Stat 134 lec 1

Today:
1) announcements
2) introduction
3) sec 1.1 Equally likely outcomes
4) sec 1.2 Interpretations
5) Sec 1.3 Distributions.

Announcements:

- Lab today !!
- Class website www.stat134.org
- Previous student advice to you:

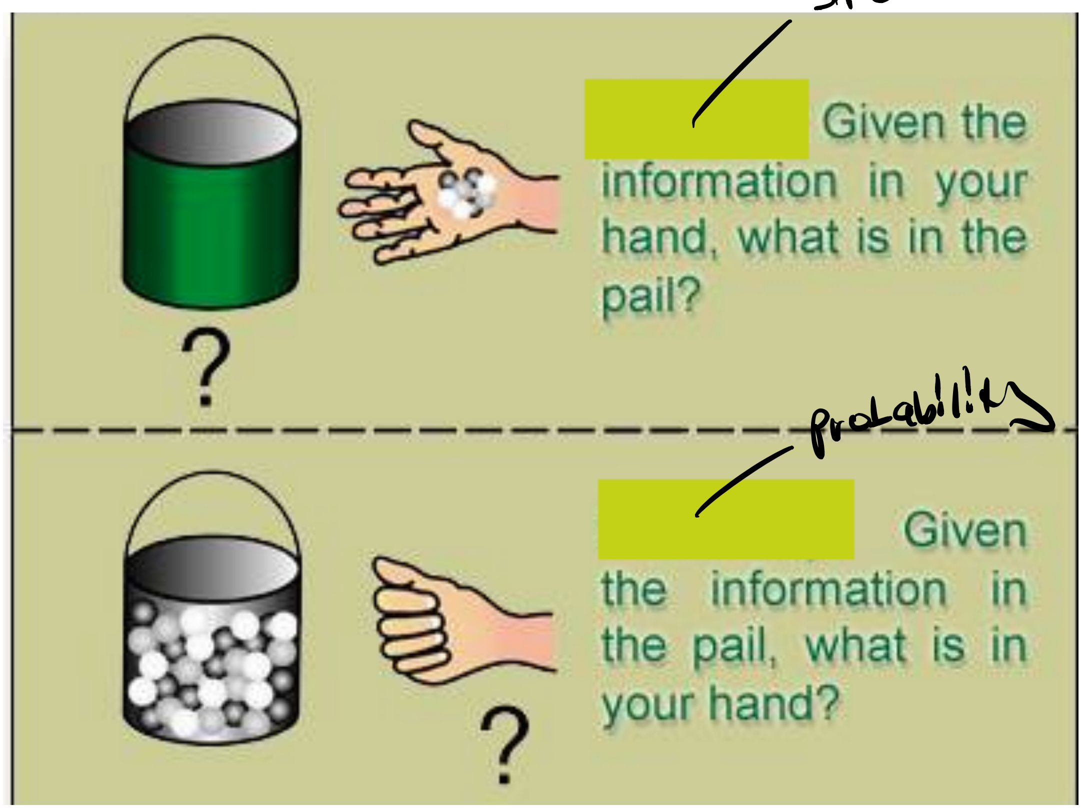
"Make sure you understand everything before even going to lecture. It helps with getting new concepts"

- See schedule on website.
- My OH is after class in SLC.
- The adjunct class 198 starts tomorrow. Contact Mike Leong mleong@berkeley.edu

Introduction

One of these pictures describes probability, which one?

statistics = inverse probability



Sec 1.1 Equally Likely Outcomes

If all outcomes of a finite set Ω are equally likely, the probability of an event A is the number of outcomes in A divided by the total number of outcomes,

$$\boxed{P(A) = \frac{\#A}{\#\Omega}}$$

Ex Imagine you have a 3 card deck J, Q, K

What is the probability that the 2nd card is a Q?

A = 2nd card is Q

Ω = 2nd card is J, Q, K

$\#A = 1$

$\#\Omega = 3$

$P(A) = 1/3$.

Deck of cards: 4 suits H, C, D, S

13 ranks Ace, 2-10, J, Q, K

52 cards.

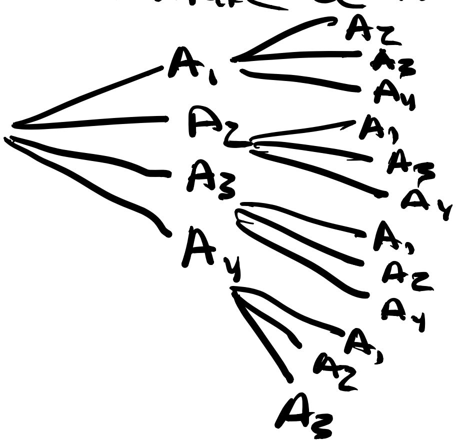
ex — #5e p10

Suppose a deck of cards is shuffled and the top 2 cards are dealt. What is chance you get at least one ace among the 2 cards?

$$A = \{ \overset{4\cdot 3}{(\text{ace}, \text{ace})}, \overset{4\cdot 48}{(\text{ace}, \text{nonace})}, \overset{48\cdot 4}{(\text{nonace}, \text{ace})} \}$$

$$\Omega = \{ \underset{52\cdot 51}{(*, *)} \}$$

To see why the chance of getting (ace, ace) is $\frac{4\cdot 3}{52\cdot 51}$ you can make a tree diagram:



There are 12 branches corresponding to each pair of aces, so the event (ace, ace) has size 12. There are $52\cdot 51$ pairs of possible cards.

$$\#A = 4(3+48+48) = 4\cdot 99$$

$$\#\Omega = 52\cdot 51$$

$$P(A) = \frac{4\cdot 99}{52\cdot 51} = .149$$

or

$$1 - P(A^c) = 1 - \frac{48\cdot 47}{52\cdot 51} = .149$$

Sec 1.2 Interpretations

frequency interpretation

$P(A)$ is the long run relative frequency of A to Ω.

(i.e. if you perform the 1st example 1000 independent times the 2nd card will be queen roughly 333 times out of 1000).

subjective interpretation

ex the probability of a particular patient surviving an operation.

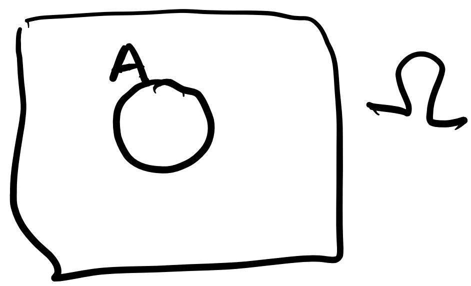
Your probabilistic opinion may change over time as you acquire new data.

Sec 1.3 Distributions

To define probability we start with an <u>outcome space</u> Ω and assign to each element a nonneg number and require that all numbers add up to 1.

<u>Axioms</u>

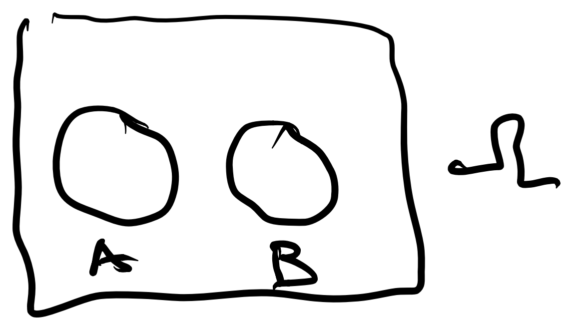
1) $P(A) \geq 0$ for all $A \subseteq \Omega$



2) $P(\Omega) = 1$

3) If A and B are mutually exclusive sets then $P(A \cup B) = P(A) + P(B)$

(addition rule)



Example 3. **Shapes.**

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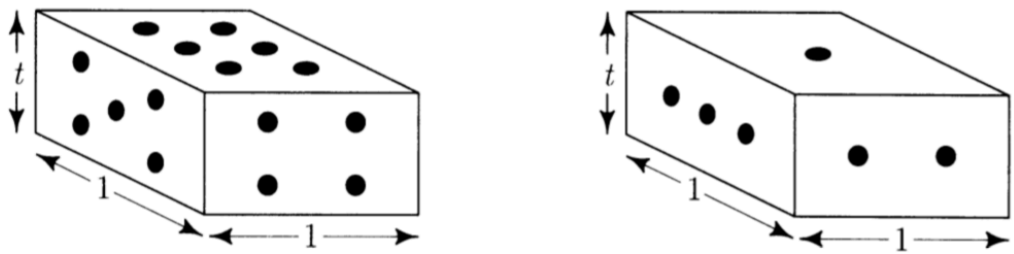
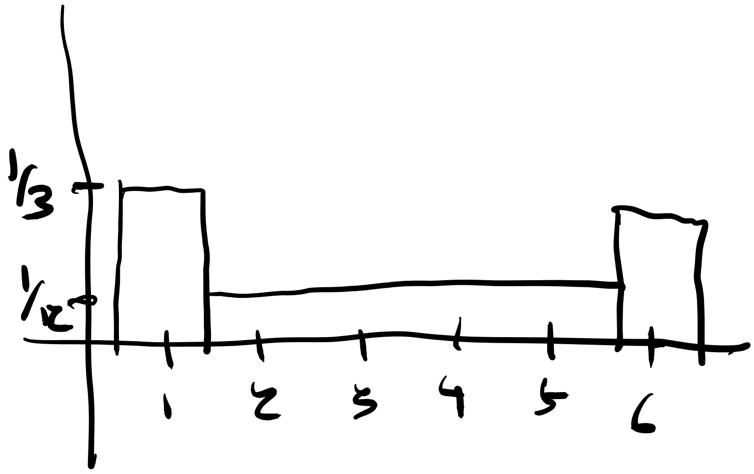
A *shape* is a 6-sided die with faces cut as shown in the following diagram:



Suppose thickness of the die, t, is such that the chance of landing flat (i.e. 1 or 6) is $2/3$.

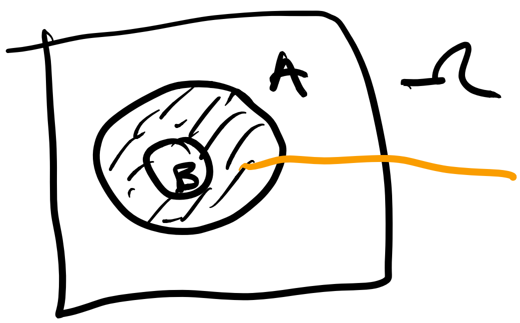
Find the prob distribution of the shape. Draw a histogram.

$P(1) = 1/3$
$P(2) = 1/12$
$P(3) = 1/12$
$P(4) = 1/12$
$P(5) = 1/12$
$P(6) = 1/3$



Difference rule

Suppose $B \subseteq A$



$A \setminus B$ or AB^c

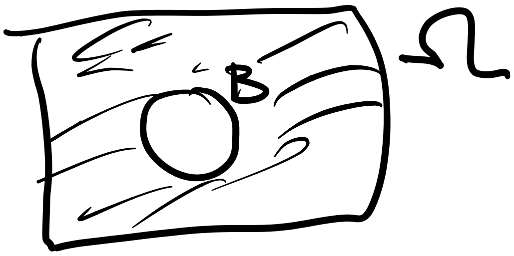
$\boxed{P(A) - P(B)}$

Give formula for $P(A \setminus B)$

$A = B \cup (A \setminus B)$ disjoint union

$P(A) = P(B) + P(A \setminus B)$ addition rule.

Complement rule



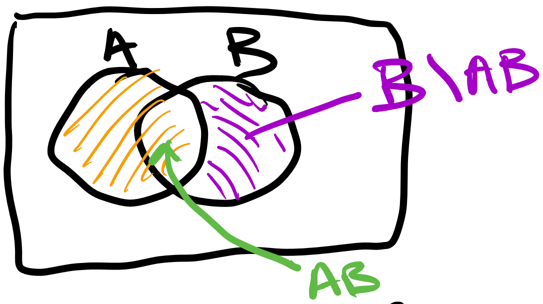
$$P(B^c) = 1 - P(B)$$

let $A = \Omega$ apply diff formula

$$B^c = \Omega \setminus B$$

$$\Rightarrow P(B^c) = \underbrace{P(\Omega)}_{=1} - P(B)$$

Inclusion Exclusion Rule



$$P(A \cup B) = P(A) + P(B) - P(AB)$$

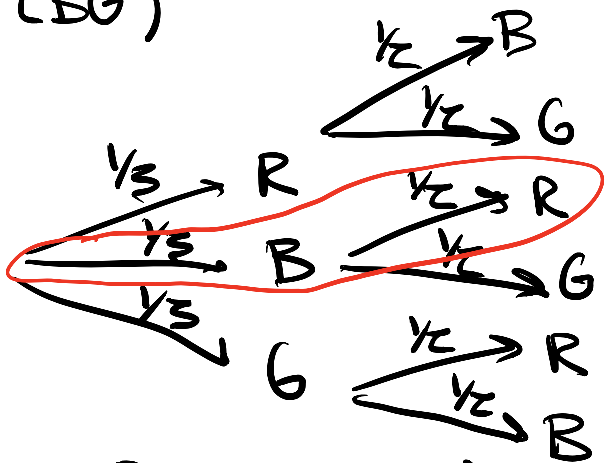
$$A \cup B = A \cup (B \setminus AB)$$ disjoint

use addn rule,

multiplication rule

draw 2 tickets without replacement.

Find $P(BG)$



$$P(BR) = P(B^{1st}) \cdot P(R^{2nd} \mid B^{1st})$$
$$= \frac{1}{3} \cdot \frac{1}{2} = 1/6$$

multiplication rule $\boxed{P(AB) = P(A)\,P(B \mid A)}$

division rule $\boxed{P(B \mid A) = \frac{P(AB)}{P(A)}}$

Boole's inequality

$$P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$$

For example for $n=3$ this says

$$P(A_1 \cup A_2 \cup A_3) \leq P(A_1) + P(A_2) + P(A_3).$$

We need to show this for any n.

Prove by induction:

verify for $n=1$
assume true for $n=m$
show true for $n=m+1$.

to be continued next time...