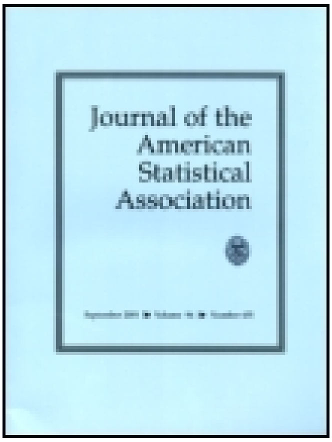

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THE UNRELATED QUESTION RANDOMIZED RESPONSE MODEL: THEORETICAL FRAMEWORK

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This paper develops a theoretical framework for the unrelated question randomized response technique suggested by Walt R. Simmons. The statistical efficiency of this technique is compared with the Warner technique under situations of both truthful and untruthful responses. Methods of allocating the total sample to each of two subsamples required by the unrelated question approach are developed. Recommendations are made concerning choices of values for those parameters which can be assigned at the discretion of the investigator.

1. INTRODUCTION

REFUSALS to respond or intentionally misleading replies are known to be two of the main sources of non-sampling bias in sample surveys of human populations [2]. Recognizing that these two sources of error are more frequent when the respondents are queried about sensitive or highly personal matters, Warner [4] developed an ingenious interviewing procedure designed to reduce or eliminate these biases. He called the technique "randomized response" because the respondent selects a question on a probability basis from two or more questions without revealing to the interviewer which of the alternative questions has been chosen. The individual reply, which must be "Yes" or "No" to each question, is of no certain meaning for a specific respondent, but a batch of replies provides useful information for estimating the proportion of the population that has the "sensitive characteristic." In the event that this characteristic is concerned with illegal behavior or socially deviant acts, such as an induced abortion, both the respondent and the interviewer are protected in the confidentiality of the reply.

Warner considered the case where the population is divided into two mutually exclusive groups—one with and the other without the stigmatizing characteristic. With the help of a randomizing device, the respondent replies only "Yes" or "No" to whichever of the two statements occurs by chance.

"I am a member of Group A",
"I am not a member of Group A".

The interviewer is not shown the statement selected and hence does not know to which statement the reply refers. If the respondent understands the method or if he has faith in the method whether he understands it completely

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or not, he might be persuaded first to participate and then to cooperate more fully than he would otherwise because his personal privacy with respect to the sensitive characteristic is maintained. Although the interviewer will never know which statement is being answered when the technique is properly administered, it seems clear that the respondent must believe this before he will answer truthfully. The interviewer, not having the confidentiality privileges of the doctor or priest, is also protected because he cannot interpret with any certain assurance the meaning of a respondent's reply.

If the survey is designed to estimate the proportion of persons with a sensitive attribute, say A, the Warner procedure may be described with the following notation.

Let

π_A = true proportion with attribute A

P = probability that the statement, "I am a member of Group A," is selected to be answered by the respondent

n = sample size

n' = total number of respondents who reply "Yes."

Warner showed that the maximum likelihood estimate is unbiased, if persons are encouraged by the technique to tell the truth, and its value is

$$(\hat{\pi}_A)_W = \frac{1}{2P - 1}\left\{P - 1 + \frac{n'}{n}\right\}, \qquad P \neq \tfrac{1}{2}. \tag{1.1}$$

The variance of the estimate, as given by Warner, is,

$$\operatorname{Var}(\hat{\pi}_A)_W = \frac{\pi_A(1 - \pi_A)}{n} + \frac{P(1 - P)}{n(2P - 1)^2}\,. \tag{1.2}$$

As can be seen from (1.2), the variance of the Warner estimate has two components. The first part is the usual binomial variance associated with a direct question and completely truthful replies by all respondents. The second part is the additional variance or cost one pays for the uncertainty associated with the randomized response.

This addition to the variance can be relatively high when P is close to 0.5 and n is not large. The survey designer must consider, therefore, under what circumstances this additional variance may be justifiable. A definitive, generalized answer to this question is not possible because the main advantage of the Warner technique may be to increase the response rate itself. When the survey involves a characteristic like induced abortion, in which it is well known that respondents will either refuse to answer or not answer truthfully, field trials are necessary to learn the effect of the Warner technique upon the response rate.

If one considers only the bias caused by evasive answers of the respondents, it is possible to arrive at a generalized answer to the question of when it is preferable to use the technique. Warner examined the ratio of the mean square errors so that both bias and variance were considered. He considered attributes with $\pi_A = 0.5$ and 0.6 because of interest in studying items such as voting behavior. These values of π_A seem unrealistically high for an attribute to be deemed truly sensitive or stigmatizing. If membership in group A is really a socially disapproved attribute, values in the neighborhood of 0.05 and 0.10 seem more reasonable in most situations. Thus, if $\pi_A = 0.05$ and using a sample of 1000, the Warner technique with $P = 0.05$ is half as efficient as the direct question method. It is only one-tenth as efficient when $P = 0.20$. These comparisons assume that all respondents tell the truth in both methods.

When only 90% of the respondents in group A are likely to admit their membership in it, the efficiency of the Warner procedure climbs to two-thirds for $P = 0.05$ and to about one-seventh for $P = 0.20$. As evasive answer bias increases, the Warner procedure begins to achieve superiority. Thus, if 25% of the members in group A would lie in response to a direct question about it, the Warner method is twice as efficient when $P = 0.05$ although still only one-half as efficient when $P = 0.20$. If as many of 50% of those in group A would deny their membership, then the Warner procedure is six times more efficient when $P = 0.05$, and four-thirds more efficient when $P = 0.20$.

Thus, based only on the evasive answer bias and not on the response rate, the Warner technique usually requires a substantial amount of lying before it becomes worthwhile. Efforts to make the randomized response technique more efficient are obviously in order when respondents do not report at all or report untruthfully. In those cases, direct questioning and the binomial variance for simple random sampling are inappropriate.

In a previous paper [1], several of the authors extended Warner's technique to the general case where there were $t \geq 2$ categories each of which was in varying degrees potentially harmful or stigmatizing. The reason behind this extension was that a multichotomous situation seemed more realistic for some attributes than the dichotomy. A by-product of this extension with a set of, say, three, rather than two possible questions, is that those authors felt it provided additional opportunity for the respondent to feel that his answer would not be self-incriminating. There is no factual basis for this inference other than an intuitive one that the respondent might be willing to report more truthfully when replying "Yes" or "No" to one of three questions rather than to just one of two queries.

Another approach to increasing the respondent's likelihood of cooperation is the one suggested by Walt R. Simmons [3] and adopted in the present paper. Simmons felt that by providing the respondent the opportunity of replying to one of two questions in which one question is completely innocuous and unrelated to the stigmatizing attribute, the respondent might be more truthful. Thus, if the main focus of the survey is on induced abortion, or illegitimacy of offspring, the neutral question might concern itself with whether the respondent was born in a particular area, or whether the respondent had ever been in a certain city, or whether the respondent was left-handed. Whether or not the unrelated question actually does increase the respondent's probability of telling the truth will have to be ascertained in actual surveys such as reported in [3]. The present paper is concerned with the theoretical framework upon which such surveys might be based and to show that the Simmons method actually reduces the resultant variance.

2. THE UNRELATED QUESTION TECHNIQUE IN THE SIMPLEST CASE (TRUTHFUL REPORTING)

For simplicity, let us assume a single sensitive characteristic, A, in the population and that because of the randomized response procedure there is completely truthful reporting by the respondent. Instead of requesting the respondent to reply affirmatively or negatively to one of the two statements,

"I am a member of Group A",
"I am not a member of Group A",

the alternatives are stated differently. The respondent is asked to reply "Yes" or "No" to one of two statements selected on a probability basis

"I am a member of Group A",
"I am a member of Group Y",

where membership in Group Y carries no possible embarrassment or condemning quality.

For instance, the two statements posed might be,

"During the past 12 months, I had an operation or did something to myself while I was pregnant which kept me from having a baby,"

"I was born in North Carolina."

In this illustration, the latter statement would carry no anticipated embarrassment nor should replying to it present any difficulty to the respondent. By introducing Group Y into the situation, we have essentially created the same estimation problem that was solved in [1] when there is more than the single sensitive characteristic. We must estimate not only π_A but also the proportion π_Y of the population having the completely independent characteristic Y. It is clear that

$$\pi_A + \pi_Y \lesseqgtr 1$$

since respondents may belong to either group alone, to both, or to neither. The situation can exist where previous information is available regarding π_Y in which case the sample estimate can be used either as a validating device, a Bayesian estimate can be derived, or one can reduce the amount of sampling involved. Use of prior information on π_Y to reduce sample size will be considered in Section 4.

Two independent, non-overlapping simple random samples of size n_1 and n_2 are drawn from the population. The sizes of n_1 and n_2 are not necessarily equal and, in fact, can be manipulated to produce minimum variance estimates according to other characteristics of the problem.

Every respondent in the two samples is asked to reply with only a "Yes" or "No" answer to the specific single statement which turns up in his case. The selection of the statement is made by a randomization device on a probability basis. In this way, the respondent's status is not revealed to the interviewer provided that the latter cannot observe the randomization process in the device.

Suppose the randomization device consists of a deck of cards, or a sealed clear plastic box containing colored beads, of two varieties. One type of card, or bead color, designates statement A and the other type designates statement Y. By cutting the cards, or shaking the plastic box to have a single bead appear in the window, the respondent selects his own statement without the interviewer's intervention.

In this particular model, two sets of the randomization device need to be employed. Set 1 is used for respondents in the first sample, and set 2 is used for respondents in the second sample. If more than one interviewer is used in either sample, every interviewer in sample 1 has a randomization device identical to set 1, and every interviewer in sample 2 has a randomization device identical to set 2. Moreover, the two sets, 1 and 2, must also be different with respect to the probability that statement A will be selected.

Let the randomization devices be such that Group A (or statement A) is represented on a probability basis by P_1 in set 1 and P_2 in set 2, and $P_1 \neq P_2$. Similarly, let Group Y (or statement Y) be represented on a probability basis by $(1-P_1)$ in set 1, and $(1-P_2)$ in set 2.

Since we have postulated that respondents are reporting 100 percent truthfully, let

$$\begin{aligned} \lambda_1 &= \text{the probability that a "Yes" answer} \\ &\quad\ \text{will be reported in the first sample,} \\ &= P_1\pi_A + (1 - P_1)\pi_Y, \\ &= P_1(\pi_A - \pi_Y) + \pi_Y. \end{aligned} \tag{2.1}$$

Similarly, let

$$\begin{aligned} \lambda_2 &= \text{the probability that a "Yes" answer} \\ &\quad\ \text{will be reported in the second sample,} \\ &= P_2\pi_A + (1 - P_2)\pi_Y, \\ &= P_2(\pi_A - \pi_Y) + \pi_Y. \end{aligned} \tag{2.2}$$

One can construct the likelihood function of the sample and obtain the ML estimate of π_A, as was done in previous papers [1], [4]. Alternatively, the identical value can be obtained by solving (2.1) and (2.2) for π_A, yielding†

$$(\pi_A)_U = \frac{\lambda_1(1 - P_2) - \lambda_2(1 - P_1)}{P_1 - P_2} \tag{2.3}$$

where $P_1(1-P_2)-P_2(1-P_1)\neq 0$ since $P_1 \neq P_2$.

A similar expression can be derived for $(\pi_Y)_U$ and is

$$(\pi_Y)_U = \frac{P_2\lambda_1 - P_1\lambda_2}{P_2 - P_1}\,.$$

Suppose we let the observed proportions of "Yes" answers reported in the first and second samples be designated by $\hat{\lambda}_1 = n_1'/n_1$ and $\hat{\lambda}_2 = n_2'/n_2$ respectively, where n_1' and n_2' are the numbers of "Yes" answers in the two corre-

† The subscript U outside the parenthesis of a parameter or its estimate will refer to the unrelated question model, and the subscript W will refer to Warner's model.

sponding samples. Then, the sample estimate, $(\hat{\pi}_A)_U$, is obtained by replacing λ_i by $\hat{\lambda}_i$ in (2.3) and it follows that

$$(\hat{\pi}_A)_U = \frac{\hat{\lambda}_1(1 - P_2) - \hat{\lambda}_2(1 - P_1)}{P_1 - P_2} \cdot \tag{2.4}$$

The observed proportions, $\hat{\lambda}_1$ and $\hat{\lambda}_2$, are binomially distributed with parameters (n_1, λ_1) and (n_2, λ_2) respectively. It therefore follows that the expression in (2.4) is unbiased and its variance is given by the following equation,

$$\operatorname{Var}(\hat{\pi}_A)_U = \frac{1}{(P_1 - P_2)^2}\left\{\frac{\lambda_1(1 - \lambda_1)(1 - P_2)^2}{n_1} + \frac{\lambda_2(1 - \lambda_2)(1 - P_1)^2}{n_2}\right\} \cdot \tag{2.5}$$

Estimates of the variance are made by substituting $\hat{\lambda}_1$ and $\hat{\lambda}_2$ in equation (2.5).

In order to compare the efficiency of $(\hat{\pi}_A)_U$ with the estimate by using Warner's method, it is well to recall that only one sample, of size n, is required to estimate π_A in the latter case. For purposes of comparing the efficiency of estimation in the two models, we will first assume that the total sample sizes are the same in both and that $n_1 = n_2 = \bar{n}$ under the unrelated question model. This restriction on equality of the two sample sizes in the unrelated question model will be relaxed later because optimal allocation of n_1 and n_2 has considerable influence on the variance given in (2.5). Initially, however, we will assume that $n = n_1 + n_2 = 2\bar{n}$, and (1.2) yields

$$\text{Var. } (\hat{\pi}_A)_W = \frac{\pi_A(1 - \pi_A)}{2\bar{n}} + \frac{P(1 - P)}{2\bar{n}(2P - 1)^2} \cdot \tag{2.6}$$

A comparison of (2.4) and (2.5) with their counterparts (1.1) and (2.6), assuming that all respondents are reporting truthfully in both models, leads to the following observations.

A. *Estimation*

1. Point estimates as well as confidence limits for (π_A) can occur outside the range (0, 1) in both models even with large sample sizes. If inefficient design parameters are chosen in the unrelated question technique, the probability for point estimates outside the range will be larger than for the Warner model. Moreover, in the latter case, crude quality control measures can more easily be applied to the individual interviewer because the reported proportion of "Yes" responses should lie, in a probability sense, between P and $(1-P)$ regardless of π_A.

2. The denominator in (2.4) can become quite small by choosing P_2 too close to P_1 with the result that the point estimate of π_A might be greater than unity in the unrelated question model. Thus, a first general rule is that P_2 should be selected as far from P_1 as possible without jeopardizing the likelihood of a respondent's cooperation. Obviously, $P_2 = 0$, or 1, would not be a randomization device at all. An easy guide to accomplish the desideratum is to make $P_1 + P_2 = 1$. This has the additional advantage of affecting both samples identically when the respondents examine the randomization device. This gen-

eral rule for P_2 turns out to be exactly the same as when the criterion is based upon the variance of $(\hat{\pi}_A)_U$ and this is considered below in Section 2B.1.

B. *Variance of the estimates*

The sampling variance, when the respondents report truthfully, can be studied theoretically as well as experimentally to deduce general rules for selection of the unrelated Y characteristic, the choice of P_1 and P_2, and the allocation of total sample size into n_1 and n_2.

We shall proceed by deducing first a general rule for selection of P_1 and P_2, and then select the level of π_Y under that condition. Finally, optimal allocation of sample size into n_1 and n_2 will be based upon the prior choice of these other design parameters. This may not be the best overall strategy for choosing values of the four variables to minimize the variance of (π_A). The results of following this procedure, however, yield a variance which is so close to the binomial that it must also be close to optimal. It has the further advantage of simplicity to calculate and in implementation for field work.

1. *Choice of P_1 and P_2.* Based upon the knowledge about selection of P in the Warner technique, we shall fix P_1 by choosing for it a value as far from 0.5 as is practicable without arousing suspicion in the respondent. (This would probably be either 0.20, or .80, $\pm$.10). The next step is to derive a value for P_2 which will yield the minimum variance of $\hat{\pi}_A$ at a given level of π_Y, n_1, and n_2 under the restraint that P_2 can not be any closer to 0 or 1 than was P_1. The reason for this restraint is that P_1 itself was initially selected to be as close to 0 or 1 as was possible without jeopardizing cooperation of the respondent.

The expression for $\text{Var}(\hat{\pi}_A)_U$ given in (2.5) is now a function of P_2, with $\lambda_2=(\pi_A-\pi_Y)P_2+\pi_Y$. Differentiating (2.5) with respect to P_2,

$$\frac{d(\text{Var }\hat{\pi}_A)_U}{dP_2} = \frac{1-P_1}{(P_1-P_2)^3}\left\{\frac{2\lambda_1(1-\lambda_1)(1-P_2)}{n_1}+\frac{(\lambda_1+\lambda_2-2\lambda_1\lambda_2)(1-P_1)}{n_2}\right\}.$$

If we set this equal to 0, a minimum cannot be attained for any P_1 except $P_1=1$, in which case any P_2 will suffice. This is a trivial case since we desire $P_1\neq 0$, or 1. For $P_1\neq 1$, $d(\text{Var}\hat{\pi}_A)_U/dP_2 \gtrless 0$ for all $P_2 \lessgtr P_1$, and the graph of $\text{Var}(\hat{\pi}_A)_U$ against P_2 resembles a double hyperbola with an asymptote at P_1. This indicates that in order to reduce $\text{Var}(\hat{\pi}_A)_U$, it is desirable to select P_2 as far away from P_1 as practicable. Thus, if $P_1<0.5$, select P_2 as close to 1 as possible without threatening the degree of cooperation by the respondents. (If $P_1>0.5$, then P_2 should be as close to 0 as possible.) A good working rule to achieve maximum separation is $P_1+P_2=1$. This has the additional advantage, as previously noted, of affecting both samples identically when the respondents examine the randomization device.

2. *Selection of the unrelated characteristic.* Accepting the finding that $P_1+P_2=1$ is a good principle under the procedure adopted for selection of the design parameters, attention can be turned to investigating how to select the unrelated, neutral characteristic with parameter π_Y. By comparing the value

TABLE 1. COMPARISON OF THE VARIANCES OF THE ESTIMATES OBTAINED BY WARNER'S RANDOMIZATION RESPONSE METHOD AND THE UNRELATED QUESTION MODEL FOR $\pi_A=.20$, $n_1=n_2=\bar{n}=n/2=500$, AND DIFFERING LEVELS OF P, P_1, P_2, AND π_Y

Proportion of Statement A in the Randomization Device			Efficiency $=\dfrac{\operatorname{Var}(\hat{\pi}_A)_W}{\operatorname{Var}(\hat{\pi}_A)_U}$ when $\pi_Y=$				
Warner	Unrelated Question Model						
P	First Sample P_1	Second Sample P_2	.1	.3	.5	.7	.9
.7	.7	.3	1.49	1.13	0.96	0.88	0.86
.8	.8	.2	1.10	0.92	0.82	0.76	0.72
.9	.9	.1	0.77	0.71	0.66	0.62	0.60

of $\operatorname{Var}(\hat{\pi}_A)_U$ with the comparable one for the Warner model, the loss (or gain) caused by introducing the unrelated characteristic into the randomized response model can be studied at the same time.

For varying levels of π_Y, a numerical investigation was carried out by assuming that the sensitive characteristic, $\pi_A=.20$, $P=P_1$, $P_1+P_2=1$, and $n_1=n_2=\bar{n}=n/2=500$. Although $n_1=n_2$ is not generally desirable, this restriction simplifies calculations here. The optimal values of n_1 and n_2 will be determined after specifying P_1, P_2, and π_Y. In Table 1, the results of the comparison are shown where the efficiency of the estimate is expressed as

$$\operatorname{Var}(\hat{\pi}_A)_W/\operatorname{Var}(\hat{\pi}_A)_U$$

and values less than unity represent loss caused by introducing the unrelated characteristic instead of using the Warner model.

As judged from Table 1 when $\pi_A=.20$, the most efficient estimate of π_A is made when π_Y is lowest. This holds true regardless of the level of $P_1(=P)$. The same condition holds true, in fact, as long as $\pi_A<0.5$. In fact, as π_A gets smaller, say 0.05, the relative efficiency which occurs when using $\pi_Y=0.10$ becomes two and one-half times greater than when $\pi_Y=0.50$ for the case when $P_1=0.7$.

When $\pi_A>0.5$, the reverse prevails because of symmetry and the minimum variance occurs when π_Y is largest.

When $\pi_A=.50$, the highest efficiency occurs for π_Y at the tails and is symmetric.

Thus, the rule for selecting an unrelated characteristic is relatively simple. Choose π_Y on the same side of 0.5 as π_A, and maximize $|\pi_Y-0.5|$ as far as practicable. When $\pi_A=0.5$, only the latter criterion is necessary.

Regardless of the level of π_A, it is not desirable to choose a characteristic with π_Y too close to 0, since such action may well contradict the whole purpose of using the unrelated question approach. Since the respondent's desire to reply truthfully may be affected if π_Y is chosen close to zero, a sensible guideline is to aim for π_Y in the neighborhood of .10 (or .90). If π_A is very small, say .01, it may not be desirable to choose π_Y even in the neighborhood of .10, although such a choice is advantageous from theoretical considerations based only on the sampling variance. In this situation, respondents who belong to Group A may argue to themselves, after examining the randomizing device, as follows:

1. Relatively few respondents will be answering "Yes" to either statement.
2. Hence, persons who answer "Yes" will be suspect, whether they belong to group A or not.
3. Since, I belong to Group A and do not wish to be even suspected of this, I will not answer "Yes" regardless of the statement I select.

If π_Y is reasonably large when π_A is small, respondents who belong to Group A may be more willing to answer truthfully since the proportion of "Yes" answers in the survey may then be considered sufficiently large to preclude pointing the finger of suspicion. Clearly, if π_A is $>.50$, then π_Y should also be large to minimize response bias. This is quite consistent with the strategy indicated by choosing π_Y to minimize the sampling variance.

A surprising result in Table 1 is that in some cases the unrelated question model can actually be more efficient than the Warner method despite the additional parameter. The Warner method is very inefficient if P is chosen close to 0.5 and therefore it is relatively easier for the unrelated characteristic procedure to be more efficient when $P=0.7$ than it is when, say $P=0.9$.

The superiority of the unrelated question model is considerably understated in Table 1 because the indicated efficiencies were calculated with $n_1=n_2$. The following section considers optimal allocation between the two samples.

3. *Optimal allocation of n_1 and n_2.* Under the unrelated question model the expression for the variance of $\hat{\pi}_A$ has two components,

$$\frac{\lambda_1(1-\lambda_1)(1-P_2)^2}{n_1},$$

and

$$\frac{\lambda_2(1-\lambda_2)(1-P_1)^2}{n_2}.$$

The numerators of these components are rarely equal and a proper choice of n_1 and n_2 can be made so as to minimize the sum of the two terms. If the numerators are not identical, the proportion of the total sample size which should be allocated to each sample is

$$\frac{n_1}{n_2}=\sqrt{\frac{\lambda_1(1-\lambda_1)(1-P_2)^2}{\lambda_2(1-\lambda_2)(1-P_1)^2}}. \tag{2.7}$$

In application of the formula (2.7), it is necessary to make a rough guess of π_A and π_Y in order to calculate λ_1 and λ_2. If the estimates of π_A and π_Y are in error, λ_1 and λ_2 are similarly distorted but any misleading effect this might have upon n_1 and n_2 is cushioned by the ratio of the λ's in (2.7). What formula (2.7) essentially does is simply to allocate a larger portion to that particular sample which has the greater probability of selecting an A statement.

When optimal allocation of n_1 and n_2 has been applied to the cases illustrated in Table 1, the efficiency of the unrelated question model always improves and almost doubles in some instances. Thus, for $P=P_1=0.7$, $\pi_Y=.1$, the efficiency mounts from 1.49 to 1.78, and even for $P=P_1=0.9$, $\pi_Y=0.1$, the efficiency goes from 0.77 to 1.30.

This means that the introduction of the unrelated characteristic does not have to be penalizing in terms of efficiency in estimating π_A *and* can actually yield a gain.

To sum up, when respondents are reporting truthfully and there is only one stigmatizing attribute A, we recommend that one select constants in the unrelated question method as follows.

a. Choose an unrelated characteristic such that $|\pi_Y-0.5|$ is a maximum and π_Y falls on the same side of 0.5 as π_A. If $\pi_A=0.5$, then $|\pi_Y-0.5|$ should be a maximum on either side. It is recognized, of course, that the magnitude of π_A is unknown to the investigator and that, in fact, the purpose of the survey is to estimate π_A. If one has no idea whatsoever on which side of 0.5 to expect π_A, then a moderate value of π_Y between 0.25 and 0.75 will at least control the loss in efficiency. Fortunately, when $\pi_A=0.5$, the curve of efficiency plotted against π_Y is rather flat and U-shaped. The difference in efficiency between the worst choice (viz. $\pi_Y=0.5$) and the best feasible (say, $\pi_Y=0.9$ (or .1)) when $P=P_1=0.7$ is only 1.00 versus 1.16. Thus, if π_A is believed close to 0.5, such as might occur in a survey of voting behavior, choosing a non-optimal value for π_Y does not cause a serious loss in efficiency.

b. Determine P_1 somewhere in the vicinity of 0.20 ± 0.10, or 0.80 ± 0.10, and choose $P_2=1-P_1$. The choice of P_1, and therefore P_2, should be made as small (or large) as the respondents are likely to accept.

c. Allocate total sample size into portions n_1 and n_2 in accord with the expression shown in (2.7) so as to minimize the sampling variance in estimating π_A.

d. The choices for π_Y, P_1, and P_2 should not be made solely on the basis of minimizing sampling variance. The unrelated characteristic and the sampling devices must be such that the respondent is induced to report accurately. Otherwise, the advantage of confidentiality will appear to be lost and the bias arising from incomplete or untruthful answers will dominate the mean square error.

The occurrence of less than completely truthful reporting is a prospect which one must be prepared to expect in dealing with highly sensitive matters such as abortion or socially deviant behavior. The effect which this will have upon the two estimates will be considered in the next section.

3. LESS THAN COMPLETELY TRUTHFUL REPORTING

To compare the unrelated question model with the Warner model when reporting is not completely truthful, we shall assume that both techniques are administered properly. Each model requires an explanation and administration of the randomization device. For instance, a respondent should be urged to shake the plastic box several times to assure himself that both types of bead may appear. Only after he has done this several times is the respondent requested to do a test run. Since both Warner and unrelated question procedures should be affected equally with respect to incorrect responses caused by genuine misunderstanding of the device, the following comparison will exclude such errors. The misreporting under consideration here stems solely from the respondent's desire to protect his self-image.

Let $T_A (0 < T_A \leq 1)$ denote the probability in the unrelated question model that respondents who belong to Group A will tell the truth when confronted with a question concerning membership therein regardless of whether they fall in sample 1 or sample 2. It is further postulated that respondents confronted with a question relating to membership in Group Y will report truthfully particularly if Y has been appropriately selected by the survey designer.* Therefore, equations (2.1) and (2.2) become

$$\lambda_1' = P_1(\pi_A T_A - \pi_Y) + \pi_Y \tag{3.1}$$

and

$$\lambda_2' = P_2(\pi_A T_A - \pi_Y) + \pi_Y. \tag{3.2}$$

The expected value of the estimate in (2.3) becomes

$$E(\hat{\pi}_A')_U = \frac{\lambda_1'(1 - P_2) - \lambda_2'(1 - P_1)}{P_1 - P_2}. \tag{3.3}$$

Also, the bias in this estimate is

$$\begin{aligned} \text{Bias}(\hat{\pi}_A')_U &= E\{\hat{\pi}_A' - \pi_A\}_U \\ &= \frac{(\lambda_1' - \lambda_1)(1 - P_2) - (\lambda_2' - \lambda_2)(1 - P_1)}{P_1 - P_2} \\ &= \pi_A(T_A - 1). \end{aligned} \tag{3.4}$$

In the Warner model, let $T_A' (0 < T_A' \leq 1)$ denote the probability that a respondent in Group A will report truthfully. We presume that T_A will be different (and hopefully greater) than T_A', even if $P_1 = 1 - P_2 = P$, since the remaining statement is different in the two techniques. The innocuous nature of the unrelated statement is to convince the respondent that his true individual situation is not detectable. In the Warner technique both statements are about membership in Group A and the respondent may still suspect a trick in the form of a "Heads, I win and Tails, you lose" kind of situation.

The probability, T_A', applies not only to respondents who draw the affirmative statement, but also to those who draw the negative statement, viz., "I am not a member of Group A." Of course, persons who are not members of Group A have no reason to fabricate by claiming that they are really members of it. Hence, the probability of obtaining a "Yes" answer for the Warner technique is

$$\begin{aligned} \lambda' &= \pi_A P T_A' + \pi_A(1 - P)(1 - T_A') + (1 - \pi_A)(1 - P) \\ &= \pi_A[P T_A' + (1 - P)(1 - T_A')] + (1 - \pi_A)(1 - P). \end{aligned} \tag{3.5}$$

* We recognize that a more general model would take into account the fact that some respondents may not reply truthfully to membership in Y for various reasons. Even if membership in Y is not stigmatizing, a person might deny membership therein because he knows that a "Yes" answer *might* be embarrassing whereas with a "No" answer there is never any possibility of embarrassment.

The value of (3.5) would be inserted for n'/n in equation (1.1) to derive the estimate of $(\hat{\pi}'_A)_W$. Therefore, the bias is measured by

$$\begin{aligned}\text{Bias}(\hat{\pi}'_A)_W &= E\{\hat{\pi}'_A - \pi_A\}_W \\ &= \frac{1}{2P-1}\{(P-1) + \pi_A P T'_A + \pi_A(1-P)(1-T'_A) \\ &\quad + (1-\pi_A)(1-P) - \pi_A(2P-1)\} \qquad (3.6)\\ &= \frac{1}{2P-1}\{(P-1)\pi_A T'_A + \pi_A P T'_A - \pi_A(2P-1)\} \\ &= \pi_A(T'_A - 1).\end{aligned}$$

The bias in both models, (3.4) and (3.6), is a negative value since T_A, $T'_A \leq 1$ and this implies that the kind of untruthful reporting postulated here has the effect of lowering the estimate of π_A. If $T_A = T'_A$, the bias is identical in both cases and any comparison involving mean square error $= \{(\text{bias})^2 + \text{variance}\}$ will give essentially similar results as in Section 2 when no bias prevailed.

When $T_A \neq T'_A$, a comparison of the efficiency of the unrelated question to the Warner procedure is obtained from a ratio of the mean square errors, as follows:

$$\text{MSEE}(\hat{\pi}'_A)_U = \frac{\text{MSE}(\hat{\pi}'_A)_W}{\text{MSE}(\hat{\pi}_A')_U}, \qquad (3.7)$$

where values greater than unity favor the unrelated question model.

A numerical comparison of the two methods can be carried out with varying values of T_A and T'_A. For the investigation here, a value of $P = P_1 = 1 - P_2 = .20$ was chosen as a reasonable one. The odds are against an investigator's willingness to risk the two questions in a ratio greater than 4:1. The value of π_A was selected at .20, $\pi_Y = .10$, and $n = 1000$. In this work, however, n_1 and n_2 were allocated optimally according to equation (2.7) using the adjusted values from (3.1) and (3.2) in the former. The results are presented in Table 2.

Examining the values *below* the diagonal in Table 2, the conclusion is obvious that the unrelated question technique offers considerable gain in efficiency if, in fact, $T_A > T'_A$. The equally interesting and not too surprising observation is that values along the diagonal are *all* greater than unity. That is, if $T_A = T'_A$, the unrelated question technique is still superior because of the optimal allocation of the total sample size into two segments. Thus, when $T_A = T'_A = 1$, the optimal allocation of the sample into n_1 and n_2 results in an increase of 56% in efficiency with the unrelated question technique.

When the probability of truthful reporting decreases for either method, for fixed π_A and n the contribution of the bias term to the mean square error mounts rapidly. Using the data for the illustration cited in Table 2, when $T_A = T'_A = .90$, the proportion of the mean square error contributed by bias is approximately 50%, but this fraction reaches about 90% for T_A or $T'_A = .70$. Thus, if the unrelated question model can improve even very slightly the probability of reporting truthfully a respondent's membership in Group A, the mean square

TABLE 2. MEAN SQUARE ERROR EFFICIENCY* OF THE UNRELATED QUESTION MODEL VERSUS THE WARNER TECHNIQUE WHEN REPORTING IS NOT COMPLETELY TRUTHFUL ($P=P_1=1-P_2=.80$, $\pi_A=.20$, $\pi_Y=.10$, $n_1+n_2=1000$ AND ALLOCATED OPTIMALLY.)

T'_A for Warner's Procedure	T_A for Unrelated Question Procedure					
	1.00	0.90	0.80	0.70	0.60	0.50
1.00	<u>1.56</u>	<u>0.79</u>	0.31	0.15	0.09	0.06
0.90	2.57	<u>1.30</u>	<u>0.51</u>	0.25	0.15	0.10
0.80	5.64	2.86	<u>1.13</u>	<u>0.56</u>	0.33	0.21
0.70	10.79	5.47	2.15	<u>1.07</u>	<u>0.62</u>	0.41
0.60	18.01	9.13	3.59	1.78	<u>1.04</u>	<u>0.68</u>
0.50	27.59	13.84	5.44	2.70	1.58	1.03

* MSE Efficiency

$$=\frac{MSE(\pi'_A)_W}{MSE(\pi'_A)_U},$$

and where mean square error $=\{(\text{bias})^2+\text{variance}\}$.

error will show gains almost out of proportion to such increase in truthfulness. The gain for the special case considered is, of course, more specifically documented in the values below the diagonal of Table 2. Other values for the parameters n, π_A, and π_Y show the same trend, viz., that the proportion of the mean square error contributed by the bias mounts rapidly as T_A or T'_A decreases from unity. For this reason alone, the potential gain in accuracy made possible by the unrelated question model makes it a most worthwhile procedure.

The foregoing advantage of the unrelated question model has been deduced under the postulate that knowledge of the frequency of the unrelated characteristic is unknown. If the prevalence of the unrelated characteristic is indeed known in advance, one can use it either to validate the survey or to improve the precision of the estimate of the unknown characteristic A. This concept is developed further in the following section.

4. WHEN THE TRUE PROPORTION OF THE UNRELATED CHARACTERISTIC IS KNOWN

Suppose that the neutral question introduced into the survey involves a characteristic Y whose true proportion π_Y is known in advance.* Thus, in a survey relating to abortions, or illegitimacy of offspring, one might use as a second question the probably unrelated statement, "I was born in the month of April," to which the respondent is supposed to reply simply "Yes," or "No". From other knowledge, such as census data or birth registration files, one might know what proportion of the respondents had been born in the month of April.

Prior knowledge of π_Y can be useful by reducing the sampling to one group

* One method of assuring this has been suggested by Mr. Richard Morton of the University of Sheffield and is now being field tested. The randomization device, say the plastic box, contains three colors of beads. If the bead appearing in the window has color L, the respondent answers "Yes" or "No" to the question about membership in Group A. If the bead color is M, the answer given is always "Yes", and if the color is N, the appropriate reply is always "No."

of size n, as in the Warner procedure, because there is now only one parameter to estimate. Thus, from equation (2.1), we have

$$\lambda_1 = P_1(\pi_A - \pi_Y) + \pi_Y$$

or

$$\lambda_1 = P_1\pi_A + \pi_Y(1 - P_1).$$

As before, the ML estimate can be obtained in the usual fashion or the equivalent value obtained by solving the foregoing equation for π_A. Thus,

$$(\pi_A \mid \pi_Y)_U = \frac{\lambda_1 - \pi_Y(1 - P_1)}{P_1}. \tag{4.1}$$

To obtain an estimate for π_A, simply substitute the observed value of $\hat{\lambda}_1$ for its expected value in the foregoing equation. The sample variance is also relatively simple in structure owing to the lack of error assumed present in π_Y. Thus,

$$\text{Var. } (\hat{\pi}_A \mid \pi_Y)_U = \frac{\lambda_1(1 - \lambda_1)}{nP_1^2}, \tag{4.2}$$

and the estimate is obtainable by use of $\hat{\lambda}_1$. The result in (4.2) could be combined with the expressions used to derive (4.1) to get its value in terms of the design parameters. Thus,

$$\text{Var}(\hat{\pi}_A \mid \pi_Y)_U = \frac{1}{nP_1^2}\{P_1\pi_A + \pi_Y(1 - P_1\}\{1 - P_1\pi_A - \pi_Y(1 - P_1)\}.$$

If the sampling was actually carried out in two portions, with sample sizes n_1 and n_2, a single estimate is still possible by the weighted combination of the estimates from each sample. Using invariances as the weighting factor for each sample, the combined estimate of π_A is

$$(\hat{\pi}_A \mid \pi_Y)_{U(\text{combined})} = \frac{[\hat{\lambda}_1 - \pi_Y(1 - P_1)]A + [\hat{\lambda}_2 - \pi_Y(1 - P_2)]B}{AP_1 + BP_2}, \tag{4.3}$$

where

$$A = n_1P_1\hat{\lambda}_2(1 - \hat{\lambda}_2)$$
$$B = n_2P_2\hat{\lambda}_1(1 - \hat{\lambda}_1).$$

The sampling variance of the estimate in (4.3) is one-half the harmonic mean of the two individual variances since the reciprocals of the latter are the weights used in deriving the combined estimate. The variance of the combined estimate attains a minimum when all the observations are alloted to the ith subsample which is greater in P_i.

To get an idea as to the gain in precision when π_Y is known, consider the cases when $P_1 = .80$, $\pi_Y = .10$, $n = 1000$, and $\pi_A = 0.05$ or 0.20. The sampling variances in the three techniques studied, in comparison to the regular binomial case, are as follows.

	$\pi_A = .20$	$\pi_A = .05$
Case 1. Standard Warner procedure	0.000604	0.000492
Case 2. Unrelated question, π_Y unknown	0.000386	0.000370
Case 3. Unrelated question, π_Y known	0.000231	0.000088
Case 4. Regular binomial	0.000160	0.000048

This is a most remarkable result because it now shows that the use of the unrelated question can actually make a sizeable improvement in precision efficiency over the Warner technique in learning about the value of π_A. Case 3 is better than case 2 because the sampling does not waste observations in that particular sample where P_i is low. In other words, if the probability is low that statement A will appear, do not sacrifice observations on it beyond the point of necessity. Both cases 2 and 3 appear better than case 1 despite the fact that the Warner technique is always asking about Group A either in an affirmative or negative sense. The negative A question turns out to be a liability rather than an asset. Case 1 is less efficient than cases 2 and 3 because an affirmative reply in the Warner technique can be either membership in A or membership in not A. As P approaches unity, the Warner procedure leaves less doubt as to what the probability of a "Yes" means and asymptotically reaches the binomial situation.

Except for the special case cited in the footnote on page 532, the assumption has been made that π_Y is known without error. In many cases, such an assumption will represent wishful thinking because the population under survey may have changed with time or may be slightly deviant from a larger group for whom the characteristic Y is known. For instance, anthropometric studies may indicate that p per cent of the general population is left-handed but it may very well be that p^* per cent ($p \neq p^*$) are left-handed in the state of North Carolina. Consideration should be given to the effect of this deviation upon the estimate of π_A and its sampling variance.

Let π_Y^* represent the estimate of π_Y used in the unrelated question method where π_Y^* is obtained from some source of information other than the survey itself. Assume that $\pi_Y = \pi_Y^* - C$. In this case, the estimate in (4.1) will become

$$(\hat{\pi}_A \mid \pi_Y^*)_U = \frac{\hat{\lambda}_1 - \pi_Y^*(1 - P_1)}{P_1}. \tag{4.4}$$

Continuing, as before, with T_A representing the probability that respondents in Group A will tell the truth,

$$\begin{aligned} \text{Bias}(\hat{\pi}_A \mid \pi_Y^*)_U &= E\{(\hat{\pi}_A \mid \pi_Y^*)_U - \pi_A\} \\ &= \frac{1}{P_1}\{P_1\pi_A T_A + \pi_Y(1 - P_1) - (\pi_Y - C)(1 - P_1) - P_1\pi_A\} \\ &= \pi_A(T_A - 1) + \frac{C(1 - P_1)}{P_1} = \beta_A + \beta_Y, \end{aligned} \tag{4.5}$$

where

β_A = bias arising from untruthful reporting of characteristic A

β_Y = bias arising from erroneous estimation of π_Y from sources external to the survey.

It follows that

$$\begin{aligned} \text{MSE}(\hat{\pi}_A \mid \overset{*}{\pi_Y})_U &= (\text{Bias})^2 + \text{Var}(\hat{\pi}_A \mid \overset{*}{\pi_Y})_U \\ &= (\beta_A + \beta_Y)^2 + \text{Var}(\hat{\pi}_A \mid \overset{*}{\pi_Y})_U \end{aligned} \tag{4.6}$$

If $T_A = 1$ (and assuming $T'_A = 1$), then the unrelated question technique (case 3) is superior to the Warner procedure (case 1) as long as

$$(\beta_Y)^2 < \text{Var}(\hat{\pi}_A)_W - \text{Var}(\hat{\pi}_A \mid \overset{*}{\pi_Y})_U.$$

In one of the examples cited where $\pi_A = .20$, $\pi_Y = .10$, $n = 1000$, and $P_1 = .80$,

$$\left(\frac{C(1 - P_1)}{P_1}\right)^2 < 0.000604 - 0.000231$$

$$|C| < \frac{.80}{.20}\sqrt{.000373}$$

or

$$|C| < 4(0.0193), \quad \text{or say } 7.7\%.$$

In other words, case 3 would still be preferable to case 1 if the value of π_Y^* ranged anywhere from 2.3% to 17.7%. The error of margin is somewhat less, however, in comparing case 3 versus case 2. Here

$$\left(\frac{C(1 - P_1)}{P_1}\right)^2 < 0.000386 - 0.000231$$

$$|C| < 4(.0124), \quad \text{or say } 5.0\%.$$

Thus, case 3 is better than case 2 as long as π_Y^* is in the range from 5% to 15%.

The considerable latitude admissible in the value of C is, of course, a function of the values used for parameters in this illustration. If $\pi_A = 0.05$, case 3 is better than case 2 as long as π_Y^* is in the range from 6.4% to 13.6%. If there is any doubt that π_Y^* is in the specified ranges, the investigator can, of course, always rely on case 2 wherein two samples are selected so as to enable an internal estimate of π_Y^* to be made, using the equations in Section 2.

When T_A and T'_A do not equal unity, consideration of equation (4.6) is of still further interest. In order for the unrelated question technique to be superior to the Warner one,

$$(\beta_A + \beta_Y)^2 < \text{MSE}(\hat{\pi}_A)_W - \text{Var}(\hat{\pi}_A \mid \overset{*}{\pi_Y})_U.$$

This new formulation has more of an advantage than a disadvantage. The value of β_A is always negative under the conditions of untruthful reporting assumed here. Therefore, if β_Y is positive and less than $|2\beta_A|$, the combined estimate is actually better than it was when $C=0$ and π_Y^* was unbiased. Hence, if one is uncertain of π_Y and $T_A \neq 1$, use should be made of an estimate which underestimates π_Y. It is better both for bias and mean square error. The rule for underestimation applies regardless of the value of π_Y with respect to 0.5.

Consider the same illustration as before where $\pi_A = .20$ except that $T_A = T_A' = .90$. The various mean square error terms are recalculated as follows.

Case 1. Warner procedure $= (0.02)^2 + 0.000592 = 0.000992$.
Case 2. Unrelated question, π_Y unknown $= (0.02)^2 + 0.000361 = 0.000761$.
Case 3. Unrelated question, π_Y known $= [-(0.02) + (C/4)]^2 + 0.000214$.

Note that the value of 0.000214 for case 3 is still derived from $\hat{\lambda}_1(1-\hat{\lambda}_1)/nP_1^2$, and since $T_A = .90$, the expected value of $\hat{\lambda}_1$ is obtained from $\lambda_1 = P_1(\pi_A T_A - \pi_Y) + \pi_Y = 0.1640$, where use is made of π_Y as it is that value operating in the population and not what value one assumes for π_Y^* to be for the estimate in equation (4.4).

Case 3 is superior to case 2 when

$$\left[-(0.02) + \frac{C}{4}\right]^2 < 0.000761 - 0.000214$$

$$\left|-0.02 + \frac{C}{4}\right| \leq \sqrt{0.000547}$$

or

$$-1.4\% < C < 17.4\%.$$

In this situation, therefore, values for π_Y^* anywhere in the interval from 0 to 11.4% would make the mean square error for case 3 lower than that for case 2. In a considerable portion of this range, the bias would also be less.

Along similar lines, the argument for case 3 against case 1 will result in a still wider zone for π_Y^*.

All this adds up to the fact that the unrelated question technique is desirable. If the frequency of the unrelated characteristic is known or guessable, use should be made of this figure with preference being given to underestimating it. If the frequency of the unrelated characteristic is completely unknown, use two samples and make an internal estimate of π_Y.

5. GENERALIZATION TO MORE THAN ONE STIGMATIZING GROUP

As indicated earlier, a previous paper [1] generalized the randomized response technique to the general case where there were $t \geq 2$ categories each of which could be stigmatizing in varying degrees. The t groups were assumed mutually exclusive and unbiased minimum variance estimates of the multinomial parameters were obtained.

The trinomial case was investigated most extensively in that paper and results showed that selection of the P_{ij} ($i=1, 2$ samples and $j=1, 2, 3$ population groups) for the two samples was not as clear-cut as in the binomial case. The efficiency of estimating π_A, π_B, and π_C fluctuated under differing combinations of P_{ij} and the single useful criterion is, perhaps, to choose that configuration of P_{ij} and n_i which minimizes sampling variance of the most stigmatizing group, say Group A. Such configurations were usually those in which the P_{ij} differed maximally from equality (or one-third in this case), but even here it was not uniformly the best for all three groups. As before, one could not permit P_{ij} to approach too closely to zero or unity without jeopardizing respondent cooperation.

Consider now the application of the unrelated question technique to the trinomial situation. We now have π_A, π_B, and π_C as well as π_Y, where

$$\pi_A + \pi_B + \pi_C = 1.$$

Three samples and three sets of P_{ij} are necessary to make estimates unless π_Y is known in advance. It is unnecessary to associate one of the three stigmatizing groups with a set of P_{ij} since estimation of the frequency of that group, say C, is by subtracting the sum of π_A and π_B from unity.

Thus, the probability of a "Yes" answer in three samples is as follows:

$$\begin{aligned} \lambda_1 &= P_{11}\pi_A + P_{12}\pi_B + P_{13}\pi_Y \\ \lambda_2 &= P_{21}\pi_A + P_{22}\pi_B + P_{23}\pi_Y \\ \lambda_3 &= P_{31}\pi_A + P_{32}\pi_B + P_{33}\pi_Y \end{aligned} \tag{5.1}$$

As before, estimates of the parameters may be obtained by solving for π_A, π_B, and π_Y and inserting the observed values of λ_i, $i=1, 2, 3$. For the equations in (5.1) to have a unique solution, it is necessary that

$$\begin{vmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{vmatrix} = \begin{vmatrix} P_{11} & P_{12} & 1 - P_{11} - P_{12} \\ P_{21} & P_{22} & 1 - P_{21} - P_{22} \\ P_{31} & P_{32} & 1 - P_{31} - P_{32} \end{vmatrix} \neq 0.$$

The optimal selection of the P_{ij}'s and n_i's will not be as easy as in the binomial case because of the above restriction plus minimization of sampling variance of all three π's without jeopardizing respondent cooperation. A comparison of these variances against those in the trichotomy developed in [1] should show, however, that the unrelated question is not penalizing and is probably a gain. If π_Y is known or estimable from an external source, the sampling can be reduced to two samples and the relative gain in efficiency even greater. Detailed investigation of these circumstances do not yet appear warranted until more is known about selecting the P_{ij}'s and n_i's in the Warner trichotomy.

6. APPLICATION OF THE METHOD TO FIELD SURVEYS

Some of the methods proposed in this paper were first experimented with in a small study in North Carolina during the fall of 1965. The design of the study did not follow the present recommendations for selection of π_Y nor for optimal allocation of n_1 and n_2 since these considerations were deduced at a later date.

A sample of 148 households was selected from birth certificates upon which was recorded the marital status of the mother. The births had occurred in August and September, 1965. The statements to which the respondent was supposed to reply "True" or "False" were:

A. "There was a baby born in this household after January 1, 1965, to an unmarried woman who was living here."
Y. "I was born in North Carolina."

The identifying letters, viz., A and Y, were not used in the actual field trial.

Results of the trial showed amazing accuracy in estimating the frequency of illegitimacy among these households. Among the 104 households involving white respondents, the true proportion of illegitimacy from birth certificate data was 7.7%, and the estimated value from the interviews was 7.4%. Among the 44 households involving nonwhite respondents, the true proportion selected was 45.4%. (This high value was not a random sample of birth certificates. The given high frequency had been chosen to observe the performance of the method when π_A was in the neighborhood of 50%.) The estimated value of illegitimacy among the 44 households was 42.3%. In both cases it will be noted that slight underestimation still prevails suggesting that T_A is still not unity.

These results and others have been discussed in more detail in the paper by Horvitz, Shah, and Simmons [3]. In that same field trial, those authors invented another device which further enhances use of the Warner technique. After the respondent had replied to the question selected, she was asked to try it over again. This provided two independent trials per respondent to the same set of questions. Measures of consistency and improved estimates are thereby ascertainable.

Having established some of the theoretical foundations for the randomized response technique, and having viewed its potential usefulness, several of the authors are now studying the application of the method in estimating the incidence of abortion in an open population. Also, several extensions of the technique are being investigated both theoretically and with field data.

One extension is derived from examination of the sampling variances considered in the four cases enumerated in Section 4, i.e., when π_Y is known or unknown in comparison with the binomial and Warner techniques. The Warner procedure uses two questions, A and $\bar{A}$, whose correlation is minus one. The randomized response procedure uses a neutral question whose correlation with A is presumed to be zero, and the sampling variance is less than Warner's. The binomial procedure can be viewed as using two questions, A and A, whose correlation is unity and which has the lowest sampling variance.

If the second question is still a neutral one but correlated with A, one would speculate its sampling variance is intermediate between cases 3 and 4. The theoretical considerations involve estimation of the correlation between A and Y when π_Y is either known or unknown.

A second extension of the technique is to apply the randomized response to quantitative data. The two questions might be:

1. "How much money did you earn last year?"
2. "How much money do you think a man of your age needs to earn in order to support a family of your size?"

This extension results in a distribution which consists of a mean whose expected value is $P\mu_A+(1-P)\mu_Y$, and whose corresponding variances can be derived therefrom. Using two samples, the objective is to estimate μ_A, $\sigma^2{}_A$, and to explore the conditions under which the latter is minimal.

Additional extensions of the method continue to be suggested as work in the field progresses. One can not but speculate that the legendary Pandora's box has been opened with the Warner technique.

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