

Stat 134 lec 8

Last time sec 2.5 hypergeometric distribution

Suppose a population of size N contains G good and B bad elements ($N = G + B$).

A sample, size n , with g good and b bad elements ($n = g + b$) is chosen at random without replacement.

$$P(\text{ } g \text{ good and } b \text{ bad}) = \frac{\binom{G}{g} \binom{B}{b}}{\binom{N}{n}}$$

Parameters : N = population size
 G = # Good in population
 n = Sample size.

abbrev. Hyper(N, G, n)

$\cong P(\text{poker hand is } 4 \text{ of a kind})$ rank $aabb$ $aabb$

N = number of cards = 52

G = number of rank a cards in deck = 4

n = sample size = 5

$$\binom{13}{1}, \frac{\binom{4}{1} \binom{48}{1}}{\binom{52}{5}} = \binom{13}{1} \binom{12}{1} \cdot \frac{\binom{4}{4} \binom{4}{1}}{\binom{52}{5}}$$

↗ choose 1 rank for a
 ↑ # poker hand
 ↗ not $\binom{13}{2}$ since $aabb \neq bbbb$

we saw last time this is equal to $13 \cdot 12 \cdot \binom{5}{4} \cdot \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot \frac{1}{49} \cdot \frac{4}{48}$

Today

- ① review student concept test responses.
- ② Sec 2.5 more hypergeometric
- ③ Sec 2.5 counting strategy

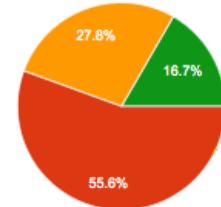
(1) The probability of being dealt a three of a kind poker hand (ranks $aaabc$ where $a \neq b \neq c$) is:

a $\binom{4}{3} \binom{4}{1} \binom{44}{1} / \binom{52}{5}$

b $\binom{13}{1} \binom{12}{2} \binom{4}{3} \binom{4}{1} \binom{4}{1} / \binom{52}{5}$

c $\binom{13}{1} \binom{12}{1} \binom{4}{3} \binom{4}{1} \binom{44}{1} / \binom{52}{5}$

d none of the above



- a
- b
- c
- d

d

the correct answer is to add (11 choose 1) to b, because for the single c, there are 11 choices for us to choose from

c

You first have 13 options for the first rank of a. Out of those 4 cards, you choose 3. Then you have 12 options left for the second rank of b. Out of those 4 cards, you choose 1. Lastly, you have $52-8=44$ options left for card c, of which you pick 1.

b

The denominator comes from choosing 5 cards out of 52 possible cards. The numerator is $(13 1)$ because you can choose any card for the first triple. It is then multiplied by $(12 2)$ because you have 12 possibilities for each single. Originally I thought it was $(12 1) & (11 1)$ but that would over count because having a poker hand of King & Ace is the same as having a hand of Ace & king.

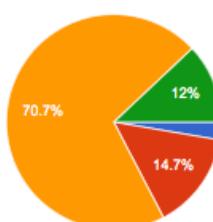
The probability of being dealt a pair in a poker hand (ranks $aabed$ where $a \neq b \neq c \neq d$) is:

a $\binom{13}{1} \binom{12}{3} \binom{4}{2} \binom{4}{1} \binom{4}{1} / \binom{52}{5}$

b $\binom{13}{2} \binom{12}{2} \binom{4}{3} \binom{4}{1} \binom{4}{1} / \binom{52}{5}$

c $\binom{13}{1} \binom{12}{3} \binom{4}{2} \binom{4}{1} \binom{4}{1} \binom{4}{1} / \binom{52}{5}$

d none of the above



- a
- b
- c
- d

d

None because should have an option for 40

c

As we can't distinguish b c d from each other, we should time 12 choose 3

(2) Practice hypergeometric

You and a friend are playing poker. If each of you are dealt 5 cards from the same deck, what is the chance that you both get a 4 of a kind (ranks $aabb$)
 $a+b$

$$\frac{\text{friend gets } 4 \text{ of a kind.}}{\binom{13}{1} \binom{12}{4} \binom{4}{1} \binom{4}{1}} \cdot \frac{\text{you get } 4 \text{ of a kind}}{\binom{11}{1} \binom{4}{4} \left[\binom{3}{1} + \binom{10}{1} \binom{4}{1} \right]} = \frac{\binom{13}{1} \binom{4}{1} \binom{48}{1}}{\binom{52}{5}} \cdot \frac{\binom{11}{1} \binom{4}{4} \binom{43}{1}}{\binom{47}{5}}$$

You can rewrite this as

$$\boxed{\frac{\binom{13}{1} \binom{4}{1} \binom{48}{1}}{\binom{52}{5}} \cdot \frac{\binom{11}{1} \binom{4}{4} \binom{43}{1}}{\binom{47}{5}}}$$

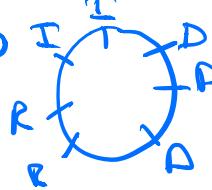
(3)

Counting strategy:

Try and break the problem into a sequence of steps and apply the multiplication rule. Above we find the chance your friend gets 4 of a kind and the probability you get 4 of a kind given that your friend got 4 of a kind.

ex There are 3 Democrats, 2 Republicans, and 2 Independents sitting around a table. What is the chance the Dem sit together, the Rep sit together and the Ind sit together?

Soln 1

There are $\binom{7}{3,2,2}$ orderings of DDDRRII. Hence the probability of getting DDDRRII is $\frac{1}{\binom{7}{3,2,2}}$. There are 7 positions around the table. The first D can go I  so the

Chance that we have DDDRRII sitting around the table is $\frac{7}{\binom{7}{3,2,2}}$. But DDDIIRR is also an acceptable ordering so the answer is

$$\frac{7 \cdot 2}{\binom{7}{3,2,2}}$$

Soln 2 (notice how we break up the problem)

First find chance Dem sit together, then Rep join table and sit together, then Indep join table and sit together.

$$\begin{array}{c} \text{Dem together} \quad \text{Rep together} \quad \text{Indep together} \\ \text{--- --- ---} \\ \text{I} \quad \text{R} \quad \text{D} \\ \text{D} \quad \text{R} \quad \text{D} \end{array} = \boxed{\frac{7 \cdot 2}{\binom{7}{3,2,2}}}$$

7 positions for first D $\frac{7}{\binom{7}{3,2,2}}$. $\frac{2}{\binom{4}{2}}$ for 2 positions for first R = $\frac{1}{\binom{2}{2}}$

Ex In a well shuffled deck, find the probability that J, Q, K appear as 12 consecutive cards and the J are grouped together, Q are grouped together, and K are grouped together?

Ex $JJJJQQQQKKKK$

Soln

First we find the chance that the 12 cards are consecutive. Second, given that the cards are consecutive, we find the chance the J, Q, K are grouped.

The chance of (12 cards all J, Q, K) followed by 40 other cards is $\frac{1}{\binom{52}{12, 40}}$. We must allow for the 12 cards

to not be first in the deck. Think of the 12 cards as one special card so the deck has 41 cards with the top card being our group of 12. There are $\binom{41}{1}$ ways we can position the special card in the deck at 41.

$$\text{Hence, } P(\text{the 12 J, Q, K are consecutive}) = \frac{\binom{41}{1}}{\binom{52}{12, 40}}$$

Given that our 12 cards are consecutive, the chance they are as $JJJJQQQQKKKK$ is $\frac{1}{\binom{12}{4, 4, 4}}$. There

are however $3!$ orderings of J, Q, K so we get $\frac{3!}{\binom{12}{4, 4, 4}}$.

Hence $P(J, Q, K \text{ are consecutive and grouped in the deck})$

$$= \boxed{\frac{\binom{41}{1}}{\binom{52}{12, 40}} \cdot \frac{3!}{\binom{12}{4, 4, 4}}}$$

ex Adam, Jess and Tom are standing in a group of 12 people. The group is randomly split into two lines of 6 people each. Find the chance that Adam, Jess and Tom are standing next to each other in one of the 2 lines.

Please think about how to do this for next time.