Stat 134: Section 23

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Conceptual Review

a. Given the joint density $f_{X,Y}$ of (X, Y) how to compute the density of X?

b. Given the joint density $f_{X,Y}$ of (X, Y) and marginal densities f_X, f_Y and a point y such that $f_Y(y) > 0$ what's the expression for the conditional distribution of X given $Y = y$ (give the density $f_X(x|Y = y)$ as a function of x).

c. Give a relation between $f_{X,Y}(x, y)$, $f_Y(y)$ and $f_X(x|Y = y)$.

Problem 1

Suppose that X, Y is distributed uniformly on the disk of center $(0, 0)$ and of radius 1. Find the conditional distribution of Y given $X = 0$ by carefully compting the density $f_Y(y|X = 0)$.

Problem 2

Suppose (X, Y) are random variables with joint density

$$f(x,y) = \begin{cases} \lambda^3 x e^{-\lambda y} & \text{for } 0 < x < y \\ 0 & \text{otherwise} \end{cases}$$

a. Find the density f_Y of Y and compute $E[Y]$

b. Find the conditional distribution of X given $Y = 1$.

c. Deduce $E[X|Y = 1]$

Ex 6.3.4 in Pitman's Probability

Problem 3

Let X, Y be independent random variables, X is uniform on $(0, 3)$ and Y is Poisson(λ) for some $\lambda > 0$. Find

a. Find $P(X < Y)$ in terms of λ

b. Find the conditional density of X given $X < Y$ (and try sketching its graph for $\lambda = 1, 2, 3$)

c. Compute $E[X|X < Y]$

Ex 6.3.13 in Pitman's Probability