Stat 134 lec 3

last time

Sec 1.4 A, B indep if $P(A|B) = P(A)$

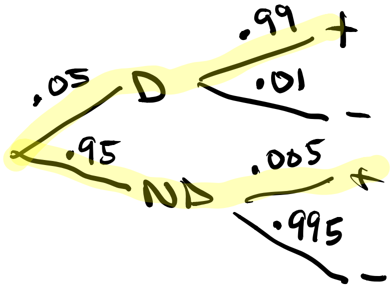
or equivalently $P(AB) = P(A)P(B)$

sec 1.5

ex 5% of the population has a disease.
(i.e $P(D) = .05$) — prior
They take a test

$P(+|D) = .99$
$P(-|ND) = .995$ } likelihoods

One person is picked at random from the population. Given test +, what is chance he has the disease? ~ $P(D|+)$



$P(+) = (.05)(.99) + (.95)(.005) = .0543$

$$\underset{\text{Posterior}}{P(D|+)} = \frac{\overset{\text{prior}}{(.05)}\overset{\text{likelihood.}}{(.99)}}{.0543} = \boxed{.9124}$$

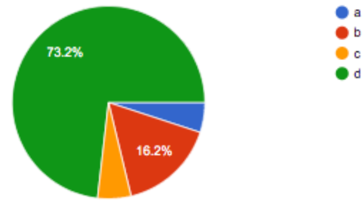
Posterior $\propto$ prior · likelihood.

$\propto$ ← proportional

Today

1) go over student responses from concept test
2) sec 1.4 mutually exclusive vs independence
3) sec 1.5 Bayes rule

1. A deck of cards is shuffled. What is the chance that the top card is the **king** of spades **or** the bottom card is the **king** of spades



a $\frac{1}{52} + \frac{1}{52} - \frac{1}{52} \times \frac{1}{52}$

b $\frac{1}{52} + \frac{1}{51}$

c $\frac{1}{52} + \frac{1}{52} - \frac{1}{52} \times \frac{1}{51}$

d none of the above

b	When you take the top card, there is a 1/52 chance of picking the K ♠. After taking the top, you would add the probability of taking the second card which is conditional that the first has already been taken (1/51). The two instances are mutually exclusive so there shouldn't be any subtraction. Since it's an "or" problem, add the two probabilities

c	The probability of the top card being king is 1/52. The probability of the bottom card being king is 1/52 as well. The probability of both cards being kings is 1/52 * 1/51. So by the inclusion exclusion principle, c is the answer

d	These events are disjoint so it's simply the sum of their individual probability

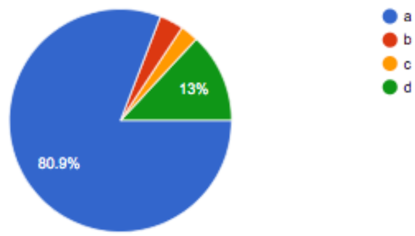
. Two separate decks of cards are shuffled. What is the chance that the top card of the first deck is the **king** of spades **or** the bottom card of the second deck is the **king** of spades

a $\frac{1}{52} + \frac{1}{52} - \frac{1}{52} \times \frac{1}{52}$

b $\frac{1}{52} + \frac{1}{51}$

c $\frac{1}{52} + \frac{1}{52} - \frac{1}{52} \times \frac{1}{51}$

d none of the above



c	The probability that the first card is king is 1/52. Bottom is 1/52 as well. We need to subtract 1/52*1/51 for inclusion exclusion

d	The events are independent so answer should be 2/52

independence implies $P(\text{top KS}, \text{bot KS}) = P(\text{top KS})P(\text{bot KS})$

a	Since they occur in 2 separate card decks, it is not longer mutually exclusive and the probability is now 1/52 + 1/52 - 1/52 * 1/52 where we subtract 1/52 * 1/52 because the events occur independently

Sec 1.4 Mutually exclusive (ME) versus Independence

let A, B be events

ME: $P(AB) = 0$

Ind: $P(A|B) = P(A)$ (conditional prob = unconditional prob)

A little set theory:

Symbol for empty set: $\emptyset$

$P(\emptyset) = 0$

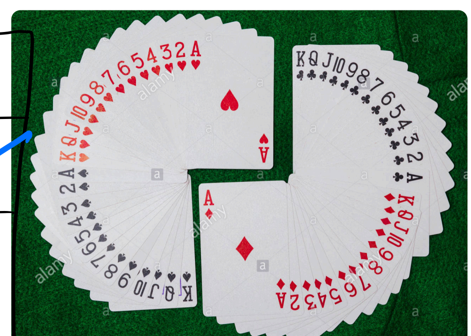
$P(\emptyset|A) = \frac{P(\emptyset, A)}{P(A)} = \frac{0}{P(A)} = 0$ (assuming $P(A) \neq 0$)

$P(A|\emptyset) = \frac{P(A, \emptyset)}{P(\emptyset)} = \frac{0}{0}$ undefined.

So we dont condition on the empty set.

ex Fill out table

A, B	ME	Ind
red, heart	$P(AB) = P(B) \neq 0$ ✗	$\underbrace{P(A\|B)}_{1} \neq \underbrace{P(A)}_{1/2}$ ✗
red, ace	$P(AB) = 2/52 \neq 0$ ✗	$\underbrace{P(A\|B)}_{1/2} = \underbrace{P(A)}_{1/2}$ ✓
red, spade	$P(AB) = 0$ ✓	$\underbrace{P(A\|B)}_{0} \neq \underbrace{P(A)}_{1/2}$ ✗
green, ace (empty set)	$P(AB) = 0$ ✓	$\underbrace{P(A\|B)}_{0} = \underbrace{P(A)}_{0}$ ✓



tinyurl.com/sept4-pt1

tinyurl.com/sept4-pt2

Stats 134

Chapter 1.4 Monday January 28 2019

1. Suppose A and B are two events with

$$P(A) = 0.5 \text{ and } P(A \cup B) = 0.8.$$

Is it possible for A and B to be both **mutually exclusive** and **independent**?

a yes

b no

c there isn't enough information to decide

if ME, $P(AB) = 0$ so $\underbrace{P(A \cup B)}_{= .8} = .5 + P(B)$

$\Rightarrow P(B) = .3$

Neither A nor B are empty so can't have ME and Ind,

2. Suppose A and B are two events with

$$P(A) = 0.8 \text{ and } P(A \cup B) = 0.8.$$

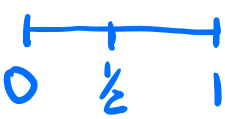
Is it possible for A and B to be both **mutually exclusive** and **independent**?

a yes

b no

c there isn't enough information to decide

Here $P(B)=0$ so possible B is null and have A, B ME and Ind.



$A = [0, \frac{1}{2}]$
$B = [\frac{1}{2}, 1]$
$AB = \{\frac{1}{2}\}$

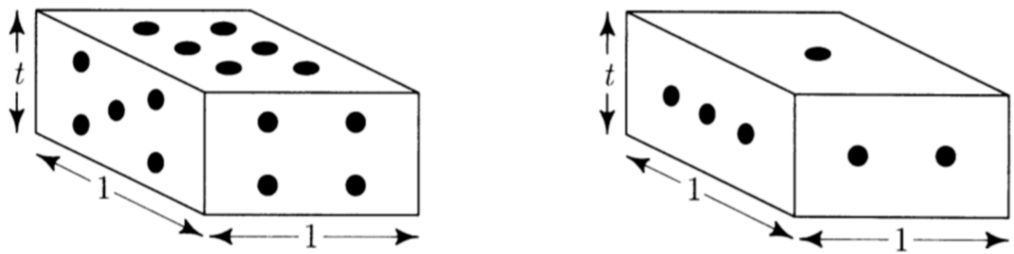
Sec 1.5

Recall from section 1.3 the example of different shaped die. We assume that it is equally likely to get a 1 or a 6.

Example 3. **Shapes.**

P 24

A *shape* is a 6-sided die with faces cut as shown in the following diagram:



ex 1.5.9 A box contains 3 shaped die, D_1, D_2, D_3 with probabilities $1/3$, $1/2$, $2/3$ respectively of landing flat (with 1 or 6 on top).

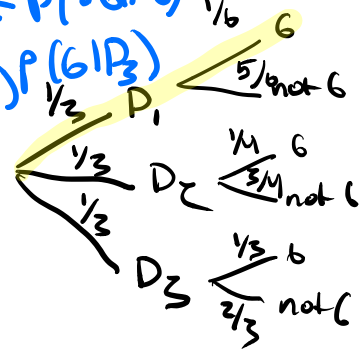
fair die ($t=1$)

a) One of the 3 shapes will be drawn at random and rolled. What is the chance the number rolled is 6?

$$P(\text{roll } 6) = P(\text{roll } 6, D_1) + P(\text{roll } 6, D_2) + P(\text{roll } 6, D_3)$$

$$= P(D_1)P(6|D_1) + P(D_2)P(6|D_2) + P(D_3)P(6|D_3)$$

$$= \tfrac{1}{3}\cdot\tfrac{1}{6} + \tfrac{1}{3}\cdot\tfrac{1}{4} + \tfrac{1}{3}\cdot\tfrac{1}{3} = .25$$



b) Given that 6 is rolled, what is the chance the fair die was chosen?

i.e find the posterior $P(D_1|6)$

$$P(D_1|6) = \frac{P(D_1, 6)}{P(6)} = \frac{\tfrac{1}{3}\cdot\tfrac{1}{6}}{.25} = .222$$

ex (Sorry not on video)

. The color of a person's eyes is determined by a single pair of genes. If they are both blue-eyed genes(bb), then the person will have blue eyes; if they are both brown-eyed genes (BB), then the person will have brown eyes; and if one is a brown-eyed gene and the other is a blue-eyed gene (Bb), then the person will have brown eyes as the brown-eyed gene is dominant over the blue-eyed gene. A newborn child independently receives one eye gene from each of its parents, and the gene it receives from a parent is equally likely to be either one of the genes of that parent. We know that Greg has brown eyes, and his dad has eye genes Bb. Given this information, what is the chance that Greg's mother has blue eyes?

M = mother bb

D = dad Bb

G = Greg brown eyes

Find $P(M \mid D, G)$

M and D are independent.

$$P(M \mid D, G) = \frac{P(M, D, G)}{P(D, G)} = \frac{P(G \mid M, D)\, P(M, D)}{P(G \mid D)\, \cancel{P(D)}}$$

$$P(M, D) = P(M)\cancel{P(D)}$$

$P(M) = 1/4$

we assume genotypes bb, Bb, bB, BB have equal frequency in the population. This follows from the highlighted text

$P(G \mid \overset{bb}{M}, \overset{Bb}{D}) = 1/2$

For Greg to have Brown eyes the dad must give Greg the B gene, which has a 50% chance.

Find $P(G \mid D)$

we will discuss at the start of next class.