Stat 134: Change of Variable and Operations Review Section

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Change of Variable and Operations Review

a. List the steps for finding the density of $Y = g(X)$, when X has a known density.

b. What is the convolution formula for a sum of two random variables: $Z = X + Y$?.

c. What is the general convolution formula for a function of two random variables: $Z = g(X, Y)$?

Problem 1

Suppose a particle is fired from the origin of the (x,y)-plane in a straight line in a direction at a random angle Θ to the x-axis, where $\Theta \sim Unif(0, \pi)$. Let Y be the y-coordinate of the place where the particle crosses either the line $x = 1$ or the line $x = -1$. Find the distribution of Y.

Inspired by ex 4.4.6 in Pitman's Probability

Problem 2

The time before a light bulb begins to flicker is exponentially dis-
tributed with rate 10. However, once it starts to flicker, it continues
to flicker for an additional amount of time before it fully burns out.
This additional amount of time is randomly distributed according to
a standard uniform distribution and is independent of the amount of
time the light bulb took to start flickering. Find the distribution of the
amount of time it takes for the light bulb to fully die out.

Problem 3

Let $V \sim Unif(0,1)$ and $W \sim Gamma(2, \lambda)$.

a. Find the distribution of Z, where $Z = VW$.

b. Find the distribution of $(XY)^V$, where X, Y and V are all indepen-
    dent standard normal random variables.