Stat 134 lec 3

warmup 11:00 – 11:10

(10 pts) An airport bus drops off 35 passengers at 7 stops. Each passenger is equally likely to get off at any stop, and passengers act independently of one another. The bus makes a stop only if someone wants to get off. Find the probability that the bus drops off passengers at every stop.

Let B_i = event drop off at least one person at stop i.

Find $P(B_1B_2B_3) = 1 - P((B_1B_2B_3)^c)$

$= 1 - P(B_1^c \cup B_2^c \cup B_3^c)$

$$= 1 - \sum_{i=1}^{3} P(B_i^c) + \sum_{i<j} P(B_i^c B_j^c) - P(B_1^c B_2^c B_3^c)$$

$$= 1 - \binom{3}{1}\left(6/7\right)^{35} + \underbrace{\frac{3\cdot 2}{2!}}_{=\binom{3}{2}}\left(5/7\right)^{35} - \binom{3}{3}\left(4/7\right)^{35}$$

Find $P(B_1B_2\cdots B_7)$

$$\binom{7}{0}\left(\frac{7}{7}\right)^{35} - \binom{7}{1}\left(6/7\right)^{35} + \binom{7}{2}\left(5/7\right)^{35} - \cdots - \binom{7}{7}\left(\frac{7-7}{7}\right)^{35}$$

$$\sum_{j=0}^{7} (-1)^j \binom{7}{j}\left(\frac{7-j}{7}\right)^{35}$$

Inclusion–exclusion formula for n events. Derive the inclusion–exclusion formula for n events

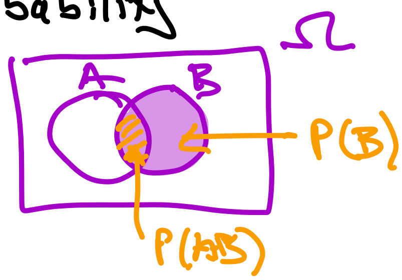
$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_i P(A_i) - \sum_{i<j} P(A_iA_j) + \sum_{i<j<k} P(A_iA_jA_k) - \cdots + (-1)^{n+1} P(A_1 \ldots A_n)$$

last time

Sec 1.4 Conditional Probability

$A|B$ "A given B"

New sample space is B,



$$P(A|B) = \frac{P(AB)}{P(B)}$$ Baye's Rule.

Or $P(AB) = P(A|B)P(B)$ multiplication Rule

$= P(A)P(B)$ (if A and B are independent).

Inclusion Exclusion Rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

ex

Two separate decks of cards are shuffled. What is the chance that the top card of the first deck is the **king** of spades **or** the bottom card of the second deck is the **king** of spades

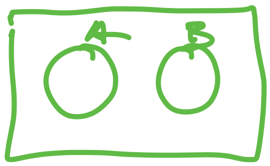
a $\frac{1}{52} + \frac{1}{52} - \frac{1}{52} \times \frac{1}{52}$
b $\frac{1}{52} + \frac{1}{51}$
c $\frac{1}{52} + \frac{1}{52} - \frac{1}{52} \times \frac{1}{51}$
d none of the above

Today

1. Sec 1.4 Mutually Exclusive versus Independent
2. Sec 1.5 Baye's Rule

① Sec 1.4 Mutually Exclusive (ME) versus Independent

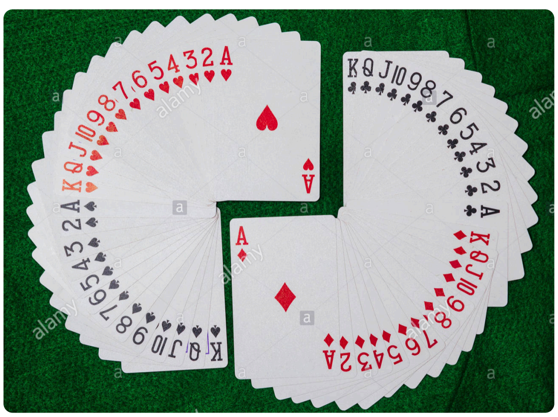
ME: $P(AB) = 0$



Ind: $P(A|B) = P(A)$

eg Consider different kinds of cards

Is red and Heart ME, Ind ?



$$\underbrace{P(R|H)}_{=1} \neq \underbrace{P(R)}_{=1/2} \Rightarrow \text{dep}$$

eg Is red and Spade ME, Ind ?

$$\underbrace{P(R|S)}_{=0} \neq \underbrace{P(R)}_{=1/2} \Rightarrow \text{dep.}$$

If A, B are nonempty sets

if A, B ME is A, B Dependent?

$AB = \phi$

$A, B \neq \phi$

$$P(A|B) = \frac{P(AB)^{=0}}{P(B)_{\neq 0}} = 0 \Rightarrow \text{dep.}$$

$P(A) \neq 0$ not equal

Sec 1.5 Baye's rule

ex A factory produces 2 models of cell phones.

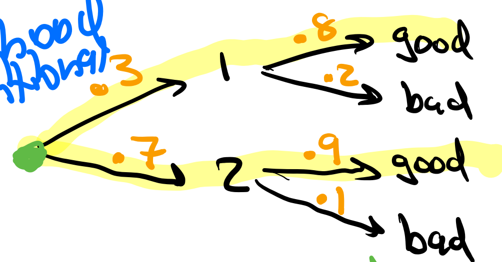
Prior Prob → Given $P(1) = .3$

forward conditional prob or likelihood → $P(\text{good} \mid 1) = .8$

$P(\text{good} \mid 2) = .9$

$P(A \text{ and } B)$
$P(AB)$
$P(A,B)$
$P(A \cap B)$
all the same

conditional



Find $P(1, \text{good})$ ← and $= P(1)P(\text{good} \mid 1) = (.3)(.8) = .24$

$P(\text{good}) = P(1,\text{good}) + P(2,\text{good})$

$= (.3)(.8) + (.7)(.9) = .87$

need Baye's rule. → $P(1 \mid \text{good})$

$$= \frac{P(1,\text{good})}{P(\text{good})} = \frac{.24}{.87} = .28$$

"Backwards conditional"

or

posterior conditional prob, ← proportional

$P(1 \mid \text{good}) \propto P(1) \cdot P(\text{good} \mid 1)$

posterior   prior   likelihood

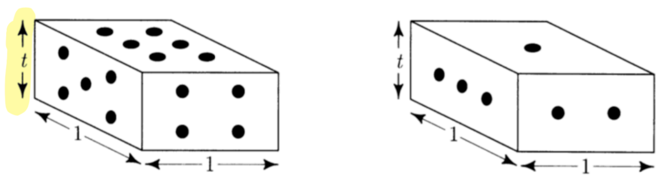
Note $P(1 \mid \text{good}) = \frac{1}{P(\text{good})} \cdot P(1)\, P(\text{good} \mid 1)$

constant.

ex

Shapes.

A *shape* is a 6-sided die with faces cut as shown in the following diagram:



A box contains 3 shaped die (see pic above), D_1, D_2, D_3, with probability $1/3$, $1/2$, $2/3$ respectively of landing flat (with 1 or 6 on top).

Note: the numbers $1/3, 1/2, 2/3$ don't add up to 1 because they are the chance of landing flat for 3 different die.

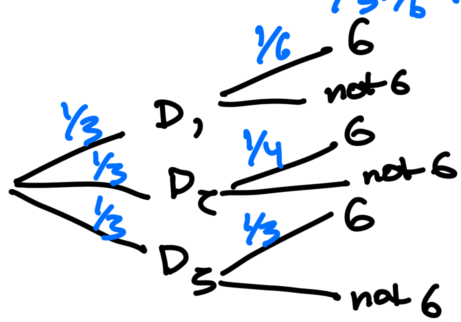
a) what is $P(\text{get } 6 \mid D_1) = \frac{1}{2}\cdot\frac{1}{3} = \frac{1}{6}$ (likelihood)

b) what is $P(\text{get } 6, D_1) = P(\text{get } 6 \mid D_1)\cdot P(D_1)$
$= \frac{1}{6}\cdot\frac{1}{3} = \frac{1}{18}$

c) what is $P(\text{get } 6)$ —

$\frac{1}{3}\cdot\frac{1}{6} + \frac{1}{3}\cdot\frac{1}{4} + \frac{1}{3}\cdot\frac{1}{3}$

$= \boxed{1/4}$



d) Find posterior $P(D_1 \mid 6) = \dfrac{P(D_1, 6)}{P(6)} = \dfrac{1/18}{1/4} = \boxed{2/9}$