Stat 134: Section 20

Adam Lucas

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Conceptual Review

a. How do we find $P(Y = y)$ from $P(Y = y|X = x)$ and $P(X = x)$?

b. What is the method for finding $\mathbb{E}(Y)$ based on another random variable X?

c. What is Bayes' Rule?

d. *Random variable or constant?* For each of the following, indicate whether it is a random variable or a constant. For instance, $\mathbb{E}(X)$ would be a constant.

  (i) $\mathbb{E}(Y \mid X)$;
  (ii) $\mathbb{E}(Y \mid X = x)$;
  (iii) $\mathbb{E}(\mathbb{E}(Y|X))$;
  (iv) $\mathbb{E}(X|X)$;
  (v) $\mathbb{E}(Y \mid X = Y)$.

Problem 1

Suppose I toss three coins. Some of them land heads and some land tails. Those that land tails I toss again. Let X be the number of heads showing after the first tossing, Y the total number showing after the second tossing, including the X heads appearing on the first tossing. So X and Y are random variables such that $0 \leq X \leq Y \leq 3$ no matter how the coins land. Write out distribution tables and sketch histograms for each of the following distributions:

a. the distribution of X;

b. the conditional distribution of Y given $X = x$ for $x = 0, 1, 2,$;

c. the joint distribution of X and Y;

Ex 6.1.1 in Pitman's Probability

Problem 2

Let $X \sim \text{Geom}(p)$ on $\{1, 2, \ldots, \}$. Let $Y \sim \text{Uniform}\{0, 1, \ldots, X\}$ (that is, given $X = x$, Y is uniformly distributed from 0 to x).

1. Find $\mathbb{E}(Y|X = k)$;
2. Find $\mathbb{E}(Y)$.

Problem 3

Let $X \sim \text{Exponential}(\lambda)$, and let $Y \sim \text{Poisson}(X)$ (that is, given $X = x$, Y follows the Pois (x) distribution).

a. Find $P(X \in dx, Y = y)$;

b. Use (a) to find the unconditional distribution of Y;

c. Given $Y = y$, what is the conditional density of X? (Hint: use Bayes' Rule).

Prepared by Neil Sharma and Yiming Shi