Stat 134: Section 13

Adam Lucas

October 15th, 2018

Problem 1

Suppose calls are arriving at a telephone exchange at an average rate of one per second, according to a Poisson arrival process. Find:

a. the probability that the fourth call after time $t = 0$ arrives within 2 seconds of the third call;

b. the probability that the fourth call arrives by time $t = 5$ seconds;

c. the expected time at which the fourth call arrives.

Ex 4.2.5 in Pitman's Probability

Problem 2

Local calls are coming into a telephone exchange according to a Poisson process with rate λ_{loc} calls per minute. Independently of this, long-distance calls are coming in at a rate of λ_{dis} calls per minute. Write down expressions for probabilities of the following events:

a. exactly 5 local calls and 3 long-distance calls come in a given minute;

b. exactly 50 calls (counting both local and long distance) come in a given three- minute period;

c. starting from a fixed time, the first ten calls to arrive are local.

Ex 4.rev.13 in Pitman's Probability

Problem 3: Gammas, Exponentials, and Moments

Consider the gamma function $\Gamma(r) = \int_0^\infty x^{r-1} e^{-x} dx, \quad r > 0$.

a. Use integration by parts to show that $\Gamma(r+1) = r\Gamma(r)$.

b. Deduce from (a) that for any positive integer n, $\Gamma(n) = (n-1)!$

c. Show that if $T \sim$ Exp (1), then $\mathbb{E}(T^n) = n!$.

d. Show that if $S = T/\lambda$, then $S \sim$ Exp (λ). (Note: from this, we can easily show that $\mathbb{E}(S^n) = n!/\lambda^n$).

Hint: Consider the expression $P(S > s)$, then substitute for S appropriately.

Ex 4.2.9 in Pitman's Probability

Prepared by Brian Thorsen and Yiming Shi