Stat 88 Midterm Practice Solutions

Question 1 (Pitman 3.4.1)

a. Binomilal Prob: $\binom{9}{5}p^5(1-p)^4$

b. $P(\text{first 6 tosses tails, 7th head}) = (1-p)^6 p$

c. $P(\text{exactly 4 heads among 1st 11 tosses and 12th toss is head}) = \binom{11}{4}p^4(1-p)^7 p$

d. $\sum_{k=0}^{5} P(\text{k heads among 1st 8 tosses and k among next 5}) = \sum_{k=0}^{5} \binom{8}{k}p^k(1-p)^{8-k} \cdot \binom{5}{k}p^k(1-p)^{5-k}$

Question 2(Pitman 3.5.12) The total number of particles reaching the counter is the sum of two independent Poisson random variables, one with parameter 3.87, the other with parameter 5.41 (these numbers come from a famous experiment by Rutherford, Chadwick, and Ellis, in the 1920's). So the total number of particles reaching the counter follows the Poisson (9.28) distribution. So the required probability is the chance that a Poisson(9.28) variable is at most 4, which is $\sum_{k=0}^{4} e^{-9.28}\frac{9.28^k}{k!}$

Question 3 (Pitman 1.6.6)

a. $p_1 = 0$
$p_2 = \frac{1}{6}$
$p_3 = \frac{5}{6} \cdot \frac{2}{6}$
$p_4 = \frac{5}{6} \cdot \frac{4}{6}\frac{2}{6}$
$p_5 = \frac{5}{6} \cdot \frac{4}{6}\frac{3}{6} \cdot \frac{4}{6}$
$p_6 = \frac{5}{6} \cdot \frac{4}{6}\frac{3}{6} \cdot \frac{2}{6}\frac{5}{6}$
$p_7 = \frac{5}{6} \cdot \frac{4}{6}\frac{3}{6} \cdot \frac{2}{6}\frac{5}{6} \cdot 1$
$p_8 = p_9 = ... = 0$

b. $p_1 + ... + p_{10} = 1$ you must stop before the tenth roll, and the events determining p_1, p_2 , etc., are mutually exclusive

c. Of course you can compute them and add them up. Here's another way. In general, let A_i be the event that the first i rolls are different, then $p_i = P(A_{i-1}) - P(A_i)$ for $i = 2, ..., 7$, with $P(A_1) = l$, and $P(A_7) = 0$. Adding them up, you can easily check that the sum is 1.

Question 4 (Pitman 3.2.6)
$X = I_1 + I_2 + \; + I_r$ where I_i is an indicator random variable indicating whether the ith card is a spade. Then $E(X) = 7P(\text{first card is a spade}) = \frac{7}{4}$

Question 5 (Pitman 1.6.4)

a.
$$\frac{1}{20} \cdot \frac{9}{20} \cdot \frac{1}{20} = \frac{9}{8000}$$

b.
$$(\frac{1}{20} \cdot \frac{9}{20} \cdot \frac{19}{20}) + (\frac{1}{20} \cdot \frac{11}{20} \cdot \frac{1}{20}) + (\frac{19}{20} \cdot \frac{9}{20} \cdot \frac{1}{20} = \frac{353}{8000}$$

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c.

$$P(jackbpot) = \frac{3}{20} \cdot \frac{1}{20} \cdot \frac{3}{20} \cdot = \frac{9}{8000}$$

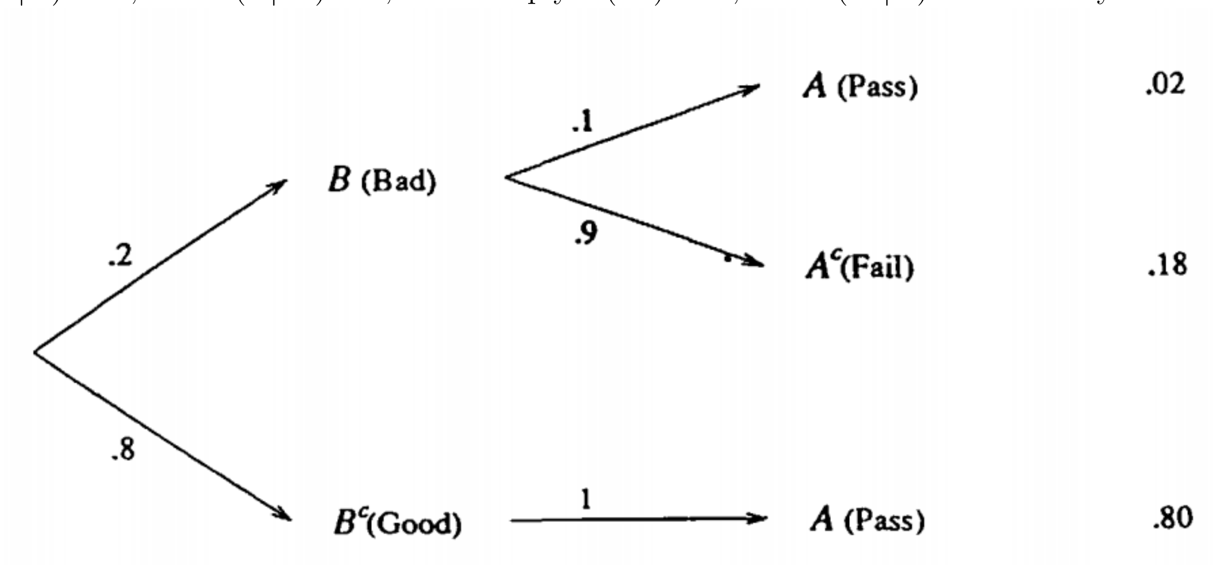
same as before

$$P(twobells) = (\frac{3}{20} \cdot \frac{1}{20} \cdot \frac{17}{20}) + (\frac{3}{20} \cdot \frac{19}{20} \cdot \frac{3}{20}) + (\frac{17}{20} \cdot \frac{1}{20} \cdot \frac{3}{20} = \frac{273}{8000}$$

The chance of the jackpot is the same on both machines, but the 1-9-1 machine encourages you to play, because you have a better chance of two bells. It will seem that you are "dose" to a jackpot more frequently.

Question 6 (Pitman 1.5.3) *Do not use complement notation with students in office hours! Replace with english versions*

Pick at chip at random. Let B = (chip is bad), let A= (chip passes the cheap test). Then $P(B) = .2$, $P(A|B) = .1$, and $P(A|B^c) = 1$, which imply $P(B^c) = .8$, and $P(A^c|B) = .9$. Thus your tree diagram becomes:



a.

$$P(Good|Pass) = \frac{P(Pass|Good)P(Good)}{P(Pass|Good)P(Good) + P(Pass|Bad)P(Bad)} = \frac{0.8}{0.02 * 0.8} = \frac{40}{41}$$

b.

$$P(Bad|Pass) = 1 - P(Good|Pass) = \frac{1}{41}$$

Question 7 (Pitman 2.5.3)

a. P(first player holds all aces) $= \frac{\binom{4}{4}\binom{48}{9}}{\binom{52}{13}}$

b. P(first player holds all the aces given that she holds the ace of hearts) means there is one less card from the 52 to choose from, and there are only 3 aces to choose from, and we want her to have all 3 of the remaining aces. This occurs with probability $\frac{\binom{3}{3}\binom{48}{9}}{\binom{51}{12}}$ (don't need to multiply by the chance that she gets the ace of hearts since its given)

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c.

$$P(\text{first player holds all the aces given that she holds at least one}) = \frac{P(\text{holds all aces})}{P(\text{at least one ace})}$$

Breaking down the denominator: the number of ways you can draw 13 cards and get at least one ace plus the number of ways you can draw 13 cards and get no aces must equal the number of ways you draw 13 cards from 52 (partitioning the outcome space into its possibilities). Thus $P(\text{at least one ace}) = P(\text{draw 13 cards from 52}) - P(\text{draw 13 cards from the 48 non-ace cards})$. The numerator is just part a. Thus

$$P(\text{first player holds all given holds at least one}) = \frac{\binom{4}{4}\binom{48}{9}}{\binom{52}{13} - \binom{48}{13}}$$

d. $P(\text{second player holds all the aces given that he holds all the hearts})$ is zero since there are 13 heart cards in the deck, and since the player holds 13 cards, the player holds **only** heart cards. There is an ace for each suite, so if he held say an ace of spades, that would violate the given that he holds only ace cards.

Question 8 (Pitman 1.4.8) Assume n cards and all $2n$ faces are equally likely to show on top.

$$P(\text{white on bottom I black on top})$$

$$= \frac{P(\text{white on bottom and black on top})}{P(\text{black on top})} = \frac{0.5 \cdot 1/2}{0.5 \cdot 1/2 + 0.2} = \frac{5}{9}$$

Question 9 (Pitman 1.3.6)

a. The length of the word can be 1, 2, 4, 6, 7, 8 (count yourself), and counting the number of words gives the distribution table

outcome	1	2	4	6	7	8
probability	1/10	1/5	3/10	1/5	1/10	1/10

b. The possible number of vowels is 1, 2, 3 and counting the possibilities leads to the distribution table

outcome	1	2	3
probability	3/5	1/5	1/5

Question 10(Pitman 3.1.9)

x	2	3	4	5
$P(X = x)$	5/35	10/35	12/35	8/35

The possible values are $2, 3, 4$, and 5 because we need at least 2 cards to get the same card twice. The maximum is 5 because once we've drawn 4 without a repeat, we've gotten one of each card and there are only two of each kind, so the next card must be one we've seen before.

To find $P(X = 2)$, we can get any card in the first draw, then we must get the one card that matches it. At that point, there are 7 cards and 1 we want, so we get $\frac{1}{7}$.

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To find $P(X = 3)$, we can get any card in the first draw, we must draw a card that does not match the card in the first draw ($\frac{6}{7}$), and then draw a card that matches either the first card or the 2nd card ($\frac{2}{6}$). Multiplying these probabilities together, we get $\frac{2}{7}$

To find $P(X = 4)$, we can get any card in the first draw, we must draw a card that does not match the card in the first draw ($\frac{6}{7}$), draw a card that does not match the card in the first or second draw ($\frac{4}{6}$), and then draw a card that matches either the first card, 2nd or 3rd card ($\frac{3}{5}$). Multiplying these probabilities together, we get $\frac{12}{35}$

To find $P(X = 5)$, we can get any card in the first draw, we must draw a card that does not match the card in the first draw ($\frac{6}{7}$), draw a card that does not match the card in the first or second draw ($\frac{4}{6}$), draw a card that does not match the card in the first, second, or third draw ($\frac{2}{5}$) and then draw a card that matches either the first card, 2nd or 3rd card (1). Multiplying these probabilities together, we get $\frac{8}{35}$

Question 11(Pitman 3.5.15)

a. The chance that a given pages has no mistakes is $e^{-0.01}\frac{0.01^0}{0!} = e^{-0.01} = .99005$ and thus the expected number of pages with no mistakes is $200 \times .99005 = 198.01$

b. The mistakes found on a given page is distributed as a Poisson(.009), so the chance that at least one mistake will be found on a given page is $1 - (e^{-0.009}\frac{0.009^0}{0!} = .00896$. The expected number of pages on which at least one mistake is found is then $200 \times .00896 = 1.79$.

c. The number of pages with mistakes can be well approximated as a Poisson(l.99) since 1.99 is the expected number of pages with mistakes. Let $X =$ number of pages with mistakes and use the Poisson approximation to get $P(X \geq 2) = 1 - (P(X = 0) + P(X = 1)) = 1 - (e^{-1.99} + 1.99e^{-1.99}) = 0.59$

Question 12((Pitman 3.2.7)
$E(X) = \sum_{i=1}^{n} p_i$, by linearity of E. No more assumptions required.

Question 13 (Pitman 1.4.10)

a. *Do not use complement notation with students in office hours! Replace with english versions*
Let A_i be the event that the ith source works.

$$P(\text{zero work}) = P(A_1^c A_2^c) = 0.6x0.5 = 0.3$$
$$P(\text{exactly one works}) = P(A_1 A_2^c) + P(A_1^c A)2) = 0.4x0.5 + 0.6x0.5 = 0.5$$
$$P(bothwork) = P(A_1 A_2) = 0.4x0.5 = 0.2$$

b. $P(enoughpower) = 0.6 \times 0.5 + l \times 0.2 = 0.5$

Question 14 (Pitman 2.1.4)

$$P(\text{2 sixes in first 5 rolls — 3 sixes in all 8 rolls})$$

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$$= \frac{P(\text{2 sixes in 1st 5, and 3 sixes in all 8})}{P(\text{3 sixes in all 8})}$$

$$= \frac{P(\text{2 sixes in 1st 5, and 1 six in next 3})}{P(\text{3 sixes in all 8})}$$

$$= \frac{\binom{5}{2}(1/6)^2(5/6)^3 \cdot \binom{3}{1}(1/6)^1(5/6)^2}{\binom{8}{3}(1/6)^3(5/6)^5} = \frac{\binom{5}{2}\binom{3}{1}}{\binom{8}{3}} = 0.535714$$

Question 15 (Pitman 3.rev.7)

a. $P(\text{more than 4 heads in 5 tosses}) = \frac{6}{32} = 0.1875$

b. $P(\text{0, 1, or 2 heads in 5 tosses}) = \frac{16}{32} = 0.5$

c. $\frac{7}{32}$

Let us consider the possible values of the number of people turned away. Since we have 10 seats and 12 people, the number of possible people turned away are 0, 1, or 2. We do not need to calculate the probability that 0 people are turned away because this probability does not matter in our calculation of the expectation.

To calculate the chance that 1 person is turned away, this means that 4 people out of the 5 who come with probability 0.5 must show up. This is the probability that a Binomial$(5, 0.5)$ takes on the value 4. This probability is equal to $\frac{5}{32}$.

To calculate the chance that 2 people are turned away, this means that 5 people out of the 5 who come with probability 0.5 must show up. This is the probability that a Binomial$(5, 0.5)$ takes on the value 5. This probability is equal to $\frac{1}{32}$.

Using our typical expected value formula, we get $1 * \frac{5}{32} + 2 * \frac{1}{32} = \frac{7}{32}$