Stat 134 lec 2

Last time

Addition rule (OR) if A, B mutually exclusive sets

$$P(A \text{ or } B) = P(A) + P(B).$$

Inclusion exclusion (OR)

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

Today

1. Sec 1.3 Distributions
2. Sec 1.4 Conditional Probability
3. sec 1.5 Baye's rule

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Boole's inequality

$$P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$$

For example for $n=3$ this says

$$P(A_1 \cup A_2 \cup A_3) \leq P(A_1)+P(A_2)+P(A_3).$$

We need to show this for any n.

Prove by induction:

verify for $n=1$
assume true for $n=m$
show true for $n=m+1$.

$n=1$ $P(A_1)=P(A_1)$ ✓

Assume true for $n=m$

$$P\left(\bigcup_{i=1}^{m+1} A_i\right) = P\left(\bigcup_{i=1}^{m} A_i \cup A_{m+1}\right)$$

$$= P\left(\bigcup_{i=1}^{m} A_i\right) + P(A_{m+1}) - P\left(\left(\bigcup_{i=1}^{m} A_i\right)\cdot A_{m+1}\right)$$

$$\leq \sum_{i=1}^{m} P(A_i) + P(A_{m+1}) = \sum_{i=1}^{m+1} P(A_i)$$

$\square$

Named distribution

can repeat

Uniform distribution on a finite set $\{x_1, \ldots, x_n\}$:

Imagine you have numbers $x_1, \ldots, x_n$ in a hat. Let X be a random draw of one of these numbers.

$$P(X = x_i) = \frac{1}{n} \quad \text{for all } i$$

We say that X has uniform distribution on $\{x_1, \ldots, x_n\}$.

7 1 4 2 8 6
~~ex~~ Suppose a word is randomly picked
4 4 8
from this sentence. What is the distribution of the length of the word picked.

<u>answ</u> $\text{unif}(\{7, 1, 4, 2, 8, 6, 4, 4, 8\})$

A deck of cards is shuffled. What is the chance that the top card is the **king** of spades **or** the bottom card is the **king** of spades

a $\frac{1}{52} \times \frac{1}{51}$

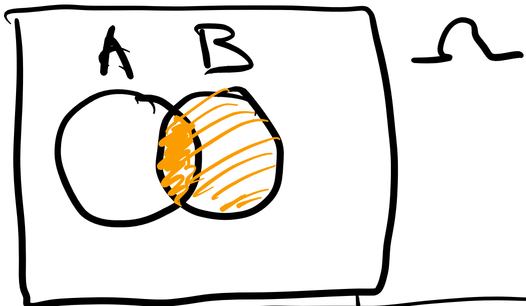
b $\frac{1}{52} + \frac{1}{51}$

c $\frac{1}{52} + \frac{1}{52} - \frac{1}{52} \times \frac{1}{51}$

d none of the above $\frac{1}{52} + \frac{1}{52}$

Sec 1.4 Conditional Probability and Independence

let A, B be subsets of Ω



given

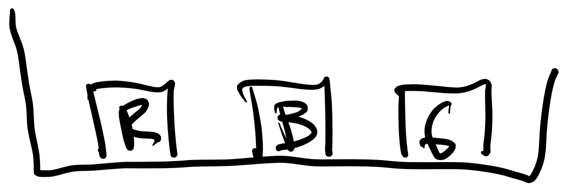
Bayes rule $P(A|B) = \dfrac{P(AB)}{P(B)}$

$\boxed{P(AB) = P(B)\cdot P(A|B)}$ multiplication rule (AND)

If A and B are independent events then $P(A|B) = P(A)$ since A doesn't depend on B

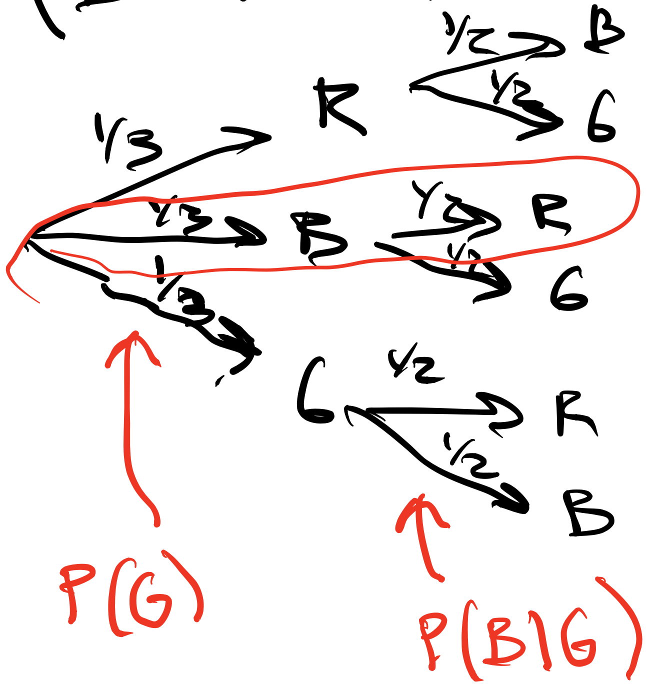
$\Rightarrow$ $\boxed{P(AB) = P(B)P(A)}$

ex



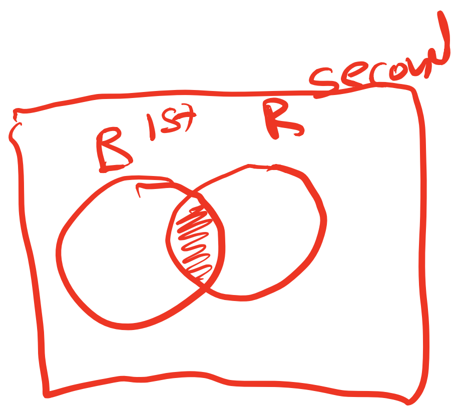
draw 2 tickets
w/o replacement

P(B first and R second) ?



$= P(B) \cdot P(R|B)$

$= \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$



sec 1.5 Bayes rule

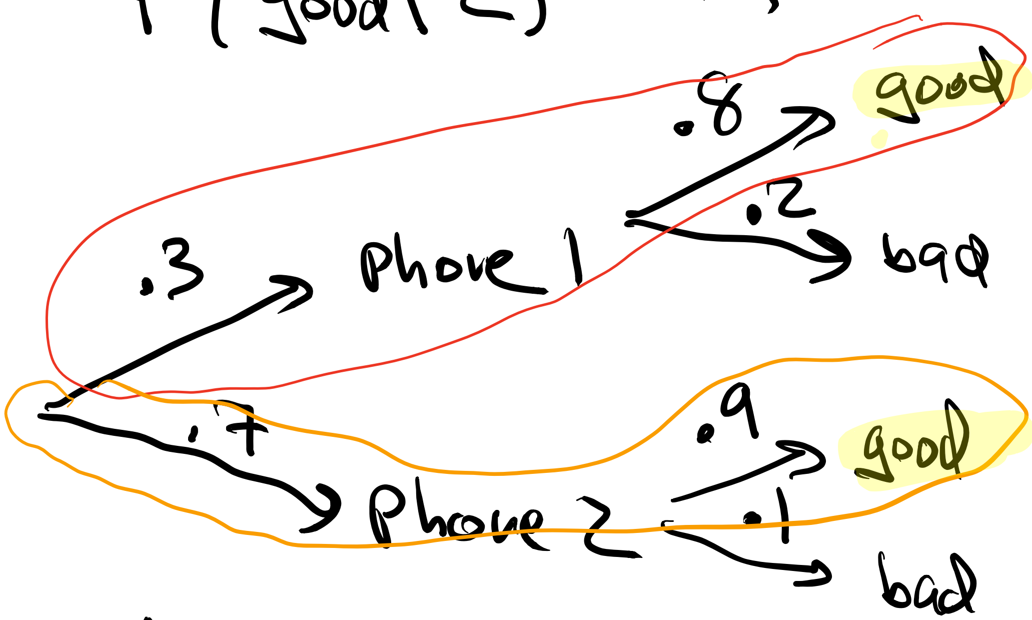
A factory produces 2 models of cell phones

Given

$P(\text{cell phone } 1) = .3$

$P(\text{good} \mid 1) = .8$

$P(\text{good} \mid 2) = .9$



Find

and

$P(1, \text{good}) = (.3)(.8) = .24$

$P(\text{good}) = P(\text{good}, 1) + P(\text{good}, 2) = .87$

$= .24 + (.7)(.9)$

$P(1 \mid \text{good}) = \dfrac{P(1, \text{good})}{P(\text{good})} = \dfrac{.24}{.87} = .28$

Another way to write Bayes rule:

$$P(1 \mid good) = \frac{P(1, good)}{P(good)} = \frac{P(good \mid 1) \cdot P(1)}{P(good)}$$

$$= \boxed{\frac{P(good \mid 1) P(1)}{P(good \mid 1) P(1) + P(good \mid 2) P(2)}}$$

$P(1)$, $P(2)$ are prior probabilities (first probability in the chain).

These are likely found from a long run frequency interpretation.

(randomly pick 100 phones and find 30 are type 1).

When we have additional information, for example whether the phone is good or bad we may update our prior, by forming a posterior probability $P(1 \mid data)$.

likelihood

Prior

Bayes's rule says $P(1 \mid data) = \frac{P(good \mid 1) \cdot P(1)}{P(good)}$.

Posterior

$P(good \mid 1)$ is called the likelihood probability.