{2}{n-1}$. For large n, $SD(T_2)$ is not much bigger than $SD(T_1)$

Question 3

$\hat{Y} = \frac{E(XY)}{E(X^2)}X$

Question 4

$MSE(a^*) = E\big((Y - a^*X)^2\big) = E(Y^2) - 2a^*E(XY) + a^{*2}E(X^2) = E(Y^2) - \frac{(E(XY))^2}{E(X^2)}$ after plugging in a^* from the previous exercise.

The result follows because $MSE(a^*) \geq 0$.

Question 5

a. if $a > 0$ then $r(X, V) = 1$.

If $a < 0$ then $r(X, V) = -1$.

b. if $a > 0$ then $r(X, W) = r(X, Y)$.

If $a < 0$ then $r(X, W) = -r(X, Y)$.

Question 6

a. 1 or -1 depending on whether $r(X, Y) > 0$ or $r(X, Y) < 0$.

b. 0

Question 7

a. $r(X, Y)$

b. $\hat{Y^*} = rX^*$ where $r = r(X, Y)$

Question 8

a. 0.779

Stat 88 Chapter 11 Exercise Solutions

b. 0.678

Question 9

Let $r(X,Y) = r$ for short. $Var(\hat{Y}) = r^2\sigma_Y^2$ and $Var(D) = (1-r^2)\sigma_Y^2$. The sum of the two is $Var(Y)$.

Question 10

a. $MSE(c) = E\big((X-c)^2\big)$

b. $\hat{c} = \mu_X$

c. $MSE(\hat{c}) = \sigma_X^2$

Question 11

a. $E(D_X D_Y) = (-1)(1-\frac{2}{3})\frac{1}{3} + 0(0-\frac{2}{3})\frac{1}{3} + 1(1-\frac{2}{3})\frac{1}{3} = 0$. Thus $r(X,Y) = \frac{1}{\sigma_X\sigma_Y}E(D_X D_Y) = 0$.

b. No