Stat 134 lec 14

Quiz 3 Wednesday sec 2.5, 3.1 – 3.3

Last time

Sec 3.3

$Var(X+Y) = Var(X) + Var(Y)$ if X, Y indep.

Central limit Theorem (CLT)

Let $S_n = X_1 + \cdots + X_n$ independent and identically distributed (i.i.d.), $E(X) = \mu$

$SD(X) = \sigma$, then

$$S_n \approx \text{Normal}\left(n\mu, \underbrace{(\sqrt{n}\,\sigma)^2}_{\text{variance}}\right) \text{ for "large" } n$$

Today

sec 3.3 example CLT

sec 3.6 (sec 3.4 next time) calculating variance of a sum of dependent indicators.

CLT

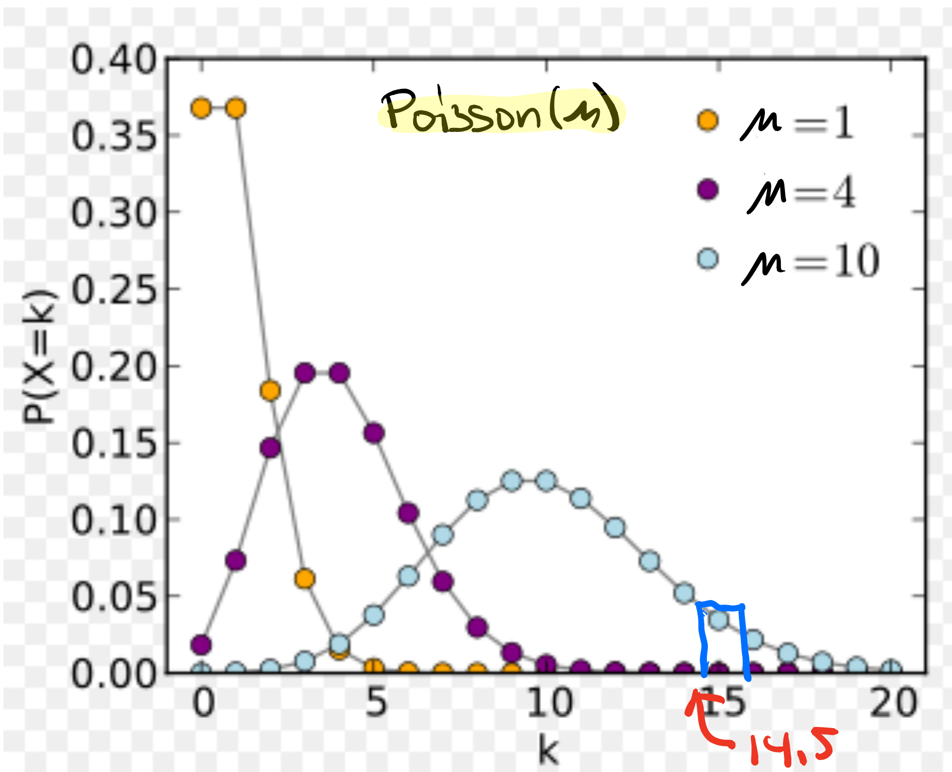
ex Let $X_1, X_2, \ldots$ be i.i.d. Poisson(1).

Let $S_{10} = X_1 + \cdots + X_{10}$. Find $P(S_{10} \geq 15)$

Facts

if $X \sim \text{Pois}(\mu)$, $E(X) = \mu$

$\text{Var}(X) = \mu$



$E(X) = 1 \quad \Rightarrow \quad E(S_{10}) = 10$

$SD(X) = 1 \qquad SD(S_{10}) = \sqrt{10} \cdot 1 = \sqrt{10}$

$$S_{10} \approx N(10, (\sqrt{10})^2)$$

$$P(S_{10} \geq 15) = 1 - \Phi\left(\frac{14.5 - 10}{\sqrt{10}}\right)$$

$$= 1 - \Phi(1.42) = .077$$

The variance of a sum of independent indicators

ex $I = \begin{cases} 1 & \text{Prob } p \\ 0 & \text{Prob } 1-p \end{cases}$

$$Var(I) = E(I^2) - (E(I))^2$$

$$E(I^2) = 1^2 \cdot p + 0^2 \cdot (1-p) = p$$

$$E(I) = p$$

$$\Rightarrow Var(I) = p - p^2 = \boxed{p(1-p)}$$

ex $X \sim Bin(n,p)$

$$X = I_1 + \dots + I_n$$

$$E(X) = nE(I_1) = \boxed{np}$$

$$Var(X) = n\,Var(I_1) = \boxed{np(1-p)}$$

sec 3.6

Look at reading guide for what to read.

Variance of a sum of dependent indicators

ex

14. A building has 10 floors above the basement. If 12 people get into an elevator at the basement, and each chooses a floor at random to get out, independently of the others, at how many floors do you expect the elevator to make a stop to let out one or more of these 12 people?

X = # buttons pressed at ground floor

1, 2, 3, …, 10

Find $E(X)$, $Var(X)$.

$p_1 = 1 - \left(\frac{9}{10}\right)^{12}$

$I_1 = \begin{cases} 1 & \text{if } 1^{st} \text{ button pressed} \\ 0 & \text{else} \end{cases}$

$X = I_1 + \dots + I_{10}$

$\boxed{E(X) = 10 p_1}$

$Var(X) = E(X^2) - E(X)^2$

100 terms

$E(X^2) = E\left((I_1 + \dots + I_{10})^2\right) = \sum_{1 \le i, j \le 10} E(I_i I_j)$

$$
\begin{array}{c|cccc}
 & I_1 & I_2 & I_3 \cdots & I_{10} \\
\hline
I_1 & I_1 I_1 & I_1 I_2 & & \\
I_2 & & I_2 I_2 & & \\
\vdots & & & \ddots & \\
I_9 & & & & \\
I_{10} & & & &
\end{array}
$$

$$I_1 = \begin{cases} 1 & \text{if } 1^{st} \text{ button pressed} \\ 0 & \text{else} \end{cases}$$

$$I_2 = \begin{cases} 1 & \text{if } 2^{nd} \text{ button pressed} \\ 0 & \end{cases}$$

P_{12}

$$I_1 I_2 = \begin{cases} 1 & \text{if } 1^{st} \text{ and } 2^{nd} \text{ button pressed} \\ 0 & \text{else} \end{cases}$$

$$P_{12} = 1 - \text{Prob}\left(1^{st} \text{ not pressed or } 2^{nd} \text{ not pressed}\right)$$

$$\parallel$$

$$\left(\frac{9}{10}\right)^{12} + \left(\frac{9}{10}\right)^{12} - \left(\frac{8}{10}\right)^{12}$$

$$P_{12} = 1 - \left[2\left(\frac{9}{10}\right)^{12} - \left(\frac{8}{10}\right)^{12}\right]$$

$I_1 I_2 = I_{12}$

$I_1 I_1 = I_1$

$E(I_1)$

$E(X^2) = 10 p_1 + 10 \cdot 9 p_{12}$

$(E(X))^2 = (10 p_1)^2$

$$\boxed{\text{var}(X) = 10 p_1 + 10 \cdot 9 p_{12} - (10 p_1)^2}$$

Recall multinomial dist

roll a die 10 times

$P(2 \text{ ones}, 3 \text{ twos}) = \binom{10}{2,3,5} \left(\frac{1}{6}\right)^2 \left(\frac{1}{6}\right)^3 \left(\frac{4}{6}\right)^5$

$\frac{10!}{2!\,3!\,5!}$

Stat 134

Monday September 24 2018

1. A fair die is rolled 14 times. Let X be the number of faces that appear exactly twice. Which of the following expressions appear in the calculation of $Var(X)$

   a $14 * 13 * \binom{14}{2,2,10}(1/6)^2(1/6)^2(4/6)^{10}$

   b $\binom{14}{2}(1/6)^2(5/6)^{12}$

   c more than one of the above

   d none of the above

$P_1 = \binom{14}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{12}$

X = # faces that appear twice

$\sim 1, 2, \ldots, 5$

$X = I_1 + \cdots + I_6$ $\quad I_1 = \begin{cases} 1 & 1^{st} \text{ face twice} \\ 0 & \text{else} \end{cases}$

$P_{12} = \binom{14}{2,2,10}\left(\frac{1}{6}\right)^2\left(\frac{1}{6}\right)^2\left(\frac{4}{6}\right)^{10}$

$I_{12} = \begin{cases} 1 & 1^{st} \text{ and } 2^{nd} \text{ face twice} \\ 0 & \text{else} \end{cases}$

$E(X^2) = 6P_1 + 6 \cdot 5 P_{12}$

$Var(X) = E(X^2) - (E(X))^2 = 6P_1 + 30P_{12} - (6P_1)^2$