Stat 88 lec 23

Warm up 2:00-2:10

exercise 7.4.5

5. The number of typos on the cover page of an exam has a distribution given by

value	0	1
probability	0.8	0.2

The number of misprints in the rest of the exam has the Poisson (3) distribution, independently of the cover page.

Find the expectation and SD of the total number of misprints on the exam.

$SD(\text{Bernoulli}(p)) = \sqrt{pq}$

$SD(\text{Poisson}(\mu)) = \sqrt{\mu}$

X_1 = # misprints cover

X_2 = # misprints exam

$X_1 \sim$ Bernoulli (.2)

$X_2 \sim$ Poisson (3)

independent

$$E(X_1+X_2) = E(X_1) + E(X_2)$$

$$= .2 + 3 = \boxed{3.2}$$

$$Var(X_1+X_2) = \underbrace{Var(X_1)}_{=(.2)(.8)} + \underbrace{Var(X_2)}_{=3} = \boxed{3.16}$$

$$SD(X_1+X_2) = \sqrt{3.16}$$

Last time

Announcement: HW#7 due Wednesday.

Sec 7.1 Sums of independent random variables

If X_1, X_2 are independent RVs

$$Var(X_1+X_2) = Var(X_1) + Var(X_2)$$

$X \sim$ Bernoulli (p)

$SD(X) = \sqrt{pq}$

$X \sim$ Binomial (n, p)

$SD(X) = \sqrt{npq}$

$X \sim$ Poisson (μ)

$SD(X) = \sqrt{\mu}$

Today

1. review correct test last time
2. Sec 7.1 Sums of indep. Geometric RVs
3. Sec 7.2 Sampling without replacement

① Concept test last time

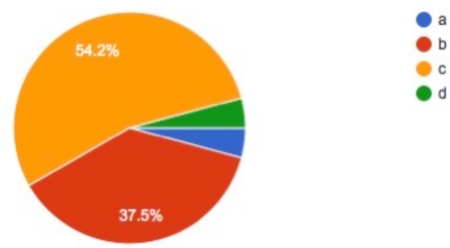
A list of non negative numbers has an average of 1 and an SD of 2. Let p be the proportion of numbers ~~over~~ at least 5. To get an upper bound for p, you should:

a Assume a binomial distribution

b Use Markov's inequality — w/ correction

c Use Chebyshev's inequality

d none of the above



M: $P(X \geq \mu + k\sigma) \leq \dfrac{E(X)}{\mu + k\sigma}$

C: $P(X \geq \mu + k\sigma) \leq \dfrac{1}{k^2}$

$X \geq 0$

$E(X) = 1$

$SD(X) = 2$

M: $p = P(X \geq \cancel{6}\,5) \leq \cancel{\dfrac{1}{6} = \dfrac{4}{24}}\ \dfrac{1}{5}$ ← better

C: $p = P(X \geq \cancel{6}\,5) \leq \cancel{\dfrac{1}{\left(\frac{5}{2}\right)^2} = \dfrac{4}{25}}\ \dfrac{1}{4}$ ← better

$6 = 1 + k \cdot 2$

$2k = 5$

$\boxed{k = 5/2}$

$5 = 1 + k \cdot 2$

$4 = k \cdot 2$

$\boxed{k = 2}$

2 Sec 7.1 <u>Sums of independent RVs</u>

recall X = # trials till a success (prob p of success) is Geometric(p).

$E(X) = \frac{1}{p}$

SD of Geometric

<u>Fact</u> If $X \sim$ Geometric(p)

$$\boxed{SD(X) = \frac{\sqrt{q}}{p}}$$

ex waiting till the 10^{th} success

Suppose you roll a die until the 10^{th} six. Let R be the number of rolls required.

Find $SD(R)$

$$\underbrace{\underbrace{\times\ \times\ \checkmark}_{X_1=3}\quad \underbrace{\times\ \times\ \checkmark}_{X_2=3}}_{\text{indep}}\ \_\ \text{etc}$$

$R = X_1 + X_2 + \cdots + X_{10}$ $\quad X_i \sim \text{Geometric}\left(\frac{1}{6}\right)$

$Var(R) = Var(X_1) + \cdots + Var(X_{10})$

$= 10 \frac{q}{p^2} = 10 \frac{\left(\frac{5}{6}\right)}{\left(\frac{1}{6}\right)^2} = 300$

$\boxed{SD(R) = \sqrt{300}}$

$\sqrt{\frac{q}{p^2} + \frac{q}{p^2}} \neq \sqrt{\frac{q}{p^2}} + \sqrt{\frac{q}{p^2}}$

ex (binomial)

10. A non-negative integer valued random variable has expectation 50 and SD 10. Could the random variable have a binomial distribution?

$E(X) = 50$

$SD(X) = 10$

if $X \sim \text{Binomial}(n, p)$ then $E(X) = np$, $SD(X) = \sqrt{npq}$

$50 = np$

$10 = \sqrt{npq}$

$100 = \underbrace{np}_{50} q \Rightarrow q = 2$ ← not possible