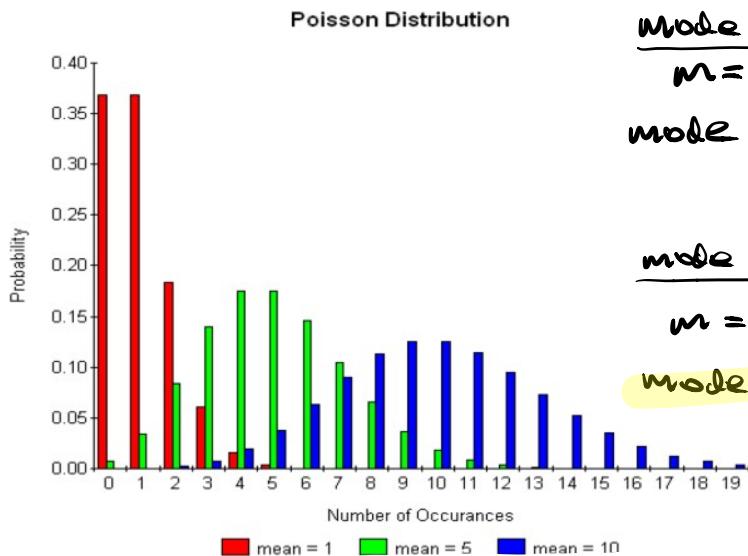


Stat 134 lec 7

Last time

sec 2.4 Poisson approx to binomial.

We saw that $\text{Pois}(n)$ is a limit of binomials for $n \rightarrow \infty$, $p \rightarrow 0$ and $np \rightarrow M$.



mode of $\text{Bin}(n,p)$:

$$m = \lfloor np + p \rfloor$$

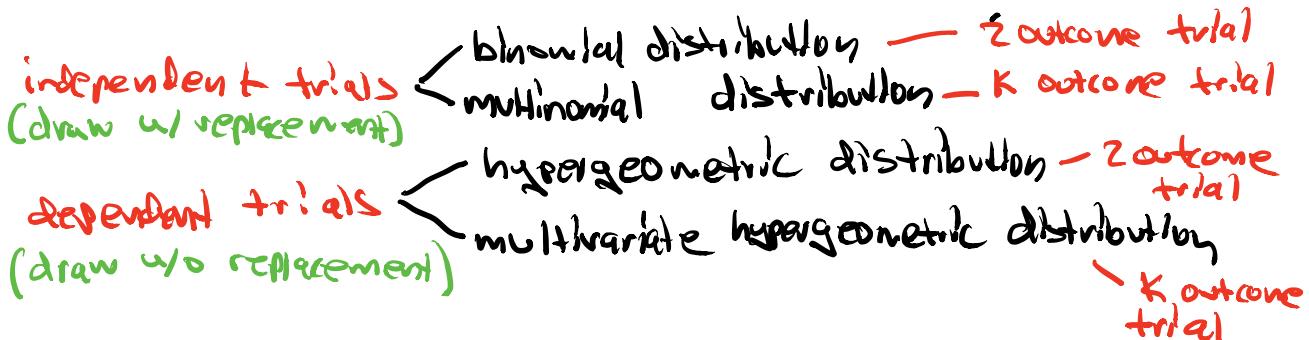
$$\text{mode} = \begin{cases} m & \text{if } np + p \notin \mathbb{Z} \\ \{m-1, m\} & \text{if } np + p \in \mathbb{Z} \end{cases}$$

mode of $\text{Pois}(M)$:

$$m = \lfloor M \rfloor \text{ since } np + p \rightarrow M$$

$$\text{mode} = \begin{cases} m & \text{if } M \notin \mathbb{Z} \\ \{m-1, m\} & \text{if } M \in \mathbb{Z} \end{cases}$$

Today sec 2.5 Random Sampling



Sec 2.5

Random sampling with replacement

Ex Class 100 students
grade distribution:

- A 50 students
- B 30 students
- C 15 students
- D 5 students

You sample 10 students with replacement.

a) What is the chance you get

AAAA BBB CC D ?

Ans $(.5)^4 (.3)^3 (.15)^2 (.05)^1$

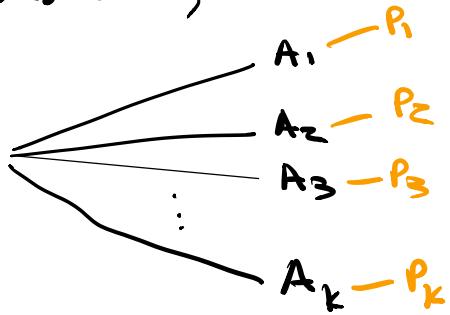
b) Find $P(4A's, 3B's, 2C's, 1D)$

$$\frac{10!}{4!3!2!1!} (.5)^4 (.3)^3 (.15)^2 (.05)^1$$

$\nwarrow \binom{10}{4,3,2,1} = \binom{10}{4} \binom{6}{3} \binom{3}{2} \binom{1}{1}$

Defⁿ Multinomial Distribution

If you have n independent trials, where each trial has K possible outcomes, A_1, A_2, \dots, A_K with probabilities P_1, P_2, \dots, P_K ,



then the probability you get n_1 outcome A_1 , n_2 outcome A_2 , ..., n_k outcome A_k is

$$P(n_1, n_2, \dots, n_k) = \binom{n}{n_1, n_2, \dots, n_k} P_1^{n_1} P_2^{n_2} \dots P_k^{n_k}$$

$\frac{n!}{n_1! n_2! \dots n_k!}$

Note Binomial distribution is a special case with $K=2$.

independent trials
(draw w/ replacement)

binomial distribution — 2 outcome trial
multinomial distribution — K outcome trial

random sample without replacement

e.g. In a very student friendly class with 100 students

the grade distribution is:

- A 70 students
- B 30 students

You sample 5 students at random without replacement (called a simple random sample (SRS))

a) Find the chance you get

A A A B B

$$\frac{70}{100} \frac{69}{99} \frac{68}{98} \frac{30}{97} \frac{29}{96}$$

b) Find $P(3A's, 2B's)$,

$$\binom{5}{2,3} \frac{70}{100} \frac{69}{99} \frac{68}{98} \frac{30}{97} \frac{29}{96}$$

$\frac{5!}{2!3!}$

(if we drew with replacement the answer would be $\binom{5}{2,3} (0.7)^3 (0.3)^2$)

$$\frac{5!}{3!2!} \frac{70}{100} \frac{69}{99} \frac{68}{98} \frac{30}{97} \frac{29}{96} = \frac{\frac{70 \cdot 69 \cdot 68}{3!} \cdot \frac{30 \cdot 29}{2!}}{\frac{100 \cdot 99 \cdot 98 \cdot 97 \cdot 96}{5!}}$$

$$= \frac{\binom{70}{3} \binom{30}{2}}{\binom{100}{5}}$$

hypergeometric formula

Defⁿ hypergeometric distribution

Suppose a population of size N contains G good and B bad elements ($N=G+B$).

A sample, size n , with g good and b bad elements ($n=g+b$) is chosen at random without replacement.

$$P(g \text{ good and } b \text{ bad}) = \frac{\binom{G}{g} \binom{B}{b}}{\binom{N}{n}}$$

This generalizes to the multivariate hypergeometric distribution

Now instead of 2 types of elements we have K with sizes G_1, G_2, \dots, G_K ($N=G_1+\dots+G_K$) and in our sample we have

$$n = g_1 + \dots + g_K.$$

$$P(g_1, g_2, \dots, g_K) = \frac{\binom{G_1}{g_1} \binom{G_2}{g_2} \dots \binom{G_K}{g_K}}{\binom{N}{n}}$$

e.g. Class 100 students grade distribution:

A 50 students

B 30 students

C 15 students

D 5 students

You sample 10 students with without replacement (SRS)

Find $P(4A'_1, 3B'_1, 2C'_1, 1D)$ $\longrightarrow \frac{(50)(30)(15)(5)}{(100)(10)}$

\Leftrightarrow A 5 card poker hand consists of
a SRS of 5 cards from a 52 Card deck,
There are $\binom{52}{5}$ poker hands.

a) Find $P(\text{poker hand has 4 aces and a king})$

$$\frac{\binom{4}{4} \binom{4}{1} \binom{44}{0}}{\binom{52}{5}}$$

b) Find $P(\text{poker hand has 4 aces})$

$$\frac{\binom{4}{4} \binom{48}{1}}{\binom{52}{5}} = \frac{\binom{12}{1} \binom{4}{4} \cdot \binom{4}{1}}{\binom{52}{5}}$$

c) Find $P(\text{poker hand has } \underline{4 \text{ or a kind}})$

$$\frac{\binom{13}{1} \binom{4}{4} \binom{48}{1}}{\binom{52}{5}}$$

13 choices for a $\binom{52}{5}$ choices for b
 13 choices for a 12 choices for a
 13 choices for b 12 choices for b
 here a and b are different.

or

$$\frac{\binom{13}{1} \binom{12}{1} \binom{4}{4} \binom{4}{1}}{\binom{52}{5}}$$

Stat 134

Chapter 2 Wednesday February 5 2019

1. The probability of being dealt a three of a kind poker hand (ranks $aaabc$ where $a \neq b \neq c$) is:

a $\binom{4}{3} \binom{4}{1} \binom{44}{1} / \binom{52}{5}$

b $\binom{13}{1} \binom{12}{2} \binom{4}{3} \binom{4}{1} \binom{4}{1} / \binom{52}{5}$

c $\binom{13}{1} \binom{12}{1} \binom{4}{3} \binom{4}{1} \binom{44}{1} / \binom{52}{5}$

d none of the above

(b)

choose a rank for three of a kind, then
choose 2 ranks for the singles. we have
 $\binom{12}{2} = \frac{12 \cdot 11}{2}$ instead of $12 \cdot 11$ since $aaabc = aacab$
in a poker hand so we double count.

Note Not $\binom{13}{3}$ since this treats the 3 of a kind
and singles as equivalent ranks. The rank assigned to a
3-of a kind is different than the rank assigned to a single.

ex if have 8 card poker hand with two 3 of
a kind and 2 singles you would have $\binom{15}{2} \binom{11}{2}$
in front, since the two 3 of a kind are equivalent,

2. The probability of being dealt a pair in a poker hand (ranks $aabcd$ where $a \neq b \neq c \neq d$) is:

a $\binom{13}{1} \binom{12}{3} \binom{4}{2} \binom{4}{1} \binom{4}{1} / \binom{52}{5}$

b $\binom{13}{2} \binom{12}{2} \binom{4}{3} \binom{4}{1} \binom{4}{1} / \binom{52}{5}$

c $\binom{13}{1} \binom{12}{3} \binom{4}{2} \binom{4}{1} \binom{4}{1} \binom{4}{1} / \binom{52}{5}$

d none of the above

Ans

choose rank of pair
choose 3 ranks of single where order
doesn't matter,

$$\frac{\binom{13}{1} \cdot \binom{12}{3} \cdot \binom{4}{2} \binom{4}{1} \binom{4}{1} \binom{4}{1}}{\binom{52}{5}}$$

because

$$\begin{aligned} aabcd &= aqbdc = \underset{2}{\text{aa}}bcd = aqcdb \\ &= aacbdc = aqcdb \end{aligned}$$

in your poker hand.