stat 134 lec 9

Last time Harder hypergeometric problems.

today sec 2.5 Binomial approx to hypergeometric.

sec 3.1 – random variables (RV)
joint distribution of 2 RVs and independence

sec 2.5 Binomial approx to hypergeometric.

Binomial — independent trials (draw with replacement)
Hypergeometric — dependent trials. (draw w/o replacement).

ex 100 person class with a grade distribution:

A grade : 70 students
B grade : 30 students.

Sample 5 students at random w/o replacement (SRS).

Find $P(3A\text{'s}, 2Bs)$

exact
hypergeometric

$$= \frac{\binom{70}{3}\binom{30}{2}}{\binom{100}{5}} = \binom{5}{3}\frac{70}{100}\frac{69}{99}\frac{68}{98}\frac{30}{97}\frac{29}{96} = .316$$

approx
binomial

$$= \binom{5}{3}(.7)^3(.3)^2 = .309$$

When N is large relative to n, $HG(5,100,70) \approx Bin(5,.7)$

why? w/o replacement = w/ replacement.

$$HG(n, N, G) \approx Bin(?) \quad n, \frac{G}{N}$$

Summary of approximations

$HG(n, N, G)$

approx by binomial
N large, n small
$p = \frac{G}{N}$

binomial(n, p)

approx by Poisson
$p \to 0,\ n \to \infty,\ np \to \mu$

Poisson(μ)

approx by normal
n large
$\mu = np,\ \sigma = \sqrt{npq}$
$0 < \mu \pm 3\sigma < n$
use continuity correction

Normal(μ, σ^2)

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Chapter 2 Friday February 7 2019

1. Adam, Jess and Tom are standing in a group of 12 people. The group is randomly split into two lines of 6 people each. The chance that Adam, Jessica, and Tom are standing next each other in one of these lines is:

a $\dfrac{\binom{3}{3}\binom{9}{3}}{\binom{12}{6}} * \dfrac{\binom{4}{1}}{\binom{6}{3}}$

b $2 * \dfrac{\binom{3}{3}\binom{9}{3}}{\binom{12}{6}} * \dfrac{\binom{4}{1}}{\binom{6}{3}}$

c none of the above

Soln

We first find the chance A, J, T are together in line 1 or together in line 2 but not necessarily consecutive.

Out of 12 we take a sample of 6.

A, J, T are good and the other 9 are bad.

we have $\dfrac{\binom{3}{3}\binom{9}{3}}{\binom{12}{6}}$.

Have 2 possible half lines so the chance that A, J, T are in the same half line is $2 \frac{\binom{3}{3}\binom{9}{3}}{\binom{12}{6}}$.

Next we find the chance they are consecutive given that they are in the same half line.

Thinking of AJT as one person there are 4 people in the line and $\binom{4}{1}$ ways to place AJT in the line.

Hence given that AJT are in the same line the chance they are consecutive is $\frac{\binom{4}{1}}{\binom{6}{3,3}}$.

Now apply the multiplication rule,

$$\boxed{2 \cdot \frac{\binom{3}{3}\binom{9}{3}}{\binom{12}{6}} \cdot \frac{\binom{4}{1}}{\binom{6}{3,3}}}.$$

Sec 3.1 Intro to Random Variables (RV)

A RV, X, is the outcome of an experiment.

What distribution is the following RV?

X = The number of aces in 5 cards drawn from a standard deck?

Write $X \sim HG(n=5, N=52, G=4)$

ex flip a prob p coin 2 times

$X = \#$ heads

$X=1$ is an event

$P(X=1) = \binom{2}{1} p^1 (1-p)^2$ binomial formula

we write $X \sim Bin(2, p)$

More precisely,

$X: \Omega \longrightarrow \mathbb{R}$ is a function

outcome space

$HH \longmapsto 2$

$HT \longmapsto 1$

$TH \longmapsto 1$

$TT \longmapsto 0$

so $X=1$ means $\{HT, TH\} \subseteq \Omega$

X has a probability distribution

X	0	1	2
$P(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

P(HT or TH)

You can find the distribution of $g(X) = |X-1|$? — range? — 0, 1

function of a RV

$\lvert X-1\rvert$	0	1
$P(x)$	$\frac{1}{2}$	$\frac{1}{2}$

What distribution is $|X-1|$? $Bin(1, \frac{1}{2})$

Joint Distribution

Let (X, Y) be the joint outcome of 2 RVs X, Y.

The event $(X=x, Y=y)$ is the intersection of events $X=x$ and $Y=y$.

ex

X: one draw from $\boxed{1}\ \boxed{2}\ \boxed{2}\ \boxed{3}$

Given $X=x$, Y = number of heads in x coin tosses.

$$P(X=x, Y=y) = P(Y=y \mid X=x) \cdot P(X=x)$$

$$P(X=1, Y=1) = \underbrace{P(Y=1 \mid X=1)}_{=1/2} \cdot \underbrace{P(X=1)}_{=1/4} = 1/8$$

What the range of values of X? — $1, 2, 3$

Y? — $0, 1, \ldots, x$

Find,

$P(1,0) = \frac{1}{2} \cdot \frac{1}{4} = 1/8$

$P(1,1) = \frac{1}{2} \cdot \frac{1}{4} = 1/8$

$P(2,0) = \frac{1}{4} \cdot \frac{1}{2} = 1/8$

$P(2,1) = \frac{1}{2} \cdot \frac{1}{2} = 1/4$

$P(2,2) = \frac{1}{4} \cdot \frac{1}{2} = 1/8$

$P(3,0) = \frac{1}{8} \cdot \frac{1}{4} = 1/32$

$P(3,1) = \frac{3}{8} \cdot \frac{1}{4} = \frac{3}{32}$

$P(3,2) = \frac{3}{8} \cdot \frac{1}{4} = \frac{3}{32}$

$P(3,3) = \frac{1}{8} \cdot \frac{1}{4} = \frac{1}{32}$

marginal prob of X

$P(x) = \sum_{y \in Y} P(x,y)$

Y \ X	1	2	3	
	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	
3	0	0	$\frac{1}{32}$	$\frac{1}{32}$
2	0	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{7}{32}$
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{32}$	$\frac{15}{32}$
0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{9}{32}$

marginal prob of Y

$P(y) = \sum_{x \in X} P(x,y)$

$X-1 \sim \text{Bin}(2, \frac{1}{2})$ since
$P(X-1=0) = \binom{2}{0}\left(\frac{1}{2}\right)^2 = \frac{1}{4}$
$P(X-1=1) = \binom{2}{1}\left(\frac{1}{2}\right)^2 = \frac{1}{2}$
$P(X-1=2) = \binom{2}{2}\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Y not a named distribution.

Is X, Y dependent? — yes.

ex
$$\left.\begin{aligned} P(Y=0 \mid X=1) &= \tfrac{1}{2} \\ P(Y=0) &= \tfrac{9}{32} \end{aligned}\right\} \Rightarrow X, Y \text{ dep}$$

$\underline{\text{Def}^n}$ Two RVs are <u>independent</u> if

$$P(Y=y \mid X=x) = P(Y=y) \quad \text{for all } x \in X,\ y \in Y$$

By the multiplication rule, if X, Y are indep,

$$P(X=x, Y=y) = \underbrace{P(Y=y \mid X=x)}_{= P(Y=y)} P(X=x)$$

so $\boxed{P(X=x, Y=y) = P(X=x)P(Y=y)}$.

Check X, Y dependent from table above.

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Chapter 3 **Monday February 11 2019**

1. The joint distribution of X and Y is drawn below:

Y \ X	0	1	2	P(y)
1	1/4	1/3	1/12	2/3
0	1/8	1/6	1/24	1/3
P(x)	3/8	1/2	1/8	

(*) you should check all x, y

a X and Y are independent

Chk $(\frac{3}{8})(\frac{2}{3}) = \frac{6}{24} = \frac{1}{4}$
$(\frac{1}{2})(\frac{2}{3}) = \frac{1}{3}$ etc

b $P(X = x|Y = 0) = P(X = x|Y = 1)$, for all x.

$P(X=0|Y=0) = \frac{P(0,0)}{P(Y=0)} = \frac{1/8}{1/3} = 3/8$

$P(X=0|Y=1) = \frac{P(0,1)}{P(Y=1)} = \frac{1/4}{2/3} = 3/8$ ✓

c More than one of the above

d None of the above

(a) ⇒ (b).

This is different from

$P(x,0) = P(x,1)$

we have $P(x,0) = \frac{1}{2}P(x,1)$