Stat 134 lec 4

Quiz 1 next Wed on Chap 1

Last time

Sec 1.4 independence

Note that if $P(AB)=P(A)P(B)$ then $P(A^cB)=P(A^c)P(B)$

since,

$\Omega B = B$

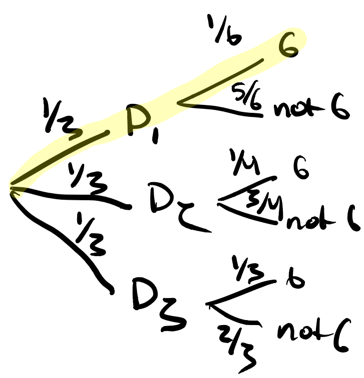
Prob of difference.

$$P(A^cB) = P((\Omega \setminus A)B) = P(B \setminus AB) = P(B) - P(AB)$$
$$= P(B) - P(A)P(B) = P(B)[1-P(A)] = P(B)P(A^c)$$
$$= P(A^c)P(B)$$

Sec 1.5 Baye's rule

Forward vs. backwards conditional

ex A box contains 3 shaped die D_1, D_2, D_3 with Prob $1/3$, $1/2$, $2/3$ of landing flat (1 or 6).



$P(6|D_1)$ is forward conditional (likelihood)

DON'T NEED BAYES TO COMPUTE

$P(D_1|6)$ is backwards conditional (Posterior)

NEED BAYES TO COMPUTE

Today

1. Go over student responses concept test
2. Sec 1.6 independence of 3 or more events
3. Sec 3.1 Binomial Distribution
4. Sec 3.1 Consecutive odds ratio

(1) Student responses to concept test.

.. Suppose A and B are two events with

$$P(A) = 0.5 \text{ and } P(A \cup B) = 0.8.$$

Is it possible for A and B to be both **mutually exclusive** and **independent**?

a yes

b no

c there isn't enough information to decide

c	d	¯_(ツ)_/¯

b	b	Mike said it's b

b	b	If the events are both mutually exclusive and independent, that means that P(A and B) = 0 which implies that P(A) = 0.5 and P(B) = 0.3. Since A and B are independent, P(A and B) = P(A) * P(B) = 0.5*0.3 which is nonzero. This contradicts the assumption that A and B are mutually exclusive.
b	b	A or B needs to be impossible to be both ME and independent.
b	b	cannot be mutually exclusive and independent if they are possible and the overlap provides that it is possible
b	b	A or B can happen. It is not impossible
b	b	The only way you can get a & b to be ME and IND is for a or b to be impossible - as the lec notes state

sec 1.6 Independence of 3 events

A, B, C are pairwise independent if

$P(AB) = P(A)P(B)$ and $P(AC) = P(A)P(C)$ and $P(BC) = P(B)P(C)$

ex 3 people

B_{ij} = the event that person i and j have the same B-day.

B_{12}, B_{13}, B_{23} are pairwise independent

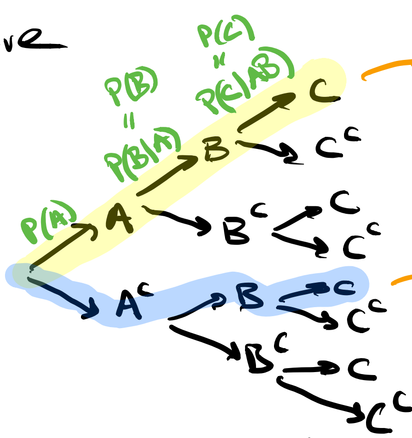
$$\underbrace{P(B_{12}B_{13})}_{=\frac{365}{365}\frac{1}{365}\frac{1}{365}} \overset{?}{=} \underbrace{P(B_{12})}_{=\frac{1}{365}}\underbrace{P(B_{23})}_{=\frac{1}{365}} \checkmark$$

A, B, C are independent if

1) A, B, C are pairwise indep

2) $P(ABC) = P(A)P(B)P(C)$. (and the same for any of the events replaced by its complement)

Picture



— $P(ABC) = P(A)P(B)P(C)$

— $P(A^cBC) = P(A^c)P(B)P(C)$

Note that we need $P(C|AB) = P(C)$ for independence but this isn't given by pairwise indep of A, B, C !!

ex

Is B_{12}, B_{13}, B_{23} independent?

$$P(B_{12}B_{13}B_{23}) \stackrel{?}{=} P(B_{12})P(B_{13})P(B_{23}) = \left(\frac{1}{365}\right)^3$$

$$P(B_{12}B_{23}B_{23}) = P(B_{12}B_{13}) = \left(\frac{1}{365}\right)^2$$

$$P(B_{12}) = \frac{1}{365}, \quad P(B_{13}) = \frac{1}{365}, \quad P(B_{23}) = \frac{1}{365}$$

No!

so just because you have pairwise indep doesnt mean you have independence.

③ <u>Sec 2.1</u> Binomial distribution.

<u>Bernoulli(p) trial (distribution)</u>

two outcomes < success — p
failure — 1−p

ex roll a die.
success is getting a 6 — 1/6
failure is not getting a 6 — 5/6

<u>Binomial (n,p) distribution (Bin (n,p))</u>

We have n independent Bernoulli (p) trials

n: fixed

p: fixed (unconditional probability)

ex we roll a die n times,

What are the possible number of successes?

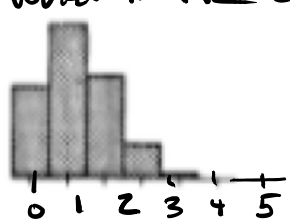
The chance of having each of these number of successes is called the Bin (n, p) distribution

Binomial formula:

$$\underbrace{P(k)}_{\text{number of successes}} = \frac{\overbrace{n}^{\#\text{ trials}}!}{k!\,(n-k)!}\,\underbrace{p}_{\text{chance of success.}}{}^{k}\,(1-p)^{n-k}$$

ex You roll a die 5 times. What is the chance of getting 2 sixes

n = ? 5
k = ? 2
= ? 1



$$P(2) = \frac{5!}{2!3!}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 = \frac{5\cdot 4\cdot \cancel{3\cdot 2\cdot 1}}{2\cdot 1\cdot \cancel{3\cdot 2\cdot 1}}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3$$

$$= \boxed{10\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3}$$

What is chance of getting

success (6)

failure (not 6)

How many of these are there?

1 1 0 0 0 ? — $\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3$

0 1 1 0 0 ? ← $\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 = \left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3$

⋮

$\frac{5!}{2!3!}$ ← there are 5 positions for the 1^{st} 1, 4 positions for the 2^{nd} 1, 3 positions for the 1^{st} 0, etc.

11000 is counted 2!3! times

01100 is counted 2!3! times

We write $\frac{5!}{2!3!}$ as $\binom{5}{2}$ or $\binom{5}{3}$ or $\binom{5}{2,3}$

$$\binom{5}{3} = \frac{5!}{3!2!}$$

Stats 134

Chapter 2 Wednesday January 30 2019

1. Ten cards are dealt off the top of a well shuffled deck. The binominal formula doesn't apply to find the chance of getting exactly three diamonds because:

   a The probability of a trial being successful changes

   b The trials aren't independent

   c There isn't a fixed number of trials

   d more than one of the above

The (unconditional) probability of getting an ace is always $1/13$. If we were drawing from two decks of cards having different number of aces then (a) would be false

The number of trials is 10.

2. Is the binomial formula applicable to find the chance that the sum of draws is **3** while drawing **5 times with replacement** from a box with 9 tickets marked **0** and one ticket marked **1**?

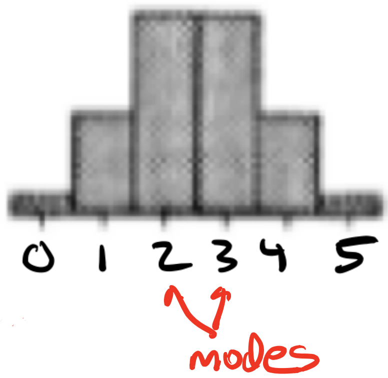
a yes

b no

This satisfies requirement for the binomial formula since we have 5 independent Bernoulli (P) trials with $p = 1/10$ and $k = 3$ since the number of successes is the sum of our draw, ④ <u>Consecutive odds ratio</u> — a tool to find the mode of the binomial distribution.

<u>mode</u>

The mode of the binomial distribution is the k such that $P(k) = \binom{n}{k} p^k (1-p)^{n-k}$ is largest



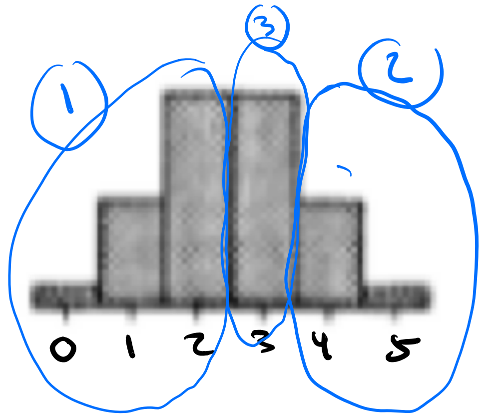
here $n=5$
$k = 0, 1, 2, 3, 4, 5$
$p = \frac{1}{2}$

$m = \lfloor np + p \rfloor$ is an important number
floor (integer part)

here $m = \lfloor 5 \cdot \frac{1}{2} + \frac{1}{2} \rfloor = \lfloor 3 \rfloor = 3$

$$\text{mode} = \begin{cases} m & \text{if } np+p \notin \mathbb{Z} \\ m, m-1 & \text{if } np+p \in \mathbb{Z} \end{cases}$$

$np + p = 3$



notice

(1) $\overset{0,1,2}{K} < \overbrace{np+p}^{=3}$ when $P(K-1) < P(K)$

(2) $\overset{4,5}{K} > np+p$ when $P(K-1) > P(K)$

(3) $\overset{3}{K} = np+p$ when $P(K-1) = P(K)$