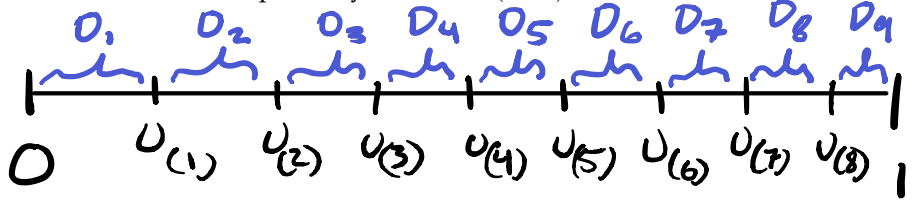
Problem 2

Let X and Y be the minimum and maximum of 8 independent uniform $(0,1)$ random variables respectively. Find $Corr(X,Y)$.



We examine X and Y in terms of the spacings between consecutive order statistics.

Let $D_1 = U_{(1)}$, $D_i = U_{(i+1)} - U_{(i)}$ $(2 \le i \le 7)$,
$D_9 = 1 - U_{(8)}$, as in the diagram above.

It can be shown that $D_1, D_2, \ldots, D_9$ have the same distribution (Ex. 5.2.13, 5.rev.25 for $k = m+1$).

Furthermore, $D_1 + D_2 + \ldots + D_9 = 1$ always.

So by the Conceptual Review, $Corr(D_1, D_9) = \frac{-1}{9-1}$.

We note $X = D_1$, $Y = 1 - D_9$.

$$\therefore \quad Corr(X,Y) = Corr(D_1, 1-D_9) = -Corr(D_1, D_9)$$
$$= -\left(-\frac{1}{9-1}\right) = \boxed{+\frac{1}{8}}.$$