STAT 134 - Instructor: Adam Lucas

Midterm

Friday, March 8, 2019

Print your name: ______________________________

SID Number: ______________________________

Exam Information and Instructions:

- You will have 45 minutes to take this exam. Closed book/notes/etc. No calculator or computer.
- We will be using Gradescope to grade this exam. Write any work you want graded on the front of each page, in the space below each question. Additionally, write your SID number in the top right corner on every page.
- Please use a dark pencil (mechanical or #2), and bring an eraser. *If you use a pen and make mistakes, you might run out of space to write in your answer.*
- Provide calculations or brief reasoning in every answer.
- Unless stated otherwise, you may leave answers as unsimplified numerical and algebraic expressions, and in terms of the Normal c.d.f. Φ. Finite sums are fine, but simplify any infinite sums.
- Do your own unaided work. Answer the questions on your own. The students around you have different exams.

I certify that all materials in the enclosed exam are my own original work.

Sign your name: ______________________________

GOOD LUCK!

1. (5 pts)

a Suppose that there is a machine that gives out a random number Y between 0 and 80. You are also told that $\mathbb{E}[Y] = 20$. Now someone proposes you a game where you win if the number that shows up is strictly smaller than 40. Assume that you always play games when you have a chance of at least $\frac{1}{2}$ of winning. Given the information you have, can you determine whether you should agree to play the game?

b Suppose that there is a machine that gives out a random number X between 20 and 100. You are also told that $\mathbb{E}[X] = 40$. Now someone proposes you a game where you win if the number that shows up is strictly smaller than 60. Assume that you always play games when you have a chance of at least $\frac{1}{2}$ of winning. Given the information you have, can you determine whether you should agree to play the game?

2. (5 pts) An airport bus drops off 35 passengers at 7 stops. Each passenger is equally likely to get off at any stop, and passengers act independently of one another. The bus makes a stop only if someone wants to get off. Find the probability that the bus drops off passengers at every stop.

3. (5 pts) Consider a grid of n^2 cups, with n rows of n cups. Toss n ping pong balls at random into these cups, where at most one ball can occupy a particular cup. Let X be the number of unoccupied rows (i.e., where every cup in that row contains no balls). Find:

   (a) $P(X \geq 1)$;

   (b) $E(X)$;

(c) (Continued from Q3): $Var(X)$.

SID: ____________________

4. (5 pts) Consider a box containing 1000 balls, of which m are gold and the remaining $1000 - m$ are blue (Go Bears!). You draw 1000 balls from the box, randomly with replacement. Let X be the number of gold balls you get.

   (a) For any $1 \leq m \leq 1000$, what is $P(X = m)$? *Your answer should be in terms of m.*

   (b) For $m = 500$, what is $P(X = 500)$, approximately?

   (c) For $m = 2$, what is $P(X = 2)$, approximately?

5. (5 pts) Suppose that Player A and Player B take turns rolling a pair of balanced dice and that the winner is the first player who obtains the sum of 7 on a given roll of the two dice. Assume that the rolls are independent. What is the probability that Player A wins the game if he rolls first?

SID: ______________________

Discrete

name and range	$P(k) = P(X = k)$ for $k \in$ range	mean	variance
uniform on $\{a, a+1, \dots, b\}$	$\frac{1}{b-a+1}$	$\frac{a+b}{2}$	$\frac{(b-a+1)^2-1}{12}$
Bernoulli (p) on $\{0, 1\}$	$P(1) = p$; $P(0) = 1-p$	p	$p(1-p)$
binomial (n, p) on $\{0, 1, \dots, n\}$	$\binom{n}{k} p^k (1-p)^{n-k}$	np	$np(1-p)$
Poisson (μ) on $\{0, 1, 2, \dots\}$	$\frac{e^{-\mu}\mu^k}{k!}$	μ	μ
hypergeometric (n, N, G) on $\{0, \dots, n\}$	$\frac{\binom{G}{k}\binom{N-G}{n-k}}{\binom{N}{n}}$	$\frac{nG}{N}$	$n\left(\frac{G}{N}\right)\left(\frac{N-G}{N}\right)\left(\frac{N-n}{N-1}\right)$
geometric (p) on $\{1, 2, 3 \dots\}$	$(1-p)^{k-1}p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
geometric (p) on $\{0, 1, 2 \dots\}$	$(1-p)^k p$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$
negative binomial (r, p) on $\{0, 1, 2, \dots\}$	$\binom{k+r-1}{r-1} p^r (1-p)^k$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$