Stat 134: Section 26

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Conceptual Review

a. What does bivariate normal distribution mean?

b. Let X, Y be i.i.d $N(0,1)$ are (X, Y) bivariate normal?

c. What is the distribution of a linear combination of independent Gaussians?

d. Give a necessary and sufficient condition for two linear combinations of independent Gaussians to be independent.

Problem 1

Let X, Y independent $N(0,1)$ random variables.

a. For a constant k, find $P(X > kY)$.

b. If $U = \sqrt{3}X + Y$ and $V = X - \sqrt{3}Y$, find $P(U > kV)$.

c. Find $P(U^2 + V^2 < 1)$

d. Find the conditional distribution of X given $V = v$.

Ex 6.5.6 in Pitman's Probability

Problem 2

Suppose that $W \sim N(\mu, \sigma^2)$ and that given $W = w$ the distribution of Z is $N(aw + b, \tau^2)$.

a. Show that the joint distribution of W, Z is bivariate normal and find its parameters

b. What is the distribution of Z.

c. Find the conditional distribution of W given $Z = z$.

Ex 6.5.9 in Pitman's Probability

Problem 3

Let (X, Y) be standard bivariate normal with correlation ρ. Show that

$$E[\max(X, Y)] = \sqrt{\frac{1-\rho}{\pi}}$$

Ex 6.5.9 in Pitman's Probability

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