

- Today:
- 1) announcements
 - 2) introduction
 - 3) Sec 1.1 Equally likely outcomes
 - 4) Sec 1.2 Interpretations
 - 5) Sec 1.3 Distributions

Announcements:

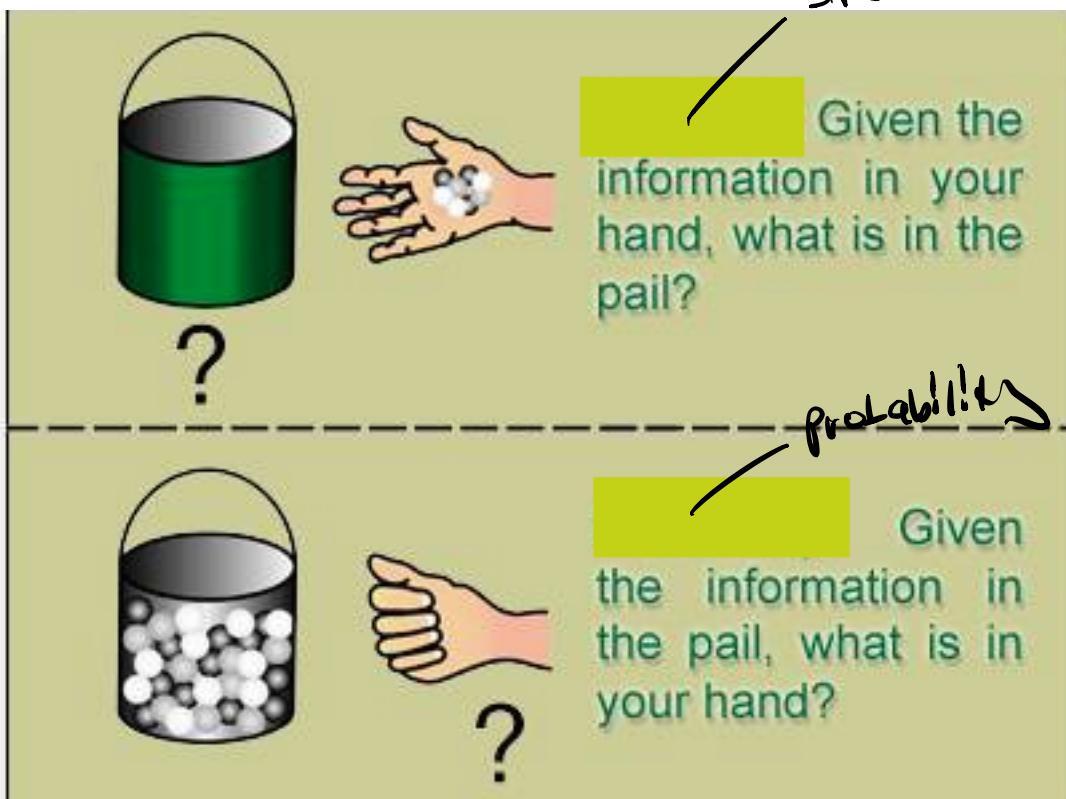
- Lab today !!
- Class website www.stat134.org
- Previous student advice to you:

"More sure you understand everything before even going to lecture. It helps with getting new concepts"

- See schedule on website.
- My OT is after class in SLC.
- The adjacent class 198 starts tomorrow. Contact Mike Leong mleong@berkeley.edu

Introduction

One of these pictures describes probability, which one?



Sec 1.1 Equally Likely Outcomes

If all outcomes of a finite set Ω are equally likely, the probability of an event A is the number of outcomes in A divided by the total number of outcomes,

$$P(A) = \frac{\#A}{\#\Omega}$$

Ex Imagine you have a 3 card deck J, Q, K

What is the probability that the 2nd card is a Q?

A = 2nd card is Q

Ω = 2nd card is J, Q, K

$$\#A = 1$$

$$\#\Omega = 3$$

$$P(A) = \frac{1}{3}.$$

Deck of cards: 4 suits H, C, D, S

13 ranks Ace, 2-10, J, Q, K
52 cards.

Ex - #5e p 10

Suppose a deck of cards is shuffled and the top 2 cards are dealt,

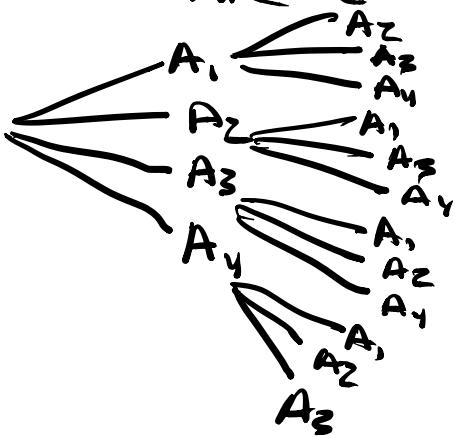
What is chance you get at least one ace among the 2 cards?

$$P(A) = \frac{4}{52} \cdot \frac{3}{51} + \frac{48}{52} \cdot \frac{4}{51} = \frac{48}{52} \cdot \frac{4}{51}$$

$$A = \left\{ \begin{array}{l} (\text{ace, ace}), (\text{ace, nonace}), (\text{nonace, ace}) \end{array} \right\}$$

$$\Omega = \left\{ \begin{array}{l} (\text{*}, \text{*}) \\ \text{52.51} \end{array} \right\}$$

To see why the chance of getting (ace, ace) is $\frac{4}{52} \cdot \frac{3}{51}$, you can make a tree diagram:



There are 12 branches corresponding to each pair of aces, so the event (ace, ace) has size 12. There are 52.51 pairs of possible cards.

$$\#A = 4(3 + 48 + 48) = 4.99$$

$$\#\Omega = 52.51$$

$$P(A) = \frac{4.99}{52.51} = .149$$

$$1 - P(A^c) = 1 - \frac{48.47}{52.51} = .149$$

Sec 1.2 Interpretations

frequency interpretation

$P(A)$ is the long run relative frequency of A to Ω .

(i.e if you perform the experiment 1000 independent times to 2nd Card will be queen roughly 333 times out of 1000).

subjective interpretation

e.g the probability of a patient patient surviving an operation.

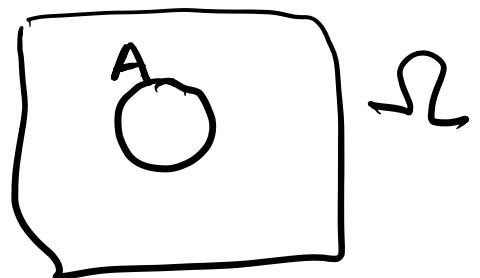
Your probabilistic opinion may change over time as you acquire new data.

Sec 1.3 Distributions

To define probability we start with an outcome space Ω and assign to each element a nonneg number and require that all numbers add up to 1.

Axioms

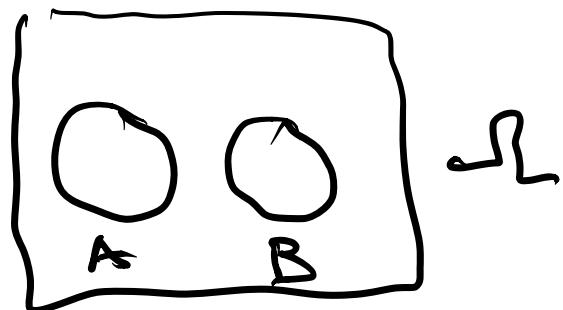
1) $P(A) \geq 0$ for all $A \subseteq \Omega$



2) $P(\Omega) = 1$

3) If A and B are mutually exclusive sets then $P(A \cup B) = P(A) + P(B)$

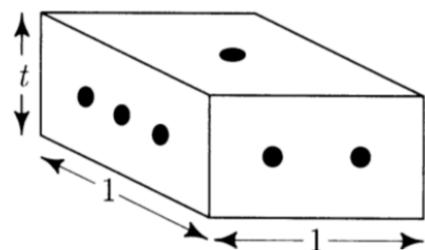
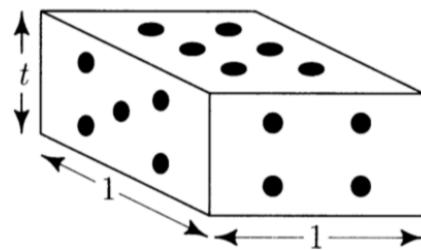
(addition rule)



Example 3. Shapes.

P 24

A *shape* is a 6-sided die with faces cut as shown in the following diagram:



Suppose thickness of the die, t , is such that the chance of landing face (i.e. 1 or 6) is $2/3$.

Find the prob distributions of the shape.
Draw a histogram.

$$P(1) = \frac{1}{3}$$

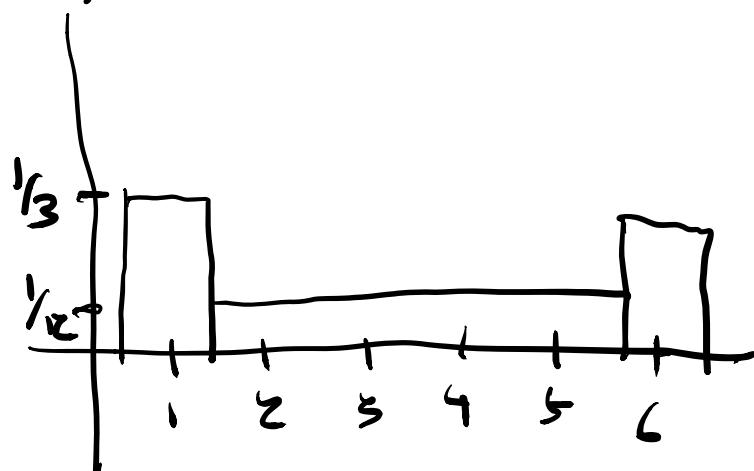
$$P(2) = \frac{1}{12}$$

$$P(3) = \frac{1}{12}$$

$$P(4) = \frac{1}{12}$$

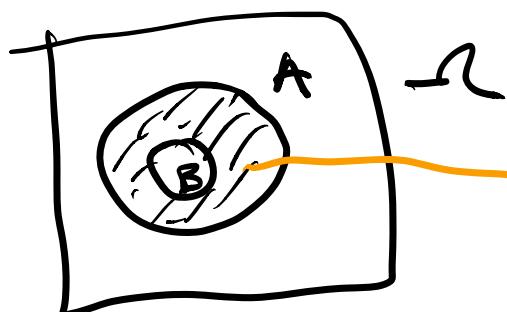
$$P(5) = \frac{1}{12}$$

$$P(6) = \frac{1}{3}$$



Difference rule

Suppose $B \subseteq A$



$A \setminus B$ or AB^c

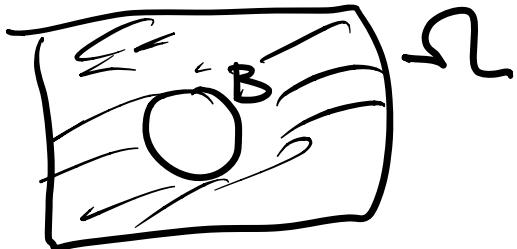
$$P(A) - P(B)$$

Give formula for $P(A \setminus B)$

$A = B \cup (A \setminus B)$ disjoint union

$P(A) = P(B) + P(A \setminus B)$ addn rule,

Complement rule



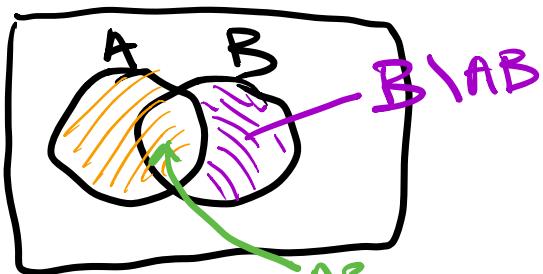
$$P(B^c) = 1 - P(B)$$

let $A = \mathcal{U}$ apply diff formula

$$\begin{aligned} B^c &= \mathcal{U} \setminus B \\ \Rightarrow P(B^c) &= P(\mathcal{U}) - P(B) \end{aligned}$$

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Inclusion Exclusion Rule



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$A \cup B = A \cup (B \setminus A \cap B)$ disjoint
use addn rule,

Boole's inequality

$$P\left(\bigcup_{i=1}^n A_i^c\right) \leq \sum_{i=1}^n P(A_i^c)$$

For example for $n=3$ this says

$$P(A_1 \cup A_2 \cup A_3) \leq P(A_1) + P(A_2) + P(A_3).$$

We need to show this for any n .

Prove by induction:

verifying for $n=1$

assume true for $n=m$

Show true for $n=m+1$.

to be continued next time ..