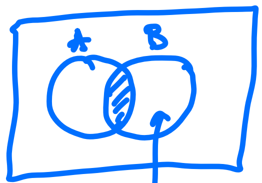
Stat 134 lec 4

warmup: 10:00 - 10:10

2. **(5 pts)** Find whether or not each of the following three expressions is equivalent to $P(A|B^c)$ and detail your solution.

- $\cancel{(a)}$ $1 - P(A|B) = P(A^c|B)$
- (b) ✓ $\frac{(1-P(B|A))P(A)}{1-P(B)} = \frac{P(B^c A)}{P(B^c)} = P(A|B^c)$
- $\cancel{(c)}$ $\frac{P(B|A^c)P(A^c)}{P(B)} = \frac{P(BA^c)}{P(B)} = P(A^c|B)$



$1 - P(A|B) = P(A^c|B)$

$P(B^c|A)P(A) = P(B^c A)$

recall multiplication rule

$P(XY) = P(X|Y)P(Y)$

Announcement: We resume in person instruction next Monday.

Last time

If A and B are indep then so is A, B^c, and A^c, B and A^c, B^c.

sec 1.5 Bayes rule

There are two types of conditional probabilities:

ex

$P(7 \mid \text{drew } 2)$ is forward conditional (likelihood conditional)
DON'T NEED BAYES TO COMPUTE

$P(\text{drew } 2 \mid 7)$ is backwards conditional (posterior conditional)
NEED BAYES TO COMPUTE

Today

① sec 1.6 independence of 3 or more events
② sec 2.1 Binomial Distribution

sec 1.6 Independence of 3 events

Def^n (pairwise independence of 3 events)

A, B, C are pairwise independent if

$P(AB)=P(A)P(B)$ and $P(AC)=P(A)P(C)$ and $P(BC)=P(B)P(C)$

ex

> One ball is drawn randomly from a bowl containing four balls numbered 1, 2, 3, and 4. Define the following three events:
>
> - Let A be the event that a 1 or 2 is drawn. That is, $A=\{1,2\}$.
> - Let B be the event that a 1 or 3 is drawn. That is, $B=\{1,3\}$.
> - Let C be the event that a 1 or 4 is drawn. That is, $C=\{1,4\}$.

Is A, B, C pairwise independent?

$\underbrace{P(AB)}_{1/4} \stackrel{?}{=} \underbrace{P(A)}_{1/2}\underbrace{P(B)}_{1/2}$ ✓ Yes

similarly $P(AC)=P(A)P(C)=1/4$

$P(BC)=P(B)P(C)=1/4$

Is $P(ABC)=P(A)P(B)P(C)$

NO $P(ABC)=1/4 \neq \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$

Def^n (mutual independence of 3 events)

A, B, C are mutually independent if

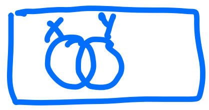
$P(ABC)=P(A)P(B)P(C)$, (and the same for any of the events replaced by its complement)

$P(A^cBC)=P(A^c)P(B)P(C)$

We require showing 8 equations is true for mutual independence. This is a strong condition.

Thm Suppose A, B, C are mutually independent. Then they are also pairwise independent.

$P(x) = P(xy) + P(xy^c)$



Pf/ We can write

$$P(AB) = P(ABC) + P(ABC^c)$$

addⁿ rule

$$= P(A)P(B)P(C) + P(A)P(B)P(C^c)$$

$$= P(A)P(B)\left[\underbrace{P(C) + P(C^c)}_{=1}\right]$$

$$= P(A)P(B).$$

similar for the other cases eg $P(AC) = P(A)P(C)$

$\square$

Note that $P(ABC) = P(A)P(B)P(C)$ by itself doesn't imply pairwise independence:

ex let $\Omega = \{1,2,3,4,5,6,7,8\}$

$A = B = \{1,2,3,4\}$ $\qquad ABC = \{1\}$

$C = \{1,5,6,7\}$

Is $\underbrace{P(ABC)}_{= 1/8} = \underbrace{P(A)}_{= 1/2}\underbrace{P(B)}_{= 1/2}\underbrace{P(C)}_{= 1/2}$? ✓ True

Is A, B, C pairwise indep?

No $\underbrace{P(AB)}_{= 1/2} \neq \underbrace{P(A)}_{= 1/2}\underbrace{P(B)}_{= 1/2}$

Thm
A, B, C are mutually independent iff
1) A, B, C are pairwise indep
2) $P(ABC) = P(A)P(B)P(C)$.

Sketch

Suppose (1) and (2) hold,

let's show $P(ABC^c) = P(A)P(B)P(C^c)$

write

$$P(ABC^c) = \underbrace{P(ABC^c) + P(ABC)}_{\substack{\| \\ P(AB) \\ \| \\ P(A)P(B)}} - \underbrace{P(ABC)}_{\substack{\| \\ P(A)P(B)P(C)}}$$

$$= P(A)P(B)\left[1 - P(C)\right]$$

$$= P(A)P(B)P(C^c)$$

similar for other cases

$\square$

(2) <u>Sec 2.1</u> Binomial distribution.

<u>Bernoulli(p) trial (distribution)</u>

two outcomes — success — p
— failure — 1-p

ex roll a die.
success is getting a 6 — 1/6
failure is not getting a 6 — 5/6

<u>Binomial (n,p) distribution (Bin (n,p))</u>

We have n independent Bernoulli(p) trials
n: fixed
p: fixed (unconditional probability)

ex we roll a die n times.

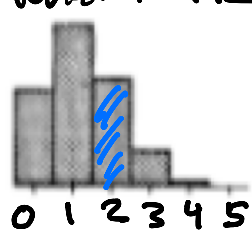
What are the possible number of successes? — 0, 1, 2, ..., n

In Bin(n,p) the chance of having k successes ($0 \le k \le n$) is given by the Binomial formula:

$$P(k) = \frac{n!}{k!\,(n-k)!} p^{k} (1-p)^{n-k}$$

n: # trials
p: chance of success.
k: number of successes

ex You roll a die 5 times. What is the chance of getting 2 sixes?
n = ? 5
k = ? 2
p = ? 1/6



$$P(2) = \frac{5!}{2!\,3!}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 = \frac{5\cdot 4\cdot \cancel{3\cdot 2\cdot 1}}{2\cdot 1\cdot \cancel{3\cdot 2\cdot 1}}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3$$

$$= \boxed{10\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3}$$

What is chance of getting

_ _ _ _ _

success (6)

failure (not 6)

How many of these are there?

$$\left\{\begin{array}{cccccc} 1 & 1 & 0 & 0 & 0 & ? \\ 0 & 1 & 1 & 0 & 0 & ? \\ & \vdots & & & & \\ & \vdots & & & & \end{array}\right.$$

5 positions for first 1

4 position for 2nd 1

3 positions for 1st 0

2 " " 2nd 0

1 " " 3rd 0

$$\frac{5!}{2!\,3!}$$

5 choose 2

We write $\frac{5!}{2!\,3!}$ as $\binom{5}{2}$ or $\binom{5}{3}$ or $\binom{5}{2,3}$

$$\binom{5}{3} = \frac{5!}{3!\,2!}$$

success failure

. A well shuffled deck is cut in half so there are 7 ~~aces~~ diamond in the first half deck and 6 ~~aces~~ diamond in the second half deck. Five cards are dealt off the top of one half deck and five cards are dealt off the top of the other half deck. The binominal formula doesn't apply to find the chance of getting exactly three diamonds total because:

a The probability of a trial being successful changes Prob of drawing a diamond changes 7/26 first half deck, 8/26 second half deck

b The trials aren't independent

we draw without replacement

c There isn't a fixed number of trials

n = 10

d more than one of the above

13/52 chance 1st card is diamond

13/52 chance 2nd card is diamond.

chance 1st and 2nd card diamond

$\frac{13}{52} \cdot \frac{12}{51}$

ex

1. Ten cards are dealt off the top of a well shuffled deck. The binominal formula doesn't apply to find the chance of getting exactly three diamonds because:

   a The probability of a trial being successful changes

   b The trials aren't independent

   c There isn't a fixed number of trials

   d more than one of the above