Stat 134: Section 11

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Problem 1: Properties of the Geometric Distribution

Let $X_1, X_2 \overset{\text{i.i.d.}}{\sim} \text{Geom}\,(p)$ on $\{0, 1, 2, \ldots\}$.

a. *Memoryless property:* Show that for all $k, m \geq 0$, $P(X_1 = m + k \mid X_1 \geq k) = P(X_1 = m)$. Provide an explanation for why this must be the case, in terms of sequences of successes and failures.

b. *Sums of geometrics:* Let $Y = X_1 + X_2$. What is the distribution of Y? Find $P(X_1 = k \mid Y = n)$, for $0 \leq k \leq n$. (What distribution does this remind you of?)

Problem 2

How many raisins per cubic centimeter must a large batch of dough contain on average for there to be at least a 99% chance that one 50 cm^3 cookie made from this dough contains at least one raisin?
From Ex 3.5.2 in Pitman's Probability

Problem 3

Suppose X, Y, and Z are independent Poisson random variables, with parameters μ_X, μ_Y, μ_Z respectively. Find:

a. $P(X+Y=4)$

b. $\mathbb{E}((X+Y+Z)^2)$

Hint: Recall the equation $Var(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$.

c. $P(\max\{X,Y,Z\} > k)$, for $k = 0, 1, 2, \ldots$

From Ex 3.5.11 in Pitman's Probability