Stat 134: Section 19

Adam Lucas

November 5th, 2018

Conceptual Review

a. Suppose $X, Y \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0,1)$. Identify the distribution of:

   (i) X^2

   (ii) $X^2 + Y^2$

   (iii) $\sqrt{X^2 + Y^2}$

   (iv) $4X + 3Y + 5$

b. Let X, Y have joint density $f_{X,Y}$. How do we find the density of $Z = X + Y$? (Write out the formula.)

c. Repeat (b), but for $W = 3X + 4Y$. (Don't forget the Jacobian term!)

Problem 1

X, Y are i.i.d. standard Normal variables. Find (without integration):

a. $P(X > 3Y + 2)$;

b. $P(0 < X < Y)$;

c. $P(|\min\{X, Y\}| < 1)$

For (b) and (c), it may be helpful to draw the relevant region in the plane.

Adapted from Ex 5.3.3, 5.3.6 in Pitman's Probability

Problem 2

The 51B bus is scheduled to arrive at 8:10am, but its arrival time is normally distributed with mean 8:10 and SD 40 seconds. Suppose Adam tries to arrive at 8:09, but his arrival time is normally distributed with mean 8:09 and SD 30 seconds. Assume arrivals are independent.

a. What percent of the time does Adam arrive before the bus arrives?

b. If Adam arrives at 8:09, and no bus shows up between 8:09 and 8:12, what is the probability he missed the bus?

Ex 5.3.7 in Pitman's Probability

Problem 3

Suppose $X \sim \text{Unif}\,(-1,1)$, and $Y \sim \text{Unif}\,(0,1)$. Find the density of $Z = X + Y$, using:

a. the convolution formula (draw a picture as well);

b. the C.D.F. of Z.