Stat 134 lec 4

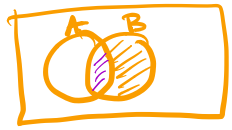
Quiz 1 next Wed on Sec 1.1-1.6

Last time

sec 1.4 independence

Note that if $P(AB)=P(A)P(B)$ then $P(A^cB)=P(A^c)P(B)$

since,



difference rule

$$P(A^cB) = P(B) - P(AB)$$

independence of A,B

$$= P(B) - P(A)P(B) = (1-P(A))P(B) = P(A^c)P(B)$$

sec 1.5 Baye's rule

Finish problem from last time.

M = mother bb

D = dad Bb

G = Greg brown eyes

Find $P(G \mid D)$

condition on mom's genes:

$$P(G \mid D) = \overset{1/2}{P(G \mid D, \text{mom bb})}\, \overset{1/4}{P(\text{mom bb})} + \overset{1}{P(G \mid D, \text{mom BB})}\, \overset{1/4}{P(\text{mom BB})} + \overset{3/4}{P(G \mid D, \text{mom bB})}\, \overset{1/4}{P(\text{mom bB})} + \overset{3/4}{P(G \mid D, \text{mom Bb})}\, \overset{1/4}{P(\text{mom Bb})}$$

Dad Bb, Mom bB } Greg: Bb BB bb bB

Chance Brown is 3/4

$$= \left(\tfrac{1}{2} + 1 + \tfrac{3}{4} + \tfrac{3}{4}\right) \cdot \tfrac{1}{4} = \tfrac{3}{4}$$

$$\Rightarrow P(M \mid D,G) = P(M \mid D,G) = \frac{P(M,D,G)}{P(D,G)} = \frac{P(G \mid M,D)\,\overset{P(M)P(D)}{P(M,D)}}{P(G \mid D)P(D)} = \frac{\tfrac{1}{2}\cdot\tfrac{1}{4}}{\tfrac{3}{4}} = \tfrac{1}{6}$$

Note

$P(G \mid M,D)$ is forward conditional (likelihood conditional)

DON'T NEED BAYES TO COMPUTE

$P(M \mid D,G)$ is backwards conditional (posterior conditional)

NEED BAYES TO COMPUTE

Today

1. Go over student responses concept test
2. sec 1.6 independence of 3 or more events
3. sec 2.1 Binomial Distribution
4. sec 2.1 Consecutive odds ratio

① Student responses

.. Suppose A and B are two events with

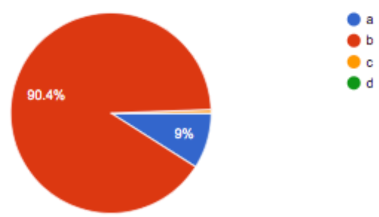
$$P(A) = 0.5 \text{ and } P(A \cup B) = 0.8.$$

Is it possible for A and B to be both **mutually exclusive** and **independent**?

a yes

b no

c there isn't enough information to decide



a	We need more information but that means it is still possible

b	We literally just discussed this, the only way that A and B can be both mutually exclusive and independent is if one of them is the empty set. Since P(A U B) > P(A), B is not the empty set. Since P(A)>0, A is not the empty set.

Suppose A and B are two events with

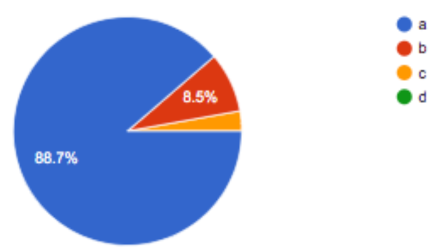
$$P(A) = 0.8 \text{ and } P(A \cup B) = 0.8.$$

Is it possible for A and B to be both **mutually exclusive** and **independent**?

a yes

b no

c there isn't enough information to decide



c	There are two scenarios in which P(A) = P(A or B): if B is an empty set or if B is a subset of A. Thus, we can't know for sure.

a	If P(B)=0, then A and B are mutually exclusive and independent

sec 1.6 Independence of 3 events

A, B, C are pairwise independent if

$P(AB)=P(A)P(B)$ and $P(AC)=P(A)P(C)$ and $P(BC)=P(B)P(C)$

ex 3 people

B_{ij} = the event that person i and j have the same B-day.

B_{12}, B_{13}, B_{23} are pairwise independent

$$P(B_{12}B_{13}) \overset{?}{=} P(B_{12})P(B_{23})$$

$$P(B_{12}B_{13}) = P(B_{12}|B_{13})P(B_{13})$$

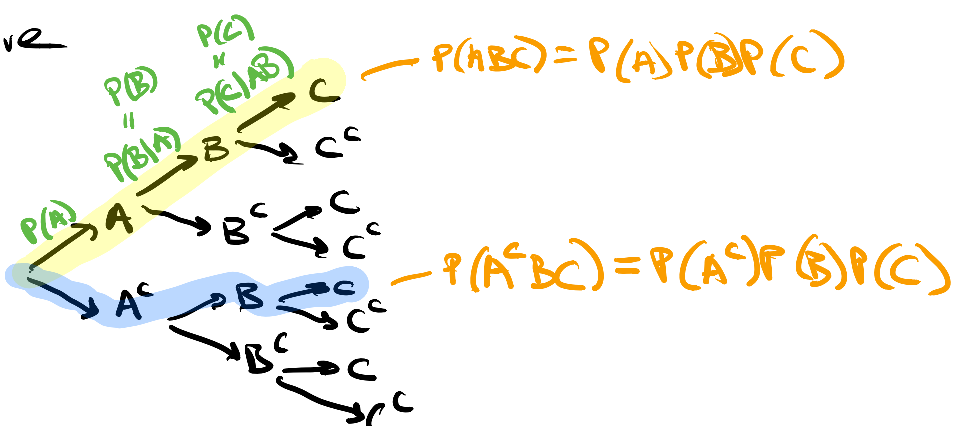
$$P(B_{12}|B_{13}) = P(B_{12})$$

A, B, C are independent if

1) A, B, C are pairwise indep

2) $P(ABC) = P(A)P(B)P(C)$. (and the same for any of the events replaced by its complement)

Picture



Note that we need $P(C|AB)=P(C)$ for independence but this isn't given by pairwise indep of A, B, C !!

ex

Is B_{12}, B_{13}, B_{23} independent? — No

we already know pairwise independent.

Check $$\underbrace{P(\underbrace{B_{12}B_{13}B_{23}}_{=\,B_{12}B_{13}})}_{=\,\frac{1}{365}\cdot\frac{1}{365}} \overset{?}{=} \underbrace{P(B_{12})}_{=\,\frac{1}{365}}\underbrace{P(B_{13})}_{=\,\frac{1}{365}}\underbrace{P(B_{23})}_{=\,\frac{1}{365}}$$

(3) <u>sec 2.1</u> Binomial distribution.

<u>Bernoulli(p) trial (distribution)</u>

two outcomes — success — p
— failure — $1-p$

ex roll a die.
success is getting a 6 — $1/6$
failure is not getting a 6 — $5/6$

<u>Binomial (n,p) distribution $(\text{Bin}(n,p))$</u>

We have n independent Bernoulli (p) trials

fixed — fixed (unconditional probability)

ex we roll a die n times.

What are the possible number of successes? — $0,1,2,\dots,n$

The chance of having each of these number of successes is called the Bin (n,p) distribution

Binomial formula:

$$P(k) = \frac{n!}{k!\,(n-k)!}\, p^{k} (1-p)^{n-k}$$

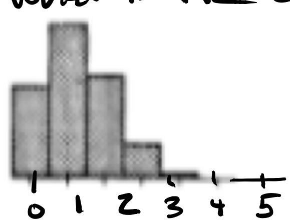
trials

chance of success.

number of successes

ex You roll a die 5 times. What is the chance of getting 2 sixes?

$n = ?$ — 5
$k = ?$ — 2
$p = ?$ — $1/6$



$$P(2) = \frac{5!}{2!3!}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 = \frac{5\cdot4\cdot\cancel{3\cdot2\cdot1}}{2\cdot1\cdot\cancel{3\cdot2\cdot1}}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3$$

$$= \boxed{10\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3}$$

What is chance of getting

success (6)

failure (not 6)

1 1 0 0 0 ? $\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3$

0 1 1 0 0 ? $\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 = \left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3$

⋮

How many of these are there? $\frac{5!}{2!3!}$

there are 5 positions for the 1st 1, 4 positions for the 2nd 1, 3 positions for the 1st 0, etc.

11000 is counted 2!3! times

01100 is counted 2!3! times

We write $\frac{5!}{2!3!}$ as $\binom{5}{2}$ or $\binom{5}{3}$ or $\binom{5}{2,3}$

$$\binom{5}{3} = \frac{5!}{3!2!}$$

tinyurl.com/sept6-pt1

tinyurl.com/sept6-pt2

Stats 134

Chapter 2 Wednesday January 30 2019

1. Ten cards are dealt off the top of a well shuffled deck. The binominal formula doesn't apply to find the chance of getting exactly three diamonds because:

   a The probability of a trial being successful changes

   b The trials aren't independent

   c There isn't a fixed number of trials

   d more than one of the above

The (unconditional) Prob of getting a diamond is always 1/4. However the trials are dependent on one another.

$P(2^{nd}$ trial is a success | 1^{st} trial is a success$)$

$= P(2^{nd}$ card is diamond | 1^{st} card is diamond$)$

$\frac{3}{51}$

$\neq P(2^{nd}$ card is diamond$)$

$\frac{4}{52}$

2. A well shuffled deck is cut in half. Five cards are dealt off the top of one half deck and five cards are dealt off the top of the other half deck. The binominal formula doesn't apply to find the chance of getting exactly three diamonds because:

a The probability of a trial being successful changes

b The trials aren't independent

c There isn't a fixed number of trials

d more than one of the above

The probability of getting a diamond is still $1/4$.

$$P(1^{st}\,D) = P\left(1^{st}D \,\middle|\, \begin{matrix}0 \text{ diamond } 1^{st} \text{ half} \\ 13 \text{ diamond } 2^{nd} \text{ half}\end{matrix}\right) \cdot P\begin{pmatrix}0 \text{ diamond } 1^{st} \text{ half} \\ 13 \text{ diamond } 2^{nd} \text{ half}\end{pmatrix} +$$

$$+ P\left(1^{st}D \,\middle|\, \begin{matrix}1 \text{ diamond } 1^{st} \text{ half} \\ 12 \text{ diamond } 2^{nd} \text{ half}\end{matrix}\right) \cdot P\begin{pmatrix}1 \text{ diamond } 1^{st} \text{ half} \\ 12 \text{ diamond } 2^{nd} \text{ half}\end{pmatrix} +$$

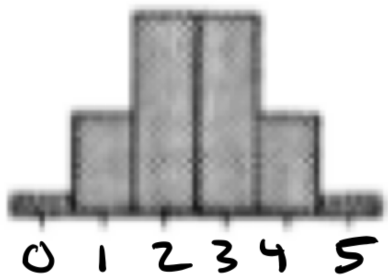
etc

If I say "A well shuffled deck is cut in half with 5 diamonds in the top half", then the prob of a card in the top half being an ace is $p = 5/26$ and the prob in the bottom half is $p = 8/26$.

④ <u>Consecutive odds ratio</u> — a tool to find the mode of the binomial distribution.

<u>mode</u>

The mode of the binomial distribution is the k such that $P(k) = \binom{n}{k} p^k (1-k)^{n-k}$ is largest

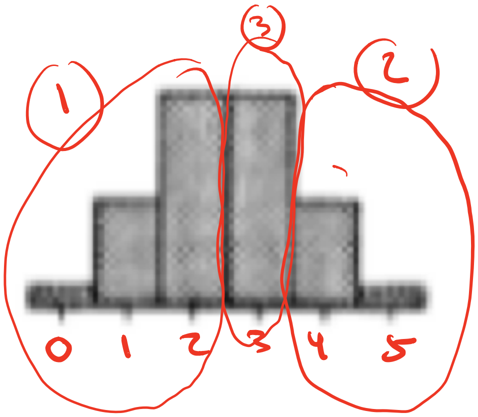


mode

here $n = 5$
$k = 0, 1, 2, 3, 4, 5$
$p = \frac{1}{2}$

$np + p$ is an important number

$np + p = 5 \cdot \frac{1}{2} + \frac{1}{2} = 3$



notice

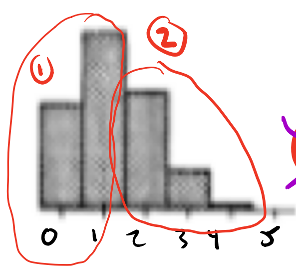
① $\overset{0,1,2}{k} < \overset{3}{np+p}$ iff $P(k-1) < P(k)$

② $\overset{4,5}{k} > np+p$ iff $P(k-1) > P(k)$

mode ③ $\overset{3}{k} = np+p$ iff $P(k-1) = P(k)$

ex $n=5$, $p = 1/4$, $k = 0,1,2,3,4,5$

$np + p = 5 \cdot \frac{1}{4} + \frac{1}{4} = 1.5$



① $\overset{0,1}{k} < np+p$ iff $P(k-1) < P(k)$

② $\overset{2,3,4,5}{k} > np+p$ iff $P(k-1) > P(k)$

~~③~~ $k = np+p$ iff $P(k-1) = P(k)$

← not an integer,

← i.e. $np+p$ is an integer

Notice when $\lfloor np+p \rfloor = np+p$ then you have case 3 and $k = \lfloor np+p \rfloor$ and $k-1 = \lfloor np+p \rfloor - 1$ is the mode

when $\lfloor np+p \rfloor < np+p$ you dont have case 3 and $k = \lfloor np+p \rfloor$ is the mode