VAR modeling illustration with real data

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The following material is taken from Fan & Yao (2015).

1 VAR Modeling

Let x be the matrix with 3 columns containing the log daily returns in 2011 of FTSE 100, FTSE MidCap, FTSE SmallCap. To fit a vector VAR(p) model with p determined by BIC (i.e. the Schwartz Criterion), we issue the following commands and obtain the summary of fitted results:

```
mydata = read.table("~/ftse2011.dat",header = T)
tt = nrow(mydata)
mydata2 <- mydata[tt:1,]
x1 = diff(log(mydata2[,2]))
x2 = diff(log(mydata2[,3]))
x3 = diff(log(mydata2[,4]))
x = data.frame(x1,x2,x3)
names(x)=c("FTSE100", "FTSE MidCap", "FTSE SmallCap")
attach(x)
library(vars)
FTSEvar = VAR(x, lag.max =3, ic="SC")
summary(FTSEvar)

##
## VAR Estimation Results:
## =========================
## Endogenous variables: FTSE100, FTSE.MidCap, FTSE.SmallCap
## Deterministic variables: const
## Sample size: 248
## Log Likelihood: 2748.593
## Roots of the characteristic polynomial:
## 0.1363 0.1363 0.04969
## Call:
## VAR(y = x, lag.max = 3, ic = "SC")
```

```
##
##
## Estimation results for equation FTSE100:
## ========================================
## FTSE100 = FTSE100.l1 + FTSE.MidCap.l1 + FTSE.SmallCap.l1 + const
##
##                    Estimate Std. Error t value Pr(>|t|)
## FTSE100.l1        0.4623839  0.1822584   2.537   0.0118 *
## FTSE.MidCap.l1   -0.5277461  0.2154000  -2.450   0.0150 *
## FTSE.SmallCap.l1  0.2438484  0.1981300   1.231   0.2196
## const            -0.0003319  0.0008497  -0.391   0.6964
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.01329 on 244 degrees of freedom
## Multiple R-Squared: 0.03624,Adjusted R-squared: 0.02439
## F-statistic: 3.058 on 3 and 244 DF,  p-value: 0.02896
##
##
## Estimation results for equation FTSE.MidCap:
## ============================================
## FTSE.MidCap = FTSE100.l1 + FTSE.MidCap.l1 + FTSE.SmallCap.l1 + const
##
##                    Estimate Std. Error t value Pr(>|t|)
## FTSE100.l1        0.6041161  0.1689675   3.575 0.000422 ***
## FTSE.MidCap.l1   -0.5709383  0.1996923  -2.859 0.004616 **
## FTSE.SmallCap.l1  0.2651595  0.1836817   1.444 0.150139
## const            -0.0005873  0.0007877  -0.746 0.456663
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.01232 on 244 degrees of freedom
## Multiple R-Squared: 0.08665,Adjusted R-squared: 0.07542
## F-statistic: 7.716 on 3 and 244 DF,  p-value: 6.044e-05
##
##
## Estimation results for equation FTSE.SmallCap:
## ==============================================
## FTSE.SmallCap = FTSE100.l1 + FTSE.MidCap.l1 + FTSE.SmallCap.l1 + const
##
##                    Estimate Std. Error t value Pr(>|t|)
## FTSE100.l1        0.2692036  0.1076958   2.500   0.0131 *
## FTSE.MidCap.l1   -0.1276323  0.1272791  -1.003   0.3170
```

```
## FTSE.SmallCap.l1  0.0778511  0.1170743   0.665   0.5067
## const            -0.0006338  0.0005021  -1.262   0.2080
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.007855 on 244 degrees of freedom
## Multiple R-Squared: 0.1037,Adjusted R-squared: 0.09271
## F-statistic: 9.414 on 3 and 244 DF,  p-value: 6.557e-06
##
##
##
## Covariance matrix of residuals:
##                 FTSE100 FTSE.MidCap FTSE.SmallCap
## FTSE100       1.767e-04   1.545e-04     8.617e-05
## FTSE.MidCap   1.545e-04   1.519e-04     8.237e-05
## FTSE.SmallCap 8.617e-05   8.237e-05     6.170e-05
##
## Correlation matrix of residuals:
##               FTSE100 FTSE.MidCap FTSE.SmallCap
## FTSE100        1.0000      0.9430        0.8252
## FTSE.MidCap    0.9430      1.0000        0.8508
## FTSE.SmallCap  0.8252      0.8508        1.0000
```

The selected order by BIC is $p = 1$. From the previous fitting result, we find that some coefficient estimates are insignificant. To refit the model by leaving out insignificant terms, we use

```
FTSEvarR = restrict(FTSEvar)
summary(FTSEvarR)

##
## VAR Estimation Results:
## =========================
## Endogenous variables: FTSE100, FTSE.MidCap, FTSE.SmallCap
## Deterministic variables: const
## Sample size: 248
## Log Likelihood: 2744.399
## Roots of the characteristic polynomial:
## 0.2057 0.2057     0
## Call:
## VAR(y = x, lag.max = 3, ic = "SC")
##
##
## Estimation results for equation FTSE100:
## ========================================
```

```
## FTSE100 = FTSE100.l1 + FTSE.MidCap.l1
##
##                Estimate Std. Error t value Pr(>|t|)
## FTSE100.l1       0.4689     0.1816   2.582   0.0104 *
## FTSE.MidCap.l1  -0.3983     0.1904  -2.092   0.0375 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.01329 on 246 degrees of freedom
## Multiple R-Squared: 0.02987,Adjusted R-squared: 0.02198
## F-statistic: 3.787 on 2 and 246 DF,  p-value: 0.02399
##
##
## Estimation results for equation FTSE.MidCap:
## ============================================
## FTSE.MidCap = FTSE100.l1 + FTSE.MidCap.l1
##
##                Estimate Std. Error t value Pr(>|t|)
## FTSE100.l1       0.6083     0.1687   3.605 0.000378 ***
## FTSE.MidCap.l1  -0.4265     0.1769  -2.411 0.016634 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.01235 on 246 degrees of freedom
## Multiple R-Squared: 0.0782,Adjusted R-squared: 0.07071
## F-statistic: 10.43 on 2 and 246 DF,  p-value: 4.469e-05
##
##
## Estimation results for equation FTSE.SmallCap:
## ==============================================
## FTSE.SmallCap = FTSE100.l1
##
##            Estimate Std. Error t value Pr(>|t|)
## FTSE100.l1  0.19508    0.03714   5.253 3.24e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
##
## Residual standard error: 0.00785 on 247 degrees of freedom
## Multiple R-Squared: 0.1005,Adjusted R-squared: 0.09684
## F-statistic: 27.59 on 1 and 247 DF,  p-value: 3.236e-07
##
##
```

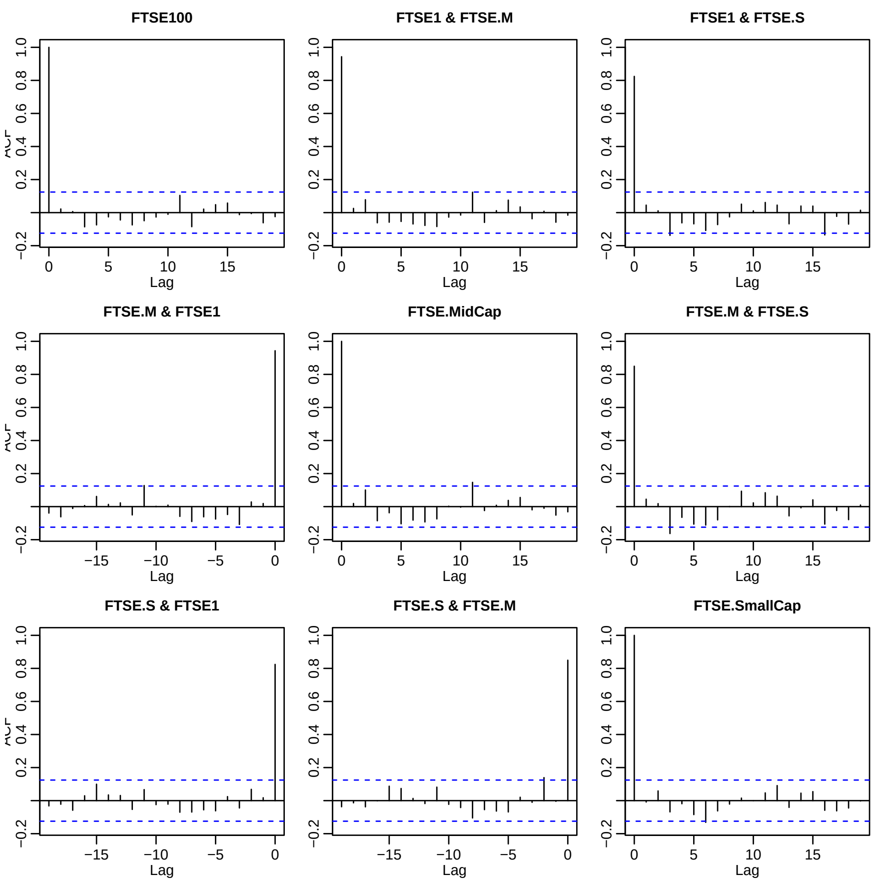
```
##
## Covariance matrix of residuals:
##                 FTSE100 FTSE.MidCap FTSE.SmallCap
## FTSE100       1.778e-04   1.557e-04     8.653e-05
## FTSE.MidCap   1.557e-04   1.532e-04     8.277e-05
## FTSE.SmallCap 8.653e-05   8.277e-05     6.197e-05
##
## Correlation matrix of residuals:
##               FTSE100 FTSE.MidCap FTSE.SmallCap
## FTSE100        1.0000      0.9433        0.8243
## FTSE.MidCap    0.9433      1.0000        0.8495
## FTSE.SmallCap  0.8243      0.8495        1.0000
```

which leads to

$$\begin{aligned}
\mathrm{FTSE100}_t &= 0.469\mathrm{FTSE100}_{t-1} - 0.398\mathrm{FTSEMid}_{t-1} + \varepsilon_{t1}, \\
\mathrm{FTSEMid}_t &= 0.608\mathrm{FTSE100}_{t-1} - 0.427\mathrm{FTSEMid}_{t-1} + \varepsilon_{t2} \\
\mathrm{FTSESmall}_t &= 0.195\mathrm{FTSE100}_{t-1} + \varepsilon_{t3}.
\end{aligned}$$

The following figure presents the cross-correlations of the residuals from the above fitted VAR(1) model, produced by R command

```
acf(residuals(FTSEvarR))
```



More diagnostic plots can be produced by calling the following R functions:

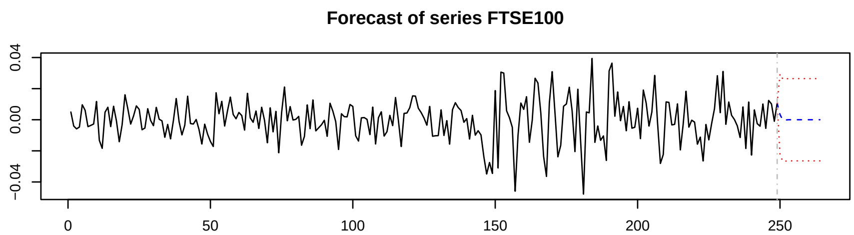
```
FTSEdiag = serial.test(FTSEvarR)
plot(FTSEdiag)
```

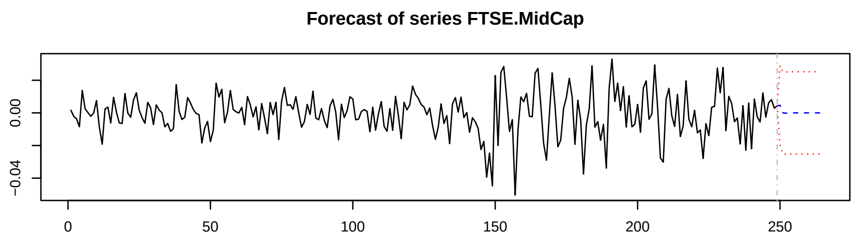
To preform the $Q_k(m)$ test for the residual with, for example, $m = 6$, run

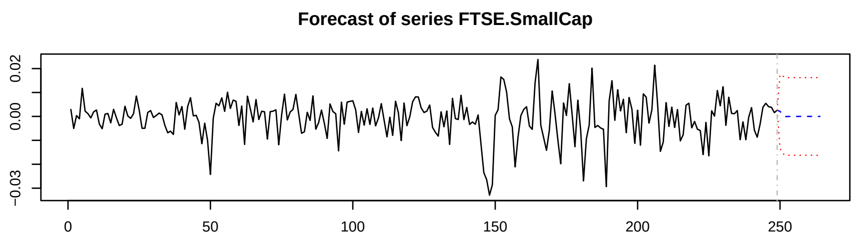
```
serial.test(FTSEvarR,lags.pt = 6,type = "PT.adjusted")

##
##  Portmanteau Test (adjusted)
##
## data:  Residuals of VAR object FTSEvarR
## Chi-squared = 81.537, df = 45, p-value = 0.0006984
```

VAR models can be used to forecast future values. For example, we may forecast the next 15 returns of those three FTSE indices by using R function predict as follow, where we specify the predictive boundaries with the coverage probability 0.95.

```
FTSEpred = predict(FTSEvarR,n.ahead = 15,ci=0.95)
plot(FTSEpred)
```







2 Granger Causality

The function causality in the package **vars** implements the Granger Causality analysis.

```
causality(FTSEvarR,cause="FTSE100")
# causality(FTSEvarR,cause="FTSE100",boot = TRUE,boot.runs = 1000)

## $Granger
```

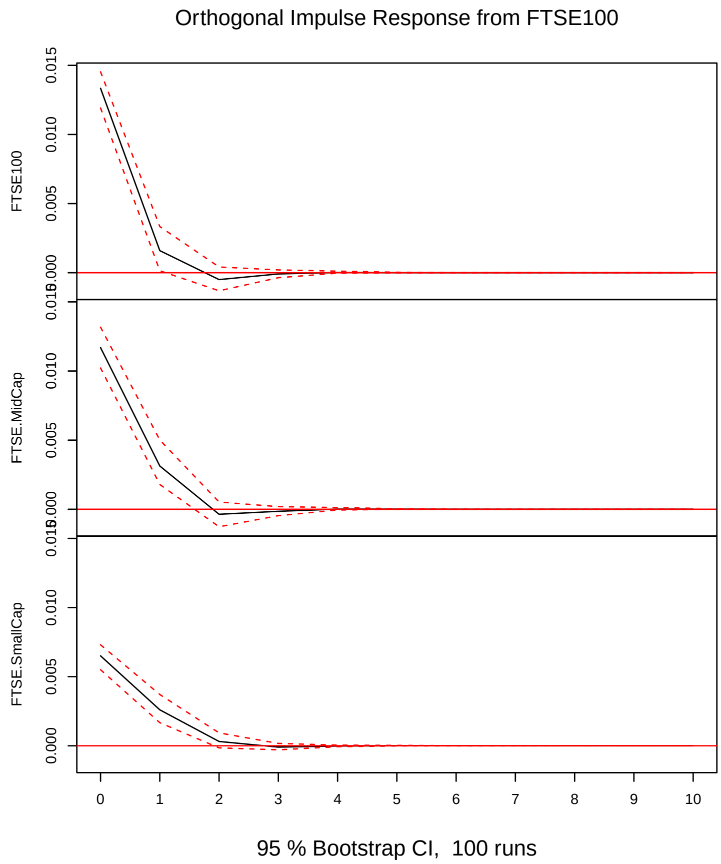
```
##
##  Granger causality H0: FTSE100 do not Granger-cause FTSE.MidCap
##  FTSE.SmallCap
##
## data:  VAR object FTSEvarR
## F-Test = 4.7663, df1 = 2, df2 = 732, p-value = 0.008778
##
##
## $Instant
##
##  H0: No instantaneous causality between: FTSE100 and FTSE.MidCap
##  FTSE.SmallCap
##
## data:  VAR object FTSEvarR
## Chi-squared = 116.87, df = 2, p-value < 2.2e-16
```

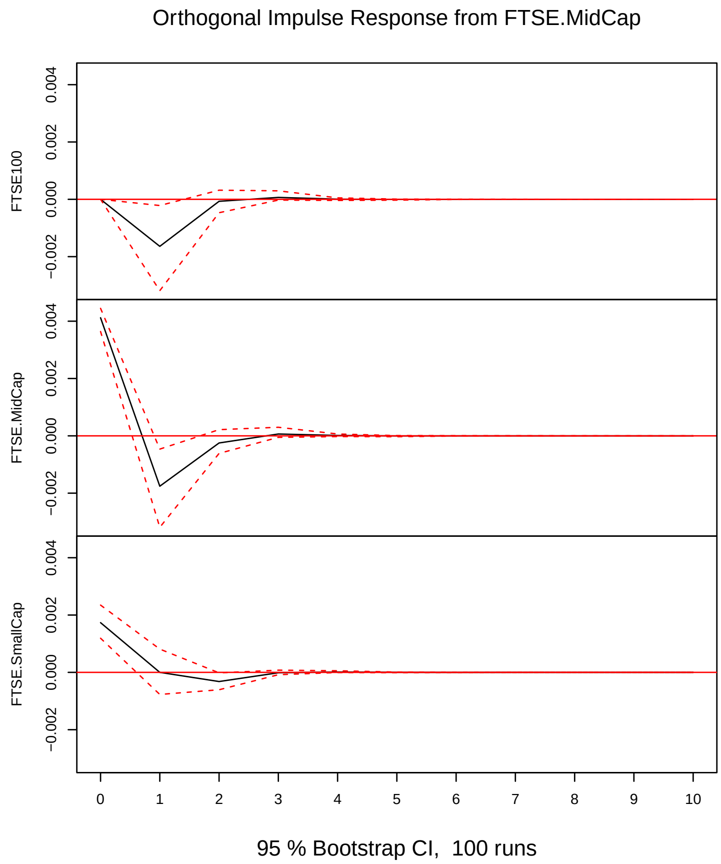
The null hypothesis of no Granger causality is rejected with the p-value 0.001, there is significant evidence indicating that FTSE100 Granger causes FTSE MidCap and FTSE SmallCap. The null hypothesis of no instantaneous causality is rejected with the p-value 0.

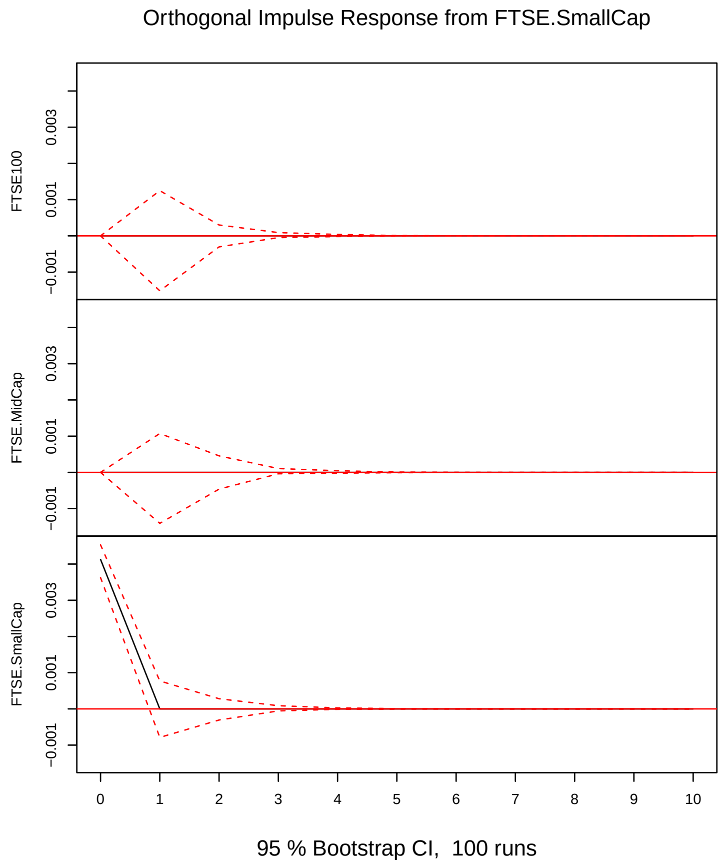
3 Impulse Response Functions

The effect of a change in one component series on the other components can be investigated via the so-called impulse response functions, which measure the resulting changes in other components at different time lags due to a unit change in one component series.

```
irfFTSE = irf(FTSEvarR)
plot(irfFTSE)
```


Orthogonal Impulse Response from FTSE100
FTSE100
FTSE.MidCap
FTSE.SmallCap
0.000
0.005
0.010
0.015
0 1 2 3 4 5 6 7 8 9 10
95 % Bootstrap CI, 100 runs


Orthogonal Impulse Response from FTSE.MidCap
FTSE100
FTSE.MidCap
FTSE.SmallCap
0.004
0.002
0.000
−0.002
0
1
2
3
4
5
6
7
8
9
10
95 % Bootstrap CI, 100 runs



4 Forecast Error Variance Decomposition

In variance decomposition, the forecast error variance is decomposed into components accounted for by innovations in the different variables of the system. The function fevd in the package **vars** implements the Forecast Error Variance Decomposition analysis.

```
fevd(FTSEvar)

## $FTSE100
##         FTSE100 FTSE.MidCap FTSE.SmallCap
##   [1,] 1.0000000  0.00000000   0.000000000
##   [2,] 0.9779315  0.01663722   0.005431299
##   [3,] 0.9778880  0.01667701   0.005434949
```

```
##  [4,] 0.9778843  0.01667973   0.005435924
##  [5,] 0.9778843  0.01667980   0.005435941
##  [6,] 0.9778843  0.01667980   0.005435941
##  [7,] 0.9778843  0.01667980   0.005435941
##  [8,] 0.9778843  0.01667980   0.005435941
##  [9,] 0.9778843  0.01667980   0.005435941
## [10,] 0.9778843  0.01667980   0.005435941
##
## $FTSE.MidCap
##         FTSE100 FTSE.MidCap FTSE.SmallCap
##  [1,] 0.8892089   0.1107911   0.000000000
##  [2,] 0.8703408   0.1225813   0.007077877
##  [3,] 0.8702947   0.1226010   0.007104352
##  [4,] 0.8702878   0.1226060   0.007106174
##  [5,] 0.8702878   0.1226060   0.007106174
##  [6,] 0.8702878   0.1226060   0.007106175
##  [7,] 0.8702878   0.1226060   0.007106175
##  [8,] 0.8702878   0.1226060   0.007106175
##  [9,] 0.8702878   0.1226060   0.007106175
## [10,] 0.8702878   0.1226060   0.007106175
##
## $FTSE.SmallCap
##         FTSE100 FTSE.MidCap FTSE.SmallCap
##  [1,] 0.6809012  0.04774647     0.2713524
##  [2,] 0.7097809  0.04508592     0.2451332
##  [3,] 0.7090695  0.04596916     0.2449613
##  [4,] 0.7090733  0.04597134     0.2449554
##  [5,] 0.7090732  0.04597146     0.2449554
##  [6,] 0.7090732  0.04597146     0.2449554
##  [7,] 0.7090732  0.04597146     0.2449554
##  [8,] 0.7090732  0.04597146     0.2449554
##  [9,] 0.7090732  0.04597146     0.2449554
## [10,] 0.7090732  0.04597146     0.2449554
```