Stat 134: Section 16

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Conceptual Review

Please discuss these short questions with those around you in section. These problems are intended to highlight concepts from lecture that will be relevant for today's problems.

- a. What is an order statistic?
- b. What have we learned in Chapter 4 so far?

Problem 1

Suppose we have a random variable X with continuous and strictly increasing CDF F_X . Find the distribution of $F_X(X)$.

Let $U_{(1)}, \ldots, U_{(n)}$ be the values of n i.i.d. Uniform (0,1) variables arranged in increasing order. For 0 < x < y < 1, find simple formulae for:

- a. $P(U_{(1)} > x, U_{(n)} < y)$
- b. $P(U_{(1)} > x, U_{(n)} > y)$
- c. $P(U_{(1)} < x, U_{(n)} < y)$
- d. $P(U_{(1)} < x, U_{(n)} > y)$

Ex 4.6.3 in Pitman's Probability

Problem 3

Four people agree to meet at a cafe at noon. Suppose each person arrives at a time chosen uniformly at random between 11:45 am and 12:15 pm, independently of the others.

- (a) What is the chance that the first person to arrive at the cafe gets there before 11:50?
- (b) What is the chance that some of the four have not arrived by 12:10?
- (c) Suppose that if all 4 people have shown up before 12:10, the waiter takes their orders as soon as the fourth person arrives. Otherwise, the waiter takes the orders of whoever is there at 12:10 pm. Let *T* represent the time at which the waiter takes their order. Find and sketch the cdf of *T*.

From Ex 4.6.1 in Pitman's Probability