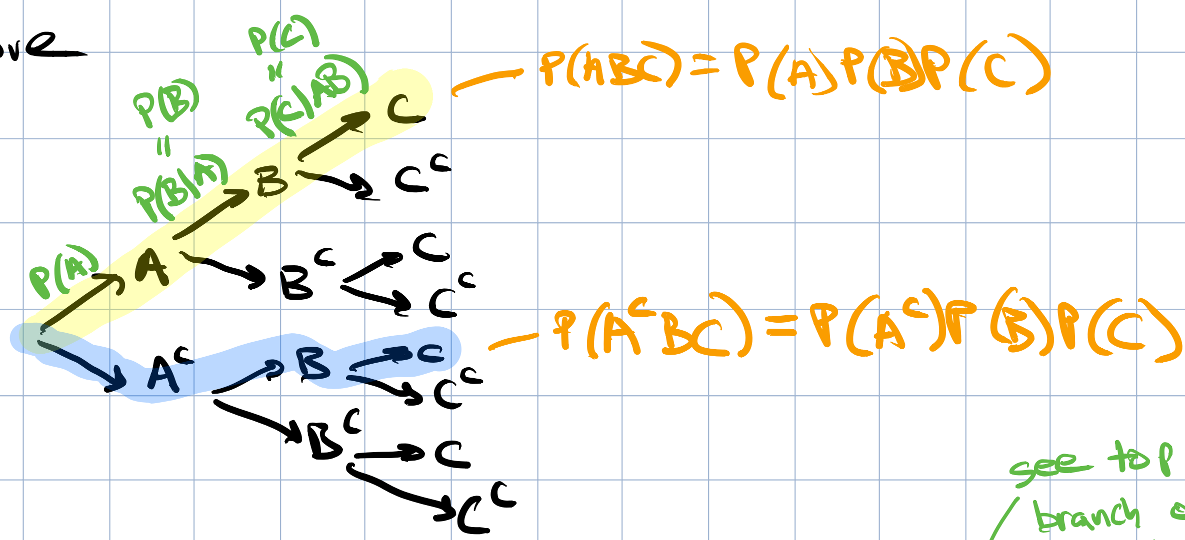
Quiz 1 next Wed on chap 1

Last time sec 1.6 multiplication for 3 indep events

$$\boxed{P(ABC)=P(A)P(B)P(C) \text{ and the same for any of the events replaced by their complements}}$$

Picture



see top branch of tree above.

Independence is stronger than pairwise independence

We need $P(C)=P(C|AB)$ for independence but this isn't given by pairwise independence which says $P(C|A)=P(C)$ and $P(C|B)=P(C)$.

For example if $A=B_{12}$, $B=B_{23}$, $C=B_{13}$ in our b-day example from last class,

$$P(B_{13}|B_{12}B_{23}) = P(B_{13}|B_{123}) = 1 \neq P(B_{13})=\frac{1}{365}$$

but $P(B_{13}|B_{23})=P(B_{13})=\frac{1}{365}$ and $P(B_{13}|B_{12})=P(B_{13})=\frac{1}{365}$

so just because you have pairwise indep doesn't mean you have independence.

Today Sec 2.1 The Binomial Distribution

Bernoulli(p) trial

two outcomes $\left\langle \begin{array}{l} 1 \leftarrow \text{prob } p \\ 0 \leftarrow \text{prob } 1-p \end{array} \right.$

ex roll a die

let 1 = get a 6 — $p = 1/6$

0 = don't get a 6 — $1-p = 5/6$

Binomial distribution

Suppose we have n independent Bernoulli(p) trials

(i.e roll die n times)

What are the possible number of successes

answ $0, 1, 2, \ldots, n$

ex what is the chance of getting 2 sixes in 5 rolls.

$1\ 1\ 0\ 0\ 0$ — Prob $\left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$

$0\ 1\ 1\ 0\ 0$ — Prob $\left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$

$\vdots$

How many of these are there? — answ $\binom{5}{2}$

Aside on counting:

How many orderings of a, b, c, d, e are there? — 5!

$\underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 5!$

How many orderings of aabbb are there?

aabbb
aabbb
⋮

$\Bigg\}$ there are $2!3! = 12$ copies of aabbb in all permutations of aabbb

Similarly there are 12 copies of ababb

Need to divide 5! by 2!3!

so $\frac{5!}{2!3!}$ orderings of aabbb

$$\frac{5!}{2!3!} = \binom{5}{2}$$

← 5 choose 2

end of aside

There are $\binom{5}{2}$ diff orderings of 11000 each with prob $\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3$

$\Rightarrow$ $P(\text{get 2 successes}) = \binom{5}{2} \cdot \underset{\frac{1}{6}}{p}^2 \underset{5/6}{(1-p)}^3$

Find chance you get exactly k successes out of n Bernoulli (p) trials?

$$\boxed{P(k \text{ successes}) = \binom{n}{k} p^k (1-p)^{n-k}}$$

Binomial formula.

ingrediants for binomial formula:

- n indep Bernoulli (p) trials
- k successes.

ex toss a fair coin 5 times

Find $P(\text{get at least 3 heads})$

hhhtt $\quad P(3 \text{ heads}) = \binom{5}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$

hhhht $\quad P(4 \text{ heads}) = \binom{5}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1$

hhhhh $\quad P(5 \text{ heads}) = \binom{5}{5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0$

$$= \sum_{k=3}^{5} \binom{5}{k}\left(\frac{1}{2}\right)^k\left(\frac{1}{2}\right)^{5-k}$$

Stats 134

Chapter 2 Wednesday August 29 2018

1. For which of the following is the binomial formula applicable?

   a the chance that the sum of draws is **3** while drawing **5 times with replacement** from a box with 9 tickets marked **0** and one ticket marked **1**.

   b the chance that there are exactly **3** diamonds among the first **10** cards dealt off the top of a shuffled deck.

(a) satisfies requirement for binomial formula since we have 5 independent Bernoulli (P) trials with $p = 1/10$ and $K = 3$ since the number of successes is the sum of our draw,

(b) doesnt satisfy requirement for binomial formula since the draws from the top of the deck are not independent, The probability of an ace is always $p = 4/52 = 1/13$ however,

Consecutive odds ratio

— a tool to find the mode of the binomial distribution.

mode

The binomial formula
$$P(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

The mode of a binomial dist is the k such that $P(k)$ is largest.

see end of lecture notes for proof.

Facts

(*)

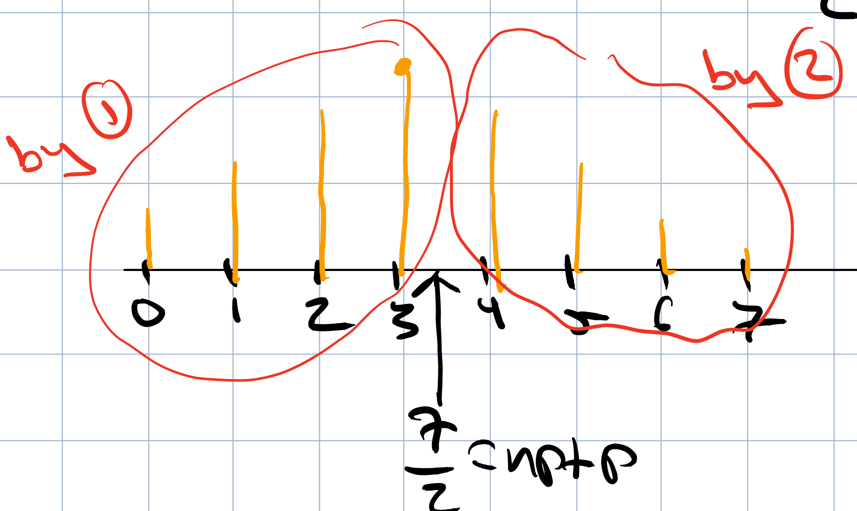
① $P(k-1) < P(k) \iff k < np+p$

② $P(k-1) > P(k) \iff k > np+p$

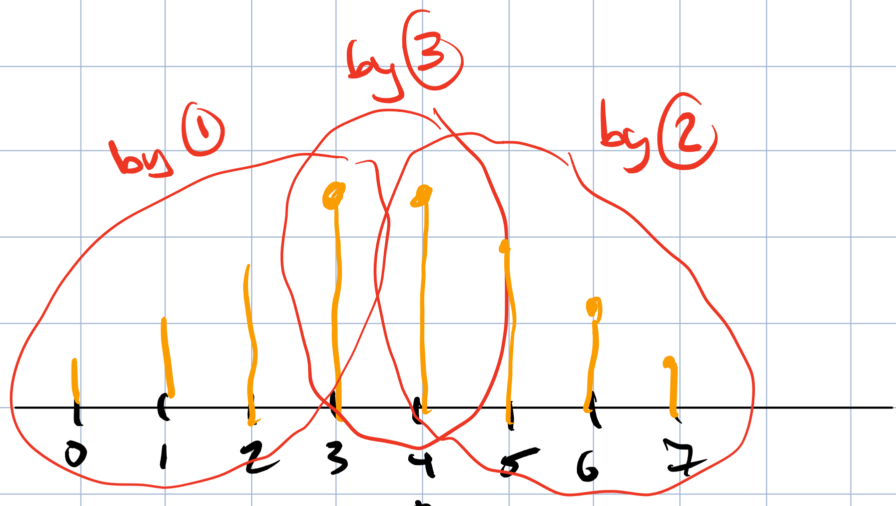
③ $P(k-1) = P(k) \iff k = np+p$

Picture

assume $np+p = \frac{7}{2}$



assume $np+p = 4$



$\uparrow$

$4 = np+p$

let $m = \lfloor np+p \rfloor$ — integer part of $np+p$

ex $\lfloor 7.2 \rfloor = 7$

$$\text{mode} = \begin{cases} m & \text{if } np+p \notin \mathbb{Z} \\ m, m-1 & \text{if } np+p \in \mathbb{Z} \end{cases}$$

Proof of Fact * above:

First note that $\dfrac{\binom{n}{k}}{\binom{n}{k-1}} = \dfrac{\dfrac{\cancel{n!}}{k!(n-k)!}}{\dfrac{\cancel{n!}}{(k-1)!(n-k+1)!}} = \boxed{\dfrac{n-k+1}{k}}$

$$\frac{P(k)}{P(k-1)} = \frac{\binom{n}{k} p^k (1-p)^{n-k}}{\binom{n}{k-1} p^{k-1} (1-p)^{n-k+1}} = \boxed{\frac{n-k+1}{k} \cdot \frac{p}{1-p}}$$

where ☺ is <, >, or =

$P(k-1)$ ☺ $P(k)$

$\iff$ 1 ☺ $\dfrac{P(k)}{P(k-1)}$

$\iff$ 1 ☺ $\dfrac{n-k+1}{k} \cdot \dfrac{p}{1-p}$

$\iff$ $k(1-p)$ ☺ $(n-k+1)p$

$\iff$ $k - \cancel{kp}$ ☺ $np - \cancel{pk} + p$

$\iff$ k ☺ $np+p$

So $P(k-1)$ ☺ $P(k)$ $\iff$ k ☺ $np+p$

where ☺ is <, > or =