Warm up 1:00 - 1:10 Stat 134 lec 4

. The color of a person's eyes is determined by a single pair of genes. If they are both blue-eyed genes(bb), then the person will have blue eyes; if they are both brown-eyed genes (BB), then the person will have brown eyes; and if one is a brown-eyed gene and the other is a blue-eyed gene (Bb), then the person will have brown eyes as the brown-eyed gene is dominant over the blue-eyed gene. A newborn child independently receives one eye gene from each of its parents, and the gene it receives from a parent is equally likely to be either one of the genes of that parent. We know that Greg has brown eyes, and his dad has eye genes Bb. Given this information, what is the chance that Greg's mother has blue eyes?

M = mother bb

D = dad Bb

G = Greg brown eyes

Assumptions:

$P(M) = 1/4$ (bb, bB, Bb, BB) are equally likely.

$P(M,D) = P(M)P(D)$

Find $P(M \mid D, G) = \frac{P(M,D,G)}{P(D,G)} = \frac{P(G \mid M,D)P(M,D)}{P(G \mid D)P(\cancel{D})}$ $P(M)P(\cancel{D})$

$P(M) = 1/4$

$P(G \mid M, D) = 1/2$ since for Greg to have Brown eyes dad gives Greg B which has a 50% chance.

bb, Bb

$$P(G \mid D) = \overset{1/2}{P(G \mid D, \text{mom } bb)} \cdot \overset{1/4}{P(\text{mom } bb)}$$
$$+ \overset{3/4}{P(G \mid D, \text{mom } bB)} \cdot \overset{1/4}{P(\text{mom } bB)}$$
$$+ \overset{3/4}{P(G \mid D, \text{mom } Bb)} \cdot \overset{1/4}{P(\text{mom } Bb)}$$
$$+ \overset{1}{P(G \mid D, \text{mom } BB)} \cdot \overset{1/4}{P(\text{mom } BB)} = 3/4$$

$$\Rightarrow P(M \mid D, G) = \frac{1/2 \cdot 1/4}{3/4} = \boxed{1/6}$$

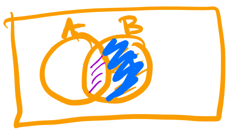
Announcement: (1) Q1 next Wednesday 8–12 am
Coverage Sec 1.1–1.6
logistics info TBA

(2) my OH: 2–2:45 after class
on lecture zoom
meeting (not SLC).

Last time

Sec 1.4 Independence

Note that if $P(AB)=P(A)P(B)$ then $P(A^cB)=P(A^c)P(B)$
since,



$P(A^cB)=P((AB)^cB) = P(B)-P(AB)$ ← difference rule

$= P(B)-P(A)P(B) = (1-P(A))P(B) = P(A^c)P(B)$ ← independence of A, B

Sec 1.5 Baye's rule

There are two types of conditional probabilities:

ex From todays warmup question

$P(G|M,D)$ is forward conditional (likelihood conditional)
DON'T NEED BAYES TO COMPUTE

$P(M|D,G)$ is backwards conditional (posterior conditional)
NEED BAYES TO COMPUTE

Today

(1) sec 1.6 independence of 3 or more events
(2) sec 2.1 Binomial Distribution
(3) sec 2.1 The shape of the binomial distribution

sec 1.6 Independence of 3 events

Def^n (Pairwise independence of 3 events)

A, B, C are pairwise independent if

$P(AB) = P(A)P(B)$ and $P(AC) = P(A)P(C)$ and $P(BC) = P(B)P(C)$

ex 3 people

B_{ij} = the event that person i and j have the same B-day.

Are B_{12}, B_{13}, B_{23} are pairwise independent?

$$P(B_{12}B_{13}) \stackrel{?}{=} \underbrace{P(B_{12})}_{=1/365}\underbrace{P(B_{13})}_{=1/365} \checkmark$$

Person 1 can have any B-day. The chance that person 2 and 3 both have that B-day is $\left(\frac{1}{365}\right)^2$.

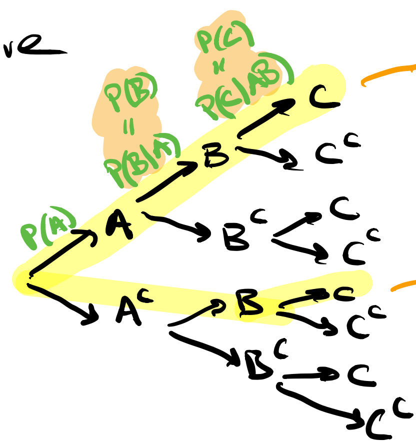
Def^n (independence of 3 events)

A, B, C are independent if

1) A, B, C are pairwise indep

2) $P(ABC) = P(A)P(B)P(C)$, (and the same for any of the events replaced by its complement)

Picture



$P(ABC) = P(A)P(B)P(C)$

$P(A^cBC) = P(A^c)P(B)P(C)$

Note that we need $P(C|AB) = P(C)$ for independence but this isn't given by pairwise indep of A, B, C !!

In class there was a question of whether independence of 3 events implies pairwise independence.
The answer is no:

ex Flip a coin 2 times,

let H_1 = event 1^{st} coin lands head

H_2 = event 2^{nd} coin lands head

S = event get H_1H_2 or T_1T_2.

The events H_1, H_2, S are pairwise independent but not independent.

ex

Are B_{12}, B_{13}, B_{23} independent?

we already know pairwise independent.

NO

Check $P(B_{12}B_{13}B_{23}) \overset{?}{=} P(B_{12})P(B_{13})P(B_{23})$

Notice that $B_{12}B_{13}B_{23} = B_{12}B_{13}$ since

$B_{12}B_{13}B_{23}$ says 3 people have the same B-day and $B_{12}B_{13}$ says 3 people have the same B-day.

So $P(B_{12}B_{13}B_{23}) = P(B_{12}B_{13}) = \left(\frac{1}{365}\right)^2$ but

$P(B_{12})P(B_{13})P(B_{23}) = \left(\frac{1}{365}\right)^3$.

② <u>Sec 2.1</u> Binomial distribution.

<u>Bernoulli(p) trial (distribution)</u>

$$\text{two outcomes} \begin{cases} \text{success} - p \\ \text{failure} - 1-p \end{cases}$$

<u>ex</u> roll a die.
success is getting a 6 — 1/6
failure is not getting a 6 — 5/6

<u>Binomial (n,p) distribution (Bin (n,p))</u>

we have n independent Bernoulli(p) trials

n: fixed

p: fixed (unconditional probability)

<u>ex</u> we roll a die n times.

What are the possible number of successes? ~0,1,2,...,n

The chance of having each of these number of successes is called the Bin (n, p) distribution

Binomial formula:

$$P(k) = \frac{n!}{k!\,(n-k)!}\, p^{k} (1-p)^{n-k}$$

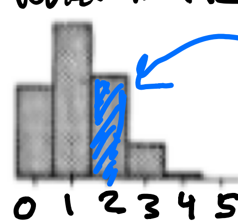
n: # trials

p: chance of success.

k: number of successes

<u>ex</u> You roll a die 5 times. What is the chance of getting 2 sixes?

$n = ? - 5$
$k = ? - 2$
$p = ? - 1/6$



$$P(2) = \frac{5!}{2!\,3!}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 = \frac{5\cdot4\cdot\cancel{3\cdot2\cdot1}}{2\cdot1\cdot\cancel{3\cdot2\cdot1}}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3$$

$$= \boxed{10\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3}$$

this is the area of the block above $k=2$.

What is chance of getting

success (6)

failure (not 6)

$$\begin{array}{cccccc} 1 & 1 & 0 & 0 & 0 & ? \\ 0 & 1 & 1 & 0 & 0 & ? \\ & & \vdots & & & \end{array}$$

How many of these are there?

$$\frac{5!}{2!\,3!}$$

5 positions for 1^{st} 1

4 positions for the 2^{nd} 1

3 positions for the 1^{st} 0

2 positions for the 2^{nd} 0

1 " " " 3^{rd} 0

5 choose 2 $= \dfrac{5!}{2!\,3!}$

We write $\dfrac{5!}{2!\,3!}$ as $\dbinom{5}{2}$ or $\dbinom{5}{3}$ or $\dbinom{5}{2,3}$

$$\binom{5}{3} = \frac{5!}{3!\,2!}$$

tinyurl.com/sept2-pt1

Stats 134

Chapter 2

1. Ten cards are dealt off the top of a well shuffled deck. The binominal formula doesn't apply to find the chance of getting exactly three diamonds because:

   a The probability of a trial being successful changes

   b The trials aren't independent

   c There isn't a fixed number of trials

   d more than one of the above

The (unconditional probability) of getting a diamond is always 1/4. However the trials are dependent on one another

$$\left.\begin{array}{l} P(1D) = 1/4 \\ P(2D) = 1/4 \end{array}\right\}$$ says 1D and 2D have same probability

$P(2D|1D) = 12/51 \neq \overset{1/4}{P(2D)}$ so 1D and 2D are dependent

To get answer d: Suppose the top card is always a diamond, and the rest of the deck is well shuffled, then $P(1D) = 1$ and $P(2D) = P(3D) = 12/51$