Stat 134 lec 8

Last time

Sec 2.5 Hypergeometric distribution

Analogue to binomial when draw w/o replacement (i.e dependent trials).

ex P(poker hand 4 of a kind) (rank aaaab)

\# poker hands with 4J and 1K

$$13 \cdot 12 \cdot \frac{\overset{J}{\binom{4}{4}}\overset{K}{\binom{4}{1}}}{\binom{52}{5}} \quad \text{or} \quad 13 \cdot 12 \cdot \binom{5}{4} \cdot \frac{\overset{J}{4}}{52} \cdot \frac{\overset{J}{3}}{51} \cdot \frac{\overset{J}{2}}{50} \cdot \frac{\overset{J}{1}}{49} \cdot \frac{\overset{K}{4}}{48}$$

\# poker hands.

diff places for J

Today

Finish chap 2
- harder hypergeometric.
- counting
- approx hypergeom by binomial

ex <u>Past quiz hypergeometric</u>

You and a friend are playing poker. If each of you are dealt 5 cards from the same deck, what is the chance that you both get a 4 of a kind?

aaaab

friend gets 4 of a kind

cccc(b or d)

you get 4 of a kind.

$$\frac{13 \cdot 12 \binom{4}{4}\binom{4}{1}}{\binom{52}{5}} \cdot \frac{11 \cdot \binom{4}{4}\left[\binom{3}{1} + 10\binom{4}{1}\right]}{\binom{47}{5}}$$

You can rewrite this as

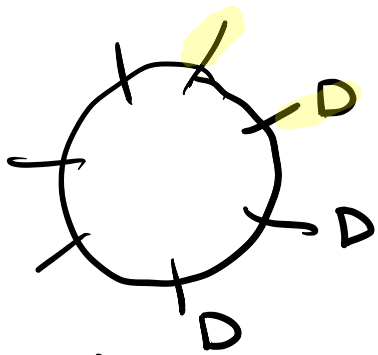
$$\boxed{\frac{13 \cdot \binom{4}{4}\binom{48}{1}}{\binom{52}{5}} \cdot \frac{11 \cdot \binom{4}{4}\binom{43}{1}}{\binom{47}{5}}}$$

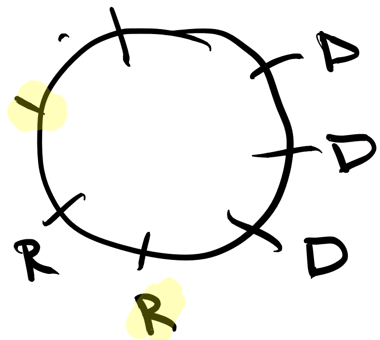
<u>Counting</u>

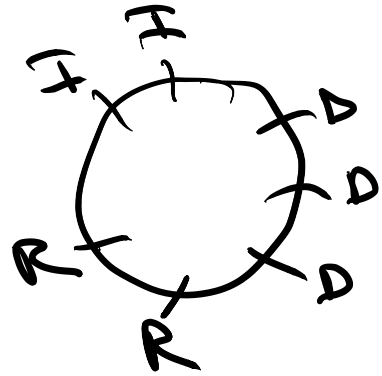
(Principal) Try and break the problem into a sequence of steps and apply multiplication rule.

ex There are 3 Democrats, 2 Republicans, and 2 Independents sitting around a table. What is the chance the Dem sit together, the Rep sit together and the Ind sit together?

— Dem together — Rep together — Inden together







$$7 \cdot \overset{D}{\frac{3}{7}}\,\overset{D}{\frac{2}{6}}\,\overset{D}{\frac{1}{5}}\,\frac{4}{4}\,\frac{3}{3}\,\frac{2}{2}\,\frac{1}{1} \qquad 2 \cdot \frac{2}{4}\,\frac{1}{3}\,\frac{2}{2}\,\frac{1}{1} \qquad \frac{2}{2}\,\frac{1}{1}$$

$$= 7 \cdot \frac{3!\,4!}{7!} \cdot 2 \cdot \frac{2!\,2!}{4!} \cdot 1$$

$$= \boxed{7 \cdot 2\, \frac{3!\,2!\,2!}{7!}}$$

ex In a well shuffled deck, find the probability that J, Q, K appear as 12 consecutive cards and the J are grouped together, Q are grouped together, and K are grouped together?

ex JJJJQQQQKKKK

___ J, Q, K consecutive ___ · ___ J together, Q together, K together ___

$$\binom{41}{1} \cdot \frac{12!\,40!}{52!} \cdot 3! \cdot \frac{\overset{J}{4!}\,\overset{Q}{4!}\,\overset{K}{4!}}{12!}$$

$$= \frac{12 \cdot 41!}{52!}$$

Stat 134

Chapter 2 Monday September 10 2018

1. Adam, Jess and Tom are standing in a group of 12 people. The group is randomly split into two lines of 6 people each. The chance that Adam, Jessica, and ~~Michael~~ Tom are standing next each other in one of these lines is:

a $\frac{4!3!}{6!} * \binom{3}{3}\binom{9}{3} / \binom{12}{6}$

b $\frac{4!3!}{6!} * 2\binom{3}{3}\binom{9}{3} / \binom{12}{6}$

c $\frac{3!3!}{6!} * 2\binom{3}{3}\binom{9}{3} / \binom{12}{6}$

d none of the above

<u>Soln</u>

Two steps:

___ A, J, T are in the same line ___ A, J, T are consecutive

$$\boxed{2 \cdot \frac{\binom{3}{3}\binom{9}{3}}{\binom{12}{6}} \cdot \binom{4}{1}\frac{3!3!}{6!}}$$

explanation.

We first find the chance A, J, T are in line 1. Out of 12 we take a sample of 6. A, J, T are good and the other 9 are bad. we have $\frac{\binom{3}{3}\binom{9}{3}}{\binom{12}{6}}$. The chance that AJT are in line 2 is the same so the chance that A, J, T are in the same line is $2\frac{\binom{3}{3}\binom{9}{3}}{\binom{12}{6}}$.

Given that A, J, T are in the same line, we find the chance that they are consecutive.

First the chance that AJT are the first 3 in any order is $\frac{3}{6}\frac{2}{5}\frac{1}{4}\frac{3}{3}\frac{2}{2}\frac{1}{1} = \frac{3!3!}{6!}$.
Thinking of AJT as one person there are 4 people in the line and $\binom{4}{1}$ ways to place AJT in the line.
Hence given that AJT are in the same line the chance they are consecutive is $\binom{4}{1}\frac{3!3!}{6!}$
Now apply the multiplication rule,

Binomial approx to hypergeometric

Binomial is drawing from your population with replacement. Hypergeometric is drawing from your population without replacement. For large population size N, we can approximate the hypergeometric formula by the binomial formula which is easier to compute.

ex Only two types of students in the class of 100.

Class 100 students, A: 70 students
B: 30 students.

Sample 5 students at random without replacement (called a simple random sample SRS)

Find $P(3A\text{'s}, 2B\text{s})$.

exact (hypergeometric) $= \dfrac{\binom{70}{3}\binom{30}{2}}{\binom{100}{5}} = \boxed{.316}$

$p = \dfrac{6}{N} = \dfrac{70}{100} = .7$

approx (binomial) $= \binom{5}{3}(.7)^3(.3)^2 = \boxed{.309}$

Next time, chapter 3!