Stat 134 lec 8

Quiz 2 Sec 2.1, 2.2, 2.4, and 2.5 (fairly standard problem)

last time sec 2.5 hypergeometric distribution

Suppose a population of size N contains G good and B bad elements ($N=G+B$). A sample, size n, with g good and b bad elements ($n=g+b$) is chosen at random without replacement.

$$P(g \text{ good and } b \text{ bad}) = \frac{\binom{G}{g}\binom{B}{b}}{\binom{N}{n}}$$

Parameters: N = population size
G = # Good in population
n = sample size.

abrev. Hyper(N, G, n)

ex P(poker hand is 4 of a kind) rank aaaab $a \neq b$

N = number of cards = 52
G = number of rank a cards in deck = 4
n = sample size = 5

$$\underbrace{\binom{13}{1}}_{\substack{\text{choose}\\ 1 \text{ rank for}\\ a}} \cdot \frac{\overset{a}{\binom{4}{4}}\overset{\text{not } a}{\binom{48}{1}}}{\underbrace{\binom{52}{5}}_{\#\text{ poker hands}}} = \binom{13}{1}\binom{12}{1} \cdot \frac{\binom{4}{4}\binom{4}{1}}{\binom{52}{5}}$$

Not $\binom{13}{2}$ since aaaab $\neq$ bbbba

we saw last time this is equal to $13 \cdot 12 \cdot \underset{\text{diff places for a}}{\binom{5}{4}} \cdot \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot \frac{1}{49} \cdot \frac{4}{48}$

Today

① review student concept test responses.

② almost finish chap 2 — more hypergeometric / counting

On Monday we will quickly cover binomial approx to hypergeometric.

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1. The probability of being dealt a three of a kind poker hand (ranks $aaabc$ where $a \neq b \neq c$) is:

a $\overset{a}{\binom{4}{3}}\overset{b}{\binom{4}{1}}\overset{\text{not } a \text{ or } b}{\binom{44}{1}} / \binom{52}{5}$

b $\binom{13}{1}\binom{12}{2}\overset{a}{\binom{4}{3}}\overset{b}{\binom{4}{1}}\overset{c}{\binom{4}{1}} / \binom{52}{5}$

c $\binom{13}{1}\binom{12}{1}\overset{a}{\binom{4}{3}}\overset{b}{\binom{4}{1}}\overset{\text{not } a \text{ or } b}{\binom{44}{1}} / \binom{52}{5}$

d none of the above

c	There's 13 options for a, 12 for b, and then there's 44 cards left over to choose from

44 cards not rank a or b, however overcount by a factor of 2.
aaabc = aaacb

a	First (4,3) to get 3 of a kind. Then multiply by (4,1) to get a random fourth card. Then multiply by (44,1) to pick a fifth card from the remaining 44 that will not contain the former two numbers.

need to choose ranks a, b

b	The trick is recognizing that 13*12 isn't correct because order does not matter. I usually break this up into choosing rank then card.

note: not $\binom{13}{3}$ since

aaabc ≠ bbbac etc.

② Practice hypergeometric

You and a friend are playing poker. If each of you are dealt 5 cards from the same deck, what is the chance that you both get a 4 of a kind (ranks aaaab, $a \neq b$)

aaaab		cccc (b or d)
friend gets 4 of a kind.	AND	you get 4 of a kind

$$\frac{\binom{13}{1}\binom{12}{1}\binom{4}{4}\binom{4}{1}}{\binom{52}{5}} \cdot \frac{\binom{11}{1}\binom{4}{4}\left[\binom{3}{1}+\binom{10}{1}\binom{4}{1}\right]}{\binom{47}{5}}$$

You can rewrite this as

$$\boxed{\frac{\binom{13}{1}\binom{4}{4}\binom{48}{1}}{\binom{52}{5}} \cdot \frac{\binom{11}{1}\binom{4}{4}\binom{43}{1}}{\binom{47}{5}}}$$

Counting strategy:

Try and break the problem into a sequence of steps and apply the multiplication rule. Above we find the chance your friend gets 4 of a kind and the probability you get 4 of a kind given that your friend got 4 of a kind.

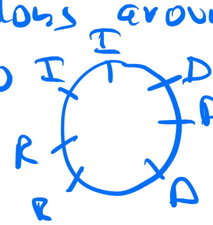
ex There are 3 Democrats, 2 Republicans, and 2 Independents sitting around a table. What is the chance the Dem sit together, the Rep sit together and the Ind sit together?

soln 1

There are $\binom{7}{3,2,2}$ orderings of DDDRRII.

Hence the probability of getting DDDRRII is $\dfrac{1}{\binom{7}{3,2,2}}$.

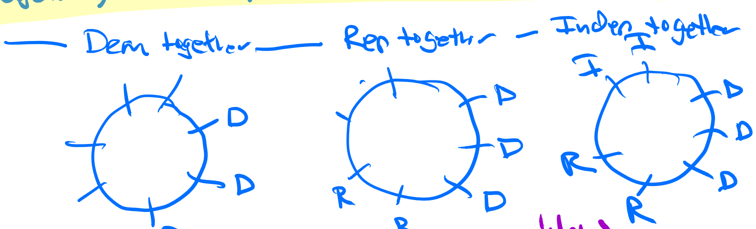
There are 7 positions around the table the first D can go



so the chance that we have DDDRRII sitting around the table is $\dfrac{7}{\binom{7}{3,2,2}}$. But DDDIIRR is also an acceptable ordering so the answer is $\boxed{\dfrac{7\cdot 2}{\binom{7}{3,2,2}}}$

soln 2 (notice how we break up the problem)

First find chance Dem sit together, then Rep join table and sit together, then indep join table and sit together.



$$\frac{7}{\binom{7}{3,4}} \cdot \frac{2}{\binom{4}{2}} \cdot \frac{1}{\binom{2}{2}} = \boxed{\frac{7\cdot 2}{\binom{7}{3,4}\binom{4}{2}\binom{2}{2}}}$$

7 positions for first D

2 positions for first R

ex In a well shuffled deck, find the probability that J, Q, K appear as 12 consecutive cards and the J are grouped together, Q are grouped together, and K are grouped together?

ex JJJJQQQQKKKK

Soln

First we find the chance that the 12 cards are consecutive. Second, given that the cards are consecutive we find the chance the J, Q, K are grouped.

The chance of (12 cards all J, Q, K) followed by 40 other cards is $\frac{1}{\binom{52}{12,40}}$. We must allow for the 12 cards to not be first in the deck. Think of the 12 cards as one special card so the deck has 41 cards with the top card being our group of 12. There are $\binom{41}{1}$ ways we can position the special card in the deck of 41.

Hence, $P(\text{the 12 J,Q,K are consecutive}) = \frac{\binom{41}{1}}{\binom{52}{12,40}}$

Given that our 12 cards are consecutive, the chance they are as JJJJQQQQKKKK is $\frac{1}{\binom{12}{4,4,4}}$. There are however 3! orderings of J, Q, K so we get $\frac{3!}{\binom{12}{4,4,4}}$.

Hence $P(\text{J,Q,K are consecutive and grouped in the deck})$

$$= \boxed{\frac{\binom{41}{1}}{\binom{52}{12,40}} \cdot \frac{3!}{\binom{12}{4,4,4}}}$$

ex Adam, Jess and Tom are standing in a group of 12 people. The group is randomly split into two lines of 12 people each. Find the chance that Adam, Jess and Tom are standing next to each other in one of the 2 lines.

Soln

We first find the chance A, J, T are together in line 1 or together in line 2 but not necessarily consecutive.

Out of 12 we take a sample of 6.

A, J, T are good and the other 9 are bad.

we have $\frac{\binom{3}{3}\binom{9}{3}}{\binom{12}{6}}$. The chance that

AJT are in line 2 is the same so the chance that A, J, T are in the same line is $2\frac{\binom{3}{3}\binom{9}{3}}{\binom{12}{6}}$.

Next we find the chance they are consecutive given that they are in the same line.

Thinking of AJT as one person there are 4 people in the line and

$\binom{4}{1}$ ways to place AJT in the line.

Hence given that AJT are in the same line the chance they are consecutive is $\frac{\binom{4}{1}}{\binom{6}{3,3}}$.

Now apply the multiplication rule.

$$\boxed{2 \cdot \frac{\binom{3}{3}\binom{9}{3}}{\binom{12}{6}} \cdot \frac{\binom{4}{1}}{\binom{6}{3,3}}}.$$