

## *Stat 134: Section 21*

*Adam Lucas*

*December 2nd, 2019*

### ***Conceptual Review***

- a. What is the Law of Iterated Expectation?
- b. Suppose two continuous random variables  $X, Y$  have joint density  $f(x, y)$ . How do we find the conditional density of  $Y$  given  $X = x$ ?

### *Problem 1: Success Runs*

I toss a coin which lands heads with probability  $p$ . Let  $W_H$  be the number of tosses until I get one head in a row (i.e., one head), and  $W_{HH}$  be the number of tosses until I get two heads in a row. Find:

- a.  $\mathbb{E}(W_H)$ ;
- b.  $\mathbb{E}(W_{HH})$ .

*Ex 6.rev.21 in Pitman's Probability*

*Problem 2*

Suppose that  $N$  is a Poisson random variable with parameter  $\mu$ . Suppose that given  $N = n$ , random variables  $X_1, X_2, \dots, X_n$  are independent with uniform  $(0,1)$  distribution. So there are a random number of  $X$ 's.

- Given  $N = n$ , what is the probability that all the  $X$ 's are less than  $t$
- What is the (unconditional) probability that all the  $X$ 's are less than  $t$
- Let  $S_n = X_1 + X_2 + \dots + X_n$ . Find  $E(S_n)$ .

*Ex 6.2.6 in Pitman's Probability*

*Problem 3*

Suppose that  $Y$  and  $Z$  are random variables with the following joint density:

$$f(y, z) = \begin{cases} k(z - y) & \text{for } 0 \leq y \leq z \leq 2, \\ 0 & \text{otherwise} \end{cases}$$

for some constant  $k$ . Find:

- the marginal distribution of  $Y$ ;
- $P(Z < \frac{2}{3} | Y = \frac{1}{2})$

*Ex 6.3.7 in Pitman's Probability*