A Bayesian Approach on Chess Openings

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Abbreviated abstract:

Chess grandmasters prepare the opening in depth. It is intriguing that even with the help of engines, they failed to reach a consensus on which the strongest opening is. Statistics have been covered to compare the strength of each opening, yet most of the approaches are from the frequentist's point of view. In this study, a Bayesian approach is suggested as an alternative.

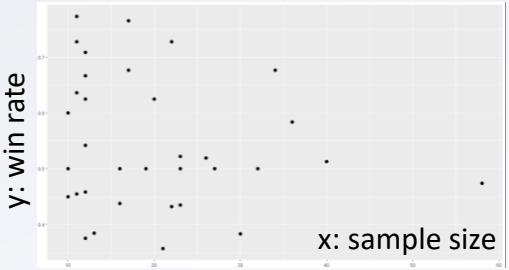
Related publications:

– Lee, J. (2021). Chess Data Analysis: A Bayesian Approach on Opening Tier. **[QR]**
– Gelman, A. (2004). Bayesian data analysis. Boca Raton, Fla: Chapman & Hall/CRC.

The Limitations of Frequentist Approach

- **The three phases of chess:**
  1. **opening** (statistics) 2. middle game 3. endgame (computation)
- **Frequentist approach:**
  – Build a multinomial model with three possible outcomes: *win*, *draw* or *loss*.
  – Count the numbers of W/D/L for each opening to estimate the true values of parameters. (MLE)
- **Data**: lichess.org game data (better use GM-level games next time ☹)

Best 2 Openings for White	Worst 2 Openings for White
B32 Open Sicilian (0.77)	C45 Scotch Game (0.36)
B30 Old Sicilian (0.76)	C50 Giuoco Piano (0.38)



- **Problem**:
  – Extreme win rates might be simply due to small sample sizes. ↗
  – The legitimacy of an opening that hasn't been played by stronger players is doubted.
  → **Try the Bayesian approach instead!**

Bayesian Multinomial Model

- **Bayesian trinomial model:**

$$Prior: \theta \sim Dirichlet(\alpha, \beta, \gamma)$$
$$Likelihood: Y|\theta \sim Trinomial(\theta)$$
$$Posterior: \theta|y \sim Dirichlet(\alpha + w, \beta + d, \gamma + l)$$

	ECO		FEN	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10
0	A00	rnbqkbnr/pppppppp/8/8/1P6/8/P1PPPPPP/RNBQKBNR ...	1/2-1/2	1/2-1/2	0-1	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2
1	A01	rnbqkbnr/pppppppp/8/8/8/1P6/P1PPPPPP/RNBQKBNR ...	0-1	1/2-1/2	0-1	1/2-1/2	1/2-1/2	1/2-1/2	0-1	1/2-1/2	1/2-1/2	1/2-1/2
2	A04	rnbqkbnr/pppppppp/8/8/5P2/8/PPPPP1PP/RNBQKBNR ...	0-1	0-1	1/2-1/2	1/2-1/2	1-0	1/2-1/2	1/2-1/2	0-1	1/2-1/2	1/2-1/2
3	A15	rnbqkb1r/pppppppp/5n2/8/2P5/8/PP1PPPPP/RNBQKBN...	1/2-1/2	1/2-1/2	1/2-1/2	1-0	1/2-1/2	1-0	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2
4	A16	rnbqkb1r/pppppppp/5n2/8/2P5/2N5/PP1PPPPP/R1BQK...	1/2-1/2	0-1	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2	1-0
5	A40	rnbqkbnr/pppp1ppp/8/4p3/3P4/8/PPP1PPPP/RNBQKBN...	1-0	1-0	1-0	1-0	1-0	1-0	1-0	0-1	1-0	1-0
6	A45	rnbqkb1r/pppppppp/5n2/8/3P4/8/PPP1PPPP/RNBQKBN...	1/2-1/2	1/2-1/2	1-0	1/2-1/2	1/2-1/2	1/2-1/2	1-0	1-0	1/2-1/2	1/2-1/2
7	A46	rnbqkb1r/pppppppp/5n2/8/3P4/5N2/PPP1PPPP/RNBQK...	1/2-1/2	0-1	1/2-1/2	1-0	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2	1-0	1/2-1/2
8	A80	rnbqkbnr/ppppp1pp/8/5p2/3P4/8/PPP1PPPP/RNBQKBN...	1-0	1/2-1/2	1/2-1/2	1/2-1/2	0-1	1/2-1/2	1/2-1/2	0-1	1/2-1/2	1-0
9	B00	rnbqkbnr/ppppp1pp/5p2/8/4P3/8/PPPP1PPP/RNBQKBN...	1-0	1-0	0-1	1/2-1/2	1-0	1/2-1/2	1-0	1/2-1/2	1-0	1-0
10	B01	rnbqkbnr/ppp1pppp/8/3p4/4P3/8/PPPP1PPP/RNBQKBN...	1/2-1/2	1/2-1/2	1/2-1/2	1/2-1/2	1-0	1/2-1/2	1-0	1/2-1/2	1-0	1-0

- **Prior belief:**
  - Get the opinion of the best player in the world: **Stockfish**.
  - Simulate 10 games of Stockfish vs Stockfish per each opening. ↗
  - Save the results as the parameters for the prior distribution!

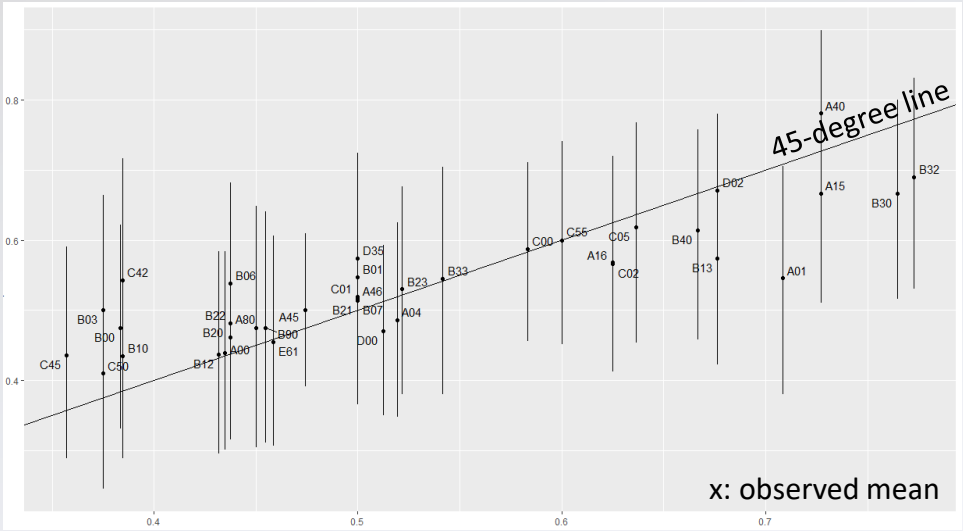
the outcome of simulated games:
Stockfish vs Stockfish

- **Monte Carlo simulation:**
  - To approximate the posterior distribution that each win rate follows.

$$E(p_w + \frac{1}{2}p_d|w,d) = \int_\Theta (p_w + \frac{1}{2}p_d)\, p(p_w, p_d|w,d) d\Theta$$ 😟

$$\frac{1}{S}\sum_{s=1}^{S}\left(p_w^{(s)} + \frac{1}{2}p_d^{(s)}\right) \to E(p_w + \frac{1}{2}p_d|w,d) \quad as\ S \to \infty$$ ☺

Posterior Intervals of Win Rates per Opening



Best Openings for White	Worst Openings for White
A40 Queen's Pawn Game (0.78)	C50 Giuoco Piano (0.41)
B32 Open Sicilian (0.69)	B10 Caro-Kann (0.43)
D02 Queen's Pawn Game (0.67)	C45 Scotch Game (0.44)
B30 Old Sicilian (0.67)	B12 Caro-Kann Defense (0.44)
A15 English (0.67)	A00 Uncommon Opening (0.44)

ECO C50
Giuoco Piano

- **Shrinkage effect:**
  - The expected value of win rate is pulled a bit from the observed mean towards the prior mean by the amount depending on the sample size.