Stat 134: Section 22

Adam Lucas

Dec 11th, 2019

Conceptual Review

a. PDF:

1. $f(x) \geq 0$
2. $\int_{x \in \mathcal{D}} f(x) x = 1$

CDF:

1. $0 \leq F(x) \leq 1$
2. $F(-\infty) = 0$, $F(\infty) = 1$, F non decreasing.
3. $F(x) = F(x^+)$
4. $F(X) \sim Unif[0,1]$ if F is continuous and increasing.

b. $P(T > t + s | T > t) = P(T > s) \quad (s \geq 0, t \geq 0)$

c. relation example:

$$P(T_4 > T_1 + 2) = P(W_2 + W_3 + W_4 > 2) = P(N_2 < 3)$$

distribution:

$$N_t \sim Poi(\lambda t)$$
$$W_r \sim Exp(\lambda)$$

$$P(T_r \in t)/dt = P(N_t = r-1)\lambda = \exp^{-\lambda t} \frac{(\lambda t)^{r-1}}{(r-1)!} \lambda \sim Gamma(r, \lambda)$$

d. $f_{(k)}(x) = n f(x) \binom{n-1}{k-1} (F(x))^{k-1} (1 - F(x))^{n-k}$

Problem 1

See *Ex 4.rev.6 in Pitman's Probability*

Hint: google "complete solution pitman probability"

Problem 2

See *Ex 4.6.5 in Pitman's Probability*

Problem 3

For this problem you will have to notice that

1. Exponential distribution is memoryless
2. if $X_1 \sim Exp(\lambda_1)$, $X_2 \sim Exp(\lambda_2)$, then $\min(X_1, X_2) \sim Exp(\lambda_1 + \lambda_2)$

Let W_i be the time between the leaving time of the customer who gets his serviced finished (i-1)th and the leaving time of the customer who gets his serviced finished ith. Then $W_1, \ldots, W_8$ are independent.

Before the customer who gets his serviced finished 7th leave the stand, the stand is always fully occupied. So $W_1, \ldots, W_6 \sim Exp(3\lambda)$. Then there are only two people in the stand, so the time for the next person to leave $W_7 \sim Exp(2\lambda)$, and similarly, $W_8 \sim Exp(\lambda)$.

i. $E(T) = \sum_{i=1}^{8} E(W_i) = 6 \cdot \frac{1}{3\lambda} + \frac{1}{2\lambda} + \frac{1}{\lambda} = \frac{7}{2\lambda}$
ii. $Var(T) = \sum_{i=1}^{8} Var(W_i) = 6 \cdot \frac{1}{(3\lambda)^2} + \frac{1}{(2\lambda)^2} + \frac{1}{(\lambda)^2} = \frac{23}{12\lambda^2}$

Prepared by Yulun Woooo