

Stat 134: Section 21

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Conceptual Review

Please discuss these short questions with those around you in section. These problems are intended to highlight concepts that will be relevant for today's problems.

- a. Come up with examples of some random variables dependent on each other.
- b. What is the rule of average conditional probabilities?

Problem 1

Let $N \sim \text{Pois}(\lambda)$, and X be a random variable such that $X|N = n$ is binomial (n, p) distributed.

1. Show that the unconditional distribution of X is Poisson and find the parameter.
2. It is known that X-rays produce chromosome breakages in cells. The number of such breakages usually follows a Poisson distribution quite closely, where the parameter depends on the time of exposure, etc. For a particular dosage and time of exposure, the number of breakages follows the Poisson (0.4) distribution. Assume that each breakage heals with probability 0.2, independently of the others. Find the chance that after such an X-ray, there are 4 healed breakages.

Ex 6.1.7 in Pitman's Probability

Problem 2

Let A_1, \dots, A_{20} be independent events each with probability $1/2$. Let X be the number of events among the first 10 which occur and let Y be the number of events among the last 10 which occur. Find the conditional probability that $X = 5$, given that $X + Y = 12$.

Ex 6.1.4 in Pitman's Probability

Problem 3

Random variables X and Y are called conditionally independent given Z if given the value of Z , X and Y are independent. That is,

$$P(X = x, Y = y | Z = z) = P(X = x | Z = z)P(Y = y | Z = z)$$

for all possible values x , y , and z . Prove that X and Y are conditionally independent given Z if and only if the conditional distribution of Y given $X = x$ and $Z = z$ is a distribution which depends only on z :

$$P(Y = y | X = x, Z = z) = P(Y = y | Z = z)$$

for all possible values x , y , and z . Give a further equivalent condition in terms of the conditional distribution of X given $Y = y$ and $Z = z$.

Ex 6.1.9 in Pitman's Probability