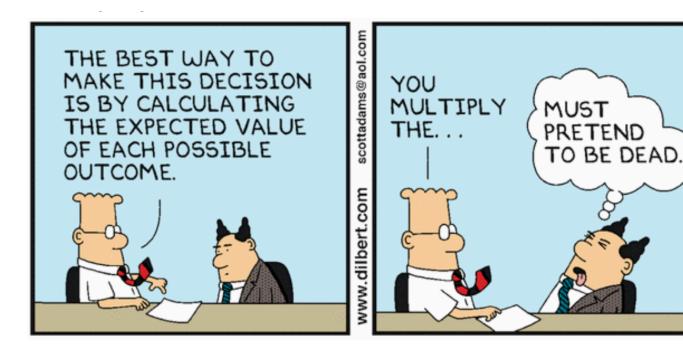
Stat 88: Probability & Mathematical Statistics in Data Science





Lecture 13: 2/19/2021
Functions of rvs, Method of indicators

Sections 5.2, 5.3

Joint distributions

- Draw two tickets without replacement from a box that has 5 tickets marked 1, 2, 2, 3, 3. Let X_1 and X_2 represent the values of the tickets drawn on the first and second draws respectively.
- Create a table of all possible outcomes for the pair $(X_{1.}, X_2)$ (which is also a random variable), and write down the probabilities using the multiplication rule.

	$X_2 = 1$	$X_2 = 2$	$X_2 = 3$
$X_1 = 1$			
$X_1 = 2$			
$X_1 = 3$			

Marginal distributions

- What is $P(X_1 = 1)$? Write down the pmf for X_1 and X_2
- Are they independent?
- Use the table to compute $P(X_1 + X_2 = 5)$
- Use the table to compute $E(g(X_1, X_2))$, where $g(X_1, X_2) = |X_1 X_2|$ (the expected distance between the two draws)

• If $S = X_1 + X_2$, find E(S).

Method of indicators

• Additivity of Expectation: This is a very useful property - no matter what the joint distribution of X and Y may be, we have:

$$E(X + Y) = E(X) + E(Y)$$

- Whether X and Y are dependent or independent, this holds, making it enormously useful.
- We also have linearity: E(aX + bY) = aE(X) + bE(Y)
- Recall that we talked about "classifying and counting" so, we divide up the outcomes into those that we are interested in (successes), and everything else (failures), and then count the number of successes.
- We can represent these outcomes as 0 and 1, where 1 marks a success and 0 and failure, so if we model the trials as draws from a box, we can count the number of success by counting up the number of times we drew a 1.
- We can represent each draw as a Bernoulli trial, where p = P(S)

Using indicators and additivity

- For example, say we roll a die 10 times, and success is rolling a 1.
- Then p=1/6, and we can define a Bernoulli rv as $X = \begin{cases} 0, & w.p.5/6 \\ 1, & w.p.1/6 \end{cases}$
- We can also define an event A: let A be the event of rolling a 1 and define a rv I_A that takes the value 1 if A occurs and 0 otherwise.
- This is a Bernoulli rv, what is its expectation?

• Now let $X \sim Bin(10, \frac{1}{6})$, so X counts the number of successes in 10 rolls. Let's find E(X) using additivity and indicators:

Using indicators

Binomial

 Hypergeometric: Did we use the independence of the trials for the binomial? If not, we can use the same method to compute the expected value of a hypergeometric rv:

Exercise 5.7.6: A die is rolled 12 times. Find the expectation of:

- a) the number of times the face with five spots appears
- b) the number of times an odd number of spots appears
- c) the number of faces that don't appear
- d) the number of faces that do appear

Example

- Let X be the number of spades in 7 cards dealt with replacement from a well shuffled deck of 52 cards containing 13 spades. Find E(X).
- 1. Write down what *X* is
- 2. Define an indicator for the kth trial: I_k
- 3. Find $p = P(I_k = 1)$
- 4. Write X as a sum of indicators
- 5. Now compute E(X) using additivity
- Do the same thing if we deal 7 cards without replacement.

Missing classes

• We can use indicators to compute the chance that something doesn't occur.

• For example, say we have a box with balls that are red, white, or blue, with 35% being red, 30% being white, and 35% blue. If we draw n times with replacement from this box, what is the expected number of colors that don't appear in the sample?

2/16/21

Examples

1. An instructor is trying to set up office hours during RRR week. On one day there are 8 available slots: 10-11, 11-noon, noon-1, 1-2, 2-3, 3-4, 4-5, and 5-6. There are 6 GSIs, each of whom picks one slot. Suppose the GSIs pick the slots at random, independently of each other. Find the expected number of slots that no GSI picks.

2. A building has 10 floors above the basement. If 12 people get into an elevator at the basement, and each chooses a floor at random to get out, independently of the others, at how many floors do you expect the elevator to make a stop to let out one or more of these 12 people?