STAT 134: Section 7

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Conceptual Review

Please discuss how the following topics are related to one another, in the context of a random variable X.

- Tail probabilities (i.e., probabilities of the form $\mathbb{P}(X \geq x)$)

- Expectation $\mathbb{E}(X)$ and, more generally, moments of a distribution (i.e., expectations of the form $\mathbb{E}(X^k)$)

- Bounds on tail probabilities, like the inequalities of Markov and Chebyshev

- Indicator random variables

Problem 1

Suppose $X = \sum_{i=1}^{n} \mathbf{1}_i$, where $\mathbf{1}_i$ indicates an event A_i. Write $\mathbb{E}(X^2)$ using terms of the form $\mathbb{P}(A_i)$ and $\mathbb{P}(A_i A_j)$.

Problem 2

Recall the problem from Section 6 which entails selecting a pool of $2k$ chopsticks uniformly–at–random from a collection of $2n$ chopsticks consisting of n unique pairs of chopsticks. We are interested in X, the number of matched pairs of chopsticks in the $2k$ drawn chopsticks. Use your answer to Problem 1 to calculate $\mathbb{E}(X^2)$.

Quiz Review: *Quiz 2 Problem 3*

Sankar and Fitch are playing a game. First, Sankar picks a parameter λ, and then he draws a random number N which has a Poisson(λ) distribution. After that, Fitch flips a fair coin N times independently. If none of the coin flips are tails, Fitch wins, else Sankar wins.

a. If $N = 3$, what is the probability that Fitch wins?

b. What is the probability that Fitch wins? You should simplify your answer as much as possible in this question. (*The final answer may involve λ.*)

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