In [280]:
```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import scipy
from scipy.io import loadmat
from numpy.random import logistic
import random
from scipy.special import expit as sigmoid
```

CS 189 Homework 4

In [9]:
```python
w0 = np.array([-2.0, 1.0, 0.0])
lam = 0.07
X = np.array([[0,3,1], [1,3,1], [0,1,1], [1,1,1]])
y = np.array([1, 1, 0, 0])

s0 = sigmoid(np.dot(X, w0))

print('s0: ', s0)

w1 = w0 + np.dot(np.linalg.inv(2*lam*np.identity(3) + np.matmul(np.dot(X.T,

print('w1: ',w1)

s1 = sigmoid(np.dot(X, w1))

print('s1: ',s1)

w2 = w1 + np.dot(np.linalg.inv(2*lam*np.identity(3) + np.dot(np.dot(X.T, np

print('w2: ',w2)
```

```
s0:  [0.95257413 0.73105858 0.73105858 0.26894142]
w1:  [-2.52944273  0.66772961  1.14165334]
s1:  [0.95870501 0.6491715  0.85928728 0.32737982]
w2:  [-3.24293197  0.19326062  2.67607706]
```

In [ ]:
```python

```

In [10]:
```python
train_x, train_y, test_x = loadmat("data.mat")['X'], loadmat("data.mat")['y
```

In [11]:
```python
#train_x[0], train_y[0]
```

Question 4: Wine Classification with Logistic Regression

4a) Batch GD

Batch GD Equation

Parameters: ϵ = 0.0001, iterations = 100000, λ = 0.00006

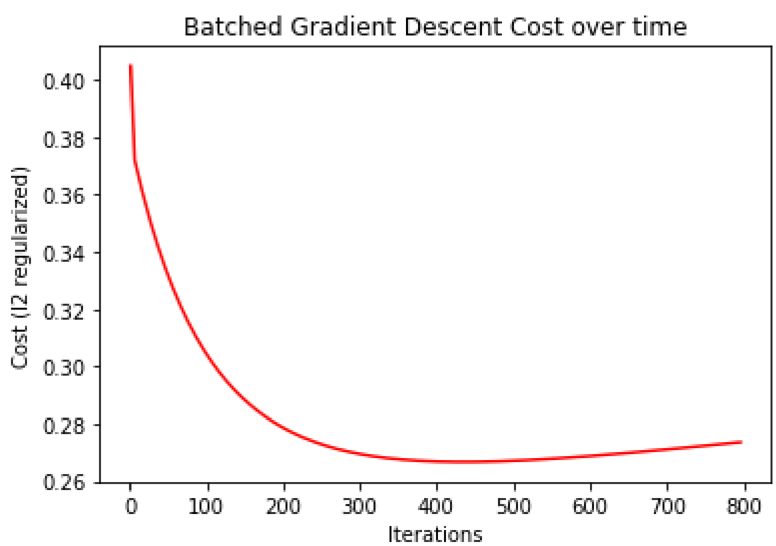
$$w = w - \epsilon(-X^T(y - s(Xw)) + 2\lambda w)$$

In [152]:
```
class GD_model:
    def __init__(self, d):
        self.w = np.array([random.random()*2 - 1 for i in range(d+1)]).resh
        self.reg_param = 1

    def train(self, X, y, epsilon, iterations):
        X = np.insert(X, 12, 1, axis=1)
        for i in range(iterations):
            term = epsilon * self.cost(X, y, batch = True)
            self.w += term

    def reformat(self, x, batch = False):
        if batch:
            if np.shape(x)[1] < 13:
                x = np.insert(x, 12, 1, axis=1)
            x = np.reshape(x, (len(x), 13, 1))
        else:
            if len(x) < 13:
                x = np.append(x, 1)
            x = x.reshape(13, 1)
        return x

    def fn(self, x, batch = False):
        x = self.reformat(x, batch)
        if batch:
            x = np.reshape(x, (len(x), 13,))
            inner_term = np.matmul(x, self.w)
        else:
            inner_term = np.dot(x.T, self.w)
        output = sigmoid(inner_term).reshape(len(inner_term), 1)
        one_rep = 1 - np.finfo(float).eps
        np.place(output, output == 1.0, [one_rep])
        np.place(output, output == 0.0, [np.finfo(float).eps])
        return output

    def classify(self, x, tie_breaker, batch = False):
        x = self.reformat(x, batch)
        #print(x)
        f_out = self.fn(x, batch)
        return np.heaviside(f_out - 0.5, tie_breaker)

    def cost(self, X, y, batch = False):
        #gradient of cost
        lam = self.reg_param
        self.w.reshape(len(self.w), )
        answer = np.dot(X.T, y - self.fn(X, batch)) - (2*lam*self.w)
        self.w.reshape(len(self.w), 1)
        return answer

    def base_cost(self, X, y, batch = True):
        #regular cost
        lam = self.reg_param
        s = 0
        i = 0
        losses = self.loss(X, y, True)
        #print('nans', self.fn(X, batch)[np.argwhere(np.isnan(losses))])
```

```
        for row in X:
            l = losses[i]
            if np.isnan(l):
                print(self.fn(row))
            w = np.reshape(self.w, 13, )
            s += l + lam*np.inner(w, w)
            i += 1
        return s / i

    def loss(self, x, y, batch):
        #single value loss
        yhat = self.fn(x, batch)
        return -y * np.log(yhat) - (1-y)*np.log(1 - yhat)

    def cost_plot(self, X, y, iterations=800):
        epsilon = 0.0000001
        sizes = np.arange(1, iterations-1, 5)
        plt_y = np.array([])
        X = np.insert(X, 12, 1, axis=1)
        for i in range(iterations):
            term = self.cost(X, y, batch = True)
            self.w += term*epsilon
            if i in sizes:
                plt_y = np.append(plt_y, self.base_cost(X, y))
        plt.plot(sizes, plt_y, color = "red")
        plt.xlabel("Iterations")
        plt.ylabel("Cost (l2 regularized)")
        plt.title("Batched Gradient Descent Cost over time")

        #return (plt_y)
```

In [902]:

```
first_model = GD_model(len(train_x[0]))
first_model.cost_plot(train_x, train_y)
```



In [230]:
```
#first_model = GD_model(len(train_x[0]))
first_model.reg_param = 1
first_model.train(train_x, train_y, .0000001, 100000)
first_model.base_cost(train_x, train_y, batch = True)
```

Out[230]:
```
array([18.03091292])
```

In [231]:
```
#Batch Gradient Descent Accuracy
e = 0
for i in range(len(train_x)):
    pred, real = first_model.classify(train_x[i], 1), train_y[i]
    #print("Real:{}, Prediction: {}".format(pred, real))
    if real == pred:
        e += 1
    else:
        pass
        #print(first_model.fn(train_x[i]), train_y[i])
e/i
```

Out[231]:
```
0.9676612768794799
```

In [176]:
```
saved_params = first_model.w
```

In [229]:
```
saved_params
```

Out[229]:
```
array([[333.76861208],
       [ 65.97028739],
       [ -7.60164012],
       [-37.23017301],
       [ 11.74687383],
       [ 11.55807342],
       [-37.59962368],
       [ 23.11108048],
       [104.71125119],
       [ 51.66538476],
       [ 15.34012231],
       [  2.2616782 ],
       [ 22.23356692]])
```

4b) Stochastic GD

Update Rule

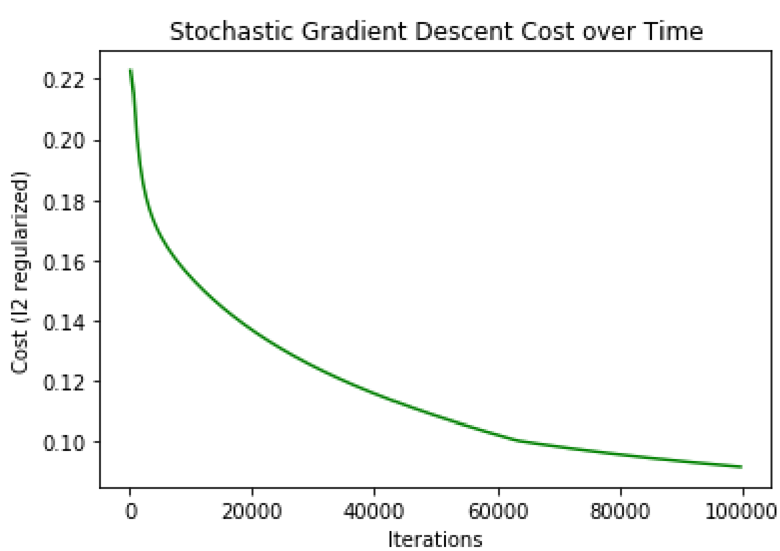
$w^{t+1} \leftarrow w^t - \epsilon \nabla_{w,i} J = w^t - \epsilon[-X_i^\intercal (y_i - s(X_i w)) + 2\lambda w]$

In [765]:
```python
class SGD_model:
    def __init__(self, d):
        self.w = np.array([0.0]*(d+1)).reshape(d+1, 1)

    def train(self, X, y, epsilon, iterations):
        X = np.insert(X, 12, 1, axis=1)
        for i in range(iterations):
            index = np.random.randint(0, len(X), 1)
            term = epsilon * self.cost(X[index], y[index], batch = False)
            self.w += term

    def reformat(self, x, batch = False):
        if batch:
            if np.shape(x)[1] < 13:
                x = np.insert(x, 12, 1, axis=1)
            x = np.reshape(x, (len(x), 13, 1))
        else:
            x = np.ndarray.flatten(x)
            if len(x) < 13:
                x = np.append(x, 1)
            #print(x)

            x = x.reshape(13, 1)

        return x

    def fn(self, x, batch = False):
        x = self.reformat(x, batch)
        if batch:
            x = np.reshape(x, (len(x), 13,))
            inner_term = np.matmul(x, self.w)
        else:
            inner_term = np.dot(x.T, self.w)
        output = sigmoid(inner_term).reshape(len(inner_term), 1)
        np.place(output, output == 1.0, [0.9999999])
        np.place(output, output == 0.0, [0.00000001])
        return output

    def classify(self, x, batch = False):
        x = self.reformat(x, batch)
        #print(x)
        f_out = self.fn(x, batch)
        return np.heaviside(f_out - 0.5, 1)

    def cost(self, X, y, batch = False, lam = 0.0001):
        self.w.reshape(len(self.w), )
        answer = np.dot(X.T, y - self.fn(X, batch)) - (2*lam*self.w)
        self.w.reshape(len(self.w), 1)
        return answer

    def base_cost(self, X, y, batch = True, lam = 0.00006):
        s = 0
        i = 0
        losses = self.loss(X, y, True)
        #print('nans', self.fn(X, batch)[np.argwhere(np.isnan(losses))])
        for row in X:
```

```
            l = losses[i]
            if np.isnan(l):
                print(self.fn(row))
            w = np.reshape(self.w, 13, )
            s += l + lam*np.inner(w, w)
            i += 1
        return s / i

    def loss(self, x, y, batch):
        #single value loss
        yhat = self.fn(x, batch)
        return -y * np.log(yhat) - (1-y) * np.log(1 - yhat)

    def cost_plot(self, X, y, iterations=100001):
        epsilon = 0.0000001
        sizes = np.arange(200, iterations-1, 500)
        plt_y = np.array([])
        X = np.insert(X, 12, 1, axis=1)
        for i in range(iterations):
            term = self.cost(X, y, batch = True)
            self.w += term*epsilon
            if i in sizes:
                plt_y = np.append(plt_y, self.base_cost(X, y))
        plt.plot(sizes, plt_y, color="green")
        plt.title("Stochastic Gradient Descent Cost over Time")
        plt.xlabel("Iterations")
        plt.ylabel("Cost (l2 regularized)")

        #return (plt_y)
```

In [766]:
```
#second_model = SGD_model(12)
#second_model.train(train_x, train_y, 0.0001, 10000)
```

In [767]:
```
second_model = SGD_model(12)
second_model.cost_plot(train_x, train_y)
```



In [772]:
```
#Stochastic Gradient Descent Training Accuracy
e = 0
for i in range(len(train_x)):
    pred, real = second_model.classify(train_x[i]), train_y[i]
    #print("Real:{}, Prediction: {}".format(pred, real))
    if real == pred:
        e += 1
e/i
```

Out[772]: 0.9736622770461744

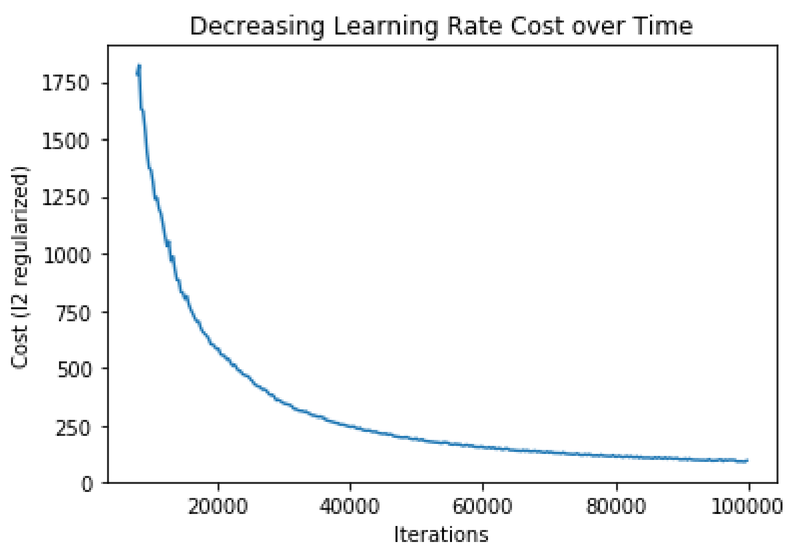
4c) Decreasing Learning Rate

In [276]:
```
class SGD_dec_model:
    def __init__(self, d):
        self.w = np.array([0.0]*(d+1)).reshape(d+1, 1)

    def train(self, X, y, iterations):
        X = np.insert(X, 12, 1, axis=1)
        for i in range(1, iterations):
            index = np.random.randint(0, len(X), 1)
            term = 1/i * self.cost(X[index], y[index], batch = False)
            self.w += term

    def reformat(self, x, batch = False):
        if batch:
            if np.shape(x)[1] < 13:
                x = np.insert(x, 12, 1, axis=1)
            x = np.reshape(x, (len(x), 13, 1))
        else:
            x = np.ndarray.flatten(x)
            if len(x) < 13:
                x = np.append(x, 1)
            #print(x)

            x = x.reshape(13, 1)

        return x

    def fn(self, x, batch = False):
        x = self.reformat(x, batch)
        if batch:
            x = np.reshape(x, (len(x), 13,))
            inner_term = np.matmul(x, self.w)
        else:
            inner_term = np.dot(x.T, self.w)
        output = sigmoid(inner_term).reshape(len(inner_term), 1)
        np.place(output, output == 1.0, [0.9999999])
        np.place(output, output == 0.0, [0.00000001])
        return output

    def classify(self, x, batch = False):
        x = self.reformat(x, batch)
        #print(x)
        f_out = self.fn(x, batch)
        return np.heaviside(f_out - 0.5, 1)

    def cost(self, X, y, batch = False, lam = 0.06):
        self.w.reshape(len(self.w), )
        answer = np.dot(X.T, y - self.fn(X, batch)) - (2*lam*self.w)
        self.w.reshape(len(self.w), 1)
        return answer

    def base_cost(self, X, y, batch = True, lam = 0.00006):
        s = 0
        i = 0
        losses = self.loss(X, y, True)
        #print('nans', self.fn(X, batch)[np.argwhere(np.isnan(losses))])
        for row in X:
```

```
            l = losses[i]
            if np.isnan(l):
                print(self.fn(row))
            w = np.reshape(self.w, 13, )
            s += l + lam*np.inner(w, w)
            i += 1
        return s / i

    def loss(self, x, y, batch):
        #single value loss
        yhat = self.fn(x, batch)
        return -y * np.log(yhat) - (1-y) * np.log(1 - yhat)

    def cost_plot(self, X, y, iterations=100001):
        epsilon = 100
        sizes = np.arange(8000, iterations-1, 300)
        plt_y = np.array([])
        X = np.insert(X, 12, 1, axis=1)
        for i in range(1, iterations):
            term = self.cost(X, y, batch = True)
            self.w += term * epsilon/i
            if i in sizes:
                plt_y = np.append(plt_y, self.base_cost(X, y))
        plt.plot(sizes, plt_y)
        plt.title("Decreasing Learning Rate Cost over Time")
        plt.xlabel("Iterations")
        plt.ylabel("Cost (l2 regularized)")
        print(plt_y[-1], '= final cost')
```

In [277]:
```
third_model = SGD_dec_model(12)
third_model.cost_plot(train_x, train_y)
```

```
95.68242719028704 = final cost
```



In [279]:

```
#Stochastic Gradient Descent with Decreasing Learning Rate Training Accurac
#third_model.train(train_x, train_y, 10000)
e = 0
for i in range(len(train_x)):
    pred, real = third_model.classify(train_x[i]), train_y[i]
    #print("Real:{}, Prediction: {}".format(pred, real))
    if real == pred:
        e += 1
e/i
```

Out[279]:

```
0.9483247207867977
```

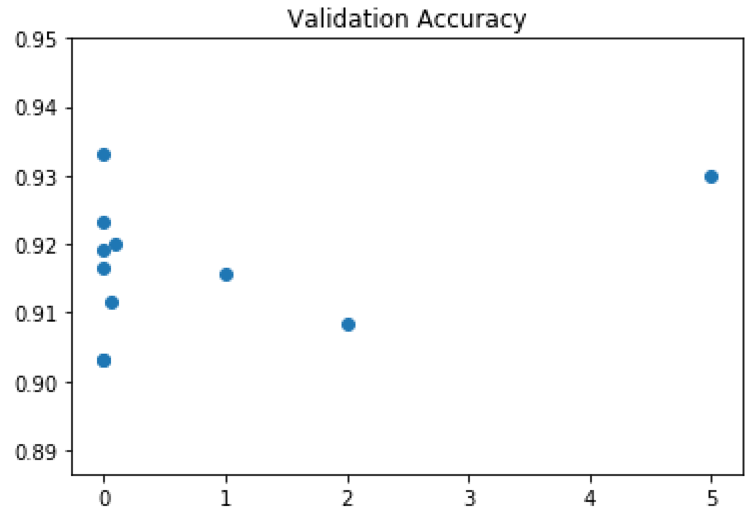
The cost seems to be converging to a larger value than the previous version of SGD. This must be due to the relatively large starting step sizes.

4d) Kaggle

In [179]:

```
#Validation for BGD
def unison_shuffled_copies(a, b):
    assert len(a) == len(b)
    p = np.random.permutation(len(a))
    return a[p], b[p]

shuffled_x, shuffled_y = unison_shuffled_copies(train_x, train_y)
split = int(len(shuffled_x)*0.8)
val_x, val_y = shuffled_x[split:], shuffled_y[split:]
temp_trainx, temp_trainy = shuffled_x[:split], shuffled_y[:split]

ploty = []
lams = [0.0000001, 0.0000005, 0.00001, 0.00002, 0.00005, 0.001, 0.07, 0.1,
for lam in lams:
    model = GD_model(12)
    model.reg_param = lam
    model.train(temp_trainx, temp_trainy, 0.0000001, 5000)

    e = 0
    for i in range(len(val_x)):
        pred, real = model.classify(val_x[i], 1), val_y[i]
        #print("Real:{}, Prediction: {}".format(pred, real))
        if real == pred:
            e += 1
    ploty.append(e/i)

    print("Done with {}".format(lam))

plt.scatter(lams, ploty)
plt.title("Validation Accuracy")
plt.show()
```

```
Done with 1e-07
Done with 5e-07
Done with 1e-05
Done with 2e-05
Done with 5e-05
Done with 0.001
Done with 0.07
Done with 0.1
Done with 1
Done with 2
Done with 5
```



In [185]:
```
# A code snippet to help you save your results into a kaggle accepted csv
import pandas as pd
import numpy as np

# Usage results_to_csv(clf.predict(X_test))
def results_to_csv(y_test, string):
    y_test = y_test.astype(int)
    df = pd.DataFrame({'Category': y_test})
    df.index += 1  # Ensures that the index starts at 1.
    df.to_csv(string + '_submission.csv', index_label='Id')

def model_export(model, test_x, string = ""):
    ans = np.array([float(model.classify(point, 1)) for point in test_x])
    results_to_csv(ans, string)
```

In [186]:
```
model_export(first_model, test_x, "up to 9846 percent")
```

In [96]:
```
```

Out[96]: 2.220446049250313e-16

In [ ]:
```
```