

# Financial Econometrics I: Assignment Two

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Due: 2020-4-21

## 1 KERNEL DENSITY ESTIMATOR

Step 1: Generate random numbers from standard Normal distribution with sample size  $T = 1000$ .<sup>1</sup> Let the generated sample be  $\{x_1, \dots, x_T\}$ .

Step 2: Let  $k(\cdot)$  be the Epanechnikov kernel, i.e.,

$$k(z) = \begin{cases} \frac{3}{4}(1 - z^2) & \text{if } |z| \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

For a fixed  $x_0$ , the kernel density estimator is given by

$$\hat{f}(x_0) = \frac{1}{Th} \sum_{t=1}^T k\left(\frac{x_t - x_0}{h}\right), \quad (1.1)$$

where  $h$  is a bandwidth. In practice, kernel estimation is sensitive to the choice of  $h$ . In Step 2, you are required to choose  $h$  based on the method called **likelihood cross-validation**, which chooses  $h$  to maximize the log likelihood function given by

$$\mathcal{L} = \sum_{t=1}^T \ln \hat{f}_{-t}(x_t), \quad (1.2)$$

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<sup>1</sup>Remember to set a seed before the random number generation.

where

$$\hat{f}_{-t}(x_t) = \frac{1}{(T-1)h} \sum_{s=1, s \neq t}^T k\left(\frac{x_s - x_t}{h}\right)$$

is the leave-one-out kernel estimator of  $f(x_t)$ .

Step 3: Using the optimal bandwidth  $h$  obtained in Step 2, estimate the density function evaluated at each sample point  $x_t$ ,  $t = 1, \dots, T$ . Compare your result with standard normal distribution.