Stat 134 lec 2

Last time

Addition rule (OR) if A, B mutually exclusive sets

$$P(A \text{ or } B) = P(A) + P(B).$$

Inclusion exclusion (OR) $P(A \text{ or } B) = P(A) + P(B) - P(AB)$

Today

1. sec 1.3 Distributions
2. sec 1.4 Conditional Probability
3. sec 1.5 Baye's rule

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Some easy facts about sets:

$$A_1 \cup A_2 \cup A_3 = (A_1 \cup A_2) \cup A_3 \Rightarrow \bigcup_{i=1}^{m+1} A_i = \left(\bigcup_{i=1}^{m} A_i\right) \cup A_{m+1}$$

$$A_1 A_3 \cup A_2 A_3 = (A_1 \cup A_2) A_3 \Rightarrow \bigcup_{i=1}^{m} (A_i A_{m+1}) = \left(\bigcup_{i=1}^{m} A_i\right) A_{m+1}$$

Sec 1.3 Distributions

Proof by induction

see #12 p31

ex Let $A_1, \dots, A_{n+1}$ be events.

Show $P\left(\bigcup_{i=1}^{n+1} A_i\right) = P\left(\bigcup_{i=1}^{n} A_i\right) + P(A_{n+1}) - P\left(\bigcup_{i=1}^{n} A_i A_{n+1}\right)$

Pf/ We prove by induction.

① base case is true, $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 A_2)$ by inclusion exclusion rule.

② Assume true for union of n events and show true for union of $n+1$ events:

$$P\left(\bigcup_{i=1}^{n+1} A_i\right) = P\left(\bigcup_{i=1}^{n} A_i \cup A_{n+1}\right)$$

$P\left(\left(\bigcup A_i\right) A_{n+1}\right)$

$\parallel$

$$= P\left(\bigcup A_i\right) + P(A_{n+1}) - P\left(\left(\bigcup_{i=1}^{n} A_i\right) A_{n+1}\right)$$

by inclusion exclusion rule.

By the principle of mathematical induction we have shown the claim for all n.

$\square$

Named distribution

can repeat

Uniform distribution on a finite set $\{x_1, .., x_n\}$:

Imagine you have numbers $x_1, .., x_n$ in a hat.
Let X be a random draw of one of these numbers.

$$P(X = x_i) = \frac{1}{n} \quad \text{for all } i$$

We say that X has uniform distribution on $\{x_1, .., x_n\}$.

~~ex~~ Suppose a word is randomly picked from this sentence. What is the distribution of the length of the word picked.

7 1 4 2 8 6 4 4 8

answ $\text{unif}(\{7, 1, 4, 2, 8, 6, 4, 4, 8\})$

Concept test pt 1

A deck of cards is shuffled. What is the chance that the top card is the **king** of spades **or** the bottom card is the **king** of spades

a $\frac{1}{52} \times \frac{1}{51}$

b $\frac{1}{52} + \frac{1}{51}$

c $\frac{1}{52} + \frac{1}{52} - \frac{1}{52} \times \frac{1}{51}$

d none of the above

King of spades and king of spades are mutually exclusive so have inclusion exclusion rule.

$\frac{1}{52} + \frac{1}{52}$

unconditional probability

Concept test pt 2

A deck of cards is shuffled. What is the chance that the top card is the **king** of spades **or** the bottom card is the **queen** of spades

a $\frac{1}{52} \times \frac{1}{51}$

b $\frac{1}{52} + \frac{1}{51}$

c $\frac{1}{52} + \frac{1}{52} - \frac{1}{52} \times \frac{1}{51}$

d none of the above

King of spades and queen of spades aren't mutually exclusive so have inclusion exclusion rule.

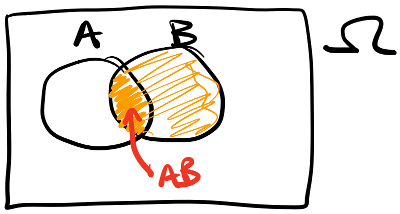
P(king of spades and queen of spades)

$= \frac{1}{52} \cdot \frac{1}{51}$

by multiplication rule

Sec 1.4 Conditional Probability and Independence

Let A, B be subsets of Ω (i.e events).



Bayes's rule says $P(A \mid B) = \dfrac{P(AB)}{P(B)}$ given

$\Leftrightarrow$ $\boxed{P(AB) = P(A \mid B)\,P(B)}$ multiplication rule. (AND)

A and B

We say A and B are <u>independent</u> if

$P(A \mid B) = P(A)$

or equivalently if $P(AB) = P(A)P(B)$

ex
A = last card is queen of spades
B = 1^{st} card is king of spades

A and B are dependent

$$P(AB) = \underset{\underset{\frac{1}{52}}{\shortparallel}}{P(B)}\ \underset{\underset{\frac{1}{51}}{\shortparallel}}{P(A \mid B)} = \boxed{\frac{1}{52} \cdot \frac{1}{51}}$$

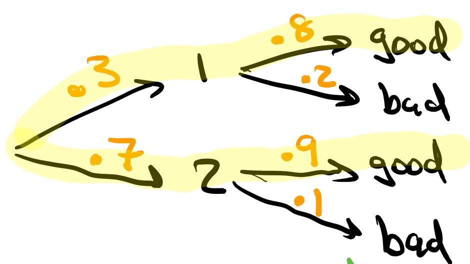
Sec 1.5 Baye's rule

ex A factory produces 2 models of cell phones.

Given $P(1) = .3$

$P(\text{good} \mid 1) = .8$

$P(\text{good} \mid 2) = .9$



Find $P(1, \text{good}) = (.3)(.8) = .24$

and

$P(\text{good}) = (.3)(.8) + (.7)(.9) = .87$

$$P(1 \mid \text{good}) = \frac{P(1, \text{good})}{P(\text{good})} = \frac{.24}{.87} = .28$$

Note

Another way to write Baye's rule

$$P(1 \mid \text{good}) = \frac{P(1, \text{good})}{P(\text{good})} = \frac{P(\text{good} \mid 1)P(1)}{P(\text{good} \mid 1)P(1) + P(\text{good} \mid 2)P(2)}$$