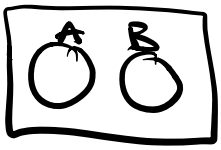
Lec 3 Stat 134

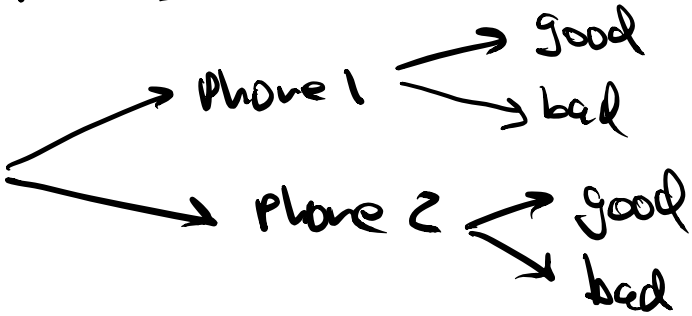
Last time sec 1.4, 1.5

A, B mutually exclusive

A, B independent



$P(AB) = P(A)P(B)$



Bayes rule:

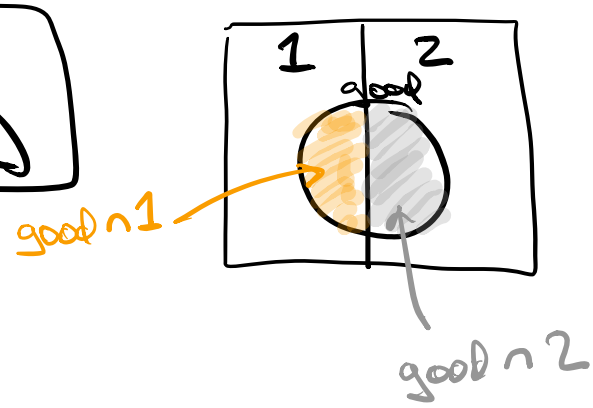
$$P(1 \mid good) = \frac{P(1, good)}{P(good)}$$

$$= \frac{P(1)P(good \mid 1)}{P(good)}$$

Posterior $\propto$ Prior · likelihood

proportional.

$$= \boxed{\frac{P(1)P(good \mid 1)}{P(1)P(good \mid 1) + P(2)P(good \mid 2)}}$$



Today

sec 1.5 Bayes rule

sec 1.6 independence vs. pairwise independence for 3 events

Sec 1.5

ex 5% of population has a disease

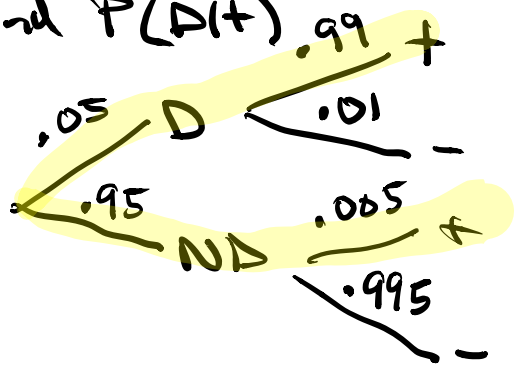
(i.e $P(D) = .05$) — Prior

They take a test

$$\left.\begin{aligned} P(+\mid D) &= .99 \\ P(-\mid ND) &= .995 \end{aligned}\right\} \text{ likelihoods}$$

One person is picked at random from the population. Given test +, what is chance he has the disease? $\sim P(D\mid +)$

- make a tree
- find $P(+)$
- find $P(D\mid +)$



$$P(+) = (.05)(.99) + (.95)(.005) = .0543$$

$$P(D\mid +) = \frac{(.05)(.99)}{.0543} = \boxed{.9124}$$

Stats 134

Chapter 1.4 Monday September 27 2018

1. Suppose A and B are two events with

$$P(A) = 0.5 \text{ and } P(A \cup B) = 0.8.$$

Is it possible for A and B to be both **mutually exclusive** and **independent**?

a yes

b no

c there isn't enough information to decide

If A and B are mutually exclusive then we must have $P(AB)=0$. Since $P(A \cup B)=.8$ and $P(A)=.5$ if A and B are mutually exclusive then $P(A \cup B)=P(A)+P(B) \Rightarrow P(B)=.3$. It is hence impossible for $0=P(AB)=P(A)P(B)$ so A, B can't be independent.

Note that the only way A and B can be both mutually exclusive and independent is for $P(A)$ or $P(B)$ to be zero. This is rare.

ex let A = event a person is 200 year old $(P(A)=0)$
B = event a person is female

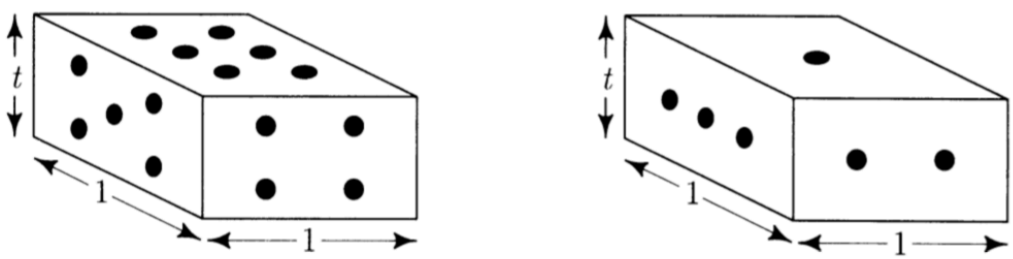
$P(AB)=0$ so A, B mutually exclusive and
$P(AB)=P(A)P(B)=0 \cdot P(B)$ ✓

Recall from section 1.3 the example of different shaped die. We assume that it is equally likely to get a 1 or a 6.

Example 3. **Shapes.**

P 24

A *shape* is a 6-sided die with faces cut as shown in the following diagram:



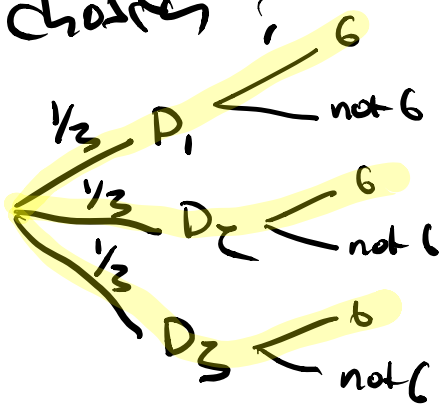
ex 1.5.9

A box contains 3 shaped die D_1, D_2, D_3 with probabilities $1/3, 1/2, 2/3$ respectively of landing flat (with 1,6 on top)

fair die

a) One of the 3 shapes will be drawn at random and rolled. What is the chance the number rolled is 6?

b) Given that 6 is rolled, what is chance the fair die was chosen?



a

$P(D_1) = P(D_2) = P(D_3) = 1/3$

$P(\text{roll } 6 \mid D_1) = \frac{1}{2} \cdot \frac{1}{3} = 1/6$

$P(\text{roll } 6 \mid D_2) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$P(\text{roll } 6 \mid D_3) = \frac{1}{2} \cdot \frac{2}{3} = 1/3$

$$P(\text{roll } 6) = P(\text{roll } 6, D_1) + P(\text{roll } 6, D_2) + P(\text{roll } 6, D_3)$$

$$= P(D_1)P(6 \mid D_1) + P(D_2)P(6 \mid D_2) + P(D_3)P(6 \mid D_3)$$

$$= 1/3 \cdot 1/6 + 1/3 \cdot 1/4 + \frac{1}{3} \cdot 1/3 = \boxed{.25}$$

b

$$P(D_1 \mid 6) = \frac{1/3 \cdot 1/6}{.25} = \boxed{.222}$$

$$\| \quad \frac{P(D_1, 6)}{P(6)}$$

<u>Sec 1.6</u> <u>Independence for 3 events.</u>

We say <u>2</u> events A, B are <u>independent</u> if $P(AB) = P(A)P(B)$.

We say <u>3</u> events A, B, C are <u>independent</u> if $P(ABC) = P(A)P(B)P(C)$.

We say 3 events A, B, C are <u>pairwise independent</u> if

$P(AB) = P(A)P(B)$ and $P(AC) = P(A)P(C)$ and $P(BC) = P(B)P(C)$.

Pairwise independence and independence for 3 events are not equivalent as the following example shows:

ex - 1.6.8

3 people

B_{ij} = person i and j have same b-day.

Lets assume that it is equally likely that a person has a birthday on any of 365 days.

a) Is B_{12} and B_{23} indep?

intuitively B_{12} and B_{13} should be indep since whether person 1 and 2 have the same b-day is indep of person 3's b-day

mathematically?

$$P(B_{12}B_{23}) \overset{?}{=} P(B_{12})P(B_{23})$$

$$P(B_{12}B_{23}) = P(B_{123}) = \frac{365}{365}\cdot\frac{1}{365}\cdot\frac{1}{365} = \left(\frac{1}{365}\right)^2$$

3 have the same b-day.

$P(B_{12}) = 1/365$

$P(B_{23}) = 1/365$

so B_{12}, B_{23} are indep.

b) Is B_{12}, B_{23}, B_{13} indep?

intuitively no since if B_{12} and B_{23} are both true then B_{13} is necessarily true.

mathematically,

$$P(B_{12}B_{23}B_{13}) = P(B_{123}) = \left(\frac{1}{365}\right)^2$$

but $P(B_{12})P(B_{23})P(B_{13}) = \left(\frac{1}{365}\right)^3$

so B_{12}, B_{23}, B_{13} are <u>dependent</u>.

c) Are B_{12}, B_{23}, B_{13} pairwise independent?

Yes since

$$P(B_{12}B_{23}) = P(B_{12})P(B_{23}) = \left(\frac{1}{365}\right)^2$$

$$P(B_{12}P_{13}) = P(B_{12})P(B_{13}) = \left(\frac{1}{365}\right)^2$$

$$P(B_{23}P_{13}) = P(B_{23})P(B_{13}) = \left(\frac{1}{365}\right)^2.$$