Stat 134: Joint Distributions Review

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Conceptual Review

Suppose X, Y are random variables with joint distribution $f_{X,Y}$ over the region $\{(x, y) \in \mathbb{R}^2 : 0 < x < y\}$.

a. Are X, Y independent?

b. Set up an integral to find each of the following:

   i. $f_X(x)$;

   ii. $F_Y(y)$;

   iii. $P(Y < X + 5)$;

   iv. $\mathbb{E}(X)$;

   v. $\mathbb{E}(g(X, Y))$.

Problem 1

Based on each of the joint densities below, with minimal calculation, identify the marginal distribution of X and Y.

a. $f_{X,Y}(x, y) = 3360x^3(y - x)^2(1 - y), \;\; 0 < x < y < 1$, (Bonus: how did we compute the constant of 3360?);

b. $f_{X,Y}(x, y) = \lambda^3 e^{-\lambda y}(y - x), \;\; 0 < x < y$, (Hint: $X \sim$ Exp (λ));

c. $f_{X,Y}(x, y) = e^{-4y}, \;\; 0 < x < 4, \;\; 0 < y$.

Problem 2

Suppose X, Y follow the standard bivariate normal distribution with correlation ρ. Find the joint density of X and Y in terms of ϕ, the standard normal PDF.
(Note we have worked a lot with these two variables, but we have not yet derived this density or used it directly!)

Problem 3

Let (X, Y) represent a point chosen uniformly at random from the region $\{(x, y) : x > 0, y > 0, x^2 + y^2 < 4\}$. Let R represent the distance from the origin to the random point (X, Y), i.e. $R = \sqrt{X^2 + Y^2}$. Find:

a. $f_{X,Y}(x, y)$;

b. $f_R(r)$;

c. $P(cX > Y)$, for some $c > 0$.