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UNIVERSITY OF ELECTRONIC SCIENCE AND TECHNOLOGY OF CHINA

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Ž Ä â x,X : !AŽ [ Ž ü Ð Ü Û Ð ßE¯> ,X-è0J ¹ 0 ž ª k,X-è0J ä p Ä B å -¹ ÈL8 Z [ (M ÿ t ¹ Û"¼ `7ÈAö ,X • ê ÈAŽ [ á Ù ÿ J ª Ž Æ4£ ¥>< ê „ mE›,X-è0J ä p È 3 á Ù ÿ 9‹ k+ $/¥ T û : ê J W -6† X,X : ! êA• :5à S*ü E›,X $ m Ä â å Ô à ¹ 0,X à « Í -è0J .,X Ï )Bõ) Æ üAŽ [ 0 Z â.B,XAÈ â J></ Aö ã Ä

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# 摘 要

# ó ûKÂNl Û A× ¹,XKÂNl#] ž /• Ž š,XKÂNl , _ V Ö Ì úG& âPR E: È :*ó ú5×A© 0 Þ È 4‡/â Ž úE×/â1 KÂNl ÄLc ê ²1( T (RRT),X Î)„ S k # ó ûKÂNl,X-è0J Ý Z Ô/¡ Ã • “ ¡ 0 ` r ‘,XA× ¹ •"© È W ü Ý ± x Z «A“5Ù/• š ,X à ÊE¬ ¤P¬ Z «A“5Ù,ó r ²1(KÂNl,X V)[ Ä Ú ‘ ‘ A× ¹ *ü ,X ‘ •"© Ä W r ‘CK 91T ) ç ¡ 0 È8¹ Ú k ' È Ã Ý ¤P¬ Au 2’ z Ä [Oj Ÿ4¡ Z-è0J # ó ûKÂNl,X Lc ê ²1( õ _ Ä J4§ Ü Ú ‘ )ÚAŽA|AŽ Z Lc ê ²1( õ _ ü Ú ‘ ,X h*ü È Au Z Warner õ _ ÃSimmons õ _ ¹ ž E¯ õ _ ü Ú ‘ ß K Ý,X # ó 2 û(M U,X!¨ _ π È4- Î Z – D Au,X><E’ ã Ä

# ó ûKÂNl ,X-è0J T T #] ž /• Ž š , ü>•A× ¹5Ù Í ) Ô,XA× ¹ á*î Z?· ,X ™ ‰ ß ÈA× ¹ k ,ó r Ý µ C ,X Ã6Ñ ûEî !¨EW ã Ä ´!8 üA× ¹ -è0J ! Æ Ý,X Z ÆC m `4£P` ¹ ž,Ì G,X Ý EY } µ C ,X , ü k,Ì 'G¡?U ÄEY } µ C ,X é 9 ` S*ü6Ñ Ý ,X E¯ ` ` X ‘ A’Au • È ¤P¬ Au,X2’ z `8V,Õ ‘ ,X C *ü1 Ä ´!8 È [ ý*üEY } µ C é 9EY } ¬G£ üLc ê>™5B ß Í þ-¹ – D Ú ÿ E¯> !¨ Au ` ² & Au ÄA|AŽ `!¨EW ü ÝEY } µ C Ã ý*ü Ê È!¨ Au ` ² & Au õ _ â s Warner õ _ ` Simmons õ _ ü1T ) ‘ ß,X AuG£ ` • Â ÄEîE› Í!¨ Au ` ² & Au,XLc ê õ _,X Ú d ` )[!¨ EW ¥)„ Ö t 9EY } ¬G£ â È ü$µC‡ Ô n,X 5 Ê ß Ã ¹ ¤P¬ AuG£,X2’ z Ä a í E¬ Ã ¹ ý*ü Æ Ý,XEY } µ C È k Î P` µ C ž P` Ú × ÈE¤*üBñ Ê ƒ4³Au | • Í – DE¯> Au Ä [A|AŽ Z Bñ Ê ƒ •"© ü # ó ûKÂNl,X h*ü Ä ý*ü P` µ C9‹ k P` Ú × È S*üBñ Ê ƒ Au " Î âP` Ú × È ¢5àAu1kÎ AuG£, 3E’ Z Ý ý*ü Æ-¹,XEY } µ C,X,Â,X Ä

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# ABSTRACT

Sensitive questions are some problems relate to private confidential, such as: whether the drivers drunken driving, whether students cheated on exams, whether the taxpayer evaded tax and so on. The appearing of the randomized response technique (RRT) brings an easy and effective method to operate and implement survey, which effectively protects the confidentiality of the respondents as well as improves the probability of respondents' truly answers. Stratified sampling is a commonly used method in the sample survey which is simple and easy to operate. If layered properly, it can effectively improve the precision of parameter estimation. This paper first introduced some randomized response models of sensitive questions, and then discussed the application of these models in stratified sampling. It estimated the proportion of sensitive property characteristics $\pi$ in stratified sampling including of Warner model, Simmons model and improved model, and given the expression of the parameter estimates.

Because the sensitivity problems often involve the private confidential in the study of sensitive questions, the investigators usually gain little information from samples when the thinking of the popularity is not in place. While the information of existing historical data and experience before the start of the investigation becomes very important. Using of auxiliary information in study can effectively refine the sampling design to improve the accuracy of the estimated and save sampling costs. Therefore, this paper pulled in auxiliary variables in randomized device to estimate the unknown parameters respectively by Ratio estimation and regression estimation. Then the article discussed and compared estimator and the variance between the original Warner and Simmons models and Ratio and regression estimation models when the auxiliary information available. And then through the analysis of these random models, the survey finds that under certain conditions. The accuracy of the estimator can be improved by adding an auxiliary variable. Furthermore, through auxiliary information the survey can also obtain prior information and prior distribution, so the paper can use Bayesian statistical inference to estimate the parameters. This paper discussed the application of the Bayesian approach to sensitive issues. We can obtain the prior distribution by Priori information, using Bayesian estimation to find the a posteriori distribution and calculate the estimated amount. This can also reach to the purpose to use known auxiliary information effectively.

**Keywords:** sensitive question, ratio estimator È regression estimator, auxiliary information, stratified sample

# 目 录

# 第一章 绪论

## 1．1 研究历史背景介绍

M6 Í á ,X4³AuA× ¹ ÍB5 È ‘ A× ¹ )„ žG¡?U,X Ô/¡A× ¹ •"© Ä W Û ¢ ' ‘ª E¯> A× ¹ È B ,XA× ¹4§ p | • ` 1k Î ',X(M U,X Ô/¡ ‘ T Ä1895 H Karier ü*2 ¿4³Au îA, ÞOj é 9 ‘ ,X V É ÈLc â ü ` ê4~ ñ ·>< ûA× ¹ E- ÔAÈ"© 3 Eä#ä>•Ž À y « Ä1909 H8Å Ñ4³Au : ŠRa 9… Î( Z É4³Au : Î. -/ß ÊÈ ü „ t 9,X ‘ "© Ô0´ Ÿ4¡ Z ‘ T üF¼ ÚNZ ³,X h*ü Ä 20 ê4~ 30 H · È2Ï4³,X ‘ )ÚAŽ ` •"© Ô Ÿ 6 ä ` ¥ ) ÄLc â Lc ‘ Ã á1 V)[ ‘ Ã2Ï4³ ‘ Ã Ú ‘ Ã H5x ‘ ` îL !‰ ‘ 1 î /¡ ‘ •"©)ÚAŽ#ä#ä ¥ ) ` ä's Ä V ž È ‘ T ,X ¥ ) Æ ,Ì ' ,X ä's ` ` X Ä ‘ A× ¹ ü Ž2O*ó# ,X _ $!¨!¨,Z È V Í Ô þ/ !å Ü*ü+ ™ ‰ ,XA× ¹ × Í å Ñ!£ H Ô õ,X < Ñ Ž · B ¹ × Í å Ñ á à ³,X4£#" ¥ )A× ¹ ¹ ž Í Ø/¡/ î çNlA× ¹ `!å ãA× ¹1 1 Ä 3 ´!8 ‘ A× ¹ ü/ î rCÉ K ÝM2 µ,X B F! û ` h*ü û Ä

å Ñ Í ‘ T ,X-è0J Ÿ b 20 ê4~ 30 H · ÈJ b 1943 H r)„ Z å Ñ1 Ô õ ‘ A× ¹ Ä<Q' å Ñ Í J-è0J,X!¨EW . È 4£E› ´ H,X ~ o È å Ñ,X ‘ T !7Eä#ä â ê+ " G yE< Ä

ü/ î rCÉ ‘ A× ¹ Ž À î.„ E- Ô o#] ž þ Ž ê ) !Ld/•,XKÂ Nl È V 4 ú5×A© Ê 0E› Þ È 4 úG& â PRPÆ ´A•PRE: È 4 ú Ý – âC . ê !¦1 á8C ° Q È 4 ú ÝE×/â$ã/â ,X ™ ‰1 KÂNlE-2OKÂNl¿ . # ó ûKÂNl Ä # ó ûKÂNl Ã ¹ Í ' Í K Ý # ó û(M U,X!¨ _,X-è0J È _ V ŠKS ú Ý<$ Y 0¹ ê ˆ o å È5×*ó ú Ý 0 Þ> È4‡/â Ž ú Ý K/â$ã/â > 1 × 3 Ã ¹ Í # ó ûKÂNl DNq î å,X-è0J È !¨ V 4‡/â Ž K/â$ã/â ,X DNq î å È Ô H ÊKÈ Y Ì E±?˜PRE:,X õ D î å È û :*ó ü õ óKÈ ( õ D 1 KÂNlÄ

Í b # ó ûKÂNl È V pA× ¹5Ù,È y ¤KÂ ÈFw «A“5Ù T T î ´ !82OKÂNl#] ž Ld/•E› b # ó5à á ã ²1( ê 0 Î á,ó r ²1( Ä FS !A× ¹ # ó ûKÂNl Ê ² 1(4§ p Î)„ # Â È Ô8 LÔ?U ÍA× ¹A'AuE¯> E¯ È S kKÂNl,X # ó/ß z#µ ê ê5Ù ¬ k á a # ó Ä 3 Ã ¹ é 9 Ô/¡ „,X E¯ 9#\L8 «A“5Ù,XNR<% È SA× ¹ k ¹

NN ýE¯> Ä

### 1.1.1 随机化回答技术（RRT）

<Q' # ó ûKÂNl ,XA× ¹ ´ D BL' ¹9‹ ª5à-è0J ÄL' È Ž À 6 6 ¥)„ Ô/¡ Ý ,X -è0J •"© È G ÖLc ê ²1( T Ä RRT ÅÄLc ê ²1( Û üA× ¹ S *ü(M n,XLc ê>™5B ÈE-/¡>™5B Ã ¹ ± x «A"5Ù,XLd/• `/¬ š È ¢5à S k «A"5Ù Ã ¹ ´NR<%,X Î,ó r ²1( ÄLc ê ²1( T 1965 H ü"W4‡[1]（*Warner*）Oj ¤ Î Z"W4‡ õ _,X Î. Þ ! ¥ )CK 9,X Ä ª ¹NX n ,X V)[A'Au Z ø þ Í0Ÿ,XKÂNl Ä $A$ ` $\bar{A}$ Åõ _ ÈJ Í K Ý # ó û(M U （2 b $A$）,X Ž ü ' 4,X!¨ _ $\pi_A$ ` •G£ $n$ . Î Z Au Ä1967 H?S:m $f$[1]ÄSimmons Å ü"W4‡,X Î. Þ ¤ Î Z?S:m $f$ õ _ È*ü ´ G,XEY }KÂNl Y · Ó Z"W4‡ õ _ ,X # ó ûKÂNl A ,X Í0ŸKÂNl Ä L! " Z>•A× ¹5Ù Í # óKÂNl,X # ó/ß z , £ å Z «A"5Ù,XNR<% È Ý ý b Ü 0 Ä y- 1973 H k 1 ŽJ\ Í?S:m $f$ õ _ $\pi_y$ þ-¹,X ™ ‰ ¤ Î5×<% â Ô þ # ó û(M U $A$ ø þ ´ G,XM2 # ó û(M UEY }KÂNl $Y_1$ ` $Y_2$ È>•A× ¹5Ù,È y ²1( $Y_1$ ` $Y_2$ ,X Ï ã Ô þ KÂNl ÈLc ê ²1( A ê = ß,X ´ GKÂNl Ä S k>•A× ¹5Ù,XLd/• k ¹ ± x È à Ê œ ¤P¬ Z2'.B û ÄLc â ÈLc ê ²1( T E¯ Ô!9,X E¯ ` ¤P¬ È Mangat ` Singh[2-3] 1 Ž Í "W4‡ õ _,XLc ê ²1( ¤ Î Z E¯ õ _ È Ralph Ã Greenberg[4]1 Ž í Í?S :m $f$ õ _E¯> Z E¯ È ¤P¬ Au2' z Ä

Lc ê ²1(E- Ô ñ Ç •"© ü Ñ ê ,X/ îA× ¹ Å k S"¯ h*ü Æ¥o H # ó ûKÂNl,XA× ¹ ü Ñ Y 3 Eä#ä« ZG¡?š Ä- E"Ñ Ã- â [5-6]1 Oj ü Ñ Y ¥>< Z Ý GLc êA× ¹ •"©,X,Ì G [0´ È4Ð Ü Z 2 û(M U # ó ûKÂNlLc êA× ¹ •"© Ä y- Ú # ó ûKÂNl,XLc êA× ¹ ¢ Ô Ô Ÿ,X `NMEÝ ½ | S îNM Û ½,X ™ ‰ [7]È 3 ¢Eä#ä+ ) ¥ ) î õ _ [8]Ä B Í ' A× ¹,X rL ™ ‰ È ¤ Î Z DG£(M U,X # ó ûKÂNl,XLc þ õ _ [9], Ú K ',X D A× ¹E@ ê 2 ûA× ¹ 1 •"© È ¢5àFS ! Z ²1( # ó û(M U,X K ' D ÄLc â Ñ Y :5Ù Í # ó ûKÂNl-è 0J,X ŽEä#ä r î ÈL 4Á Î)„ Z ( ÷ ` 3*ó Î,X t"© õ _ [10]È ,"© õ _ [11]Ä â 9 E¤*üLc ê ²1( T ?· ‡ Í DG£(M U # ó ûKÂNl,XA× ¹)ÚAŽ 3C^ 9C^ î ÄNRLÛ)$f$[12] ÚLc G4§ õ _ h*ü b"W4‡ õ _ `?S:m $f$ õ _ 6 ä ZLc "W4‡ þ õ _ `Lc ?S:m $f$ þ õ _ È á Ý ,X ¤P¬ ZA× ¹,X ± š û È 3 ¤P¬ Z AuG£,X2' z Ä%Â k ó [13]Ãì7¾J?[14]1 Ž í ÚLc ê õ _+ Ô þ # ó ûKÂNl ø þ*î7Ç î þ # ó û KÂNl õ _ Ä

## 1.1.2 复杂抽样下的敏感性问题

E¥´ H È\ î Ñ Y ê-è0J Ž , Ô Ÿ ^ # óKÂNl â rL ,X ‘ )ÚAŽ4§ ÜCK 9 JE¤ *ü á ‘ A’Au ' Ä Z ¤P¬2’ z È "þ « # ,K¿ ü ü[15] Ú"W4‡ õ _ h*ü b Ú ‘ ÄAö G ` ` )_ ` î Ï 16 Ð ¤ Î Z Ú ‘ ß,X?S:m ƒLc ê ²1( õ _ ÄP¬! ` 8×)]"¶ [17–18] í Ú"W4‡ õ _ h*ü Ú H5x ‘ ß È 8C 9 Ï 19 Ð í A|AŽ Z DG£(M U ,X # ó ûKÂNl ü Ú H5x ‘ ß,X ™ ‰ È ª ÀFÑ*ü r _ . Z Ú d Ä × ` Ã Um Ÿ[20] Ú ) ÔL !‰ ‘ ß,X Warner Lc ê ²1( õ _ | S `L !‰ ‘ , • Â ž J • Â AuvA| Z '!¨ _,X ´ # Au , • Â ž J • Â AuG£ ÄCl ž ‹ [21] í = ) H5x ‘ Ä

E- ,X-è0JE¬ Ý Ö # óKÂNl E¯,XLc h1( T õ _ Ú H5x ‘ -è0J ž h *ü[28], EY ¹ Ú ‘ ,X1 •G£ H5x ‘ •"© [29], î Ú ‘ G£,X ÚG! [30] È DG£(M U # óKÂNl Ú øL !‰ H5x ‘ ,X4³Au •"© [31]1 Ä

## 1.1.3 敏感问题辅助信息的使用

# ó ûKÂNl ,X µ CG£Eî !¨EW ã,X Ä 'A× ¹#] ž þ ŽLd/• ž Ž A‡ Ã î š êN¢ « [ ê Ê È Î bNR<% ` ™ — È \ î Ž T T ± Õ"]T¬ 5àá ã ²1( Ä 1u üE› • í # óKÂNl,X-è0J Î)„ ZAŒ îLc ê ²1(>™5B È EY } µ C,X ý*ü üE- þNZ ³ H \ å#] ž ` h*ü Ä,Â ! Æ Ý,X-è0J,X Ý Ö TT##=` K¿ ü ü Ï 21 Ð1 Ž ü-è0J # óKÂNl Ê t 9EY } µ C È ý*ü!¨ Au ` ² & Au È E¯ Au •"© È ¤P¬ Au2’ z ` )[ Ä ô Ï 22 Ð üEY } µ C þ-¹,X ™ ‰ ß ý*ü `NM ‘ 1k ÎEY } ¬G£A’Au Z Lc êA× ¹,X!¨ Au õ _ ÄEY } µ C 0 ý*üA©P` !9‹ k,X4£P` ` Z ÆC m k ,X P` µ C È 3 Î)„ Z Ô2Ï ë,XBñ Ê ƒ | • ÄWinkler ` Franklin Ï 23 ÐÈ Kim ` Tebbs Ï 24–25 ÐÈZawar Hussaina ` Migon [26–27] Í Warner ž E¯ õ _ é 9Bñ Ê ƒ •"© , | Ð Î E¥ Au ÄUnnikrishnan ` Kunte Ï 28 Ð í ü"W4‡ `?S:m ƒ,X4³ Ô õ _ é 9Bñ Ê ƒ È S*ü Ý × ƒ ‘ •"© ` MCMC •"© 1k – D ÄL L¾ V[29] Ã é0Ã î ` Y)£*ó Ï 30 ÐÃ d ù Y ` é ) Ï 31 Ð1 Ž ÚBñ Ê ƒ Au h*ü b Simmons ž J E¯ õ _,X – D AuÄ ,Â ! !6Bñ Ê ƒ Au üE- + # ó ûKÂNl,X-è0J â h*ü,Ì ÍE¬ \ å,X Ä

## 1.2 本文研究工作

[,X-è0J Y • ?U Ù À Ö

Ä1 Å Ÿ4¡ # óKÂNl,X ,Ì G V É È £EÄ Ñ Y ê Í!8KÂNl,X)ÚAŽ-è0J Ä

Ä2 Å Ÿ4¡Lc ê ²1( T Ä RRT Å ` L _,X Lc ê ²1( õ _ È ¹ ž E¯,X,Ì G õ _ È à Ê4- Î!¨ Au È ² & Au `Bñ Ê ƒ Au,X,Ì G)ÚAŽ-¹Aš Ä

Ä3 ÅA|AŽ Ú ‘ ß !¨ Au ` ² & Au È ¹ ž"W4‡ õ _ È?S:m ƒ õ _ ` E¯ ,X ´ G õ _ ü Ú ‘ ß,X h*ü Ä

Ä4 Å Ú!¨ Au ` ² & Au h*ü "W4‡ õ _ `?S:m ƒ õ _ ÈA|AŽ Î W À,X AuG£ ` • Â È Ô âEîE› )[!¨EW.B n ü Ô n 5 Ê ß2’ z Ý ¤P¬ Ä

Ä5 ÅÚ # ó ûKÂNlEîE›Bñ Ê ƒ Au Î – D AuG£ È 4- Î – DA’ n `EÝ ª Û š Ä

# 第二章 典型的随机化回答模型和贝叶斯估计

## 2.1 随机化回答模型

A× ¹ # óKÂNl Ê S*üLc ê>™5B È ¹NX A’Au Q,X V)[ A}>•A× ¹5Ù ²1( # ó KÂNl ÈFw >•A× ¹5Ù,XLd/• 6Ñ k ¹ ± x È ¢5à9‹ ª «A“5Ù,X µ Ï È k š.B,ó r,X D B Ä

### 2.1.1 沃纳（Warner）模型

1 Å Ý ² 1T ) ‘ ß,X"W4‡ õ _

1965 H"W4‡Warner) Oj ¤ Î Z1 Ô þLc ê õ _ ü ü "W4‡ õ _(Warner model) Ä 1T )Lc Ý ² ‘ ß ‘ ª •G£ $n$ ,X È A’Au ø þ Í0Ÿ,X # ó ûKÂ Nl $A$ ` $\overline{A}$ È ÍE- $n$ þ E¯> Lc ê ²1(A× ¹ È ¹!8 9 Au ' K Ý # ó 2 û ,X Ž 4,X!¨ _ $\pi_A$ ÄA’Au V ßLc ê>™5B Öš Û ø + ` < Ô ,X 5( È Ô + 5( Þ m 4 2 b $A$ ë Û È º Ô + 5( Þ m 4 2 b $\overline{A}$ ë Û È Ú ø + 5( ¹A’ n,X!¨ _ $p$ ` $1-p$ #Ë Ü Ô â 9 à Ô,¦ $ È>•A× ¹5ÙLc ‘ ª 5( È J B 5( Þ,X Y • ²1( â ú ÈA„ ß1( â Ú 5( ²,¦ $ ÄE- >•A× ¹5Ù Ã ¹ —,X ²1( È5à á*ü ™ — î ˆM Ld/• Ä

A’ _ Ê $A$ # óKÂNl È $\overline{A}$ # óKÂNl $A$,X Í0Ÿ _ Ê È $p$ m Ý 4 2 b $A$ ë Û ,X 5( ,X!¨ _ È $\pi_A$ ' K Ý # ó û(M U,X Ž 4,X!¨ _ Ä

A„ $X_i = \begin{cases} 1\text{È1 } i\text{ þ>•A× ¹5Ù ²1(} \\ 0\text{È1 } i\text{ þ>•A× ¹5Ù ²1( ú} \end{cases}$ $i = 1, 2, 3, \quad , n$ Ä

$\hat{\lambda}$ ²1( ,X!¨ _ È è Ý $\hat{\lambda} = \frac{1}{n}\sum_{i=1}^{n} X_i$ Ä $\lambda$ ' ²1( ,X !¨ _ Ä ü Ý>•A× ¹5ÙFÑ,ó r ²1(,X ! ¤ ß È+ < V)[ @ ã Ý Ö

$\lambda = p\times\pi_A + (1-p)(1-\pi_A)$ È $1-\lambda = (1-p)\times\pi_A + p(1-\pi_A)$

´!8 Ý $\hat{\pi}_A = \frac{[\hat{\lambda}-(1-p)]}{2p-1}$ È $(p \neq \frac{1}{2})$ Ä é)Ú 2. 1 Ï 32 Ð Ö ü Ý ²1T )Lc ' ß ÈA' ' •G£ $N$ È G£ $n$ È ' LÔ?U Au,X ' K Ý ¤ Ô(M U,X þ ' ) !!¨ _ $\lambda$ Ê È $M$ ' K ÝA¹(M U ,X þ ' D È ¸

$$Z_i = \begin{cases} 1, & 1\ \text{þ}\ i\ \text{þ ' ) K ÝA¹(M U} \\ 0, & 1\ \text{þ}\ i\ \text{þ ' ) á K ÝA¹(M U} \end{cases} \qquad i = 1, 2, 3, \quad , n$$ Ä

+ 1T ) Au"© È K ÝA¹(M U,X þ D!¨ _ $\hat{\lambda}$ $\lambda$ ´ # AuG£ È J • Â

$$V\left(\hat{\lambda}\right) = E(\hat{\lambda}-\lambda)^2 = \frac{N-1}{N}\frac{S_Z{}^2}{n} = \frac{N-1}{nN}\frac{[\sum_{i=1}^{N} Z_i^2 - \frac{1}{N}(\sum_{i-1}^{N} Z_i)^2]}{N-1}$$
$$= \frac{1}{n}\frac{1}{N}(M - \frac{1}{N}M^2) = \frac{1}{n}\frac{M}{N}(1-\frac{M}{N})$$
$$= \frac{1}{n}\lambda(1-\lambda)$$

J $S_Z{}^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Z_i - \bar{Z})^2$ Ä

n)Ú 2. 1 ÖA' ' •G£ $N$ , *ü1T )Lc ' ² •"© ' ª •G£ $n$ ,X , *ü"W4‡ õ _>™5B Ã-¹ Ö$\hat{\pi}_A$ $\pi_A$ ,X ´ # Au È è $\hat{\pi}_A$ ,X • Â

$$V(\hat{\pi}_A) = \frac{\pi_A(1-\pi_A)}{n} + \frac{p(1-p)}{(2p-1)^2 n}$$

A• â Ö $E\left(\hat{\pi}_A\right) = \frac{E[\hat{\lambda}-(1-p)]}{2p-1} = \frac{1}{2p-1}[\frac{1}{n}E(\sum_{i=1}^{n} X_i) - (1-p)]$

$$= \frac{1}{2p-1}[p\pi_A + (1-p)(1-\pi_A) - (1-p)]$$
$$= \pi_A$$

¹ $\hat{\pi}_A$ $\pi_A$ ,X ´ # Au Ä

œ+ é)Ú 2. 1. 1 • Â $V(\hat{\pi}_A) = \frac{V\left(\hat{\lambda}\right)}{(2p-1)^2} = \frac{1}{(2p-1)^2}\frac{N-1}{N}\frac{S_Z{}^2}{n}$

$$= \frac{1}{(2p-1)^2 n}\lambda(1-\lambda) = \frac{\pi_A(1-\pi_A)}{n} + \frac{p(1-p)}{(2p-1)^2 n}$$

¢ Þ ã Ã ¹,ß Î , ' $p$ C^C_E¥ b$\frac{1}{2}$ ÈG ø + 5( !¨ _,Ì Â á û Ê È $\hat{\pi}_A$ ,X • Â C^ û Ä ' $p$ C_E¥ b0 ê 5Ù1 Ê È G Ô + 5( !¨ _ # û Ê È $V(\hat{\pi}_A)$ !¨ EW ãÄ $p$ y E¥$\frac{1}{2}$ ÈLc ê>™5B,X 0*ü á û Ä' $p$ yE¥ 0 ê 1 Ê ÈÍ>•A× ¹5Ù,X ± x z î £ È ,È yL! " Z >•A× ¹5Ù,X Ü 0 z ÈLc ê ²1( T 3 "u Ý Z ã Ä Ô8 $p$ ,X ª ü $0.7 \sim 0.8$ KÈ Ä

' $\pi_A = \frac{1}{2}$ Ê È $\pi_A(1-\pi_A)$E' Ô û ÈNX 4- n2' z ã b $\varepsilon$ È í

$$V(\hat{\pi}_A) = \frac{\pi_A(1-\pi_A)}{n} + \frac{p(1-p)}{n(2p-1)^2} \le \frac{p(1-p)}{(2p-1)^2 n} + \frac{1}{4n} \le \varepsilon$$

$$\Rightarrow n \ge \frac{p(1-p)}{(2p-1)^2 \varepsilon} + \frac{1}{4\varepsilon}$$

´!8 È ü2' z ÞL$ $\varepsilon$ ,X ™ ‰ ß È G£ $n \ge [\frac{1}{4\varepsilon} + \frac{p(1-p)}{(2p-1)^2 \varepsilon}] + 1$ Ä

2) ´ ² 1T ) ' ß,X"W4‡ õ _

"W4‡ õ _ ü1T )Lc Ý ²,X ' ßE¯> A× ¹ È' 5à È ü,Ì à G£ ß È ²1T )Lc ' ,X • Â û b á ² ' ,X • Â È ´!8 [ 3A|AŽ"W4‡ õ _ ü1T )Lc ´ ² ' ß,X ™ ‰ Ä ' ¢ ' á ² ' ª ) E¯> Lc ê ²1( A× ¹ Ê È å À Ý ´ ² ã,X"W4‡ õ _ V ß Au Ö

é)Ú 2. 2Ï 32 ÐÖ ü ´ ²1T )Lc ' ß ÈA' ' •G£ $N$ È G£ $n$ È ' LÔ?U Au,X ' K Ý ¤ Ô(M U,X þ ' ) !!¨ _ $\lambda$ Ê È ¸

$$Z_i = \begin{cases} 1, & 1\ \text{þ}\ i\ \text{þ ' ) K ÝA¹(M U} \\ 0, & 1\ \text{þ}\ i\ \text{þ ' ) á K ÝA¹(M U} \end{cases} \quad \text{È}\ i = 1, 2, 3, \quad , n$$

$M$ ' K ÝA¹(M U,X þ ' D Ä Ý1T ) Au"© È K ÝA¹(M U,X þ D!¨ _ $\hat{\lambda}$ $\lambda$ ´ # AuG£ È J • Â

$$V(\hat{\lambda}) = \frac{1}{n}(1-\frac{n}{N})S^2 = \frac{1}{n}(1-\frac{n}{N})\frac{[\sum_{i=1}^{N} Z_i^2 - \frac{1}{N}(\sum_{i-1}^{N} Z_i)^2]}{N-1}$$

$$= \frac{1}{n(N-1)}(1-\frac{n}{N})(M - \frac{1}{N}M^2) = \frac{1}{n}\frac{N-n}{N-1}\frac{M}{N}(1-\frac{M}{N})$$

$$= \frac{1}{n}\frac{N-n}{N-1}\lambda(1-\lambda) \approx \frac{1}{n}(1-f)\lambda(1-\lambda)$$ Ä ' N Ú û Ê Å

n)Ú 2. 2 ÖA' ' •G£ $N$ , *ü ´ ²1T )Lc ' •"© ' ª •G£ $n$ ,X , *ü"W4‡ õ _>™5B Ã-¹ Ö$\hat{\pi}_A$ $\pi_A$ ,X ´ # Au È è $\hat{\pi}_A$ ,X • Â

$$V(\hat{\pi}_A) = \frac{N-n}{N-1}[\frac{\pi_A(1-\pi_A)}{n} + \frac{p(1-p)}{n(2p-1)^2}] \approx (1-f)[\frac{\pi_A(1-\pi_A)}{n} + \frac{p(1-p)}{n(2p-1)^2}]$$ È J

$f=\frac{n}{N}$ ‘ !¨ Ä

A• â Ö+ é)Ú n)Ú 2.1.2 çA• Ä

Ô8 ' ' •G£ $N\to\infty$ \ û Ê È!8 Ê Ý ² ‘ â ´ ² ‘ Ý,Ì à,X • Â Au È G )[,Ì1 Ä

## 2.1.2 西蒙斯（Simmons）模型

1 Å ² 1T ) ‘ ß,X ?S:m $f$ õ _

1967 H?S:m $f$ ü"W4‡ õ _,X Î. ÞE¯> Z E¯ Ä?S:m $f$ Ú"W4‡ õ _ $A$,X Í 0ŸKÂNl 6 ä M2 # óKÂNl$B$ Ä V Ö # ó ûKÂNl 4 ÝE›G& âPRE: ë ü"W4‡ õ _ ,X Í0ŸKÂNl 4"u ÝE›G& âPRE: ë È5à?S:m $f$ õ _ Ã ¹*ü ´ GKÂNl 4 2<" ë 9 · Ó Ä A’Au V ßLc ê>™5B Öš Û ø + ` < Ô ,X 5( È Ô + 5( 1 Þ m 4 2 b $A$ ë Û È º Ô + 5( 2 Þ m Ý 4 2 b $B$ ë Û È Ú 5( 1 ` 5( 2 ¹A’ n,X!¨ _ $p$ ` $1-p$ #Ë Ü Ô â 9 à Ô,¦ $ È>•A× ¹5ÙLc ‘ ª 5 ( È J B 5( Þ,X Y • ²1( â ú ÈA„ ß1( â Ú 5( ²,¦ $ Ä

A„ Ý # ó ûKÂNl $A$ È$\pi_A$ ' K Ý # ó û(M U,X Ž 4,X!¨ _ È $\pi_B$ ' K ÝM2 # óKÂNl$B$ 2 û (M U,X!¨ _ È Æ-¹,X Ä$p$ ‘ # óKÂNl 4!¨ _ × $n$ •G£ È $m$ ²1( ,X Ž D Ä$\lambda$ ' ²1( ,X!¨ _ × $\hat{\lambda}=\frac{m}{n}$ ²1( Ž D,X V)[ È

A„ $X_i=\begin{cases}1\text{ È1 } i\text{ þ>•A× ¹5Ù ²1(}\\0\text{ È1 } i\text{ þ>•A× ¹5Ù ²1( ú}\end{cases}$ È $i=1,2,3,\ \ ,n$

í Ý $\sum_{i=1}^{n}X_i=m$ Ä ü Ý ² ‘ ã Ý>•A× ¹5ÙFÑ,ó r ²1(,X ! ¤ ß È+ < V)[ @ ã k Ö$\lambda=p\times\pi_A+(1-p)\pi_B$ $1-\lambda=(1-\pi_A)p+(1-p)(1-\pi_B)$

¹ Ý $\hat{\pi}_A=\frac{[\hat{\lambda}-(1-p)\pi_B]}{p}$ Ä

+ b?S:m $f$ õ _,X ´ GKÂNl Ú Æ-¹ ` þ-¹ ø/¡ ™ 6 È Æ-¹,X ™ 6 ß!¨ þ -¹ ™ 6 2’ z?UP¬ È ¹ È [ ¾A|AŽ ´ GKÂNl$B$ Æ-¹,X ™ 6 Ä

n)Ú 2. 3 ÖA’ ' •G£ $N$ , *ü ²1T )Lc ‘ •"© ‘ ª •G£ $n$ ,X , *ü?S:m $f$ õ _>™5B Ö $\hat{\pi}_A$ $\pi_A$ ,X ´ # AuG£ Ä J • Â

$$V(\hat{\pi}_A)=\frac{\pi_A(1-\pi_A)}{n}-\frac{(1-p)(\pi_A+\pi_B-2\pi_A\pi_B)}{pn}+\frac{(1-p)^2\pi_B(1-\pi_B)}{p^2n}$$

A• â Ö $E(\hat{\pi}_A)=E\frac{[\hat{\lambda}-(1-p)\pi_B]}{p}=\frac{E(\hat{\lambda})-(1-p)\pi_B}{p}$

$$=\frac{\lambda-(1-p)\pi_B}{p}=\frac{\pi_A p+(1-p)\pi_B-(1-p)\pi_B}{p}$$

$=\pi_A$ Ä

¢5à È $\hat{\pi}_A$ $\pi_A$,X ´ # AuG£ Ä

$$V(\hat{\pi}_A)=\frac{[\hat{\lambda}-(1-p)\pi_B]}{p}=\frac{V(\hat{\lambda})}{p^2}=\frac{\lambda(1-\lambda)}{p^2n}$$

$$=\frac{1}{p^2n}[p^2\pi_A(1-\pi_A)+p(1-p)(\pi_A+\pi_B-2\pi_A\pi_B)+(1-p)^2\pi_B(1-\pi_B)]$$

$$=\frac{\pi_A(1-\pi_A)}{n}+\frac{(1-p)(\pi_A+\pi_B-2\pi_A\pi_B)}{pn}+\frac{(1-p)^2\pi_B(1-\pi_B)}{p^2n}$$ ÄA•!© Ä

+ ¹ Þ1 ã Ã ¹,ß Î Ö ' $p$ C^û È $\hat{\pi}_A$,X • Â C^ ã Ä ' $p$ C_ b 1 Ê È $\hat{\pi}_A$,X • Â E' Ô ã È Lc ê>™5B 0*ü 3 È A× ¹ ÚL' ¹ ) Ô Ä $p$ C_ b 0 Ê È • Â $V(\hat{\pi}_A)$ \ û È2' z \ Â ÄÔ8 $p$ ü 0.7 0.8 KÈ ª!¨EW ÜEÖÄ'4- nA× ¹2' z $\varepsilon$ Ê È+ b $\lambda(1-\lambda)\le\frac{1}{4}$ È G£ $n\ge[\frac{1}{4p^2\varepsilon}]+1$ Ä

2 Å ´ ² • ã ' ß,X ?S:m $f$ õ _

n)Ú 2.4 ÖA' ' •G£ $N$ , *ü ´ ²1T )Lc ' •"© ' ª •G£ $n$ ,X , í?S:m $f$ õ _ Au Ö $\hat{\pi}_A$ $\pi_A$,X ´ # AuG£ Ä J • Â

$$V(\hat{\pi}_A)=(1-f)[\frac{\pi_A(1-\pi_A)}{n}+\frac{(1-p)(\pi_A+\pi_B-2\pi_A\pi_B)}{pn}+\frac{(1-p)^2\pi_B(1-\pi_B)}{p^2n}]$$

J $f=\frac{n}{N}$ ' !¨ Ä

A• â Ö+ é)Ú 2.1 ` n)Ú 2.3 ç A• Ä

### 2.1.3 改进模型

A' Ý Ý # ó û ûKÂNl $A$ È$\pi_A$ ' K Ý # ó û(M U,X Ž 4,X!¨ _ È $\pi_B$ K ÝM2 # óKÂNl$B$ 2 û Ž,X!¨ _ È Æ-¹,X ÄLc ê>™5B + E- Ô + 5( 4˜ ä Ö

A’Au V ßLc ê>™5B ŐÝ + ` < ,Ì à ,X 5( È Ô + 5( 1 Þ m 4 2 b $A$ ë Û È Ô + 5( 2 Þ m Ý 4 2 b $\overline{A}$ ë Û Ô â Ô + 5( 3 Þ m Ý 4 2 b $B$ ë Û È Ú 5( 1 È5( 2 ` 5( 3 ¹A’ n,X!¨ _ $p_1$ È$p_2$ ` $p_3$#Ë Ü Ô â 9 à Ô,¦ $ È $p_1+p_2+p_3=1, p_1>p_2$ È>•A× ¹5ÙLc ‘ ª 5( È J B 5( Þ,X Y • ²1( â ú ÈA„ ß1( â Ú 5( ²,¦ $ Ä

$$\text{A„} X_i = \begin{cases} 1 \text{È1 } i \text{ þ>•A× ¹5Ù ²1(} \\ 0 \text{È1 } i \text{ þ>•A× ¹5Ù ²1( ú} \end{cases} \quad \text{È } i=1,2,3,\quad,n \text{ È}$$

¸ $\sum_{i=1}^{n} X_i = m$ Ä ü Ý ² ‘ ã Ý>•A× ¹5ÙFÑ,ó r ²1(,X ! ¤ ß È+ < V)[ @

ã k $\lambda = p(X_i=1) = p_1\times\pi_A + p_2(1-\pi_A) + p_3\pi_B$

¹ ç-¹ $\hat{\pi}_A = \dfrac{\hat{\lambda}-p_2-p_3\pi_B}{p_1-p_2}$ $\pi_A$,X ´ # Au Ä $p_1>p_2$ ÅÄ

è Ý $V(\hat{\pi}_A) = \dfrac{V(\hat{\lambda})}{(p_1-p_2)^2} = \dfrac{\lambda(1-\lambda)}{(p_1-p_2)^2 n}$

$$= \frac{\pi_A(1-\pi_A)}{n} + \frac{p_3(\pi_A+\pi_B-2\pi_A\pi_B)}{(p_1-p_2)n} + \frac{p_3^2\pi_B(1-\pi_B)+p_2(p_1+p_3)}{(p_1-p_2)^2 n}$$

' $p_2=0$ Ê È í Ý $p_3=1-p_1$ È E¯,X õ _ ¬ ä?S:m $f$ õ _ È ´!8 È?S:m $f$ õ _ !8 õ _,X(M!^ ™ ‰ Ä

' $p_3=0$ Ê È í Ý $p_3=1-p_2$ È E¯,X õ _ ¬ ä "W4‡õ _ È ´!8 È "W4‡õ _ 3 !8 õ _,X(M!^ ™ ‰ Ä

+ Þ ã Ã-¹ È?U $V(\hat{\pi}_A)$ \ ã È í $p_1-p_2$ hA¹EW û È ¹ $p_1$?U û È $p_2$?U ã È ¢5à $p_3=1-p_1-p_2$ 3?UEW ãÄ

'4- nA× ¹2’ z $\varepsilon$ Ê È+ b $\lambda(1-\lambda)\le\dfrac{1}{4}$ È G£ $n \ge [\dfrac{1}{4(p_1-p_2)^2\varepsilon}]+1$ Ä

à)Ú È ü ´ ² ‘ ß È

$$V(\hat{\pi}_A) = \frac{V(\hat{\lambda})}{(p_1-p_2)^2} = \frac{1-f}{n}\frac{\lambda(1-\lambda)}{(p_1-p_2)^2}$$

$$= (1-f)[\frac{\pi_A(1-\pi_A)}{n} + \frac{p_3(\pi_A+\pi_B-2\pi_A\pi_B)}{(p_1-p_2)n} + \frac{p_3^2\pi_B(1-\pi_B)+p_2(p_1+p_3)}{(p_1-p_2)^2 n}]$$

## 2.2 贝叶斯估计

å À ü Í # ó ûKÂNlE¯> -è0J Ê È 'A× ¹#] ž þ ŽLd/• ž Ž A‡ Ã î š êN¢ « [ ê Ê È Î bNR<% ` ™ — È \ î Ž T T ± Õ"]T¬ È á ã ²1( È5à k ,X µ CG£Eî !¨EW ã,X Ä ´!8 -è0J # ó ûKÂNl9‹ k D B,XA©P` !9‹ k,X 4£P` ` Z ÆC m k,Ì 'G¡?U Ä ü ' | • Ý Ý/¡ µ C Ö ' µ C È µ C ` P` µ C Ä5à P` µ C ü ' ! , ü,X Ô/¡ µ C È Ô8 rP` ! ,X Z ÆC m ê5Ù D B È4£E› Ú d È H)Ú ` t ¹ Ã ¹ k – D,X Ý*ü µ C Ä Ú P` µ C ý*üCK 9-è0J # ó ûKÂNl ü,Â ! 9AÈE¬ \ \ å,X Ä ,Â ! !6Bñ Ê ƒ • "©,X h*ü ü Ñ êEW î ü"W4‡ ž J E¯ õ _ ßE¯> | Ð È V Winkler ` Franklin È Migon ÈKim ` Tebbs 1 Ä5à Ñ Y :5Ù é0Ã î ` Y)£*ó Í?S:m ƒ õ _ – DE¯> ZBñ Ê ƒ Au J*ü Ú ' ÞM6 • ÄL L¾ V Ã d ù Y ` é ) í ÚBñ Ê ƒ •"© h*ü E¯,X?S:m ƒ õ _ ÄBñ Ê ƒ •"© Ý ,X ý*ü P` µ C Í þ-¹ – DE¯> Au È = û Z Æ-¹,X Ý ,X µ C ý*ü)[ Ä

Lc ¬G£ X ,X š z Ñ D $p(x;\theta)$ ÈLc ' ª Ô þ $X_1,\cdots,X_n$ È B P` µ C k Î Y – D $\theta$,X P` Ú × $\pi(\theta)$ ÈE-G Ô8 EÝ*üBñ ( Ú ×

$$\pi(\theta)=B(a,b)^{-1}\theta^{a-1}(1-\theta)^{b-1},0\le\theta\le 1,a<0,b>0$$ Ä

J $$B(a,b)=\int_0^1 x^{a-1}(1-x)^{b-1}dx=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)},a>0,b>0$$

$$\Gamma(s)=\int_0^{\infty}x^{s-1}e^{-x}dx,s>0,\Gamma(n+1)=n!$$

í âP` Ú × $$\pi(\theta|x_1,\cdots,x_n)=\frac{p(x_1,\cdots,x_n,\theta)}{p(x_1,\cdots,x_n)}=\frac{p(x_1,\cdots,x_n|\theta)\pi(\theta)}{\int p(x_1,\cdots,x_n|\theta)\pi(\theta)d\theta}$$

5à $p(x_1,\cdots,x_n)=\int p(x_1,\cdots,x_n|\theta)\pi(\theta)d\theta$ ,XE•L Ú × Ä âP` Ú ×,X ó ï $\hat{\theta}_E$ /Ä $\theta$,X âP` ó ï Au Ä

$\hat{\theta}$,X âP` • Â , $MSE(\hat{\theta}_E|x)=E^{\theta|x}(\theta-\hat{\theta}_E)^2=Var(\theta|x)+(\hat{\theta}_E-\hat{\theta})^2$

' $\hat{\theta}=\hat{\theta}_E$ Ê, Ã S âP` • ÂE' Ô ã , rL ª âP` 0 $\theta$,XBñ Ê ƒ Au Ä G Ý Ö $\hat{\theta}_E=E(\theta|x_1,\cdots,x_n)=\int p\pi(\theta|x_1,\cdots,x_n)d_p$

Bñ Ê ƒ Au á à b J ª – D Au •"© È W Ú þ-¹ – D $\theta$ ?–,X ,ß á ¢ ¤ V )[ Ú ×,XLc ¬G£ Ä5à þ-¹ – D,X V)[ Ú × í Î b rP`5Ù ¢ ! ó,X Z ÆC m ` î H,X/Ã3 4£P` k ,X Ô/¡ ?– Ú × Ä' â+ k ,X µ C È Í P` Ú × t ¹ È „ È „,X Ú × $\theta$,X âP` Ú × Ä

### 2.2.1 沃纳模型下的贝叶斯方法

A' _ Ê $A$ # óKÂNl È $\overline{A}$ # óKÂNl $A$,X Í0Ÿ _ Ê È $p$ m Ý 4 2 b $A$ ë Û ,X 5( ,X!¨ _ È $\pi_A$ ' K Ý # ó û(M U,X Ž 4,X!¨ _ Ä $m$ >•A× ¹5Ù ²

1( ,X Ž D Ä $\lambda$ ' ²1( ,X!¨ _ È í $\lambda = \frac{m}{n}$ ÄA' Ý>•A× ¹5ÙFÑ,ó r ²1( Ä

n $\pi_A$ ,X P` Ú × Bñ ( Ú × È

$f(\pi_A) = B(a,b)^{-1} \pi_A{}^{a-1} (1-\pi_A)^{b-1}$ È $0 \le \pi_A \le 1, a < 0, b > 0$ È

$B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1} dx, a > 0, b > 0$

í $\pi_A$ ,X âP` Ú ×

$$h(\pi_A | x_1, \cdots, x_n) = t\pi_A{}^{a-1}(1-\pi_A)^{b-1}[(p\pi_A + (1-p)(1-\pi_A)]^m [1-(2p-1)\pi_A - (1-p)]^{n-m}$$
$$= t\pi_A{}^{a-1}(1-\pi_A)^{b-1}[(p\pi_A + (1-p)(1-\pi_A)]^m [(2p-1)(1-\pi_A) + (1-p)]^{n-m}$$ Ä

J È $t = [\int_0^1 \pi_A{}^{a-1}(1-\pi_A)^{b-1}[(2p-1)\pi_A + (1-p)]^m [(2p-1)(1-\pi_A) + (1-p)]^{n-m} d_{\pi_A}]^{-1}$

$$= [\sum_{i=0}^{m} \sum_{j=0}^{n-m} C_m^i C_{n-m}^j (2p-1)^{i+j} (1-p)^{n-i-j} \int_0^1 \pi_A{}^{a+i-1} (1-\pi_A)^{b+j-1} d_{\pi_A}]^{-1}$$

$$= [\sum_{i=0}^{m} \sum_{j=0}^{n-m} C_m^i C_{n-m}^j (2p-1)^{i+j} (1-p)^{n-i-j} B(a+i, b+j)]^{-1}$$

¹ $\pi_A$ ,XBñ Ê $f$ Au Ö

$$\hat{\pi}_{ABves} = E(\theta | x_1, \cdots, x_n) = \int_0^1 \pi_A h(\theta | x_1, \cdots, x_n) d_{\pi_A}$$

$$= t\int_0^1 \pi_A{}^a (1-\pi_A)^{b-1}[(2p-1)\pi_A + (1-p)]^m [(2p-1)(1-\pi_A) + (1-p)]^{n-m} d_{\pi_A}$$

$$= \sum_{i=0}^{m} \sum_{j=0}^{n-m} t C_m^i C_{n-m}^j (2p-1)^{i+j} (1-p)^{n-i-j} \int_0^1 \pi_A{}^{a+i} (1-\pi_A)^{b+j-1} d_{\pi_A}$$

$$= \sum_{i=0}^{m} \sum_{j=0}^{n-m} t C_m^i C_{n-m}^j (2p-1)^{i+j} (1-p)^{n-i-j} B(a+i+1, b+j)$$

$$= \frac{\sum_{i=0}^{m} \sum_{j=0}^{n-m} C_m^i C_{n-m}^j (2p-1)^{i+j} (1-p)^{n-i-j} B(a+i+1, b+j)}{\sum_{i=0}^{m} \sum_{j=0}^{n-m} C_m^i C_{n-m}^j (2p-1)^{i+j} (1-p)^{n-i-j} B(a+i, b+j)}$$

### 2.2.2 西蒙斯模型下的贝叶斯方法

A„ Ý # ó û ûKÂNI $A$ È$\pi_A$ ' K Ý # ó û(M U,X Ž 4,X!¨ _ È $\pi_B$ K ÝM2 # óKÂNI$B$ 2 û Ž,X!¨ _ È Æ-¹,X Ä $p$ ' # óKÂNI 4!¨ _ × $n$

•G£ È $m$ ²1( ,X Ž D Ä $\lambda$ ' ²1( ,X!¨ _ × $\hat{\lambda} = \frac{m}{n}$

²1( Ž D,X V)[ È í Ý $\sum_{i=1}^{n} X_i = m$ Ä

n $\pi_A$ ,X P` Ú × Bñ ( Ú × È

$$f(\pi_A) = B(a,b)^{-1} \pi_A{}^{a-1} (1-\pi_A)^{b-1} \text{ È } 0 \le \pi_A \le 1, a<0, b>0 \text{ È}$$

$$B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1} dx, a>0, b>0$$

í $\pi_A$ ,X âP` Ú ×

$$h(\pi_A | x_1, \cdots, x_n) = t\pi_A{}^{a-1}(1-\pi_A)^{b-1}[p\pi_A + (1-p)\pi_B]^m [1 - p\pi_A - (1-p)\pi_B]^{n-m}$$

J , $t = [\int_0^1 \pi_A{}^{a-1}(1-\pi_A)^{b-1}[p\pi_A + (1-p)\pi_B]^m [1-p\pi_A - (1-p)\pi_B]^{n-m} d_{\pi_A}]^{-1}$

$$= [\int_0^1 \pi_A{}^{a-1}(1-\pi_A)^{b-1}[p\pi_A + (1-p)\pi_B]^m [p(1-\pi_A) + (1-p)(1-\pi_B)]^{n-m} d_{\pi_A}]^{-1}$$

$$= [\sum_{i=0}^{m}\sum_{j=0}^{n-m} C_m^i C_{n-m}^j p^{i+j}(1-p)^{n-i-j} \pi_B{}^{m-i}(1-\pi_B)^{n-m-j} \int_0^1 \pi_A{}^{a+i-1}(1-\pi_A)^{b+j-1} d_{\pi_A}]^{-1}$$

$$= [\sum_{i=0}^{m}\sum_{j=0}^{n-m} C_m^i C_{n-m}^j p^{i+j}(1-p)^{n-i-j} \pi_B{}^{m-i}(1-\pi_B)^{n-m-j} B(a+i, b+j)]^{-1}$$

¹ $\pi_A$ ,XBñ Ê ƒ Au Ö

$$\hat{\pi}_{ABYes} = E(\theta | x_1, \cdots, x_n) = \int_0^1 \pi_A h(\theta | x_1, \cdots, x_n) d_{\pi_A}$$

$$= t\int_0^1 \pi_A{}^{a}(1-\pi_A)^{b-1}[p\pi_A + (1-p)\pi_B]^m [1-p\pi_A - (1-p)\pi_B]^{n-m} d_{\pi_A}$$

$$= t\int_0^1 \pi_A{}^{a}(1-\pi_A)^{b-1}[p\pi_A + (1-p)\pi_B]^m [p(1-\pi_A) + (1-p)(1-\pi_B)]^{n-m} d_{\pi_A}$$

$$= \sum_{i=0}^{m}\sum_{j=0}^{n-m} tC_m^i C_{n-m}^j p^{i+j}(1-p)^{n-i-j} \pi_B{}^{m-i}(1-\pi_B)^{n-m-j} \int_0^1 \pi_A{}^{a+i}(1-\pi_A)^{b+j-1} d_{\pi_A}$$

$$= \sum_{i=0}^{m}\sum_{j=0}^{n-m} tC_m^i C_{n-m}^j p^{i+j}(1-p)^{n-i-j} \pi_B{}^{m-i}(1-\pi_B)^{n-m-j} B(a+i+1, b+j)$$

$$= \frac{\sum_{i=0}^{m}\sum_{j=0}^{n-m} C_m^i C_{n-m}^j p^{i+j}(1-p)^{n-i-j} \pi_B{}^{m-i}(1-\pi_B)^{n-m-j} B(a+i+1, b+j)}{\sum_{i=0}^{m}\sum_{j=0}^{n-m} C_m^i C_{n-m}^j p^{i+j}(1-p)^{n-i-j} \pi_B{}^{m-i}(1-\pi_B)^{n-m-j} B(a+i, b+j)}$$

Í b # ó ûKÂNl $\pi_A$ ,XBñ Ê ƒ Au ' $n$ ` $m$ \ û Ê ÈAu1k!¨EW4 )ä ÈE- Ê å À Ã ¹ ó }Au1k " kE¥ ?· Ä

# 第三章　分层抽样下敏感性问题

## 3.1 比估计和回归估计

ü ‘ A× ¹,X rL ¹ 0 ÈL8 ZA× ¹,X,Â ÛG£ ¹ ê ÈE¬ Ý Ô o â J2û š,Ì G ,XEY } ¬G£ ÈE- o ÛEY } ¬G£ SEQ,X ,Ì G,XEY } µ C È å À à ï ý*üEY } ¬G£ â,Â ÛG£ KÈ,X G2Ï 9 ¤P¬ Au,X2’ z Ä

### 3.1.1 比估计

1T )Lc ‘ ß È ) ! þ ',Â ÛG£ $Y$ Ý JEY }G£$X$ È J G2Ï $Y = RX$ ê $\bar{Y} = R\bar{X}$ È8¹?U Au $R$ Ã*ü !¨ $r = \bar{y}/\bar{x}$ 9 Ä

1 Å Í1T ) Lc ² ‘ !¨ Au ， ¹ $r = \bar{y}/\bar{x}$ 9 $R$ E¥ ´ #,X È J • # ÂE¥ $E(r-R)^2 = \frac{1}{nN}\frac{1}{\bar{X}^2}\sum_{i=1}^{N}(Y_i - RX_i)^2$ Ä

' $\bar{X}$ Æ-¹ Ê È í Ý $\bar{y}$ ,X • # Â

$$V\left(\bar{y}_R\right) \approx \frac{\sum_{i=1}^{N}(Y_i - RX_i)^2}{nN}$$
$$= \frac{1}{nN}\sum_{i=1}^{N}[(Y_i - \bar{Y}) - R(X_i - \bar{X})]^2$$
$$= \frac{1}{nN}[S_Y^2 - 2RS_{XY} + R^2 S_X^2]$$ Ä

2 Å Í b1T )Lc ´ ² ‘ !¨ Au È¹ $r = \bar{y}/\bar{x}$ 9 $R$ E¥ ´ #,X ÈJ • # ÂE¥ $E(r-R)^2 = \frac{1-f}{n(N-1)}\frac{1}{\bar{X}}\sum_{i=1}^{N}(Y_i - RX_i)^2$ Ä

' $\bar{X}$ Æ-¹ Ê È í Ý

$$V\left(\bar{y}_R\right) \approx \frac{1-f}{n(N-1)}\sum_{i=1}^{N}(Y_i - RX_i)^2$$
$$= \frac{1-f}{n(N-1)}\sum_{i=1}^{N}[(Y_i - \bar{Y}) - R(X_i - \bar{X})]^2$$
$$= \frac{1-f}{n(N-1)}[S_Y^2 - 2RS_{XY} + R^2 S_X^2]$$ Ä

J $S_Y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$，$S_X^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$，$S_{XY} = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})(Y_i - \bar{Y})$ Ä

### 3.1.2 回归估计

EY } ¬G£$X$ Í,Â Û ¬G£ $Y$ ,X ,X º Ô/¡ Au ² & Au È $\bar{y}_{lr}=\bar{y}-\beta(\bar{x}-\bar{X})$,
J $\beta$ ² &2Ï D È Ã ¹ D È 3 Ã ¹+ ‡ n Ä

' $\beta=0$ Ê È ² & AuG£ G 1T ) AuG£ Ä

8¹ $\beta=r=\bar{y}/\bar{x}$ È í ² & AuG£ !¨)[ Au Ä

' $\beta=\beta_0$ D Ê È Í1T )Lc ² ‘ E¯> ² & Au Ê È $\bar{y}_{lr}$ $\bar{Y}$ ,X ´ #
Au$E(\bar{y}_{Lr})=\bar{Y}$ È J • Â $V(\bar{y}_{Lr})=\dfrac{{\sigma_Y}^2-2\beta\sigma_{XY}+\beta^2{\sigma_X}^2}{n}$ Ä1T )Lc á ² ‘ ß í Ý
$V\left(\bar{y}_{Lr}\right)=\dfrac{1-f}{n}[S_Y^2-2\beta_0 S_{XY}+{\beta_0}^2 S_X^2]$ Ä

' $\beta$+ ‡ n È 3 ² &2Ï D $\beta=\dfrac{s_{xy}}{s_x^2}=\dfrac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$ È
à Ê ' $n$ Ú û Ê È Í1T )Lc ² ‘ , ² & AuG£#äE¯´ # È G $E(\bar{y}_{Lr})\approx\bar{Y}$ È
J •# Â $V(\bar{y}_{Lr})=\dfrac{S_Y^2(1-\rho^2)}{n}+o\left(\dfrac{1}{n^{3/2}}\right)$ × Í1T )Lc á ² ‘ , J •# Â
$V(\bar{y}_{Lr})=\dfrac{1-f}{n}\left(1-\rho^2\right)S_Y^2+o\left(\dfrac{1}{n^{3/2}}\right)$, $\rho$ $X$ â $Y$ ,X,Ì G G2Ï Ä

## 3.2 分层抽样下的比估计和回归估计

### 3.2.1 定义及符号说明

Ú ‘ ‘ A× ¹ h*ü,X,Ì ' S"¯,X Ô/¡ ‘ T ÄE-/¡ A× ¹ •"© Ú
' Ý Ô n,X?˜ í Ú ä 8¹ F þ f á,Ì x,X $ ' È E¯5à Í!£ Ô $ ' ‘ ª
E¯> A× ¹ Ä Ú ‘ ,X )[EWP¬ È å À á ™ Ã ¹ k Ø Û Û,X AuG£ ÈE¬6Ñ ¤
P¬ ,X ·>< ûE¯5à ¤P¬ Au2’ z Ä Ø Û Û AuG£,X • Â â KÈ • Â ´ G È ¾
â Y • Â Ý G Ä Í!£ Ô $ ' È ,X ‘ •"© Ã ¹ B á à,X ™ ‰E¯> á
à • ã,X ‘ È Ý ý b ‘ ¹ 0,X Ô ) ` ¤P¬ Y Au2’ z ÈE¯5à Í ',X Au
Ý,ž Ä Y,X ‘ •"© Ý \ î È [ üE-G q' G› ª1T )Lc ‘ ,X •"© Ä

A’ ' G£ $N$ È D $L$ È Ë $k$ È $k=1,2,\cdots,L$ È Ø $ D $N_k$ Èü
!£ Ô (À0Ÿ ‘ ª È!£ Ô ,X •G£ $n_k$ È ,X G£ $n=\sum_{k=1}^{L}n_k$ È1 $k$

' $W_k = \dfrac{N_k}{N}$ È1 $k$ ' !¨ $f_k = \dfrac{n_k}{N_k}$ Ä

### 3.2.2 分层抽样下简单抽样的比估计

n)Ú 3. 1 Ï 32 Ð Ö ü Ø (À0Ÿ ' ª1T )Lc È!£ FÑ*ü!¨ Au È è Ø

G£$n_k$ C‡ ó û Ê È $\bar{y}_{RS} = \sum_{k=1}^{L} W_k r_k \bar{X}_k$ $\bar{Y}$ ,XE¥ ´ # Au Ä J • # ÂE¥

$$V\left(\bar{y}_{RS}\right) = V\left(\hat{R}X\right) = \sum_{k=1}^{L} W_k^2 \frac{1-f_k}{n_k} \frac{1}{N_k - 1} \sum_{i=1}^{N_k} (Y_{ki} - R_k X_{ki})^2$$

$$= \sum_{k=1}^{L} W_k^2 \frac{1-f_k}{n_k} \left( S_{Y_k}{}^2 - 2R_k S_{X_k Y_k} + R_k{}^2 S_{X_k}{}^2 \right)$$ Ä

### 3.2.3 分层抽样下简单抽样的回归估计

n)Ú 3. 2 Ï 32 Ð Ö ü Ø (À0Ÿ ' ª1T )Lc È!£ FÑ*ü ² & Au È è Ø

G£ $n_k$ C‡ ó û Ê È $\bar{y}_{lr} = \sum_{k=1}^{L} W_k^2 \bar{y}_{lrk} = \sum_{k=1}^{L} W_k^2 [\bar{y}_k + \beta_k (\bar{X}_k - \bar{x}_k)]$ Ä

' $\beta = \beta_0$ D Ê Ý $E(\bar{y}_{lr}) = \bar{Y}$， $V\left(\bar{y}_{lr}\right) = \sum_{k=1}^{L} W_k^2 \frac{1-f_k}{n_k} [S_Y^2 - 2\beta_0 S_{XY} + \beta_0{}^2 S_X^2]$

' $\beta$ + ‡ n È 3 ² &2Ï D $\beta = \dfrac{s_{xy}}{s_x^2} = \dfrac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$ È ' $n$ Ú

û Ê È ² & AuG£#äE¯´ # È $V\left(\bar{y}_{lr}\right) \approx \sum_{k=1}^{L} W_k^2 \frac{1-f_k}{n_k} S_{Y_k}^2 (1-\rho_k{}^2)$， $\rho_k$ $X_k$ â $Y_k$ ,X,Ì

G G2Ï Ä

## 3.3 分层抽样下的敏感性问题

Ú ' ' A× ¹ *ü ,X ' •"© Ä W r 'CK 91T ) ç ¡ 0 È8¹ Ú k ' È Ã Ý ¤P¬ Au 2' z Ä ´!8 Ú # ó ûKÂNlE¤*ü Ú ' È Ú rL KÂNl â á ' 4§ Ü È ¤P¬ ' )[,X à Ê Ã Ý E¯2' z Ä [A|AŽ!£ Ô FÑ 1T )Lc ' ,X ™ ‰ Ä

### 3.3.1 分层抽样下的沃纳模型

Ú ' $N$ Ú f á Ç x,X $L$ È !£ Ô $ ' G£ $N_k$, $k=1,2,3$ È È $L$, ü!£ Ô (À0ŸE¯> 1T )Lc ' È!£ Ô ,X •G£ $n_k$ È1 $k$ $ '

$W_k = \frac{N_k}{N}$ È 1 $k$ ²1( ,X Ž D $m_k$ È $\lambda_k$ 1 $k$ $ ' ²1(

,X V)[ È1 $k$ $ ' K Ý # óKÂNl,X Ž,X!¨ _ $\pi_{Ak}$ È $\pi_A$ ' K Ý # ó

2 û,X Ž 4!¨ _ Èí Ý $\pi_A = \sum_{k=1}^{L} W_k \pi_{Ak}$ Èü Ø FÑ S*ü"W4‡Lc ê>™5B Èí Ý Ö

$\lambda_k = \pi_{Ak} p + (1-\pi_{Ak})(1-p)$ È $\pi_{Ak} = \frac{\lambda_k - (1-p)}{2p-1}$ Ä

¢5à AuG£ $\hat{\pi}_A = \sum_{k=1}^{L} W_k \hat{\pi}_{Ak} = \frac{\sum_{k=1}^{L} W_k [\hat{\lambda}_k - (1-p)]}{2p-1}$ Ä

n)Ú 3. 3 Ú ' È ' !£ Ô ,X FÑ ¹ Ý ²1T )Lc ' • ã ' ª

Ê È $\hat{\pi}_A$ $\pi_A$ ,X ´ # AuG£ È è $\hat{\pi}_A$ ,X • Â

$V(\hat{\pi}_A) = \sum_{k=1}^{L} W_k^2 [\frac{\pi_{Ak}(1-\pi_k)}{n_k} + \frac{p(1-p)}{n_k(2p-1)^2}]$ È $k = 1, 2, 3$ È È $L$ Ä

A• â Ö $E(\hat{\pi}_A) = \sum_{k=1}^{L} W_k E(\hat{\pi}_{Ak}) = \sum_{k=1}^{L} W_k E(\frac{\hat{\lambda}_k - (1-p)}{2p-1})$

$$= \sum_{k=1}^{L} W_k \frac{E(\hat{\lambda}_k) - (1-p)}{2p-1}$$

$$= \sum_{k=1}^{L} W_k \frac{\pi_{Ak} p + (1-\pi_{Ak})(1-p) - (1-p)}{2p-1}$$

$$= \sum_{k=1}^{L} W_k \hat{\pi}_{Ak} = \hat{\pi}_A$$

¹ $\hat{\pi}_A$ $\pi_A$ ,X ´ # AuG£ Ä

\+ é)Ú 2. 1 -¹ $V(\hat{\lambda}_k) = \frac{N_k - 1}{N_k} \frac{S_{Y_k}{}^2}{n_k} = \frac{\sigma_{Y_k}^2}{n_k} = \frac{\lambda_k(1-\lambda_k)}{n_k}$ È J È

$S_{Y_k}{}^2 = \frac{1}{N_k - 1} \sum_{i=1}^{N_k} (Y_{ki} - \bar{Y}_k)^2$ È $\sigma_{Y_k}{}^2 = \frac{1}{N_k} \sum_{i=1}^{N_k} (Y_{ki} - \bar{Y}_k)^2$ Ä

$$V(\hat{\pi}_A) = V(\sum_{k=1}^{L} W_k \hat{\pi}_{Ak}) = \sum_{k=1}^{L} W_k^2 V(\hat{\pi}_{Ak})$$

$$= \sum_{k=1}^{L} W_k^2 V(\frac{\hat{\lambda}_k - (1-p)}{2p-1}) = \sum_{k=1}^{L} W_k^2 \frac{V(\hat{\lambda}_k)}{(2p-1)^2}$$

$$=\sum_{k=1}^{L}W_k^2\cdot\frac{\sigma_k^2}{(2p-1)^2 n_k}=\sum_{k=1}^{L}W_k^2\cdot\frac{\lambda_k(1-\lambda_k)}{(2p-1)^2 n_k}$$

Ú $\lambda_k=\pi_{Ak}p+(1-\pi_{Ak})(1-p)$ ú 9 Þ ã Ý

$$V(\hat{\pi}_k)=\sum_{k=1}^{L}W_k^2[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k}+\frac{p(1-p)}{n_k(2p-1)^2}]$$ Ä

n)Ú 3. 4 Ú ‘ È ' Ø ,X ¹ ´ ²1T )Lc ‘ • ã ‘ ª Ê È AuG£ $\hat{\pi}_A$ $\pi_A$ ,X ´ # AuG£ È J • Â

$$V(\hat{\pi}_A)=\sum_{k=1}^{L}\frac{W_k^{\ 2}(N_k-n_k)}{N_k-1}[\frac{\pi_{Ak}(1-\pi_k)}{n_k}+\frac{p(1-p)}{n_k(2p-1)^2}]$$ È

J $\hat{\pi}_A=\sum_{k=1}^{L}W_k\hat{\pi}_{Ak}=\dfrac{\sum_{k=1}^{L}W_k[\hat{\lambda}_k-(1-p)]}{2p-1}$ Ä

A• â Ö+ é)Ú 2.2 -¹ $V(\hat{\lambda}_k)=\dfrac{N_k-n_k}{n_k(N_k-1)}\cdot\lambda_k(1-\lambda_k)$

Ú $\lambda_k=\pi_{Ak}p+(1-\pi_{Ak})(1-p)$ ú 9 Þ ã Ý

$$V(\hat{\pi}_A)=V(\sum_{k=1}^{L}W_k\hat{\pi}_{Ak})$$

$$=\sum_{k=1}^{L}W_k^2V(\frac{\hat{\lambda}_k-(1-p)}{2p-1})=\sum_{k=1}^{L}W_k^2\cdot\frac{V(\hat{\lambda}_k)}{(2p-1)^2}$$

$$=\sum_{k=1}^{L}W_k^2\frac{\frac{N_k-n_k}{n_k(N_k-1)}\cdot\lambda_k(1-\lambda_k)}{(2p-1)^2}$$

$$=\sum_{k=1}^{L}\frac{W_k^{\ 2}(N_k-n_k)}{N_k-1}[\frac{\pi_k(1-\pi_k)}{n_k}+\frac{p(1-p)}{n_k(2p-1)^2}]$$

J $f_k=\dfrac{n_k}{N_k}$ 1 $k$ ‘ !¨ Ä

### 3.3.2 分层抽样下的西蒙斯模型

Ú ' Ú f á Ç x,X $L$ È Ø $ ' G£ $N_k$ , $k=1, 2, 3$ È È $L$ È ü!£ Ô (À0ŸE¯> 1T )Lc ‘ È!£ Ô ,X •G£ $n_k$ È1 $k$ $ '

$W_k=\dfrac{N_k}{N}$ È1 $k$ ²1( ,X Ž D $m_k$ È $\lambda_k$ 1 $k$ $ ' ²1( ,X

V)[ È1 $k$ $ ' K Ý # óKÂNl $A$,X Ž,X!¨ _ $\pi_{Ak}$ È $\pi_A$ ' K Ý # ó 2 û,X Ž 4!¨ _ Èí Ý $\pi_A = \sum_{k=1}^{L} W_k \pi_{Ak}$ È$B$ â $A$ ´ G,X M2 # óKÂNlÈ$\pi_{Bk}$ 1 $k$ K Ý 2 û $B$,X Ž 4!¨ _ È è $\pi_{Bk}$ Æ-¹ Ä ü Ø FÑ S*ü?S:m ƒLc ê>™5B Ä

í $\lambda_k = \pi_{Ak} \times p + (1-p)\pi_{Bk}$ È $\hat{\pi}_{Ak} = \dfrac{[\hat{\lambda}_k - (1-p)\pi_{Bk}]}{p}$ Ä

¢5à $\hat{\pi}_A = \sum_{k=1}^{L} W_k \hat{\pi}_{Ak} = \dfrac{1}{p}\sum_{h=1}^{L} W_h \left[\lambda_k - (1-p)\pi_{Bk}\right]$ Ä

n)Ú 3. 5 ü Ú ‘ È' Ø ,X ¹ Ý ²1T )Lc ‘ • ã ‘ ª Ê ,
$\hat{\pi}_A$ $\pi_A$ ,X ´ # AuG£ È J • Â

$$V(\hat{\pi}_A) = \sum_{k=1}^{L} W_k^2 [\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k} + \frac{(1-p)(\pi_{Ak}+\pi_{Bk}-2\pi_{Ak}\pi_{Bk})}{pn_k} + \frac{(1-p)^2\pi_{Bk}(1-\pi_{Bk})}{p^2 n_k}]$$

A• â : $E(\hat{\pi}_A) = E(\sum_{k=1}^{L} W_k \hat{\pi}_{Ak}) = \sum_{k=1}^{L} W_k E(\hat{\pi}_{Ak}) = \sum_{k=1}^{L} W_k \pi_{Ak} = \pi_A$

$\hat{\pi}_A$ $\pi_A$ ,X ´ # AuG£ Ä

$$\begin{aligned} V(\hat{\pi}_A) &= \sum_{k=1}^{L} W_k^2 V(\hat{\pi}_{Ak}) = \sum_{k=1}^{L} W_k^2 \cdot \frac{V(\hat{\lambda}_k)}{p^2} \\ &= \sum_{k=1}^{L} W_k^2 \cdot \frac{\sigma_{Z_k}^2}{p^2 n_k} = \sum_{k=1}^{L} W_k^2 \cdot \frac{\lambda_k(1-\lambda_k)}{n_k p^2} \\ &= \sum_{k=1}^{L} \frac{W_k^2}{p^2 n_k}[p^2\pi_{Ak}(1-\pi_{Ak}) + p(1-p)(\pi_{Ak}+\pi_{Bk}-2\pi_{Ak}\pi_{Bk}) + (1-p)^2\pi_{Bk}(1-\pi_{Bk})] \\ &= \sum_{k=1}^{L} W_k^2 [\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k} + \frac{(1-p)(\pi_{Ak}+\pi_{Bk}-2\pi_{Ak}\pi_{Bk})}{pn_k} + \frac{(1-p)^2\pi_{Bk}(1-\pi_{Bk})}{p^2 n_k}] \end{aligned}$$

n)Ú 3. 6 ü Ú ‘ È Ø ,X ¹ ´ ²1T )Lc ‘ • ã ‘ ª Ê È
J • Â $V(\hat{\pi}_A)$

$$= \sum_{k=1}^{L} \frac{W^2{}_k (N_k - n_k)}{(N_k - 1)} [\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k} + \frac{(1-p)(\pi_{Ak}+\pi_{Bk}-2\pi_{Ak}\pi_{Bk})}{pn_k} + \frac{(1-p)^2\pi_{Bk}(1-\pi_{Bk})}{p^2 n_k}]$$

J È $f_k = \dfrac{n_k}{N_k}$ 1 $k$ ,X ‘ !¨ Ä

A• â Ö Æ-¹ $\lambda_k = \pi_{Ak} \times p + (1-p)\pi_{Bk}$ È $\hat{\pi}_{Ak} = \dfrac{[\hat{\lambda}_k - (1-p)\pi_{Bk}]}{p}$

$$\hat{\pi}_A = \sum_{k=1}^{L} W_k \hat{\pi}_{Ak} = \frac{1}{p}\sum_{h=1}^{L} W_h \left[\lambda_k - (1-p)\pi_{Bk}\right]$$

+ é)Ú 2. 2 -¹ $V(\hat{\lambda}_k = \dfrac{N_k - n_k}{n_k(N_k - 1)} \cdot \lambda_k(1-\lambda_k)$

J $f_k = \frac{n_k}{N_k}$ 1 $k$ ' !¨ È $S_{Y_k}{}^2 = \frac{1}{N_k - 1}\sum_{i=1}^{N_k}(Y_{ki} - \bar{Y}_k)^2$

¢5à $V(\hat{\pi}_A) = \sum_{k=1}^{L} W_k^2 V(\hat{\pi}_{Ak}) = \frac{1}{p^2}\sum_{k=1}^{L} W_k^2 V\left[\hat{\lambda}_k - (1-p)\pi_{Bk}\right]$

$$= \frac{1}{p^2}\sum_{k=1}^{L} W_k^2 V(\hat{\lambda}_k) = \sum_{k=1}^{L}\frac{W_k{}^2(1-f_k)}{n_k p^2} S_Y{}^2 = \sum_{k=1}^{L}\frac{W_k{}^2(1-f_k)}{n_k(N_k-1)p^2}\left(m_k - \frac{m_k{}^2}{n_k}\right)$$

$$= \sum_{k=1}^{L}\frac{W^2{}_k(N_k - n_k)}{n_k(N_k-1)p^2}\cdot\frac{m_k}{n_k}\left(1 - \frac{m_k}{n_k}\right) = \sum_{k=1}^{L}\frac{W^2{}_k(N_k - n_k)}{n_k(N_k-1)p^2}\cdot\lambda_k(1-\lambda_k)$$

$$= \sum_{k=1}^{L}\frac{W^2{}_k(N_k - n_k)}{n_k(N_k-1)p^2}[p^2\pi_{Ak}(1-\pi_{Ak}) + p(1-p)(\pi_{Ak} + \pi_{Bk} - 2\pi_{Ak}\pi_{Bk}) + (1-p)^2\pi_{Bk}(1-\pi_{Bk})]$$

$$= \sum_{k=1}^{L}\frac{W^2{}_k(N_k - n_k)}{(N_k-1)}[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k} + \frac{(1-p)(\pi_{Ak} + \pi_{Bk} - 2\pi_{Ak}\pi_{Bk})}{pn_k} + \frac{(1-p)^2\pi_{Bk}(1-\pi_{Bk})}{p^2 n_k}]$$

A•!© Ä

### 3.3.3 分层抽样下的改进模型

Ú ' Ú f á Ç x,X $L$ È Ø ' G£ $N_k$ , ü!£ Ô (À0ŸE¯> 1T ) Lc ' È!£ Ô ,X •G£ $n_k$ Ã $k$ =1, 2, 3 È È $L$ Å, 1 $k$ '

$W_k = \frac{N_k}{N}$ 1 $k$ ²1( ,X Ž D $m_k$ È $\lambda_k$ 1 $k$ þ ' ²1( ,X V È

)[ È1 $k$ K Ý # óKÂNI,X Ž,X!¨ _ $\pi_{Ak}$ È $\pi_{Bk}$ Æ-¹ Ä $\pi_A$ Ú ' K Ý # ó 2 û,X Ž 4!¨ _ È ü Ø FÑ S*ü E¯,X ´ G õ _ Lc ê>™5B È í Ý

$\lambda_k = p_1 \times \pi_{Ak} + p_2(1-\pi_{Ak}) + p_3\pi_{Bk}$ , $\hat{\pi}_{Ak} = \frac{\hat{\lambda} - p_2 - p_3\pi_{Bk}}{p_1 - p_2}$

¢5à $\hat{\pi}_A = \sum_{k=1}^{L} W_k\hat{\pi}_{Ak} = \sum_{k=1}^{L} W_k\frac{\hat{\lambda} - p_2 - p_3\pi_{Bk}}{p_1 - p_2}$ Ä

n)Ú 3. 7 Öü Ú ' È Ø ,X ¹ Ý ²1T )Lc ' • ã ' ª Ê , $\hat{\pi}_A$ $\pi_A$ ,X ´ # AuG£ È J • Â

$$V(\hat{\pi}_A) = \sum_{k=1}^{L} W_k^2[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k} + \frac{p_3(\pi_{Ak} + \pi_{Bk} - 2\pi_{Ak}\pi_{Bk})}{(p_1 - p_2)n_k} + \frac{p_3^2\pi_{Bk}(1-\pi_{Bk}) + p_2(p_1 + p_3)}{(p_1 - p_2)^2 n_k}]$$

A• â Ö $\hat{\pi}_A = \sum_{k=1}^{L} W_k \hat{\pi}_{Ak} = \dfrac{\sum_{k=1}^{L} W_k (\hat{\lambda}_k)}{p_1 - p_2}$

$$V(\hat{\pi}_A) = \sum_{k=1}^{L} W_k{}^2 V(\hat{\pi}_{Ak}) = \frac{\sum_{k=1}^{L} W_k{}^2 V(\hat{\lambda}_k)}{(p_1 - p_2)^2}$$

$$= \sum_{k=1}^{L} W_k^2 [\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k} + \frac{p_3(\pi_{Ak} + \pi_{Bk} - 2\pi_{Ak}\pi_{Bk})}{(p_1 - p_2)n_k} + \frac{p_3^2 \pi_{Bk}(1-\pi_{Bk}) + p_2(p_1 + p_3)}{(p_1 - p_2)^2 n_k}]$$

n)Ú 3. 8 Öü Ú ‘ È Ø ,X ¹ ´ ²1T )Lc ‘ • ã ‘ ª Ê ,

$\hat{\pi}_A$ $\pi_A$ ,X ´ # AuG£ È J • Â $V(\hat{\pi}_A) =$

$\sum_{k=1}^{L} \frac{W^2{}_k (N_k - n_k)}{(N_k - 1)} [\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k} + \frac{p_3(\pi_{Ak} + \pi_{Bk} - 2\pi_{Ak}\pi_{Bk})}{(p_1 - p_2)n_k} + \frac{p_3^2 \pi_{Bk}(1-\pi_{Bk}) + p_2(p_1 + p_3)}{(p_1 - p_2)^2 n_k}]$ Ä

A• â Ö à)Ú ÃA• $\hat{\pi}_A = \sum_{k=1}^{L} W_k \hat{\pi}_{Ak} = \dfrac{\sum_{k=1}^{L} W_k (\hat{\lambda}_k)}{p_1 - p_2}$

$$V(\hat{\pi}_A) = \sum_{k=1}^{L} W_k{}^2 V(\hat{\pi}_{Ak}) = \frac{\sum_{k=1}^{L} W_k{}^2 V(\hat{\lambda}_k)}{(p_1 - p_2)^2}$$

$$= \sum_{k=1}^{L} \frac{W^2{}_k (N_k - n_k)}{(N_k - 1)} [\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k} + \frac{p_3(\pi_{Ak} + \pi_{Bk} - 2\pi_{Ak}\pi_{Bk})}{(p_1 - p_2)n_k} + \frac{p_3^2 \pi_{Bk}(1-\pi_{Bk}) + p_2(p_1 + p_3)}{(p_1 - p_2)^2 n_k}]$$

# 第四章 分层抽样中敏感性问题的比估计

## 4.1 定义及符号说明

ü rL A× ¹ È+ b # ó ûKÂNl-è0J#] ž /• Ž š , JA× ¹,X,Ì G µ CG£Eî !¨EW ã,X Ä ´!8 Í # ó ûKÂNl ' ª E¯> A× ¹-è0J ! È Æ Ý,X Z ÆC m ` Ž À9‹ k,X 4£P` î k,Ì 'G¡?U ÄL8 ZA× ¹,Â Û Û Û Y ê È E¬ Ã ¹ t 9 â Û Û Y P¬ z,Ì G,X Æ-¹EY } ¬G£ X È ¢5à ý*ü EY } ¬G£ 9 ¤P¬ Í,Â Û Au G£,X2' z Ä ü rL KÂNl ÈEY } ¬G£ T T ! ó Z ÆC m （ _ V Þ Ô õ B ¹C m ） ê)„ Ý,X2k +9 Au1 Ä ´AŽ )/¡ ™ 6 X ,X ' ê ` Ô8 ™NO Æ-¹,X Ä â1T ) Au!¨ EW È!¨ AuG£ M24“ û,XEW á ,X AuG£ Ä ¾?UA× ¹ Û Û âEY } ¬G£ KÈ Ý8C Q,X4“ û,Ì G G2Ï È í!¨ Au,X2' z!¨1T ) Au,X2' zP¬ ÄE- S*ü!¨ Au ,X ?U s ´ Ä áE›!¨ Au,X S*ü p,X Q # ª ‡ bEY } ¬G£,XEÝ ½ È 3 ?U Ã6ÑEÝ ½ âA× ¹ Û Û,Ì G/ß z û,X Ä

Ú ' G£ $N$ áG¡ á$ã Ú $L$ È Ë $k$ È $k=1,2,\cdots,L$ È Ø $ ,X $ '

$N_k$ È ¸ $Y_{ki}=\begin{cases}1 \quad 1\ k\ 1\ i\ þ\ þ\ '\ ) \quad K\ Ý\ \#\ ó\ 2\ û\ A\\ 0 \quad 1\ k\ 1\ i\ þ\ þ\ '\ ) \quad á\ K\ Ý\ \#\ ó\ 2\ û\ A\end{cases}$ $\begin{pmatrix}k=1,2,\cdots,L\\ i=1.2.\cdots,N_k\end{pmatrix}$ È $\pi_{Ak}$ ><

/ 1 $k$ $ ' K Ý # ó 2 û,X Ž 4!¨ _ È í $\pi_{Ak}=\bar{Y}_k=\frac{1}{N_k}\sum_{i=1}^{N_k}Y_{ki}$ È1 $k$ $

' $W_k=\frac{N_k}{N}$, í ' K Ý # ó 2 û,X Ž 4!¨ _ $\pi_A=\sum_{i=1}^{L}W_k\pi_{Ak}$ Ä

¢ Ø $ Ú ÿ ' ª,X G£ $n_k$ 4˜ ä G£ $n$ È1 $k$ ' !¨

$f_k=\frac{n_k}{N_k}$ È$y_{ki}$ ></ 1 $k$ 1 $i$ þ ) Û Û È $\bar{y}_k$ ></ 1 $k$ K Ý # ó 2 û

,X Ž 4!¨ _ È n ' • Â $\sigma_Y{}^2=\frac{1}{N}\sum_{i=1}^{N}(Y_i-\bar{Y})^2$ È $S_Y{}^2=\frac{1}{N-1}\sum_{i=1}^{N}(Y_i-\bar{Y})^2$ Ä

## 4.2 分层抽样下沃纳模型的比估计

### 4.2.1 模型介绍和参数估计

Ú ' $N$ áG¡ á$ã,X Ú $L$ È Ø ' G£ $N_k$, Ø •G£ $n_k$ È $k=1,2,3$ È È $L$ È ü1T )Lc ‘ ß È Ø FÑ S*ü"W4‡ õ _ È $p$ m Ý 4 2 b $A$ ë Û ,X 5( ,X!¨ _ È $\bar{Z}_k$ 1 $k$ ' ) ²1( ,X V)[ È ¸

$$Z_{ki}=\begin{cases}1 & 1\ \mathrm{k}\ 1\ \mathrm{i}\ \text{þ>•A×¹ 5Ù² 1( “ ”}\\ 0 & 1\ \mathrm{k}\ 1\ \mathrm{i}\ \text{þ>•A×¹ 5Ù² 1( “ ú ”}\end{cases}\begin{pmatrix}k=1,2,\cdots,L\\ i=1.2.\cdots,n\end{pmatrix}$$

Ä $X_{ki}$ ><⁄ 1 $k$ 1 $i$ þ þ ' ) ,XEY } Û Û È $x_{ki}$ ><⁄ 1 $k$ 1 $i$ þ ) ,XEY } Û Û È $\bar{x}_k$ Æ-¹,X Ä J $Z_{ki}$ â $X_{ki}$ K ÝP¬ z!7,Ì G û Ä A’ Ý>•A× ¹5ÙFÑ ,ó r ²1( Ä

í Ý $\bar{Z}_k=P(Z_{ki}=1)=\pi_{Ak}\times p+(1-p)(1-\pi_{Ak})=\dfrac{\bar{Z}_k}{\bar{X}_k}\bar{X}_k$ È $k=1,2,3$ È È $L$, ¸

$Q_k=\dfrac{\bar{Z}_k}{\bar{X}_k}$ b Ý $\pi_{Ak}$ ,X!¨ Au Ö $\hat{\pi}_{Ak}=\dfrac{\hat{\bar{Z}}_k-(1-p)}{(2p-1)}=\dfrac{\hat{Q}_k\bar{X}_k-(1-p)}{(2p-1)}$

Ý $\hat{\pi}_A=\sum_{k=1}^{L}W_k\hat{\pi}_{Ak}=\sum_{k=1}^{L}W_k\dfrac{\hat{Q}_k\bar{X}_k-(1-p)}{(2p-1)}$ Ä

1 Å ' Ø ¹ Ý ²1T )Lc ‘ • ã ‘ ª Ê :

**é)Ú** 4. 1Ï 33 ÐÖü1T )Lc Ý ² ‘ ßE¯> !¨)[ Au Ê È' G£ $n$ C‡ ó û Ê È $\bar{y}_R$ ` $\hat{R}$ #äE¯´ #,X È G $E(\bar{y}_R)\approx\bar{Y}$ È $E(\hat{R})\approx R$ È è $\bar{y}_R$ ` $\hat{R}$,X • # Â Ö

$$V(\hat{R})\approx\frac{\left(\sigma_Y{}^2-2R\sigma_{XY}+R^2\sigma_X{}^2\right)}{n\bar{X}_k{}^2}\ \text{È}\ V(\bar{y}_R)=V(\hat{R}X)\approx\frac{\left(\sigma_Y{}^2-2R\sigma_{XY}+R^2\sigma_X{}^2\right)}{n}$$

J $\sigma_Y{}^2$ ' • Â È $\sigma_X{}^2$ EY } ¬G£ $X$ ,X • Â È $\sigma_{XY}$ EY } ¬G£ $X$ â $Y$ ,X # • Â È $\rho$ $X$ â $Y$ ,X,Ì G2Ï D Ä

**n)Ú** 4. 1 ÖÚ ‘ Èü!£ Ô S*ü1T )Lc Ý ² ‘ !¨)[ Au È Ø *ü "W4‡ õ _>™5B È '$n_k$ C‡ ó û È $\hat{\pi}_{AR}$ #äE¯´ #,X , è Ý $\pi_{AR}$,XE¥ # Â $MSE(\hat{\pi}_{AR})\approx$

$$V(\hat{\pi}_{AR})=\sum_{k=1}^{L}W_k{}^2\{\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k}+\frac{p(1-p)}{(2p-1)^2n_k}-\frac{2[(2p-1)\pi_{Ak}+(1-p)]\pi_{Ak}}{(2p-1)n_k}C_{Y_kX_k}$$

$$+\frac{[(2p-1)\pi_{Ak}+(1-p)]^2}{(2p-1)^2n_k}C_{X_k}{}^2\}\ \text{Ä}$$

J È $C_{X_k}=\sigma_{X_k}/\bar{X}_k$ È $C_{Y_kX_k}=\sigma_{Y_kX_k}/\bar{Y}_k\bar{X}_k=\sigma_{Y_kX_k}/\pi_{Ak}\bar{X}_k$ È $k=1,2,3$ È È $L$ Ä

$\sigma_{Y_k}=\dfrac{1}{N_k}\sum_{i=1}^{N_k}(Y_{ki}-\bar{Y}_k)^2$ È $\sigma_{X_k}=\dfrac{1}{N_k}\sum_{i=1}^{N_k}(X_{ki}-\bar{X}_k)^2$ È $\sigma_{Y_kX_k}=\dfrac{1}{N_k}\sum_{i=1}^{N_k}(Y_{ki}-\bar{Y}_k)(X_{ki}-\bar{X}_k)$ È

A• â Ö+ é)Ú 2. 1 ` é)Ú 4. 1 -¹ È ' $n_k$ C‡ ó û Ê í Ý

$$E\left(\hat{\pi}_{Ak}\right)=E[\frac{\hat{\bar{Z}}_k-(1-p)}{(2p-1)}]=\frac{E(\hat{Q}_k\,\overline{X}_k)-(1-p)}{(2p-1)}\approx\frac{Q_k\,\overline{X}_k-(1-p)}{(2p-1)}=\pi_{Ak}$$

1

$$E(\hat{\pi}_{AR})=\sum_{k=1}^{L}W_k E(\hat{\pi}_{Ak})=\sum_{k=1}^{L}W_k\frac{\hat{Q}_k\,\overline{X}_k-(1-p)}{(2p-1)}\approx\sum_{k=1}^{L}W_k\frac{Q_k\,\overline{X}_k-(1-p)}{(2p-1)}=\pi_A$$

´!8 $\hat{\pi}_{AR}$ $\pi_{AR}$ ,X#äE¯´ # AuG£ Ä

$$V\left(\hat{Q}_k\bar{X}_k\right)\approx\frac{\sum_{i=1}^{N_k}\left(Z_{ki}-Q_kX_{ki}\right)^2}{n_kN_k}$$

$$=\frac{\sum_{i=1}^{N_k}[(Z_{ki}-\bar{Z}_k)-Q_k(X_{ki}-\bar{X}_k)]^2}{n_kN_k}$$

$$=\frac{\left(\sigma_{Z_k}{}^2-2Q_k\sigma_{Z_kX_k}+Q_k{}^2\sigma_{X_k}{}^2\right)}{n_k}$$

œ $Q_k=\frac{\bar{Z}_k}{\bar{X}_k}=\frac{\left(2p-1\right)\pi_{Ak}+(1-p)}{\bar{X}_k}$

$$\frac{\sigma^2_{Z_k}}{n_k}=\frac{1}{n_k}\bar{Z}_k(1-\bar{Z}_k)=\frac{(2p-1)^2\pi_{Ak}(1-\pi_{Ak})}{n_k}+\frac{p(1-p)}{(2p-1)^2n_k}$$

$$\sigma_{Z_kX_k}=n_kE[\left(\bar{z}_k-\bar{Z}_k\right)\left(\bar{x}_k-\bar{X}_k\right)]=n_kE[(2p-1)(\bar{y}_k-\bar{Y}_k)\left(\bar{x}_k-\bar{X}_k\right)]=\left(2p-1\right)\sigma_{Y_kX_k}$$

b $V\left(\bar{Z}_k\right)=V\left(\hat{Q}_k\bar{X}_k\right)$

$$\approx\frac{1}{n_k}\{\sigma^2_{Z_k}-2[\frac{\left(2p-1\right)\pi_{Ak}+(1-p)}{\bar{X}_k}]\left(2p-1\right)\sigma_{Y_kX_k}+[\frac{\left(2p-1\right)\pi_{Ak}+(1-p)}{\bar{X}_k}]^2\sigma_{X_k}{}^2\}$$

$$=\frac{\sigma^2_{Z_k}}{n_k}-\frac{2\left(2p-1\right)[\left(2p-1\right)\pi_{Ak}+(1-p)]\pi_{Ak}}{n_k}C_{Y_kX_k}+\frac{[\left(2p-1\right)\pi_{Ak}+(1-p)]^2}{n_k}C_{X_k}{}^2$$

J È $C_{X_k}=\sigma_{X_k}\big/\bar{X}_k$ È $C_{Y_k}=\sigma_{Y_k}\big/\pi_{Ak}$ È $C_{Y_kX_k}=\sigma_{Y_kX_k}\big/\bar{Y}_k\bar{X}_k=\sigma_{Y_kX_k}\big/\pi_{Ak}\bar{X}_k$ Ä

' â È $V(\hat{\pi}_{Ak})=V[\frac{\hat{Z}_k-(1-p)}{(2p-1)}]=\frac{V(\hat{Z}_k)}{(2p-1)^2}$

$$\approx\frac{\sigma^2_{Z_k}}{\left(2p-1\right)^2n_k}-\frac{2[\left(2p-1\right)\pi_{Ak}+(1-p)]\pi_{Ak}}{\left(2p-1\right)n_k}C_{Y_kX_k}+\frac{[\left(2p-1\right)\pi_{Ak}+(1-p)]^2}{\left(2p-1\right)^2n_k}C_{X_k}{}^2$$

$$=\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k}+\frac{p(1-p)}{(2p-1)^2n_k}-\frac{2[\left(2p-1\right)\pi_{Ak}+(1-p)]\pi_{Ak}}{\left(2p-1\right)n_k}C_{Y_kX_k}$$

$$+\frac{[\left(2p-1\right)\pi_{Ak}+(1-p)]^2}{\left(2p-1\right)^2n_k}C_{X_k}{}^2$$

¹ È $MSE(\hat{\pi}_{AR}) \approx V(\hat{\pi}_{AR}) = \sum_{k=1}^{L} W_k^{\ 2} V(\hat{\pi}_{Ak})$

$$\approx \sum_{k=1}^{L} W_k^{\ 2} \{ \frac{\pi_{Ak}(1-\pi_{Ak})}{n_k} + \frac{p(1-p)}{(2p-1)^2 n_k} - \frac{2[\left(2p-1\right)\pi_{Ak} + (1-p)]\pi_{Ak}}{\left(2p-1\right) n_k} C_{Y_k X_k}$$

$$+ \frac{[\left(2p-1\right)\pi_{Ak} + (1-p)]^2}{\left(2p-1\right)^2 n_k} C_{X_k}^{\ 2} \}$$

ü2' z ÞL$ $\varepsilon$ ,X ™ ‰ ß ÈW4‡ õ _1T ) "©,X G£

$n_0 \geq [\frac{1}{4\varepsilon} + \frac{p(1-p)}{(2p-1)^2 \varepsilon}] + 1$ ÄE- Í • Â Ý4- n,X2' z 9 ‡ n G£,X ™ ‰ Ä í Í"W

4‡ õ _ ß!¨ Au"© G£,X.B n n $n = n_0 \cdot \frac{V\left(\hat{\pi}_R\right)}{V\left(\hat{\pi}\right)}$ Ä

2 Å ' Ø ¹ ´ ²1T )Lc ' • ã ' ª Ê :

é)Ú 4. 2Ï 33 ÐÖü1T )Lc ´ ² ' ßE¯> !¨)[ Au Ê È G£ $n$ C‡ ó û Ê È

$\bar{y}_R$ ` $\hat{R}$ #äE¯ ´ #,X È G $E\left(\bar{y}_R\right) \approx \bar{Y}$ È $E\left(\hat{R}\right) \approx R$ È è $\bar{y}_R$ ` $\hat{R}$ ,XE¥ # Â Ö

$$V\left(\hat{R}\right) \approx \frac{\left(1-f\right)\left(S_Y^{\ 2} - 2RS_{XY} + R^2 S_X^{\ 2}\right)}{n\bar{X}_k^{\ 2}}$$

$$V\left(\bar{y}_R\right) = V\left(\hat{R}X\right) \approx \frac{\left(1-f_k\right)\left(S_Y^{\ 2} - 2RS_{XY} + R^2 S_X^{\ 2}\right)}{n}$$

J $S_Y^{\ 2}$ ' • Â È $S_X^{\ 2}$ EY } ¬G£ $X$ ,X • Â È $S_{XY}$ EY } ¬G£ $X$ â $Y$ ,X # • Â È $\rho$ $X$ â $Y$ ,X,Ì G2Ï D Ä

n)Ú 4. 2 Ö Ú ' È ü!£ Ô S*ü1T )Lc ´ ² ' ßE¯> !¨)[ Au È

Ø *ü"W4‡ õ _>™5B È ' $n_k$ C‡ ó û È Ý $\pi_{AR}$ ,XE¥ # Â È $MSE(\hat{\pi}_{AR}) \approx V(\hat{\pi}_{AR}) =$

$$\sum_{k=1}^{L} W_k^{\ 2} \{ \frac{N_k - n_k}{n_k(N_k - 1)} [\pi_{Ak}(1-\pi_{Ak}) + p(1-p)] - \frac{1-f_k}{(2p-1)^2 n_k} \{ [2(2p-1)^2 \pi_{Ak}$$

$$+2\left(2p-1\right)(1-p)]\pi_{Ak} C_{Y_k X_k} + [\left(2p-1\right)\pi_{Ak} + (1-p)]^2 C_{X_k}^{\ 2} \}\}$$ Ä

J È $C_{X_k} = S_{X_k} \big/ \bar{X}_k$ È $C_{Y_k X_k} = S_{Y_k X_k} \big/ \pi_{Ak} \bar{X}_k$ È $S_{Y_k}^{\ 2} = \frac{1}{N_k - 1} \sum_{i=1}^{N_k} (Y_{ki} - \bar{Y}_i)^2$

$S_{X_k}^{\ 2} = \frac{1}{N_k - 1} \sum_{i=1}^{N_k} (X_{ki} - \bar{X}_i)^2$ È $S_{Y_k X_k} = \frac{1}{N_k - 1} \sum_{i=1}^{N_k} (Y_{ki} - \bar{Y}_k)(X_{ki} - \bar{X}_k)$ È $k = 1, 2, 3$ È È $L$ Ä

A• â Ö+ é)Ú 4. 2 -¹ Ö ' $N_k$ Ú û Ê

$$V(\hat{Z}_k) \approx \frac{1-f_k}{n_k} \frac{1}{N_k - 1} \sum_{i=1}^{N_k} (Z_{ki} - Q_k X_{ki})^2$$

$$=\frac{1-f_k}{n_k}\frac{1}{N_k-1}\sum_{i=1}^{N_k}[(Z_{ki}-\bar{Z}_k)-Q_k(X_{ki}-\bar{X}_k)]^2$$

$$=\frac{1-f_k}{n_k}[S_{Z_k}{}^2-2Q_kS_{Z_kX_k}+Q_k{}^2S_{X_k}{}^2]$$

$$=\frac{1-f_k}{n_k}[S_{Z_k}{}^2-2(2p-1)\frac{(2p-1)\pi_{Ak}+(1-p)}{\bar{X}_k}S_{Y_kX_k}+\frac{[(2p-1)\pi_{Ak}+(1-p)]^2}{\bar{X}_k{}^2}S_{X_k}{}^2]$$

J $Q_k=\frac{\bar{Z}_k}{\bar{X}_k}=\frac{(2p-1)\pi_{Ak}+(1-p)}{\bar{X}_k}$ Ä

œ$S_{Z_kX_k}=\frac{n_k}{1-f_k}E[(\bar{z}_k-\bar{Z}_k)(\bar{x}_k-\bar{X}_k)]=\frac{n_k}{1-f_k}E[p(\bar{y}_k-\bar{Y}_k)(\bar{x}_k-\bar{X}_k)]=(2p-1)S_{Y_kX_k}$

$C_{X_k}=S_{X_k}/\bar{X}_k$ È $C_{Y_kX_k}=S_{Y_kX_k}/\bar{Y}_k\bar{X}_k=S_{Y_kX_k}/\pi_{Ak}\bar{X}_k$ È

+ é)Ú 2.2 -¹

$$\frac{(1-f_k)S_{Z_k}^2}{n_k}\approx\frac{1}{n_k}(1-f_k)\lambda(1-\lambda)=\frac{(1-f_k)}{n_k}[(2p-1)^2\pi_{Ak}(1-\pi_{Ak})+p(1-p)]$$

¹ Ý $V(\hat{Z}_k)\approx\frac{1-f_k}{n_k}\{[(2p-1)^2\pi_{Ak}(1-\pi_{Ak})+p(1-p)]-$

$$[2(2p-1)^2\pi_{Ak}+2(2p-1)(1-p)]\pi_{Ak}C_{Y_kX_k}+$$

$$[(2p-1)\pi_{Ak}+(1-p)]^2C_{X_k}{}^2\}$$

$$V(\hat{\pi}_{Ak})=V[\frac{\hat{Z}_k-(1-p)}{(2p-1)}]=\frac{V(\hat{Z}_k)}{(2p-1)^2}$$

$$=\frac{1}{n_k}\{[\pi_{Ak}(1-\pi_{Ak})+\frac{p(1-p)}{(2p-1)^2}-2[\pi_{Ak}+\frac{(1-p)}{(2p-1)}]\pi_{Ak}C_{Y_kX_k}+[\pi_{Ak}+\frac{1-p}{2p-1}]^2C_{X_k}{}^2\}$$

$$MSE(\hat{\pi}_{AR})\approx V(\hat{\pi}_{AR})=\frac{1}{(2p-1)^2}\sum_{k=1}^{L}W_k{}^2V\left(\hat{Q}_k\bar{X}_k\right)$$

$$\approx\sum_{k=1}^{L}W_k{}^2\frac{(1-f_k)}{n_k}\{[\pi_{Ak}(1-\pi_{Ak})+\frac{p(1-p)}{(2p-1)^2}-2[\pi_{Ak}+\frac{(1-p)}{(2p-1)}]\pi_{Ak}C_{Y_kX_k}+[\pi_{Ak}+\frac{1-p}{2p-1}]^2C_{X_k}{}^2\}$$

A•!© Ä

### 4.2.2 效率比较

EîE›!¨EWØ "W4‡ õ _$V(\hat{\pi}_{Ak})$ ,X1T ) AuG£ `!¨ AuG£,X><E’ ã ¥)„ : ü Ô n 5 Ê ß È!¨ AuG£,X2’ z?UP¬ b s"W4‡ õ _,X1T ) AuG£ Ä G Ý Ö ' $n_k$ Ú û Ê È!¨ Au"© ì b1T ) "©,X 5 Ê

$$\frac{2[(2p-1)\pi_{Ak}+(1-p)]\pi_{Ak}}{(2p-1)n_k}C_{Y_kX_k}\geq\frac{[(2p-1)\pi_{Ak}+(1-p)]^2}{(2p-1)^2n_k}C_{X_k}{}^2$$

$$\Leftrightarrow 2(2p-1)\pi_{Ak}C_{Y_kX_k} \geq [(2p-1)\pi_{Ak}+(1-p)]C_{X_k}{}^2$$

$$\Leftrightarrow 2(2p-1)\pi_{Ak}\rho_k C_{Y_k}C_{X_k} \geq [(2p-1)\pi_{Ak}+(1-p)]C_{X_k}{}^2$$

$$\Leftrightarrow \rho_k \geq \frac{(2p-1)\pi_{Ak}+(1-p)}{2(2p-1)\pi_{Ak}}\frac{C_{X_k}}{C_{Y_k}}$$

$\rho_k = C_{Y_kX_k}/C_{Y_k}C_{X_k}$ $X_k$ â $Y_k$ ,X,Ì G2Ï D Ä

' $C_{X_k}=C_{Y_k}$ È $X$ â $Y$ Ý,ÌE¥,X ¬ Ö2Ï D È G $X$ $Y$ ! ó Z Æ D B È $\rho_k \geq \frac{1}{2}\frac{(2p-1)\pi_{Ak}+(1-p)}{(2p-1)\pi_{Ak}}$ Ê È!¨ Au"© â 2' z ì b1T ) Au Ä

## 4.3 分层抽样下西蒙斯模型的比估计

### 4.3.1 模型介绍和参数估计

Ú ' $N$ áG¡ á$ã,X Ú $L$ È Ø ' G£ $N_k$, Ø •G£ $n_k$ È $k=1,2,3$ È È $L$ È ü1T )Lc ‘ ß È Ø FÑ S*ü?S:m ƒLc ê>™5B È $p$ m Ý 4 2 b $A$ ë Û ,X 5( ,X!¨ _ È $1-p$ m Ý 4 2 b $B$ ë Û ,X 5( ,X!¨ _ È $B$ â # óKÂNl ´ G,XKÂNl È è $\pi_{Bk}$ Æ-¹ Ä$\bar{Z}_k$ 1 $k$ ' ) ²1( ,X V )[ È ¸ $Z_{ki}=\begin{cases}1 & 1\ k\ 1\ i\ \text{þ>•A×¹ 5Ù² 1( “ ”} \\ 0 & 1\ k\ 1\ i\ \text{þ>•A×¹ 5Ù² 1( “ ú ”}\end{cases}\begin{pmatrix}k=1,2,\cdots,L \\ i=1.2.\cdots,n\end{pmatrix}$ Ä$X_{ki}$ ></ 1 $k$ 1 $i$ þ þ ' ) ,XEY } Û Û È $x_{ki}$ ></ 1 $k$ 1 $i$ þ ) ,XEY } Û Û È Æ-¹,X Ä J $Z_{ki}$ â $X_{ki}$ K ÝP¬ z!7,Ì G û Ä A' Ý>•A× ¹5ÙFÑ ,ó r ²1( Ä í

$\bar{Z}_k = P(Z_{ki}=1)=\pi_{Ak}\times p+(1-p)\pi_{Bk}=\frac{\bar{Z}_k}{\bar{X}_k}\overline{X}_k = Q_k\overline{X}_k$ Ä

b Ý Ö $\hat{\pi}_{Ak}=\frac{\hat{\bar{Z}}_k-(1-p)\pi_{Bk}}{p}=\frac{\hat{Q}_k\overline{X}_k-(1-p)\pi_{Bk}}{p}$ È

$$\hat{\pi}_{AR}=\sum_{k=1}^{L}W_k\hat{\pi}_{Ak}=\frac{1}{p}\sum_{h=1}^{L}W_k\left[\hat{Q}_k\overline{X}_k-(1-p)\pi_{Bk}\right]$$

1 Å ' Ø ¹ Ý ²1T )Lc ‘ • ã ‘ ª Ê Ö

**n)Ú** 4. 3 **Ö** Ú ‘ È ü!£ Ô S*ü1T )Lc ² ‘ ßE¯> !¨)[ Au È Ø *ü?S:m ƒ õ _>™5B È ' $\pi_{Bk}$ Æ-¹ Ê È $n_k$ C‡ ó û È Ý $\pi_{AR}$ ,XE¥ # Â

$$MSE(\hat{\pi}_{AR}) \approx \sum_{k=1}^{L}W_k^2\{[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k}-\frac{(1-p)(\pi_{Ak}+\pi_{Bk}-2\pi_{Ak}\pi_{Bk})}{pn_k}+\frac{(1-p)^2\pi_{Bk}(1-\pi_{Bk})}{p^2n_k}$$

$$-\frac{2[p\pi_{Ak}+(1-p)\pi_{Bk}]\pi_{Ak}C_{Y_kX_k}}{pn_k}+\frac{[p\pi_{Ak}+(1-p)\pi_{Bk}]^2\pi_{Ak}{}^2C_{X_k}{}^2}{p^2n_k}\}$$

A• â Ö ´ $Q_k = \dfrac{\bar{Z}_k}{\bar{X}_k} = \dfrac{\pi_{Ak} \times p + (1-p)\pi_{Bk}}{\bar{X}_k}$

$$\sigma_{Z_k X_k} = n_k E[\left(\bar{z}_k - \bar{Z}_k\right)\left(\bar{x}_k - \bar{X}_k\right)] = n_k E[\left(p(\bar{y}_k - \bar{Y}_k)\right)\left(\bar{x}_k - \bar{X}_k\right)] = p\sigma_{Y_k X_k}$$

¹ È $V\left(\bar{Z}_k\right) = V\left(Q_k \bar{X}_k\right) \approx \dfrac{\left(\sigma_{Z_k}{}^2 - 2Q_k \sigma_{Z_k X_k} + Q_k{}^2 \sigma_{X_k}{}^2\right)}{n_k}$

$$= \frac{1}{n_k}\{\sigma_{Z_k}{}^2 - 2[p\pi_{Ak} + (1-p)\pi_{Bk}]p\frac{\sigma_{Y_k X_k}}{\bar{X}_k} + [p\pi_{Ak} + (1-p)\pi_{Bk}]^2 \frac{\sigma_{X_k}{}^2}{\bar{X}_k^2}\}$$

$$= \frac{1}{n_k}\{\sigma_{Z_k}{}^2 - 2[p\pi_{Ak} + (1-p)\pi_{Bk}]\pi_{Ak} p C_{Y_k X_k} + [p\pi_{Ak} + (1-p)\pi_{Bk}]^2 C_{X_k}{}^2\}$$

5à È $MSE(\hat{\pi}_{AR}) \approx V(\hat{\pi}_{AR}) = \sum_{k=1}^{L} W^2{}_k V(\hat{\pi}_{Ak})$

$$= \sum_{k=1}^{L} W^2{}_k \frac{V[\hat{Q}_{Sk} X_k - (1-p)\pi_{Bk}]}{p^2} = \sum_{k=1}^{L} \frac{W_k^2}{p^2} V\left(\hat{Q}_k X_k\right)$$

$$\approx \sum_{k=1}^{L} \frac{W_k^2}{p^2 n_k}\{\sigma_{Z_k}{}^2 - 2[p\pi_{Ak} + (1-p)\pi_{Bk}]\pi_{Ak} p C_{Y_k X_k} + [p\pi_{Ak} + (1-p)\pi_{Bk}]^2 C_{X_k}{}^2\}$$

$$= \sum_{k=1}^{L} W_k^2 \{[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k} - \frac{(1-p)(\pi_{Ak} + \pi_{Bk} - 2\pi_{Ak}\pi_{Bk})}{pn_k} + \frac{(1-p)^2 \pi_{Bk}(1-\pi_{Bk})}{p^2 n_k}$$

$$- \frac{2[p\pi_{Ak} + (1-p)\pi_{Bk}]\pi_{Ak} C_{Y_k X_k}}{pn_k} + \frac{[p\pi_{Ak} + (1-p)\pi_{Bk}]^2 C_{X_k}{}^2}{p^2 n_k}\}$$ A•!© Ä

2 Å ' Ø ¹ ´ ²1T )Lc ' • ã ' ª Ê :

**n)Ú** 4. 4 **Ö**Ú ' È ü!£ Ô S*ü1T )Lc ² ' ßE¯> !¨)[ Au È Ø

*ü?S:m ƒ õ _>™5B È ' $\pi_{Bk}$ Æ-¹ Ê È $n_k$ C‡ ó û È Ý $\pi_{AR}$ ,XE¥ # Â $V(\hat{\pi}_{AR})$

$$= \sum_{k=1}^{L} W_k{}^2 \frac{1-f_k}{n_k}[\pi_{Ak}(1-\pi_{Ak}) - \frac{(1-p)(\pi_{Ak} + \pi_{Bk} - 2\pi_{Ak}\pi_{Bk})}{p} + \frac{(1-p)^2 \pi_{Bk}(1-\pi_{Bk})}{p^2}$$

$$-2[\pi_{Ak} + \frac{(1-p)\pi_{Bk}}{p}]\pi_{Ak} C_{Y_k X_k} + [\pi_{Ak} + \frac{(1-p)\pi_{Bk}}{p}]^2 C_{X_k}{}^2]$$ Ä

A• â Ö ´ $S_{Z_k X_k} = pS_{Y_k X_k}$

¹ $$V(\hat{\pi}_{AR}) = \sum_{k=1}^{L} W^2{}_k V(\hat{\pi}_{Ak}) = \sum_{k=1}^{L} W^2{}_k \frac{V[\hat{Q}_{Sk} X_k - (1-p)\pi_{Bk}]}{p^2}$$

$$= \sum_{k=1}^{L} \frac{W_k^2}{p^2} V\left(\hat{Q}_k X_k\right) \approx \sum_{k=1}^{L} W_k{}^2 \frac{1-f_k}{p^2 n_k}[S_{Z_k}{}^2 - 2Q_{Sk} S_{Z_k X_k} + Q_{Sk}{}^2 S_{X_k}{}^2]$$

$$= \sum_{k=1}^{L} W_k{}^2 \frac{1-f_k}{p^2 n_k}[S_{Z_k}{}^2 - 2p\frac{p\pi_{Ak} + (1-p)\pi_{Bk}}{\bar{X}_k} S_{Y_k X_k} + \frac{[p\pi_{Ak} + (1-p)\pi_{Bk}]^2}{\bar{X}_k{}^2} S_{X_k}{}^2]$$

$$= \sum_{k=1}^{L} W_k{}^2 \frac{1-f_k}{n_k}[S_{Y_k}{}^2 - 2[\pi_{Ak} + \frac{(1-p)\pi_{Bk}}{p}]\pi_{Ak} C_{Y_k X_k} + [\pi_{Ak} + \frac{(1-p)\pi_{Bk}}{p}]^2 C_{X_k}{}^2]$$

$$=\sum_{k=1}^{L} W_k^{2} \frac{1-f_k}{n_k}[\pi_{Ak}(1-\pi_{Ak})-\frac{(1-p)(\pi_{Ak}+\pi_{Bk}-2\pi_{Ak}\pi_{Bk})}{p}+\frac{(1-p)^2\pi_{Bk}(1-\pi_{Bk})}{p^2}$$

$$-2[\pi_{Ak}+\frac{(1-p)\pi_{Bk}}{p}]\pi_{Ak}C_{Y_kX_k}+[\pi_{Ak}+\frac{(1-p)\pi_{Bk}}{p}]^2 C_{X_k}{}^{2}]$$ ÄA•!© Ä

### 4.3.2 效率比较

EîE›!¨EWØ ?S:m ƒ õ _ $V(\hat{\pi}_{Ak})$ ,X1T ) Au `!¨ Au,X><E’ ã ¥)„ : ü Ô n 5 Ê ß È!¨ AuG£,X2’ z?UP¬ b s?S:m ƒ õ _,X1T ) AuG£ Ä G Ý Ö ' $n_k$ Ú û Ê È!¨ Au"© ì b1T ) "©,X 5 Ê

$$2\pi_{Ak}[\pi_{Ak}+\frac{(1-p)\pi_{Bk}}{p}]C_{Y_kX_k}\ge[\pi_{Ak}+\frac{(1-p)\pi_{Bk}}{p}]^2 C_{X_k}{}^{2}$$

$$\Leftrightarrow 2\pi_{Ak}C_{Y_kX_k}\ge[\pi_{Ak}+\frac{(1-p)\pi_{Bk}}{p}]C_{X_k}{}^{2}$$

$$\Leftrightarrow 2\rho_k C_{Y_k}\ge[1+\frac{(1-p)\pi_{Bk}}{p\pi_{Ak}}]C_{X_k}$$

$$\Leftrightarrow \rho_k \ge \frac{1}{2}\frac{C_{X_k}}{C_{Y_k}}\frac{p\pi_{Ak}+(1-p)\pi_{Bk}}{p\pi_{Ak}}$$

J $\rho_k = C_{Y_kX_k}\big/C_{Y_k}C_{X_k}$ $X_k$ â $Y_k$ ,X,Ì G2Ï D Ä

' $C_{X_k}=C_{Y_k}$ È $X$ â $Y$ Ý,Ì E¥,X ¬ Ö2Ï D È G $X$ $Y$ ! ó Z Æ D B È $\rho_k \ge \frac{1}{2}\frac{p\pi_{Ak}+(1-p)\pi_{Bk}}{p\pi_{Ak}}$ Ê È!¨ Au"© â 2’ z ì b1T ) Au Ä

## 4.4 分层抽样下改进模型的比估计

### 4.4.1 模型介绍和参数估计

Ú ' $N$ áG¡ á$ã,X Ú $L$ ÈØ ' G£ $N_k$, Ø •G£ $n_k$ È $k=1,2,3$ È È $L$ È ü1T )Lc ‘ ß È Ø FÑ S*ü ´ G õ _ Lc ê>™5B È $p_1$ m Ý 4 2 b $A$ ë Û ,X 5( ,X!¨ _ È $p_2$ m Ý 4 2 b $\bar{A}$ ë Û ,X 5( ,X!¨ _ È $p_3$ m Ý 4 2 b $B$ ë Û ,X 5( ,X!¨ _ È $p_1+p_2+p_3=1, p_1>p_2$, $B$ â # óKÂ Nl ´ G,XKÂNl È è $\pi_{Bk}$ Æ-¹ Ä $\bar{Z}_k$ 1 $k$ ' ) ²1( ,X V)[ Ä

¸ $Z_{ki}=\begin{cases} 1 & 1\ \mathrm{k}\ \ 1\ \mathrm{i}\ \text{þ>•A×¹ 5Ù² 1( “ ”} \\ 0 & 1\ \mathrm{k}\ \ 1\ \mathrm{i}\ \text{þ>•A×¹ 5Ù² 1( “ ú ”} \end{cases}\begin{pmatrix} k=1,2,\cdots,L \\ i=1.2.\cdots,n \end{pmatrix}$

$X_{ki}$ ></ 1 $k$ 1 $i$ þ þ ' ) ,XEY } Û Û È $x_{ki}$ ></ 1 $k$ 1 $i$ þ ) ,XEY } Û Û È Æ-¹,X Ä J $Z_{ki}$ â $X_{ki}$ K ÝP¬ z!7,Ì G û Ä A' Ý>•A× ¹5ÙFÑ ,ó r ²1( Ä í $\bar{Z}_k = P(Z_{ki}=1) = \pi_{Ak}\times p_1 + p_2(1-\pi_{Ak}) + p_3\pi_{Bk} = \frac{\bar{Z}_k}{\bar{X}_k}\bar{X}_k = Q_k\bar{X}_k$ Ä

$$\hat{\pi}_{Ak} = \frac{\hat{\bar{Z}}_k - p_2 - p_3\pi_{B_k}}{p_1 - p_2},\quad \text{¢5à}\ \hat{\pi}_A = \sum_{k=1}^{L} W_k\hat{\pi}_{Ak} = \sum_{k=1}^{L} W_k \frac{\hat{\bar{Z}}_k - p_2 - p_3\pi_{Bk}}{p_1 - p_2}$$

$$V(\hat{\pi}_{AR}) = \sum_{k=1}^{L} W_k^{\ 2}\frac{V(\hat{\bar{Z}}_k)}{(p_1-p_2)^2}$$

1 Å ' Ø ¹ Ý ²1T )Lc ' • ã ' ª Ê Ö

**n)Ú** 4. 5 **Ö**Ú ' È ü!£ Ô S*ü1T )Lc ² ' ßE¯> !¨)[ Au È Ø *ü E¯ õ _>™5B È ' $\pi_{Bk}$ Æ-¹ Ê È $n_k$ C‡ ó û È Ý $\pi_{AR}$ ,XE¥ # Â $V(\hat{\pi}_{AR})$

$$= \sum_{k=1}^{L}\frac{W_k^{\ 2}}{n_k}\{\pi_A(1-\pi_A) + \frac{p_3(\pi_A+\pi_B-2\pi_A\pi_B)}{(p_1-p_2)} + \frac{p_3^2\pi_B(1-\pi_B)+p_2(p_1+p_3)}{(p_1-p_2)^2}$$

$$-\frac{2\pi_{Ak}[\pi_{Ak}\times p_1 + p_2(1-\pi_{Ak}) + p_3\pi_{Bk}]C_{Y_kX_k}}{(p_1-p_2)} + \frac{[\pi_{Ak}\times p_1 + p_2(1-\pi_{Ak}) + p_3\pi_{Bk}]^2 C_{X_k}^{\ 2}}{(p_1-p_2)^2}\}\ \text{Ä}$$

A• â Ö $Q_{Sk} = \frac{\bar{Z}_k}{\bar{X}_k} = \frac{\pi_{Ak}\times p_1 + p_2(1-\pi_{Ak}) + p_3\pi_{Bk}}{\bar{X}_k}$

$$\sigma_{Z_kX_k} = n_kE[\left(z_k - \bar{Z}_k\right)\left(x_k - \bar{X}_k\right)] = (p_1-p_2)\sigma_{Y_kX_k}$$

¹ È

$$V\left(\bar{Z}_k\right) = V\left(Q_{Sk}\bar{X}_k\right) \approx \frac{\left(\sigma_{Z_k}^{\ 2} - 2Q_{Sk}\sigma_{Z_kX_k} + Q_{Sk}^{\ 2}\sigma_{X_k}^{\ 2}\right)}{n_k}$$

$$= \frac{1}{n_k}\{\sigma_{Z_k}^{\ 2} - 2[\pi_{Ak}\times p_1 + p_2(1-\pi_{Ak}) + p_3\pi_{Bk}](p_1-p_2)\frac{\sigma_{Y_kX_k}}{\bar{X}_k}$$

$$+[\pi_{Ak}\times p_1 + p_2(1-\pi_{Ak}) + p_3\pi_{Bk}]^2\frac{\sigma_{X_k}^{\ 2}}{\bar{X}_k^2}\}$$

$$= \frac{1}{n_k}\{\sigma_{Z_k}^{\ 2} - 2\pi_{Ak}(p_1-p_2)[\pi_{Ak}\times p_1 + p_2(1-\pi_{Ak}) + p_3\pi_{Bk}]C_{Y_kX_k}$$

$$+[\pi_{Ak}\times p_1 + p_2(1-\pi_{Ak}) + p_3\pi_{Bk}]^2 C_{X_k}^{\ 2}\}$$

¹ $V(\hat{\pi}_{AR}) = \sum_{k=1}^{L} W^2{}_k V(\hat{\pi}_{Ak}) = \sum_{k=1}^{L} W_k^{\ 2}\frac{V(\hat{\bar{Z}}_k)}{(p_1-p_2)^2}$

$$= \sum_{k=1}^{L}\frac{W_k^{\ 2}}{n_k}\{\pi_A(1-\pi_A) + \frac{p_3(\pi_A+\pi_B-2\pi_A\pi_B)}{(p_1-p_2)} + \frac{p_3^2\pi_B(1-\pi_B)+p_2(p_1+p_3)}{(p_1-p_2)^2}$$

$$-\frac{2\pi_{Ak}[\pi_{Ak}\times p_1 + p_2(1-\pi_{Ak}) + p_3\pi_{Bk}]C_{Y_kX_k}}{(p_1-p_2)} + \frac{[\pi_{Ak}\times p_1 + p_2(1-\pi_{Ak}) + p_3\pi_{Bk}]^2 C_{X_k}^{\ 2}}{(p_1-p_2)^2}\}$$

2 Å ' Ø ´ Ý ²1T )Lc ‘ • ã ‘ ª Ê Ö

n)Ú 4. 6 Ö Ú ‘ È ü!£ Ô S*ü1T )Lc ´ ² ‘ ßE¯> !¨)[ Au È Ø *ü E¯ õ _>™5B È ' $\pi_{Bk}$ Æ-¹ Ê È $n_k$ C‡ ó û È Ý $\pi_{AR}$ ,XE¥ # Â $V(\hat{\pi}_{AR})$

$$= \sum_{k=1}^{L} W_k^{\,2}\,\frac{(1-f_k)}{n_k}\{\pi_A(1-\pi_A)+\frac{p_3(\pi_A+\pi_B-2\pi_A\pi_B)}{(p_1-p_2)}+\frac{p_3^2\pi_B(1-\pi_B)+p_2(p_1+p_3)}{(p_1-p_2)^2}$$

$$-\frac{2\pi_{Ak}[\pi_{Ak}\times p_1+p_2(1-\pi_{Ak})+p_3\pi_{Bk}]C_{Y_kX_k}}{(p_1-p_2)}+\frac{[\pi_{Ak}\times p_1+p_2(1-\pi_{Ak})+p_3\pi_{Bk}]^2 C_{X_k}^{\ 2}}{(p_1-p_2)^2}\}$$ Ä

A• â Ö à n)Ú 4. 5 çA• Ä

## 4.4.2 效率比较

EîE›!¨EWØ E¯ õ _ $V(\hat{\pi}_{Ak})$ ,X1T ) Au `!¨ Au,X><E’ ã ¥)„ : ü Ô n 5 Ê ß È!¨ AuG£,X2’ z?UP¬ b s õ _,X1T ) AuG£ Ä G Ý Ö

' $n_k$ Ú û Ê È!¨ Au"© ì b1T ) "©,X 5 Ê

$$\frac{2\pi_{Ak}[\pi_{Ak}\times p_1+p_2(1-\pi_{Ak})+p_3\pi_{Bk}]C_{Y_kX_k}}{(p_1-p_2)}\geq\frac{[\pi_{Ak}\times p_1+p_2(1-\pi_{Ak})+p_3\pi_{Bk}]^2 C_{X_k}^{\ 2}}{(p_1-p_2)^2}$$

$$\Leftrightarrow 2\rho_k\pi_{Ak}(p_1-p_2)C_{Yk}\geq[\pi_{Ak}\times p_1+p_2(1-\pi_{Ak})+p_3\pi_{Bk}]C_{X_k}$$

$\Leftrightarrow \rho_k \geq \frac{1}{2}\frac{C_{X_k}}{C_{Y_k}}[1+\frac{p_2+p_3\pi_{Bk}}{\pi_{Ak}(p_1-p_2)}]$ Ä J $\rho_k = C_{Y_kX_k}/C_{Y_k}C_{X_k}$ $X_k$ â $Y_k$ ,X,Ì G2Ï D Ä

' $C_{X_k}=C_{Y_k}$ È $X$ â $Y$ Ý ,Ì E¥ ,X ¬ Ö 2Ï D È G $X$ $Y$ ! ó Z Æ D B È $\Leftrightarrow \rho_k \geq \frac{1}{2}\frac{C_{X_k}}{C_{Y_k}}[1+\frac{p_2+p_3\pi_{Bk}}{\pi_{Ak}(p_1-p_2)}]$ Ê È!¨ Au"© â 2’ z ì b1T ) Au Ä

# 第五章 分层抽样中敏感性问题的回归估计

## 5.1 定义及符号说明

â!¨ Au2O È ² & Au 3LÔ?U , ü âA× ¹,X ?U ¬G£P¬ z,Ì G,X EY } ¬G£ ,X Ý µ C Èý*ü Ý EY } µ C ¤P¬ Au,X2' z È X AuG£,X p Äá à,X È !¨ Au ó } ø þ AuG£,X!¨ G2Ï È5à ² & Au í ó } ø þ AuG£,X4" û,Ì G G2Ï ÄA',Â Û ¬G£$Y$ È , ü Æ-¹,XEY } ¬G£$X$ â Û Û ¬G£$Y$ K ÝP¬ z4" û,Ì G G2Ï È è $Y$ Í $X$ ,X ² &4" áE› s&• Ê È í Ã ¹ ý*ü $Y$ Í $X$ ,X4" û ² & G2Ï 9 Au # ó ûKÂNl Ä

Ú ' G£ $N$ áG¡ á$ã Ú $L$ È Ë $k$ È $k=1,2,\cdots,L$ È Ø $ ,X $ ' $N_k$ Ä ¸ $Y_{ki}=\begin{cases}1 & \text{1 k 1 i þ þ ' ) K Ý \# ó 2 ûA} \\ 0 & \text{1 k 1 i þ þ ' ) á K Ý \# ó 2 ûA}\end{cases}\begin{pmatrix}k=1,2,\cdots,L \\ i=1.2.\cdots,N_k\end{pmatrix}$ È $\pi_{Ak}$ ></ 1 $k$ ' K Ý # ó 2 û,X Ž 4!¨ _ È í $\pi_{Ak}=\overline{Y}_k=\frac{1}{N_k}\sum_{i=1}^{N_k}Y_{ki}$ È1 $k$ ' $W_k=\frac{N_k}{N}$, ' K Ý # ó 2 û,X Ž 4!¨ _ $\pi_A=\sum_{i=1}^{L}W_k\pi_{Ak}$ Ä

¢ Ø $ Ú ÿ ' ª,X G£ $n_k$ 4˜ ä G£ $n$ È1 $k$ ) $w_k=\frac{n_k}{n}$ È1 $k$ ' !¨ $f_k=\frac{n_k}{N_k}$ Ä$y_{ki}$ ></ 1 $k$ 1 $i$ þ ) Û Û Èn ' • Â $\sigma_Y^2=\frac{1}{N}\sum_{i=1}^{N}(Y_i-\overline{Y})^2$ È $S_Y^2=\frac{1}{N-1}\sum_{i=1}^{N}(Y_i-\overline{Y})^2$ Ä

## 5.2 分层抽样下沃纳模型的回归估计

Ú ' $N$ áG¡ á$ã,X Ú $L$ È Ø ' G£ $N_k$, Ø •G£ $n_k$ È $k=1,2,3$ È È $L$ È ü1T )Lc ' ß È Ø S*ü"W4‡ õ _ È $p$ m Ý 4 2 b $A$ ë Û ,X 5( ,X!¨ _ È $\overline{Z}_k$ 1 $k$ ' ) ²1( ,X V)[ Ä ¸

$$Z_{ki}=\begin{cases}1 & \text{1 k 1 i þ >•A×¹ 5Ù² 1( " "} \\ 0 & \text{1 k 1 i þ >•A×¹ 5Ù² 1( " ú "}\end{cases}\begin{pmatrix}k=1,2,\cdots,L \\ i=1.2.\cdots,n\end{pmatrix}$$

Ä $X_{ki}$ ></ 1 $k$ 1 $i$ þ þ

' ) ,XEY } Û Û È $x_{ki}$ ></ 1 $k$ 1 $i$ þ ) ,XEY } Û Û È Æ-¹,X ÄJ $Z_{ki}$ â $X_{ki}$ K ÝP¬ z!7,Ì G û Ä A' Ý>•A× ¹5ÙFÑ ,ó r ²1( Ä í Ý ² & AuG£

$\bar{Z}_k = \pi_{Ak} \times p + (1-p)(1-\pi_{Ak}) = \hat{\bar{Z}}_k - \beta_k\left(\overline{x_k} - \overline{X}_k\right)$ È ¢5à Ý

$\hat{\pi}_{Ak} = \dfrac{\hat{\bar{Z}}_k - \beta_k\left(\overline{x_k} - \overline{X}_k\right) - (1-p)}{(2p-1)}$ È $\hat{\bar{Z}}_k = (2p-1)\pi_{Ak} + (1-p) + \beta_k\left(\overline{x_k} - \overline{X}_k\right)$ È

$\hat{\pi}_{ALr} = \sum_{k=1}^{L} W_k \dfrac{\hat{\bar{Z}}_k - \beta_k\left(\overline{x_k} - \overline{X}_k\right) - (1-p)}{(2p-1)}$ Ä

' $\beta_k$ LÊ Ê È!8 Ê $\hat{\pi}_{Alr}$ ,X ² & AuG£ G "W4‡ õ _,X1T ) Au Ä

' $\beta_k = \dfrac{\overline{y_k}}{\overline{\overline{x_k}}}$ Ê È!8 Ê,X ² & Au õ _ "W4‡ õ _,X!¨ Au Ä AuG£ Ã –5×1 ¯ 0´ Ä+ !8 Ã?• È1T ) AuG£ â!¨)[ AuG£ Ã?š ² & Au,X(M!^ ™ ‰ Ä

### 5.2.1 $\beta_k$ 为确定的常数

1 Å ' Ø ¹ Ý ²1T )Lc ‘ • ã ‘ ª Ê :

**n)Ú** 5. 1 **Ö**Ú ‘ È ü!£ Ô S*ü1T )Lc ² ‘ ßE¯> ² & Au È Ø *ü"W4‡ õ _>™5B È' $n_k$ C‡ ó û È$\hat{\pi}_{ALr}$ $\pi_{ALr}$ ,X ´ # Au Èè $\pi_{ALr}$ ,X • Â $V(\hat{\pi}_{ALr})$

$$= \sum_{k=1}^{L} W_k^2 \left[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k} + \frac{p(1-p)}{(2p-1)^2 n_k} - \frac{2\beta_k \rho_k \sigma_{X_k Y_k}}{(2p-1)n_k} + \frac{\beta_k^2 \sigma_{X_k}^{\ 2}}{(2p-1)^2 n_k}\right]$$ Ä

**A• â Ö** $E(\hat{\pi}_{ALr}) = \sum_{k=1}^{L} W_k \dfrac{E(\hat{\bar{Z}}_k) - \beta_k\left(\overline{x_k} - \overline{X}_k\right) - (1-p)}{(2p-1)}$

$$= \sum_{k=1}^{L} W_k \frac{E(\hat{\bar{Z}}_k) - \beta_k\left(\overline{x_k} - \overline{X}_k\right) - (1-p)}{(2p-1)}$$

$$= \sum_{k=1}^{L} W_k \frac{(2p-1)\pi_{Ak} + (1-p) + \beta_k\left(\overline{x_k} - \overline{X}_k\right) - \beta_k\left(\overline{x_k} - \overline{X}_k\right) - (1-p)}{(2p-1)}$$

$$= \sum_{k=1}^{L} W_k \pi_{Ak} = \pi_{ALr}$$

¹ $\hat{\pi}_{ALr}$ $\pi_{ALr}$ ,X ´ # Au Ä

$$\hat{\bar{Z}}_k = (2p-1)\pi_{Ak} + (1-p) + \beta_k\left(\overline{x_k} - \overline{X}_k\right)$$

$$\sigma_{Z_k X_k} = n_k E[\left(\bar{z}_k - \bar{Z}_k\right)\left(\bar{x}_k - \bar{X}_k\right)] = n_k E[((2p-1)(\bar{y} - \bar{Y}_k)\left(\bar{x}_k - \bar{X}_k\right)] = (2p-1)\sigma_{Y_k X_k}$$

$$V(\hat{\pi}_{Ak}) = V[\frac{\hat{\hat{Z}}_k - \beta_k\left(\overline{x_k} - \overline{X_k}\right) - (1-p)}{(2p-1)}]$$

$$= \frac{V(\hat{\hat{Z}}_k)}{(2p-1)^2} = \frac{\sigma_{Z_k}{}^2 - 2\beta_k \sigma_{X_k Z_k} + \beta_k{}^2 \sigma_{X_k}{}^2}{(2p-1)^2 n_k}$$

$$= \frac{\sigma_{Z_k}{}^2}{\left(2p-1\right)^2 n_k} - \frac{2\beta_k \sigma_{X_k Y_k}}{(2p-1)n_k} + \frac{\beta_k{}^2 \sigma_{X_k}{}^2}{\left(2p-1\right)^2 n_k}$$

$$= \frac{\pi_{Ak}(1-\pi_{Ak})}{n_k} + \frac{p(1-p)}{(2p-1)^2 n_k} - \frac{2\beta_k \sigma_{X_k Y_k}}{(2p-1)n_k} + \frac{\beta_k{}^2 \sigma_{X_k}{}^2}{\left(2p-1\right)^2 n_k}$$

¹ $V(\hat{\pi}_{ALr}) = \sum_{k=1}^{L} W_k{}^2 V\left(\hat{\pi}_{Ak}\right)$

$$= \sum_{k=1}^{L} W_k{}^2 [\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k} + \frac{p(1-p)}{(2p-1)^2 n_k} - \frac{2\beta_k \sigma_{X_k Y_k}}{(2p-1)n_k} + \frac{\beta_k{}^2 \sigma_{X_k}{}^2}{\left(2p-1\right)^2 n_k}]$$

2 Å ' Ø ¹´ ²1T )Lc ' • ã ' ª Ê :

n)Ú 5. 2 Ö Ú ' È ü!£ Ô S*ü1T )Lc ² ' ßE¯> ² & Au È Ø *ü"W4‡ õ _>™5B È ' $n_k$ C‡ ó û È Ý $\pi_{ALr}$ ,X • # Â

$$V(\hat{\pi}_{ALr}) = \sum_{k=1}^{L} \frac{W_k{}^2\left(1-f_k\right)}{n_k} \{S_{Y_k}{}^2 - \frac{2\beta_k S_{X_k Y_k}}{2p-1} + [\frac{\beta_k}{\left(2p-1\right)}]^2 S_{X_k}{}^2\}$$ Ä

A• â Ö $V(\hat{\pi}_{Ak}) = V[\frac{\hat{\hat{Z}}_k - \beta_k\left(\overline{x_k} - \overline{X_k}\right) - (1-p)}{(2p-1)}] = \frac{V(\hat{\hat{Z}}_k)}{(2p-1)^2}$

$$= \frac{1-f_k}{(2p-1)^2 n_k}\left(S_{Z_k}{}^2 - 2\beta_k S_{X_k Z_k} + \beta_k{}^2 S_{X_k}{}^2\right)$$

$$= \frac{1-f_k}{(2p-1)^2 n_k}\left(S_{Z_k}{}^2 - 2\beta_k S_{X_k Z_k} + \beta_k{}^2 S_{X_k}{}^2\right)$$

$$= \frac{1-f_k}{(2p-1)^2 n_k}\left(S_{Z_k}{}^2 - 2\beta_k (2p-1) S_{X_k Y_k} + \beta_k{}^2 S_{X_k}{}^2\right)$$

$$V(\hat{\pi}_{ALr}) = \sum_{k=1}^{L} W_k{}^2 V\left(\hat{\pi}_{Ak}\right) = \sum_{k=1}^{L} \frac{W_k{}^2\left(1-f_k\right)}{n_k}\{\frac{S_{Z_k}{}^2}{\left(2p-1\right)^2} - \frac{2\beta_k \rho_k S_{X_k Y_k}}{2p-1} + [\frac{\beta_k}{\left(2p-1\right)}]^2 S_{X_k}{}^2\}$$

## 5. 2. 2 $\beta_k$ 为样本回归系数

A’ $\beta_k = \frac{s_{xz}}{s_x{}^2} = \sum_{i=1}^{n_k}\left(z_{ki} - \overline{z}_k\right)\left(x_{ki} - \overline{x}_k\right) \Big/ \sum_{i=1}^{n_k}\left(x_{ki} - \overline{x}_k\right)^2$ ² &2Ï D Ä

1）' Ø ¹ Ý ²1T )Lc ‘ • ã ‘ ª Ê Ö

**n)Ú** 5. 3 **Ö** Ú ‘ È ü!£ Ô S*ü1T )Lc ² ‘ ßE¯> ² & Au È Ø *ü"W4‡ õ _>™5B È' $n_k$ C‡ ó û È$\pi_{ALr}$ ,X AuG£ #äE¯´ #,X È $\pi_{ALr}$ ,XE¥ # Â

$$MSE(\hat{\pi}_{ALr}) \approx V(\hat{\pi}_{ALr}) = \sum_{k=1}^{L} W_k^{\,2}[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k} + \frac{p(1-p)}{(2p-1)^2 n_k} - \frac{\rho_k^2\sigma_{Y_k}^2}{n_k}]$$

A• â Ö ' $n_k$ C‡ ó û È Ý $\overline{x}_k \approx \overline{X}_k$ È

$$E(\hat{\pi}_{ALr}) = \sum_{k=1}^{L} W_k \frac{E(\hat{\hat{Z}}_k) - \beta_k\left(\overline{x_k} - \overline{X}_k\right) - (1-p)}{(2p-1)}$$

$$\approx \sum_{k=1}^{L} W_k \frac{(2p-1)\pi_{Ak} + (1-p) - (1-p)}{(2p-1)}$$

$$= \sum_{k=1}^{L} W_k \pi_{Ak} = \pi_{ALr}$$

¹ $\pi_{ALr}$ ,X AuG£ #äE¯´ #,X Ä

$$V(\hat{\pi}_{Ak}) = V[\frac{\hat{\hat{Z}}_k - \beta_k\left(\overline{x_k} - \overline{X}_k\right) - (1-p)}{(2p-1)}] = \frac{V(\hat{\hat{Z}}_k)}{(2p-1)^2}$$

$$= \frac{\left(\sigma_{Z_k}{}^2 - 2\beta_k\sigma_{X_kZ_k} + \beta_k{}^2\sigma_{X_k}{}^2\right)}{(2p-1)^2 n_k}$$

+ $\beta_k = \frac{s_{x_kz_k}}{s_{x_k}{}^2} \approx \frac{\sigma_{Z_kX_k}}{\sigma_{X_k}{}^2} = (2p-1)\frac{\sigma_{Y_kX_k}}{\sigma_{X_k}{}^2}$, $\sigma_{Z_kX_k} = (2p-1)\sigma_{Y_kX_k}$ ú 9 Þ ã k Ö

$$V(\hat{\pi}_{Ak}) = \frac{V(\hat{\hat{Z}}_k)}{(2p-1)^2} = \frac{\left(\sigma_{Z_k}{}^2 - 2\beta_k\sigma_{X_kZ_k} + \beta_k{}^2\sigma_{X_k}{}^2\right)}{(2p-1)^2 n_k}$$

$$= \frac{1}{(2p-1)^2 n_k}\left(\sigma_{Z_k}{}^2 - 2(2p-1)^2\frac{\sigma_{Y_kX_k}{}^2}{\sigma_{X_k}{}^2} + (2p-1)^2\frac{\sigma_{Y_kX_k}{}^2}{\sigma_{X_k}{}^2}\right)$$

$$= \frac{\sigma_{Z_k}{}^2 - (2p-1)^2\rho_k^2\sigma_{Y_k}^2}{(2p-1)^2 n_k} = \frac{\pi_{Ak}(1-\pi_{Ak})}{n_k} + \frac{p(1-p)}{(2p-1)^2 n_k} - \frac{\rho_k^2\sigma_{Y_k}^2}{n_k}$$

J $\rho_k = \sigma_{Y_kX_k}/\sigma_{Y_k}\sigma_{X_k}$ $X_k$ â $Y_k$ ,X,Ì G2Ï D Ä

$$V(\hat{\pi}_{ALr}) = \sum_{k=1}^{L} W_k{}^2 V\left(\hat{\pi}_{Ak}\right) = \sum_{k=1}^{L} W_k{}^2[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k} + \frac{p(1-p)}{(2p-1)^2 n_k} - \frac{\rho_k^2\sigma_{Y_k}^2}{n_k}]$$

2）' Ø ¹´ ²1T )Lc ‘ • ã ‘ ª Ê

n)Ú 5. 4 ÖÚ ‘ È ü !£ Ô S*ü1T )Lc ² ‘ ßE¯> ² & Au ÈØ

*ü"W4‡ õ _>™5B È ' $n_k$ C‡ ó û È Ý $\hat{\pi}_{ALr}$ ,X • # Â

$$MSE(\hat{\pi}_{ALr}) \approx V(\hat{\pi}_{ALr}) = \sum_{k=1}^{L} W_k^{\ 2}(1-f_k)[\frac{(1-\rho_k^2)\pi_{Ak}(1-\pi_{Ak})}{n_k} + \frac{p(1-p)}{(2p-1)^2 n_k}]$$ Ä

A• â Ö $V(\hat{\pi}_{Ak}) = V[\frac{\hat{\hat{Z}}_k - \beta_k\left(\overline{x_k} - \overline{X}_k\right) - (1-p)}{(2p-1)}] = \frac{V(\hat{\hat{Z}}_k)}{(2p-1)^2}$

$$= (1-f_k)\frac{\left(S_{Z_k}^{\ 2} - 2\beta_k S_{X_k Z_k} + \beta_k^{\ 2} S_{X_k}^{\ 2}\right)}{(2p-1)^2 n_k}$$

+ $\beta_k = \frac{s_{x_k z_k}}{s_{x_k}^{\ 2}} \approx \frac{S_{Z_k X_k}}{S_{X_k}^{\ 2}} = (2p-1)\frac{S_{Y_k X_k}}{S_{X_k}^{\ 2}}$ ú 9 Þ ã k Ö

$$V(\hat{\pi}_{Ak}) = \frac{V(\hat{\hat{Z}}_k)}{(2p-1)^2} = \frac{1-f_k}{(2p-1)^2 n_k}\left(S_{Z_k}^{\ 2} - 2\beta_k S_{X_k Z_k} + \beta_k^{\ 2} S_{X_k}^{\ 2}\right)$$

$$= \frac{1-f_k}{(2p-1)^2 n_k}[S_{Z_k}^{\ 2} - 2(2p-1)^2\frac{S_{Y_k X_k}^{\ \ 2}}{S_{X_k}^{\ 2}} + (2p-1)^2\frac{S_{Y_k X_k}^{\ \ 2}}{S_{X_k}^{\ 2}}]$$

$$= \frac{1-f_k}{(2p-1)^2 n_k}[S_{Z_k}^{\ 2} - (2p-1)^2\frac{S_{X_k Y_k}^2}{S_{X_k}^2}]$$

$$= (1-f_k)[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k} + \frac{p(1-p)}{(2p-1)^2 n_k} - \frac{\rho_k^2\sigma_{Y_k}^2}{n_k}]$$

$$V(\hat{\pi}_{ALr}) = \sum_{k=1}^{L} W_k^{\ 2} V\left(\hat{\pi}_{Ak}\right) = \sum_{k=1}^{L} W_k^{\ 2}(1-f_k)[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k} + \frac{p(1-p)}{(2p-1)^2 n_k} - \frac{\rho_k^2\sigma_{Y_k}^2}{n_k}]$$

### 5. 2. 3 效率比较

+ "W4‡ õ _$V(\hat{\pi}_A)$ ,X1T ) Au ` ² & Au,X ø þ><E’ ã!¨EW ¥)„ :

a) ' $\beta_k$ .B n,X D Ê Èü Ô n 5 Ê ß È ² & AuG£,X2’ z?UP¬ b1T ) Au G£Ä G Ý Ö ' $n_k$ Ú û Ê È ² & Au ì b1T ) Au,X 5 Ê

$$\frac{2\beta_k S_{X_k Y_k}}{(2p-1)} \geq \frac{\beta_k^{\ 2} S_{X_k}^{\ 2}}{\left(2p-1\right)^2} \Leftrightarrow 2\rho_k S_{Y_k} \geq \frac{\beta_k S_{X_k}}{\left(2p-1\right)}$$

$$\Leftrightarrow \rho_k \geq \frac{1}{2}\frac{S_{X_k}}{S_{Y_k}}\frac{\beta_k}{\left(2p-1\right)} \Leftrightarrow \rho_k \geq \frac{1}{2}\frac{S_{X_k}}{S_{Y_k}}\frac{\beta_k}{\left(2p-1\right)}$$ Ä

b) ' $\beta_k$ ‡ n Ê È ² & AuG£,X2’ z ?UP¬ b1T ) AuG£ Ä

## 5.3 分层抽样下西蒙斯模型的回归估计

Ú ' $N$ áG¡ á$ã,X Ú $L$ È Ø ' G£ $N_k$, Ø •G£ $n_k$ È $k$ $=1,2,3$ È È $L$ È ü1T )Lc ‘ ß È Ø S*ü?S:m ƒLc ê>™5B È $p$ m Ý 4 2 b $A$ ë Û ,X 5( ,X!¨ _ È $1-p$ m Ý 4 2 b $B$ ë Û ,X 5( ,X!¨ _ È $B$ â # óKÂNl ´ G,XKÂNl È$\overline{Z}_k$ 1 $k$ ' ) ²1( ,X V)[ È ¸

$$Z_{ki}=\begin{cases}1 & 1\ \mathrm{k}\ \ 1\ \mathrm{i}\ \text{þ>•A×¹ 5Ù² 1( “ ”}\\ 0 & 1\ \mathrm{k}\ \ 1\ \mathrm{i}\ \text{þ>•A×¹ 5Ù² 1( “ ú ”}\end{cases}\begin{pmatrix}k=1,2,\cdots,L\\ i=1.2.\cdots,n\end{pmatrix}$$

Ä $X_{ki}$ ></ 1 $k$ 1 $i$ þ þ ' ) ,XEY } Û Û È $x_{ki}$ ></ 1 $k$ 1 $i$ þ ) ,XEY } Û Û È Æ-¹,X ÄJ $Z_{ki}$ â $X_{ki}$ K ÝP¬ z!7,Ì G û Ä A’ Ý>•A× ¹5ÙFÑ ,ó r ²1( Ä í ² & AuG£

$$\overline{Z}_{Lrk}=\pi_{Ak}\times p+(1-p)\pi_{Bk}=\hat{\overline{Z}}_k-\beta_k\left(\overline{x_k}-\overline{X}_k\right)$$ È

$$\hat{\overline{Z}}_k=p\pi_{Ak}+\beta_k\left(\overline{x_k}-\overline{X_k}\right)+(1-p)\pi_{Bk}$$ È $$\hat{\pi}_{Ak}=\frac{\hat{\overline{Z}}_k-\beta_k\left(\overline{x_k}-\overline{X_k}\right)-(1-p)\pi_{Bk}}{p}$$

' $\beta_k$ LÊ Ê È!8 Ê ² & õ _ G ?S:m ƒ õ _,X1T ) Au Ä ' $\beta_k=\frac{\overline{y_k}}{\overline{\overline{x_k}}}$ Ê È!8 Ê ,X ² & Au õ _ ?S:m ƒ!¨ Au Ä AuG£ Ã –5×1 ¯0´ Ä+ !8 Ã?• È1T ) AuG£ â!¨)[ AuG£ Ã?š ² & Au,X(M!^ ™ ‰ Ä

### 5.3.1 $\beta_k$ 为确定的常数

1 Å ' Ø ¹ Ý ²1T )Lc ‘ • ã ‘ ª Ê Ö

**n)Ú 5.5 Ö** Ú ‘ È ü!£ Ô S*ü1T )Lc ² ‘ ßE¯> ² & Au È Ø *ü?S:m ƒ õ _>™5B È' $\pi_{Bk}$ Æ-¹ Ê È$n_k$ C‡ ó û È$\hat{\pi}_{ALr}$ $\pi_{ALr}$ ,X ´ # Au È è $\hat{\pi}_{ALr}$ ,X • Â

$$V(\hat{\pi}_{ALr})=\sum_{k=1}^{L}W_k^{2}\left[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k}+\frac{(1-p)(\pi_{Ak}+\pi_{Bk}-2\pi_{Ak}\pi_{Bk})}{pn_k}+\frac{(1-p)^2\pi_{Bk}(1-\pi_{Bk})}{p^2n_k}-\frac{2\beta_k\sigma_{X_kY_k}}{pn_k}+\frac{\beta_k^{2}\sigma_{X_k}^{2}}{p^2n_k}\right]$$ Ä

A• â Ö $$E(\hat{\pi}_{ALr})=\sum_{k=1}^{L}W_kE(\hat{\pi}_{Ak})=\sum_{k=1}^{L}W_k\frac{E(\hat{\overline{Z}}_k)-\beta_k\left(\overline{x_k}-\overline{X_k}\right)-(1-p)\pi_{Bk}}{p}$$

$$=\sum_{k=1}^{L}W_k\frac{p\pi_{Ak}+\beta_k\left(\overline{x_k}-\overline{X_k}\right)+(1-p)\pi_{Bk}-\beta_k\left(\overline{x_k}-\overline{X_k}\right)-(1-p)\pi_{Bk}}{p}$$

$$=\sum_{k=1}^{L}W_k\pi_{Ak}=\pi_{ALr}$$

1 $\hat{\pi}_{ALr}$ $\pi_{ALr}$ ,X ´ # Au Ä

œ´ $\hat{\hat{Z}}_k=\pi_{Ak}\times p+(1-p)\pi_{Bk}+\beta_k\left(\overline{x_k}-\overline{X}_k\right)$

$$\sigma_{Z_kX_k}=n_kE[\left(\bar{z}_k-\bar{Z}_k\right)\left(\bar{x}_k-\bar{X}_k\right)]=n_kE[(p(\bar{y}-\bar{Y}_k)\left(\bar{x}_k-\bar{X}_k\right)]=p\sigma_{Y_kX_k}$$

1 $$V(\hat{\pi}_{Ak})=\frac{V(\hat{\hat{Z}}_k)}{p^2}=\frac{\sigma_{Z_k}{}^2-2\beta_k\sigma_{X_kZ_k}+\beta_k{}^2\sigma_{X_k}{}^2}{p^2n_k}=\frac{\sigma_{Z_k}{}^2}{p^2n_k}-\frac{2\beta_k\sigma_{X_kY_k}}{pn_k}+\frac{\beta_k{}^2\sigma_{X_k}{}^2}{p^2n_k}$$

$$=\frac{p^2\pi_A(1-\pi_A)}{p^2n_k}+\frac{p(1-p)(\pi_A+\pi_B-2\pi_A\pi_B)}{p^2n_k}+\frac{(1-p)^2\pi_B(1-\pi_B)}{p^2n_k}-\frac{2\beta_k\sigma_{X_kY_k}}{pn_k}+\frac{\beta_k{}^2\sigma_{X_k}{}^2}{p^2n_k}$$

$$V(\hat{\pi}_{ALr})=\sum_{k=1}^{L}W_k{}^2V\left(\hat{\pi}_{Ak}\right)=\sum_{k=1}^{L}W_k{}^2[\frac{\sigma_{Z_k}{}^2}{p^2n_k}-\frac{2\beta_k\sigma_{X_kY_k}}{pn_k}+\frac{\beta_k{}^2\sigma_{X_k}{}^2}{p^2n_k}]$$

$$=\sum_{k=1}^{L}W_k{}^2[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k}+\frac{(1-p)(\pi_{Ak}+\pi_{Bk}-2\pi_{Ak}\pi_{Bk})}{pn_k}+$$

$$\frac{(1-p)^2\pi_{Bk}(1-\pi_{Bk})}{p^2n_k}-\frac{2\beta_k\sigma_{X_kY_k}}{pn_k}+\frac{\beta_k{}^2\sigma_{X_k}{}^2}{p^2n_k}]$$

2 Å ' Ø ¹ ´ ²1T )Lc ‘ • ã ‘ ª Ê Ö

**n)Ú** 5. 6 **Ö** Ú ‘ È ü!£ Ô S*ü1T )Lc ´ ² ‘ ßE¯> ² & Au È Ø *ü?S:m ƒ õ _>™5B È ' $\pi_{Bk}$ Æ-¹ Ê È ' $n_k$ C‡ ó û È Ý $\hat{\pi}_{ALr}$ ,XE¥ # Â

$$V(\hat{\pi}_{ALr})\ =\sum_{k=1}^{L}W_k{}^2\frac{1-f_k}{n_k}[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k}+\frac{(1-p)(\pi_{Ak}+\pi_{Bk}-2\pi_{Ak}\pi_{Bk})}{pn_k}$$

$$+\frac{(1-p)^2\pi_{Bk}(1-\pi_{Bk})}{p^2n_k}-\frac{2\beta_k\sigma_{X_kY_k}}{pn_k}+\frac{\beta_k{}^2\sigma_{X_k}{}^2}{p^2n_k}]$$ Ä

A• â Ö $$V(\hat{\pi}_{ALr})=\sum_{k=1}^{L}W_k{}^2V\left(\hat{\pi}_{Ak}\right)=\sum_{k=1}^{L}W_k{}^2\frac{1-f_k}{p^2n_k}(\sigma_{Z_k}{}^2-2p\beta_kS_{X_kY_k}+\beta_k{}^2S_{X_k}{}^2)$$

$$=\sum_{k=1}^{L}W_k{}^2\frac{1-f_k}{n_k}[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k}+\frac{(1-p)(\pi_{Ak}+\pi_{Bk}-2\pi_{Ak}\pi_{Bk})}{pn_k}$$

$$+\frac{(1-p)^2\pi_{Bk}(1-\pi_{Bk})}{p^2n_k}-\frac{2\beta_k\sigma_{X_kY_k}}{pn_k}+\frac{\beta_k{}^2\sigma_{X_k}{}^2}{p^2n_k}]$$ A•!© Ä

### 5. 3. 2 $\beta_k$ 为样本回归系数

1) ' Ø ¹ Ý ²1T )Lc ‘ • ã ‘ ª Ê :

**n)Ú** 5.7 **Ö** Ú ‘ È ü!£ Ô S*ü1T )Lc ² ‘ ßE¯> ² & Au È Ø *ü?S:m ƒ õ _>™5B È' $n_k$ C‡ ó û È$\pi_{ALr}$ ,X AuG£E¥ ´ # È è $\pi_{ALr}$ ,XE¥ # Â $V(\hat{\pi}_{ALr})$

$$=\sum_{k=1}^{L}W_k^2[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k}+\frac{(1-p)(\pi_{Ak}+\pi_{Bk}-2\pi_{Ak}\pi_{Bk})}{pn_k}+\frac{(1-p)^2\pi_{Bk}(1-\pi_{Bk})}{p^2n_k}-\frac{\rho_k^2\sigma_{Y_k}^2}{n_k}]$$ Ä

A• â Ö ' $n_k$ C‡ ó û È Ý $\overline{x}_k \approx \overline{X}_k$ È

$$E(\hat{\pi}_{ALr}) \approx \sum_{k=1}^{L}W_k\frac{(2p-1)\pi_{Ak}+(1-p)-(1-p)}{(2p-1)}=\sum_{k=1}^{L}W_k\pi_{Ak}=\pi_{ALr}$$

$\pi_{ALr}$ ,X AuG£ E¥ ´ # ,X Ä

$$V(\hat{\pi}_{Ak})=V[\frac{\hat{\bar{Z}}_k-\beta_k\left(\overline{x_k}-\overline{X_k}\right)-(1-p)\pi_{Bk}}{p}]=\frac{V(\hat{\bar{Z}}_k)}{p^2}$$

$$=\frac{\left(\sigma_{Z_k}^2-2\beta_k\sigma_{X_kZ_k}+\beta_k^2\sigma_{X_k}^2\right)}{p^2n_k}$$

+ $\beta_k=\frac{s_{x_kz_k}}{s_{x_k}^2}\approx\frac{\sigma_{Z_kX_k}}{\sigma_{X_k}^2}=p\frac{\sigma_{Y_kX_k}}{\sigma_{X_k}^2}$ , $\sigma_{Z_kX_k}=p\sigma_{Y_kX_k}$ ú 9 Þ ã k Ö

$$V(\hat{\pi}_{Ak})=\frac{\left(\sigma_{Z_k}^2-2\beta_k\sigma_{X_kZ_k}+\beta_k^2\sigma_{X_k}^2\right)}{p^2n_k}=\frac{1}{p^2n_k}\left(\sigma_{Z_k}^2-2p^2\frac{\sigma_{Y_kX_k}^2}{\sigma_{X_k}^2}+p^2\frac{\sigma_{Y_kX_k}^2}{\sigma_{X_k}^2}\right)$$

$$=\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k}+\frac{(1-p)(\pi_{Ak}+\pi_{Bk}-2\pi_{Ak}\pi_{Bk})}{pn_k}+\frac{(1-p)^2\pi_{Bk}(1-\pi_{Bk})}{p^2n_k}-\frac{\sigma_{Y_kX_k}^2}{\sigma_{X_k}^2n_k}$$

J $\rho_k=\sigma_{Y_kX_k}/\sigma_{Y_k}\sigma_{X_k}$ $X_k$ â $Y_k$ ,X,Ì G2Ï D Ä

$$V(\hat{\pi}_{ALr})=\sum_{k=1}^{L}W_k^2V\left(\hat{\pi}_{Ak}\right)$$

$$=\sum_{k=1}^{L}W_k^2[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k}+\frac{(1-p)(\pi_{Ak}+\pi_{Bk}-2\pi_{Ak}\pi_{Bk})}{pn_k}+\frac{(1-p)^2\pi_{Bk}(1-\pi_{Bk})}{p^2n_k}-\frac{\rho_k^2\sigma_{Y_k}^2}{n_k}]$$

2) ' Ø ¹ ´ ²1T )Lc ' • ã ' ª Ê :

**n)Ú 5.8 Ö** Ú ' È ü!£ Ô S*ü1T )Lc ´ ² ' ßE¯> ² & Au È Ø *ü?S:m ƒ õ _>™5B È$\pi_{ALr}$ ,X AuG£ Ý #,X È ' $n_k$ C‡ ó û ÈÝ $\pi_{ALr}$ ,XE¥ # Â

$$V(\hat{\pi}_{ALr})=\sum_{k=1}^{L}W_k^2(1-f_k)[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k}+\frac{(1-p)(\pi_{Ak}+\pi_{Bk}-2\pi_{Ak}\pi_{Bk})}{pn_k}$$

$$+\frac{(1-p)^2\pi_{Bk}(1-\pi_{Bk})}{p^2n_k}-\frac{\sigma_{Y_kX_k}^2}{\sigma_{X_k}^2}]$$ Ä

A• â Ö $V(\hat{\pi}_{Ak}) = V[\frac{\hat{\bar{Z}}_k - \beta_k\left(\overline{x_k} - \overline{X_k}\right) - (1-p)\pi_{Bk}}{p}] = \frac{V(\hat{\bar{Z}}_k)}{p^2}$

$$= \frac{(1-f_k)}{p^2 n_k}\left(\sigma_{Z_k}{}^2 - 2\beta_k \sigma_{X_k Z_k} + \beta_k{}^2 \sigma_{X_k}{}^2\right)$$

$$= \frac{(1-f_k)}{p^2 n_k}\left(\sigma_{Z_k}{}^2 - 2p^2 \frac{\sigma_{Y_k X_k}{}^2}{\sigma_{X_k}{}^2} + p^2 \frac{\sigma_{Y_k X_k}{}^2}{\sigma_{X_k}{}^2}\right)$$

$$= (1-f_k)\frac{(\sigma_{Z_k}{}^2 - p^2 \rho_k^2 \sigma_{Y_k}^2)}{p^2 n_k}$$

J $\rho_k = \sigma_{Y_k X_k} / \sigma_{Y_k}\sigma_{X_k}$ $X_k$ â $Y_k$ ,X,Ì G2Ï D Ä

$$V(\hat{\pi}_{ALr}) = \sum_{k=1}^{L} W_k{}^2 V\left(\hat{\pi}_{Ak}\right)$$

$$= \sum_{k=1}^{L} W_k{}^2 (1-f_k)[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k} + \frac{(1-p)(\pi_{Ak} + \pi_{Bk} - 2\pi_{Ak}\pi_{Bk})}{p n_k}$$

$$+ \frac{(1-p)^2 \pi_{Bk}(1-\pi_{Bk})}{p^2 n_k} - \frac{\rho_k \sigma_{Y_k}{}^2}{n_k}]$$

### 5. 3. 3 效率比较

+ ?S:m ƒ õ _ $V(\hat{\pi}_A)$ ,X1T ) Au ` ² & Au,X ø þ><E' ã!¨EW ¥)„ :

a) ' $\beta_k$ .B n,X D Ê È ü Ô n 5 Ê ß È ² & AuG£,X2' z?UP¬ b1T ) Au G£Ä G Ý Ö ' $n_k$ Ú û Ê È ² & Au ì b1T ) Au,X 5 Ê

$$\frac{2\beta_k S_{X_k Y_k}}{p} \ge \frac{\beta_k{}^2 S_{X_k}{}^2}{p^2} \Leftrightarrow 2p\rho_k S_{Y_k} \ge \beta_k S_{X_k} \Leftrightarrow \rho_k \ge \frac{1}{2}\frac{S_{X_k}}{S_{Y_k}}\frac{\beta_k}{p}$$ Ä

b) ' $\beta_k$ ‡ n Ê È ² & AuG£,X2' z ?UP¬ b1T ) AuG£ Ä

## 5.4 分层抽样下改进模型的回归估计

### 5. 4. 1 $\beta_k$ 为确定的常数

1 Å ' Ø ¹ Ý ²1T )Lc ' • ã ' ª Ê Ö

**n)Ú** 5. 9 **Ö** Ú ' È ü!£ Ô S*ü1T )Lc ² ' ßE¯> ² & Au È Ø

*ü E¯ õ _>™5B È ' $\pi_{Bk}$ Æ-¹ Ê È ' $n_k$ C‡ ó û È Ý $\hat{\pi}_{ALr}$ ,XE¥ # Â

$$V(\hat{\pi}_A) = \sum_{k=1}^{L} W_k{}^2 [\frac{\pi_A(1-\pi_A)}{n_k} + \frac{p_3(\pi_A + \pi_B - 2\pi_A\pi_B)}{(p_1 - p_2)n_k} +$$

$$\frac{p_3^2\pi_B(1-\pi_B)+p_2(p_1+p_3)}{(p_1-p_2)^2 n_k}-\frac{2\beta_k\sigma_{X_kY_k}}{(p_1-p_2)n_k}+\frac{\beta_k{}^2\sigma_{X_k}{}^2}{(p_1-p_2)^2 n_k}]$$ Ä

A• â Ö $\bar{Z}_k = P(Z_{ki}=1)=\pi_{Ak}\times p_1+p_2(1-\pi_{Ak})+p_3\pi_{Bk}=\hat{\bar{Z}}_k-\beta_k\left(\overline{x_k}-\overline{X}_k\right)$

$$\hat{\bar{Z}}_k=(p_1-p_2)\pi_{Ak}+p_2+p_3\pi_{Bk}+\beta_k\left(\overline{x_k}-\overline{X}_k\right)$$

$$\sigma_{Z_kX_k}=n_kE[\left(\bar{z}_k-\bar{Z}_k\right)\left(\bar{x}_k-\bar{X}_k\right)]=(p_1-p_2)\sigma_{Y_kX_k}$$

¹ $$V(\hat{\pi}_{Ak})=\frac{V(\hat{\bar{Z}}_k)}{(p_1-p_2)^2}=\frac{\sigma_{Z_k}{}^2-2\beta_k\sigma_{X_kZ_k}+\beta_k{}^2\sigma_{X_k}{}^2}{(p_1-p_2)^2n_k}$$

$$=\frac{\sigma_{Z_k}{}^2}{(p_1-p_2)^2n_k}-\frac{2\beta_k\sigma_{X_kY_k}}{(p_1-p_2)n_k}+\frac{\beta_k{}^2\sigma_{X_k}{}^2}{(p_1-p_2)^2n_k}$$

$$V(\hat{\pi}_A)=\sum_{k=1}^{L}W_k{}^2V(\hat{\pi}_{Ak})=\frac{\sum_{k=1}^{L}W_k{}^2V(\hat{\bar{Z}}_k)}{(p_1-p_2)^2}$$

$$=\sum_{k=1}^{L}W_k{}^2[\frac{\pi_A(1-\pi_A)}{n_k}+\frac{p_3(\pi_A+\pi_B-2\pi_A\pi_B)}{(p_1-p_2)n_k}+$$

$$\frac{p_3^2\pi_B(1-\pi_B)+p_2(p_1+p_3)}{(p_1-p_2)^2n_k}-\frac{2\beta_k\sigma_{X_kY_k}}{(p_1-p_2)n_k}+\frac{\beta_k{}^2\sigma_{X_k}{}^2}{(p_1-p_2)^2n_k}]$$

2 Å ' Ø ¹ ´ ²1T )Lc ‘ • ã ‘ ª Ê Ö

**n)Ú** 5.10 **Ö**Ú ‘ Èü!£ Ô S*ü1T )Lc ´ ² ‘ ßE¯> ² & Au È Ø *ü E¯ õ _>™5B È ' $\pi_{Bk}$ Æ-¹ Ê È ' $n_k$ C‡ ó û È Ý $\hat{\pi}_{ALr}$ ,XE¥ # Â

$$V(\hat{\pi}_{ALr})=\sum_{k=1}^{L}W_k{}^2\frac{1-f_k}{p^2n_k}(S_{Z_k}{}^2-2p\beta_kS_{X_kY_k}+\beta_k{}^2S_{X_k}{}^2)$$ Ä

A• â Ö à)Ú ÃA• $V(\hat{\pi}_{ALr})=\sum_{k=1}^{L}W_k{}^2V\left(\hat{\pi}_{Ak}\right)=\sum_{k=1}^{L}W_k{}^2\frac{V(\hat{\bar{Z}}_k)}{p^2}$

$$=\sum_{k=1}^{L}W_k{}^2(1-f_k)[\frac{\pi_A(1-\pi_A)}{n_k}+\frac{p_3(\pi_A+\pi_B-2\pi_A\pi_B)}{(p_1-p_2)n_k}+$$

$$\frac{p_3^2\pi_B(1-\pi_B)+p_2(p_1+p_3)}{(p_1-p_2)^2n_k}-\frac{2\beta_k\sigma_{X_kY_k}}{(p_1-p_2)n_k}+\frac{\beta_k{}^2\sigma_{X_k}{}^2}{(p_1-p_2)^2n_k}]$$

## 5.4.2 $\beta_k$ 为样本回归系数

1) ' Ø ¹ Ý ²1T )Lc ‘ • ã ‘ ª Ê :

**n)Ú** 5.11 **Ö**Ú ‘ Èü!£ Ô S*ü1T )Lc ´ ² ‘ ßE¯> ² & Au È Ø *ü E¯ õ _>™5B È ' $\pi_{Bk}$ Æ-¹ Ê È ' $n_k$ C‡ ó û È Ý $\hat{\pi}_{ALr}$ ,XE¥ # Â

$$V(\hat{\pi}_{ALr})=\sum_{k=1}^{L}W_k^{\ 2}[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k}+\frac{p_3(\pi_{Ak}+\pi_{Bk}-2\pi_{Ak}\pi_{Bk})}{(p_1-p_2)n_k}+$$

$$\frac{p_3^2\pi_B(1-\pi_B)+p_2(p_1+p_3)}{(p_1-p_2)^2n_k}-\frac{\rho_k^{\ 2}\sigma_{Y_k}^{\ \ 2}}{n_k}]$$ Ä

A• â : $\beta_k=\frac{s_{x_kz_k}}{s_{x_k}^{\ 2}}\approx\frac{\sigma_{Z_kX_k}}{\sigma_{X_k}^{\ 2}}=(p_1-p_2)\frac{\sigma_{Y_kX_k}}{\sigma_{X_k}^{\ 2}}$ , $\sigma_{Z_kX_k}=(p_1-p_2)\sigma_{Y_kX_k}$

$$V(\hat{\pi}_{Ak})=\frac{V(\hat{\hat{Z}}_k)}{(p_1-p_2)^2}=\frac{\sigma_{Z_k}^{\ \ 2}-2\beta_k\sigma_{X_kZ_k}+\beta_k^{\ 2}\sigma_{X_k}^{\ \ 2}}{(p_1-p_2)^2n_k}$$

$$=\frac{\sigma_{Z_k}^{\ \ 2}}{(p_1-p_2)^2n_k}-\frac{\sigma_{X_kY_k}^2}{\sigma_{X_k}^{\ \ 2}n_k}$$

$$V(\hat{\pi}_A)=\sum_{k=1}^{L}W_k^{\ 2}V(\hat{\pi}_{Ak})=\frac{\sum_{k=1}^{L}W_k^{\ 2}V(\hat{\hat{Z}}_k)}{(p_1-p_2)^2}$$

$$=\sum_{k=1}^{L}W_k^{\ 2}[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k}+\frac{p_3(\pi_{Ak}+\pi_{Bk}-2\pi_{Ak}\pi_{Bk})}{(p_1-p_2)n_k}+$$

$$\frac{p_3^2\pi_B(1-\pi_B)+p_2(p_1+p_3)}{(p_1-p_2)^2n_k}-\frac{\rho_k^{\ 2}\sigma_{Y_k}^{\ \ 2}}{n_k}]$$

1 Å ' Ø ´ Ý ²1T )Lc ‘ • ã ‘ ª Ê Ö
n)Ú 5. 12 ÖÚ ‘ Èü!£ Ô S*ü1T )Lc ´ ² ‘ ßE¯> !¨)[ Au È
Ø *ü E¯ õ _>™5B È ' $\pi_{Bk}$ Æ-¹ Ê È $n_k$ C‡ ó û È Ý $\pi_{AR}$ ,XE¥ # Â

$$V(\hat{\pi}_{ALr})=\sum_{k=1}^{L}W_k^{\ 2}(1-f_k)[\frac{\pi_{Ak}(1-\pi_{Ak})}{n_k}+\frac{p_3(\pi_{Ak}+\pi_{Bk}-2\pi_{Ak}\pi_{Bk})}{(p_1-p_2)n_k}+$$

$$\frac{p_3^2\pi_B(1-\pi_B)+p_2(p_1+p_3)}{(p_1-p_2)^2n_k}-\frac{\rho_k^{\ 2}\sigma_{Y_k}^{\ \ 2}}{n_k}]$$ Ä

## 5. 4. 3 效率比较

+ E¯ õ _ $V(\hat{\pi}_A)$ ,X1T ) Au ` ² & Au,X ø þ><E’ ã!¨EW ¥)„ :

a) ' $\beta_k$ .B n,X D Ê È ü Ô n 5 Ê ß È ² & AuG£,X2’ z?UP¬ b1T ) Au G£Ä G Ý Ö ' $n_k$ Ú û Ê È ² & Au ì b1T ) Au,X 5 Ê

$$\frac{2\beta_k\sigma_{X_kY_k}}{(p_1-p_2)n_k}\geq\frac{\beta_k^{\ 2}\sigma_{X_k}^{\ \ 2}}{(p_1-p_2)^2n_k}\Leftrightarrow 2(p_1-p_2)\rho_k\sigma_{Y_k}\geq\beta_k\sigma_{X_k}\Leftrightarrow\rho_k\geq\frac{1}{2}\frac{\sigma_{X_k}}{\sigma_{Y_k}}\frac{\beta_k}{(p_1-p_2)}$$ Ä

b) ' $\beta_k$ ‡ n Ê È ² & AuG£,X2’ z ?UP¬ b1T ) AuG£ Ä

# 第六章 总结和展望

## 6.1 本文总结

# ó ûKÂNl D• K Ý,X # ó z S kA× ¹,X ) Ô!¨ Ô8 ,XKÂNlA× ¹8…C ,X ÊKÈ `C *üFÑ?U î Ä AŽ [ ?U-è0J Z # ó ûKÂNl ü Ý Ã ý*ü,XEY } µ C Ê,XLc ê ²1( •"© ÄOj ü Î ,XLc ê ²1( õ _ ü ü "W4‡ õ _ `?S:m ƒ õ _,X Î. Þ È Ú ÿEîE› ÍEY } µ C,X Ý ¤ ª È r tEY } ¬G£ È ¤ Î Z 2 û(M ULc ê ²1(,X !¨ Au •"© ` ² & Au •"© È k Î AuG£ ž J • Â Ä' â â"W4‡ õ _ `?S:m ƒ õ _ 0 Í!¨ È ¥)„!¨ Au ` ² & Au ü Ô n,X 5 Ê ß È Ã ¹ ¤P¬ Y – D,X2' z ,X Ä J õ ý*üBñ Ê ƒ Au È B P` µ C4- Î ?– P` Ú × È4Ð Ü µ C È k Î âP` Ú × È¢5àE' ÍEY } µ C6Ñ Ý ý*ü,X p ÄÔ â ZE¯ Ô!9 ¤P¬2' z È [ Ú Ø/¡ •"© h*ü | S Z Ú E- Ô ' 6 ã Ä

## 6.2 前景展望

4£E› ¯ î H,X ¥ ) È # ó ûKÂNl,X-è0J+ b J(M!^ û È éCK Z Ñ Y êC^ 9 C^ î,X :5Ù G"¼ Ä<Q' Lc ê ²1( T ,X)ÚAŽ ` •"© Æ4£ Ý á å-è0J È E¬ Ý \ îLÔ?U?· ‡,XKÂNl Ä

ü Æ Ý,X-è0J È \ îFÑ Î b,ó r ²1( ß È ü rL A× ¹ # óKÂNl È´ B ž ñ Ç á ! ê J ª s ´ È Ã6Ñ Ð7È>•A× ¹5Ù Î)„ á,ó r ²1( ê ´ ²1()„B5 È ¢5àEô ä Lš ,X4§ p ÝAÃ Â,X Ä ´!8 Í b á,ó r ²1( ` ´ ²1(5à ÝAÃ Â,X ™ ‰ È 3 å ÀLÔ?U-è0J,XG¡?U • å Ä

,Â ! # ó ûKÂNlE¤*ü á ' ß û î Í – D,X Au È ÍKÂ KA× ¹A'Au ,X Ã r ' û È G£,X ÚG! ¹ ž4£#"C *ü,X5×<%1 ™ 6-è0J í!¨EW å È _ V Ú ' ß Í ,X æ Ú È Ø G£,X ÚG!KÂNl1 Ä

Bñ ý*üEY } µ C,X!¨ Au Ã ² & Au `Bñ Ê ƒ Au,Â !FÑ h*ü b 2 û(M U ,XLc ê-è0J È Í DG£(M U,X # ó ûKÂNl-è0J H \ å Ä

# 致 谢

-è0J*ó : 4*ó#ƒ G Ú4§ 3 ÈÝ H Èå : 4 \ î-¹Aš È 9‹ Z \ î ð?S È
5à å 3 hA¹ óAö \ î,X Ž ÄOj óAö,X å,X Ð Ü é )5Õ Ü È M á ™ A• å : 4
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>K —,X óAö = Ü Í å,X -6† ` Õ Ä J õ È åE¬?U óAö Ò4£,XEY Ð , ì'©5Õ Ü `
ô Û5Õ Ü ¹ ž : õ,X Ø !5Õ Ü ` NZ Ð È óAö ª À ü*ó# Þ ` : 4 Þ ´ , á7Ç,X G
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Tç … å ÈA} å6Ñ ó Ý µ — `!™ o • ` ä å,X : î Ä

Ô â È>K —,X óAö Ý G — ` Õ å,X Ž ÈAöAö ½

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