Stat 134 lec 7

Last time sec 2.4 Poisson approx to binomial.

we saw Poisson is a limit of binomials for large n and small p ($np \to \mu$)

Things we know about binomials such as mode $= \lfloor np+p \rfloor$ for non integer $np+p$ translate to Poisson by letting $np \to \mu$ and $p \to 0$.

(i.e mode $= \lfloor \mu \rfloor$ for Poisson(μ) for non integer μ).

Today sec 2.5 Random Sampling

independent trials (draw w/ replacement)
- binomial distribution — 2 outcome trial
- multinomial distribution — k outcome trial

dependent trials (draw w/o replacement)
- hypergeometric distribution — 2 outcome trial
- multivariate hypergeometric distribution — k outcome trial

sec 2.5

Sampling w/ replacement

Class 100 students

A: 50 students
B: 30 students
C: 15 students
D: 5 students

Sample 10 students at random w/ replacement

Find $P(4\ A's,\ 3\ B's,\ 2\ C's,\ 1\ D)$

$\underline{A}\ \underline{A}\ \underline{A}\ \underline{A}\ \underline{B}\ \underline{B}\ \underline{B}\ \underline{C}\ \underline{C}\ \underline{D}$

$$\underbrace{\frac{10!}{4!3!2!1!}}_{\binom{10}{4,3,2,1}} \underbrace{(.5)^4(.3)^3(.15)^2(.05)^1}_{\text{multinomial formula}}$$

Def^n Multinomial Distribution

We have,

n independent trials where each trial has k possible outcomes, $A_1, A_2, \dots, A_k$ with probability $p_1, p_2, \dots, p_k$.

The probability we get n_1 outcome A_1, n_2 outcome A_2, $\dots$, n_k outcome A_k is

$$P(n_1, n_2, \dots, n_k) = \underbrace{\binom{n}{n_1, n_2, \dots, n_k}}_{\frac{n!}{n_1!n_2!\dots n_k!}} p_1^{n_1} p_2^{n_2} \cdots p_k^{n_k}$$

Sample without Replacement

ex Only two types of students in the class of 100.

Class 100 students, A: 70 students, B: 30 students,

Sample 5 students at random without replacement (called a simple random sample SRS)

Find $P(3A\text{'s}, 2B\text{s})$

$$\frac{70!}{3!67!}$$

A A A B B

$$\binom{5}{2}\frac{70}{100}\cdot\frac{69}{99}\cdot\frac{68}{98}\cdot\frac{30}{97}\cdot\frac{29}{96} = \frac{\frac{70\cdot 69\cdot 68}{3!}\cdot\frac{30\cdot 29}{2!}}{\frac{100\cdot 99\cdot 98\cdot 97\cdot 96}{5!}}$$

$$\binom{5}{2} = \frac{5!}{2!3!}$$

$$= \frac{\binom{70}{3}\binom{30}{2}}{\binom{100}{5}}$$

If we drew w/ replacement answer would be

$$\binom{5}{2}\left(\frac{70}{100}\right)^3\left(\frac{30}{100}\right)^2$$

binomial formula

hypergeometric formula.

Def^n Hypergeometric Distribution

Suppose a population of size N contains G good and N bad elements ($N = G + B$).

A sample size n with g good and b bad elements ($n = g + b$) is chosen at random without replacement.

$$P(g \text{ good and } b \text{ bad}) = \frac{\binom{G}{g}\binom{B}{b}}{\binom{N}{n}}$$

This generalizes like the ~~the~~ multinomial distribution to the

multivariate hypergeometric distribution

Now instead of 2 types of elements we have K with sizes $G_1, \ldots, G_K$ ($N = G_1 + \cdots + G_K$) and in our sample we have $n = g_1 + \cdots + g_K$.

$$P(g_1 \text{ type } 1, \ldots, g_K \text{ type } K) = \frac{\binom{G_1}{g_1}\binom{G_2}{g_2}\cdots\binom{G_K}{g_K}}{\binom{N}{n}}$$

ex Lets redo our first example
but now take our sample w/o replacement,

Class 100 students

A: 50 students
B: 30 students
C: 15 students
D: 5 students

Sample 10 students at random w/o replacement

Find P(4 A's, 3 B's, 2 C's, 1 D)

Answ

$$\boxed{\frac{\binom{50}{4}\binom{30}{3}\binom{15}{2}\binom{5}{1}}{\binom{100}{10}}}$$

ex A 5 card poker hand consists of a SRS of 5 cards from a 52 card deck.

there are $\binom{52}{5}$ poker hands.

Find P(Poker hand has 4 aces and a king)

$$\frac{\overbrace{\binom{4}{4}\binom{4}{1}\binom{4}{0}\binom{4}{0}\cdots\binom{4}{0}}^{11}}{\binom{52}{5}} = \frac{\binom{4}{4}\binom{4}{1}}{\binom{52}{5}}$$

Find P(Poker hand has 4 aces)

$$= 12 \cdot \frac{\binom{4}{4}\binom{4}{1}}{\binom{52}{5}} = \frac{\binom{4}{4}\binom{48}{1}}{\binom{52}{5}}$$

Find P(Poker hand has 4 of a kind)

rank(aaaab)

$$13 \cdot 12 \, \frac{\binom{4}{4}\binom{4}{1}}{\binom{52}{5}}$$

have a and b are different.

Unfortunately the concept test from class had a misprint

(I forgot to write 11 !)

Stat 134

Chapter 2 Wednesday September 7 2018

1. The probability of being dealt a three of a kind poker hand (ranks aaabc) is:

a $\binom{4}{3}\binom{4}{1}\binom{44}{1}/\binom{52}{5}$

b $13 * 12 * 11 \binom{4}{3}\binom{4}{1}\binom{4}{1}/\binom{52}{5}$

c $(13 * 12 * 11/2)\binom{4}{3}\binom{4}{1}\binom{4}{1}/\binom{52}{5}$

d none of the above

Sorry!

Here is the question again

↓

Stat 134

Chapter 2 Wednesday September 7 2018

1. The probability of being dealt a three of a kind poker hand (ranks aaabc) is:

a $\binom{4}{3}\binom{4}{1}\binom{44}{1}/\binom{52}{5}$

b $13*12*11\binom{4}{3}\binom{4}{1}\binom{4}{1}/\binom{52}{5}$

c $13*\binom{12}{2}\binom{4}{3}\binom{4}{1}\binom{4}{1}/\binom{52}{5}$

d none of the above

It is not (a) since this is the probability of 3 aces, one king and one of anything else.

It is not (b) because in a poker hand aaabc = aaacb (i.e the order of b and c doesnt matter). Hence when we say there are 13 diff a, 12 diff b, 11 diff c we are double counting since aaabc = aaacb. Hence (c) is correct.

ex To test your understanding, what is chance to get a pair (ranks aabcd)?

Ansu

$$\frac{13 \cdot \binom{12}{3} \cdot \binom{4}{2}\binom{4}{1}\binom{4}{1}\binom{4}{1}}{\binom{52}{5}}$$

because

aabcd = aabdc = aacbd = aacdb

= aacbd = aacdb

in your poker hand.