# A Bayesian Approach on Chess Openings

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### **Abbreviated abstract:**

Chess grandmasters prepare the opening in depth. It is intriguing that even with the help of engines, they failed to reach a consensus on which the strongest opening is. Statistics have been covered to compare the strength of each opening, yet most of the approaches are from the frequentist's point of view. In this study, a Bayesian approach is suggested as an alternative.

## **Related publications:**

- Lee, J. (2021). Chess Data Analysis: A Bayesian Approach on Opening Tier. [QR]
- Gelman, A. (2004). Bayesian data analysis. Boca Raton, Fla: Chapman & Hall/CRC.





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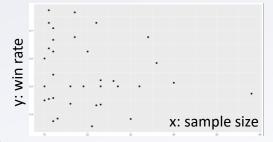
## The Limitations of Frequentist Approach

The three phases of chess:

1. **opening** 2. middle game 3. endgame statistics computation

- Frequentist approach:
  - Build a multinomial model with three possible outcomes: win, draw or loss.
  - Count the numbers of W/D/L for each opening to estimate the true values of parameters. (MLE)
- Data: lichess.org game data (better use GM-level games next time (x))

Best 2 Openings for White	Worst 2 Openings for White					
B32 Open Sicilian (0.77)	C45 Scotch Game (0.36)					
B30 Old Sicilian (0.76)	C50 Giuoco Piano (0.38)					



- Problem:
  - Extreme win rates might be simply due to small sample sizes. ↗
  - The legitimacy of an opening that hasn't been played by stronger players is doubted.
    - → Try the Bayesian approach instead!



## Bayesian Multinomial Model

## **Bayesian trinomial model:**

$$Prior: \theta \sim Dirichlet(\alpha, \beta, \gamma)$$
  
  $Likelihood: Y | \theta \sim Trinomial(\theta)$   
  $Posterior: \theta | y \sim Dirichlet(\alpha + w, \beta + d, \gamma + l)$ 

ECO	FEN	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10
A00	$rnbqkbnr/pppppppp/8/8/1P6/8/P1PPPPPP/RNBQKBNR \dots \\$	1/2-1/2	1/2-1/2	0-1	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2
A01	$rnbqkbnr/pppppppp/8/8/8/1P6/P1PPPPPP/RNBQKBNR \dots \\$	0-1	1/2-1/2	0-1	1/2-1/2	1/2-1/2	1/2-1/2	0-1	1/2-1/2	1/2-1/2	1/2-1/2
A04	$rnbqkbnr/pppppppp/8/8/5P2/8/PPPP1PP/RNBQKBNR \dots \\$	0-1	0-1	1/2-1/2	1/2-1/2	1-0	1/2-1/2	1/2-1/2	0-1	1/2-1/2	1/2-1/2
A15	rnbqkb1r/pppppppp/5n2/8/2P5/8/PP1PPPPP/RNBQKBN	1/2-1/2	1/2-1/2	1/2-1/2	1-0	1/2-1/2	1-0	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2
A16	rnbqkb1r/pppppppp/5n2/8/2P5/2N5/PP1PPPPP/R1BQK	1/2-1/2	0-1	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2	1-0
A40	rnbqkbnr/pppp1ppp/8/4p3/3P4/8/PPP1PPPP/RNBQKBN	1-0	1-0	1-0	1-0	1-0	1-0	1-0	0-1	1-0	1-0
A45	rnbqkb1r/pppppppp/5n2/8/3P4/8/PPP1PPPP/RNBQKBN	1/2-1/2	1/2-1/2	1-0	1/2-1/2	1/2-1/2	1/2-1/2	1-0	1-0	1/2-1/2	1/2-1/2
A46	rnbqkb1r/pppppppp/5n2/8/3P4/5N2/PPP1PPPP/RNBQK	1/2-1/2	0-1	1/2-1/2	1-0	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2	1-0	1/2-1/2
A80	rnbqkbnr/ppppp1pp/8/5p2/3P4/8/PPP1PPPP/RNBQKBN	1-0	1/2-1/2	1/2-1/2	1/2-1/2	0-1	1/2-1/2	1/2-1/2	0-1	1/2-1/2	1-0
B00	rnbqkbnr/ppppp1pp/5p2/8/4P3/8/PPPP1PPP/RNBQKBN	1-0	1-0	0-1	1/2-1/2	1-0	1/2-1/2	1-0	1/2-1/2	1-0	1-0
B01	rnbqkbnr/ppp1pppp/8/3p4/4P3/8/PPPP1PPP/RNBQKBN	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2	1-0	1/2-1/2	1-0	1/2-1/2	1-0	1-0
	A00 A01 A04 A15 A16 A40 A45 A46 A80 B00	A00 mbokbn/ppppppppppβ/8/8/1P6/8/P1PPPPP/RNBGKBNR   A01 mbokbn/ppppppppβ/8/8/1P6/9/1PPPPPP/RNBGKBNR   A04 mbokbn/r/pppppppp/8/8/8/1P6/P1PPPPPPRNBGKBNR   A05 mbokb1/r/ppppppp5/8/8/2P5/2P/8/PPPPPPPPRNBGKBNR   A15 mbokb1/r/ppppppp5/8/28/2P5/2N5/PP1PPPPPRNBGKBN   A40 mbokb1/r/ppppp1pp/8/3/33/44/8/PP1*PPPP/RNBGKBN   A40 mbokb1/r/ppppppp5/8/28/34/48/PP1*PPPP/RNBGKBN   A46 mbokb1/r/pppppp5/8/28/34/48/PP1*PPPPPRNBGKBN   A46 mbokb1/r/ppppp1pp/8/28/3/48/PPP1*PPPP/RNBGKBN   A80 mbokb1/r/ppppp1pp/8/28/32/34/48/PPP1*PPPPPRNBGKBN   B00 mbokbn/r/pppp1pp/8/28/4P9/3/8/PPP1*PPPPPRNBGKBN	A00 mbokbnr/pppppppββ/8/HP6/8/HPPPPPP/RNBQKBNR 1/2-1/2   A01 rnbokbnr/pppppppβ/8/8/HP6/HPPPPPPRNBQKBNR 0-1   A04 mbokbnr/pppppppβ/8/8/B/P2/8/PPPPPPPPRNBQKBNR 0-1   A15 mbokbnr/pppppppβ/8/8/B/P2/8/PPPPPPPRNBQKBNR 1/2-1/2   A16 mbokbnr/ppppppβ/8/18/18/PPF1PPPPPRNBQKBN 1/2-1/2   A40 mbokbnr/pppp1ppβ/8/14/3/3P4/8/PPF1PPPPPRNBQKBN 1/2-1/2   A41 mbokbnr/ppppp1ppβ/8/12/8/2P4/8/PPF1PPPPPRNBQKBN 1/2-1/2   A45 mbokbnr/ppppp1ppβ/8/15/2/8/2P4/8/PPF1PPPPRNBQKBN 1/2-1/2   A46 mbokbnr/ppppp1pp/8/15/2/8/2P4/8/PPF1PPPPPRNBQKBN 1-0   B00 mbokbnr/ppppp1pp/8/5p2/3/4/4/3/8/PPPF1PPPPRNBQKBN 1-0   B00 mbokbnr/ppppp1pp/8/5p2/8/4P3/8/PPPFPPPPPPPRNBQKBN 1-0	A00 mbqkbnr/ppppppppp/8/8/1P6/9/P1PPPPPPRNBGKBNR 1/2-1/2 1/2-1/2   A01 rnbqkbnr/pppppppgp/8/8/1P6/P1PPPPPPPPRNBGKBNR 0-1 1/2-1/2   A04 rnbqkbnr/pppppppgp/8/8/8/1P6/P1PPPPPPPPPRNBGKBNR 0-1 0-1   A15 rnbqkb1r/pppppppp/5/2/8/2P5/9/PPPPPPPPPPPRNBGKBN 1/2-1/2 1/2-1/2   A16 rnbqkb1r/ppppppp/5/2/8/2P5/2P5/PPPPPPPPPRNBGKBN 1/2-1/2 0-1   A40 rnbqkb1r/ppppppp/5/2/8/3P3/3P4/8/PPP1PPPPPRNBGKBN 1/2-1/2 0-1   A45 rnbqkb1r/ppppppp/5/2/8/3P3/3P4/8/PPP1PPPPPRNBGKBN 1/2-1/2 1/2-1/2   A46 rnbqkb1r/ppppppp/5/2/8/3P4/8/PPP1PPPPRNBGKBN 1/2-1/2 1/2-1/2   A46 rnbqkb1r/ppppppp/5/2/8/3P4/8/PPP1PPPPRNBGKBN 1/2-1/2 1/2-1/2   A48 rnbqkb1r/ppppppppppppppp/5/2/8/3P4/8/PPP1PPPPRNBGKBN 1-0 1/2-1/2   B00 rnbqkb1r/ppppppppppppppppppppp/5/2/8/3P4/8/PPP1PPPPRNBGKBN 1-0 1/2-1/2	A00 mbokbn/ipppppppβi8/1F6/8IPPPPPPPPRINBGKBNR 1/2-1/2 1/2-1/2 0-1   A01 mbokbn/ipppppppβi8/8/8/1F6/FIPPPPPPPRINBGKBNR -0-1 1/2-1/2 0-1   A04 mbokbn/ipppppppβi8/8/8/1F6/FIPPPPPPPRINBGKBNR -0-1 1/2-1/2 0-1   A15 mbokb1/ipppppppp5/6/8/8/F5/8/FP1PPPPRINBGKBN 1/2-1/2 1/2-1/2 1/2-1/2   A16 mbokb1/ipppppppi5/6/8/F5/8/FP1PPPPPRINBGKBN 1/2-1/2 0-1 1/2-1/2   A40 mbokb1/ippppppp15/6/8/F3/8/F3/8/FP1/FPPPRINBGKBN 1/2-1/2 0-1 1/2-1/2   A45 mbokb1/ipppppppp5/6/8/F3/8/F3/8/F91/FPP1PPPRINBGKBN 1/2-1/2 1/2-1/2 1-0   A46 mbokb1/ipppppppp3/F3/8/F3/8/F4/8/FP1/FPPPPRINBGKBN 1/2-1/2 1/2-1/2 1-0   A46 mbokb1/ippppppp1/F3/8/F3/8/F3/8/FP1/FPPPPRINBGKBN 1/2-1/2 0-1 1/2-1/2   A80 mbokb1/ipppppp1/F3/8/F3/8/F91/FPPPPPPRINBGKBN 1-0 1/2-1/2 0-1   B00 mbokb1/ipppppp1/F3/8/F3/8/F91/FPPPPPPPPPRINBGKBN 1-0 1/2-1/2 0-1   B00 mbokb1/ipppppp1/F3/8/F3/8/F91/FPPPPPPPPPPPPPPPPPP	A00 mbokkbn//ppppppppp/8/8/1P6/8/P1PPPPPPRNBOKBNR 1/2-1/2 1/2-1/2 0-1 1/2-1/2   A01 mbokbn//pppppppp/8/8/8/1P6/P1PPPPPPRNBOKBNR 0-1 1/2-1/2 0-1 1/2-1/2 0-1 1/2-1/2 0-1 1/2-1/2 0-1 1/2-1/2 0-1 1/2-1/2 0-1 1/2-1/2 0-1 1/2-1/2 0-1 1/2-1/2	A00 mbokbn/rppppppppp/8/8/1P6/8/P1PPPPPPRNBOKBNR 1/2-1/2 1/2-1/2 0-1 1/2-1/2	A00 mbokkon/poppoppopp/8/8/1P6/8/1PPPPPPR/NBOKBNR 1/2-1/2 21-21/2 0-1 1/2-1/2	A00 mbokbn/rpppppppppp/8/8/IP68/IPFPPPPPRINBOKBNR 1/2-1/2 1/2-1/2 0-1 1/2-1/2	A00 mbokkbn/ippppppppppk8/lPP6/8IPIPPPPPRNBGKBNR 1/2-1/2 1/2-1/2 0-1 1/2-1/2	A00 mbokbn/rpppppppppp/8/8/IP6/8/PTPPPPPRINBGKBNR 12-1/2 1/2-1/2 0-1 1/2-1/2

#### **Prior belief:**

- Get the opinion of the best player in the world: **Stockfish**.
- Simulate 10 games of Stockfish vs Stockfish per each opening. ↗
- Save the results as the parameters for the prior distribution!

### the outcome of simulated games:

Stockfish vs Stockfish



#### **Monte Carlo simulation:**

- To approximate the posterior distribution that each win rate follows.

$$E(p_w+rac{1}{2}p_d|w,d)=\int_{\Omega}(p_w+rac{1}{2}p_d)\,p(p_w,p_d|w,d)d\Theta$$

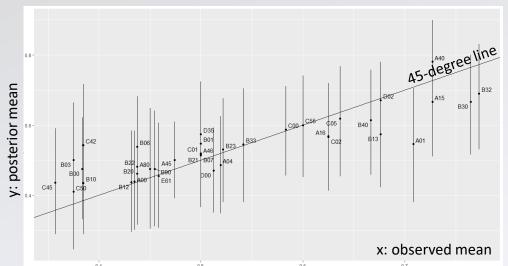


$$E(p_w + \frac{1}{2}p_d|w,d) = \int_{\Theta} (p_w + \frac{1}{2}p_d) \ p(p_w,p_d|w,d) d\Theta \qquad \boxed{\underbrace{\phantom{a}}} \qquad \qquad \frac{1}{S} \sum_{w=1}^S \left(p_w^{(s)} + \frac{1}{2}p_d^{(s)}\right) \rightarrow E(p_w + \frac{1}{2}p_d|w,d) \quad as \ S \rightarrow \infty \qquad \boxed{\underbrace{\phantom{a}}} \qquad \boxed{\underbrace{\phantom{a}}} \qquad \underbrace{\phantom{a}}$$





## Posterior Intervals of Win Rates per Opening



Best Openings for White	<b>Worst Openings for White</b>
A40 Queen's Pawn Game (0.78)	C50 Giuoco Piano (0.41)
B32 Open Sicilian (0.69)	B10 Caro-Kann (0.43)
D02 Queen's Pawn Game (0.67)	C45 Scotch Game (0.44)
B30 Old Sicilian (0.67)	B12 Caro-Kann Defense (0.44)
A15 English (0.67)	A00 Uncommon Opening (0.44)



ECO C50 Giuoco Piano

## Shrinkage effect:

- The expected value of win rate is pulled a bit from the observed mean towards the prior mean by the amount depending on the sample size.

