Stat 134 Lec 9

Quiz 2 Monday – sec 2.1, 2.2, 2.4, 2.5

Last time finished chap 2

hypergeometric (N, G, n)

$\downarrow$ approx by binomial
N large
$p = \frac{G}{N}$

binomial (n, p)

$\swarrow$ approx by Poisson
$p \to 0,\ n \to \infty,\ np \to \mu$
$\mu - 3\sigma < 0$

Poisson(μ)

$\searrow$ approx by normal
n large
$\mu = np,\ \sigma = \sqrt{npq}$
$0 < \mu \pm 3\sigma < n$
use continuity correction

normal(μ, σ^2)

Today sec 3.1
- random variable (RV)
- joint distribution of 2 RVs.
- distribution of the sum of two RVs.

Sec 3.1 Intro to RVs.

A random variable, X, is the outcome of an experiment.

ex flip a prob p coin 2 times

$X = \#$ heads

$X=1$ is an event

$P(X=1) = \binom{n}{1} p^1 (1-p)^{2-1}$ binomial formula.

We write $X \sim Bin(2, p)$

n

More precisely,

X : outcome space $\longrightarrow \mathbb{R}$ is a function

$HH \longmapsto 2$

$HT \longmapsto 1$

$TH \longmapsto 1$

$TT \longmapsto 0$

So $X=1$ is a subset of all outcomes

(here, $X=1$ means $\{HT, TH\}$)

X has a probability distribution

X	0	1	2
$P(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
$g(x) = \lvert X-1 \rvert$	1	0	1

$P(HT \text{ or } TH)$

You can find the distribution of

$g(X) = |X-1|$ — range 0, 1

$\lvert X-1 \rvert$	0	1
$P(x)$	$\frac{1}{2}$	$\frac{1}{2}$

$|X-1| \sim Ber\left(\frac{1}{2}\right)$
$= Bin(1, 1/2)$

Joint Distribution

Let (X, Y) be the joint outcome of 2 RVs X, Y.

The event $(X=x, Y=y)$ is the intersection of events $X=x$ and $Y=y$.

ex X: one draw from [1] [2] [2] [3]

Given $X=x$, $Y=$ number of heads in x coin tosses.

Find $P(X=1, Y=1)$

$$= \underbrace{P(Y=1 \mid X=1)}_{=1/2} \underbrace{P(X=1)}_{=1/4} = 1/8$$

ex A fair coin is tossed twice.

Let X = the number of heads on first toss

Y = the number of heads on first 2 tosses.

Y \ X	0	1	$P(y)$
2	0	$\frac{1}{4}$	$\frac{1}{4}$
1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$
0	$\frac{1}{4}$	0	$\frac{1}{4}$
$P(x)$	$\frac{1}{2}$	$\frac{1}{2}$	

marginal probability of X

$$P(x) = \sum_{y \in Y} P(x,y)$$

ex

$$P(0) = \frac{1}{4} + \frac{1}{4} + 0$$

$$P(1) = 0 + \frac{1}{4} + \frac{1}{4}$$

Marginal prob of Y

$$P(y) = \sum_{x \in X} P(x,y)$$

ex

$$P(0) = \frac{1}{4} + 0$$

$$P(1) = \frac{1}{4} + \frac{1}{4}$$

$$P(2) = 0 + \frac{1}{4}$$

$$X \sim Bin(1, \tfrac{1}{2})$$

$$Y \sim Bin(2, \tfrac{1}{2})$$

$P(x)$ and $P(y)$ are marginal probabilities

The conditional $P(Y=0 \mid X=0) = \dfrac{P(Y=0, X=0)}{P(X=0)} = \dfrac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$

$$P(Y=0 \mid X=1) = 0$$

so Y is dependent on X

Two RVs are <u>independent</u> if

$$P(Y=y \mid X=x) = P(Y=y) \quad \text{for all } x \in X,\ y \in Y$$

By the multiplication rule

If X, Y indep,

$$P(X=x, Y=y) = \underbrace{P(Y=y \mid X=x)}_{= P(Y=y)} \cdot P(X=x)$$

so $$\boxed{P(X=x, Y=y) = P(Y=y)\,P(X=x)}$$

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Chapter 3 Wednesday September 12 2018

1. The joint distribution of X and Y is drawn below:

	3/8	1/2	1/8	P(x) / P(y)
1	1/4	1/3	1/12	2/3
0	1/8	1/6	1/24	1/3
Y \ X	0	1	2	

a X and Y are independent

b $P(X = x|Y = 1) = P(X = x|Y = 0)$, $= P(X=x)$
for all x.

c More than one of the above

d None of the above

<u>soln</u> (a) is true since you can check that $P(X=x, Y=y) = P(X=x)P(Y=y)$ for every x, y. For example

$$\frac{1}{8} = P(X=0, Y=0) = P(X=0)P(Y=0) = 3/8 \cdot \frac{1}{3} \checkmark$$

(b) is also true since

$$P(X=0 \mid Y=0) = \frac{P(X=0, Y=0)}{P(Y=0)} = \frac{1/8}{1/3} = 3/8$$

$$P(X=0 \mid Y=1) = \frac{P(X=0, Y=1)}{P(Y=1)} = \frac{1/4}{2/3} = 3/8$$

or you can say X, Y indep $\Rightarrow$

$$P(X=0 \mid Y=0) = P(X=0 \mid Y=1) = P(X=0)$$

i.e X, Y independence implies that the conditional probability $P(X=x \mid Y=y)$ equals the unconditional probability $P(X=x)$.

Sum of 2 RVs.

$X_1 \sim Bin(1000, \frac{1}{1000}) \approx Pois(1)$

$X_2 \sim Bin(2000, \frac{1}{1000}) \approx Pois(2)$

indep.

$X_1 + X_2 \sim Bin(3000, \frac{1}{1000}) \approx Pois(3)$

$X+Y$ is the # of heads in $1000 + 2000 = 3000$ prob $p = \frac{1}{1000}$ coin tosses.

So sum of two independent binomials is binomial and this example suggests that the sum of 2 indep. Poisson is Poisson