Stat 134 lec 3

last time

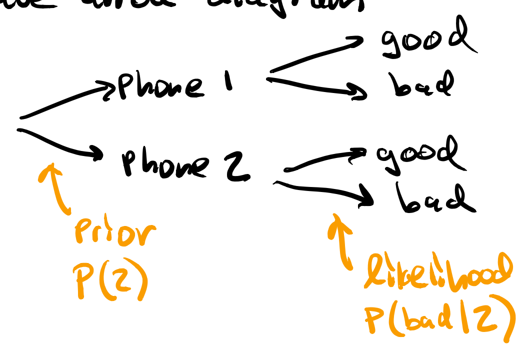
sec 1.4 A, B indep if $P(A|B) = P(A)$

or equivalently $P(AB) = P(A)P(B)$

sec 1.5

A factory produces two models of phones

have arrow diagram



Bayes's rule says $P(2 \mid bad) = \dfrac{P(2, bad)}{P(bad)} = \dfrac{P(2)\,P(bad \mid 2)}{P(bad)}$

Posterior

(updated prior)

Posterior $\propto$ prior $\cdot$ likelihood.

proportional

Today

1) go over student responses from concept test

2) sec 1.4 mutually exclusive vs independence

3) sec 1.5 Bayes rule

① Concept test

A deck of cards is shuffled. What is the chance that the top card is the **king** of spades **or** the bottom card is the **king** of spades

a $\frac{1}{52} \times \frac{1}{51}$
b $\frac{1}{52} + \frac{1}{51}$
c $\frac{1}{52} + \frac{1}{52} - \frac{1}{52} \times \frac{1}{51}$
d none of the above

1/25/201	b	b	For the first draw there is a 1/52 chance the correct card is drawn. If the first card isn't correct and you draw a second card from the remaining pool of cards then the chance of drawing the correct card is 1/51	

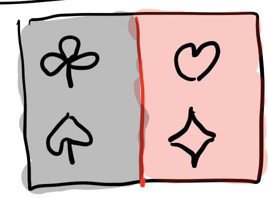
$P(A \cup B) = P(A) + P(B)$

1/25/2019	d	d	with replacement b is correct. without replacement d is correct	

1/25/201	c	c	i didnt have a neighbor :(	

1/25/2019	d	d	Since they are both the same cards and cannot happen at the same time, it should be 1/52 + 1/52	

2) Sec 1.4 Mutually exclusive (ME) vs independence



Note red cards — hearts, diamonds
black cards — clubs, spades

dependent $P(A|B) \neq P(A)$
ME $P(AB) = 0$

Draw a card from a deck of cards.
~~ex~~ let A = card is red
B = card is a heart.

$\underset{1}{\underset{\shortparallel}{P(A|B)}} \neq \underset{1/2}{\underset{\shortparallel}{P(A)}}$ so dep.

A, B	ME	Dep
red, heart	$P(AB) = P(B) \neq 0$ ✗	✓
red, ace	$P(AB) = \frac{2}{52} \neq 0$ ✗	$\underset{1/2}{\underset{\shortparallel}{P(A|B)}} = \underset{1/2}{\underset{\shortparallel}{P(A)}}$ ✗
red, spade	$P(AB) = 0$ ✓	$\underset{0}{\underset{\shortparallel}{P(A|B)}} \neq \underset{1/2}{\underset{\shortparallel}{P(A)}}$ ✓
green, ace	$P(AB) = 0$ ✓	$\underset{0}{\underset{\shortparallel}{P(A|B)}} = \underset{0}{\underset{\shortparallel}{P(A)}}$ ✗

Note $P(\text{ace} | \text{green})$ doesn't make sense since green is the empty set and you can't condition on the empty set.

The only way you can get A and B to be ME and indep is for A or B to be impossible.

Stats 134

Chapter 1.4 Monday January 28 2019

1. Suppose A and B are two events with

$$P(A) = 0.5 \text{ and } P(A \cup B) = 0.8.$$

Is it possible for A and B to be both **mutually exclusive** and **independent**?

a yes
b no
c there isn't enough information to decide

if ME, $P(AB) = 0$ so $\underset{\substack{\| \\ .8}}{P(A \cup B)} = .5 + P(B)$

$\Rightarrow P(A) = .5$
$P(B) = .3$

but $\underset{\substack{\| \\ 0}}{P(AB)} \neq (.5)(.3)$

2. Suppose A and B are two events with

$$P(A) = 0.8 \text{ and } P(A \cup B) = 0.8.$$

Is it possible for A and B to be both **mutually exclusive** and **independent**?

a yes

b no

c there isn't enough information to decide

Here $P(B)=0$ so possible to have A, B both mutually exclusive and indep.

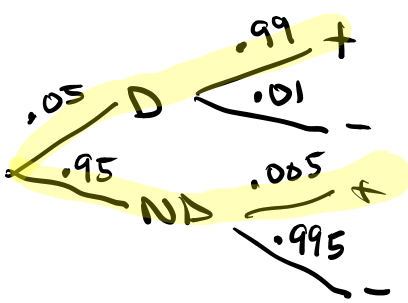
③ Sec 1.5

ex 5% of the population has a disease.
(i.e $P(D) = .05$) — prior
They take a test

$$\left.\begin{aligned} P(+ \mid D) &= .99 \\ P(- \mid ND) &= .995 \end{aligned}\right\} \text{ likelihoods}$$

One person is picked at random from the population. Given test +, what is chance he has the disease? ~ $P(D \mid +)$

- make a tree
- find $P(+)$
- find $P(D \mid +)$



$P(+) = (.05)(.99) + (.95)(.005) = .0543$

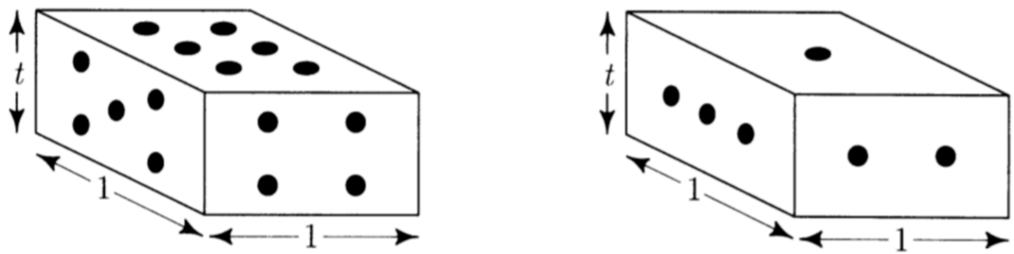
$$\underset{\text{Posterior}}{P(D \mid +)} = \frac{\overset{\text{prior}}{(.05)}\overset{\text{likelihood.}}{(.99)}}{.0543} = \boxed{.9124}$$

Recall from section 1.3 the example of different shaped die. We assume that it is equally likely to get a 1 or a 6.

Example 3. **Shapes.**

P 24

A *shape* is a 6-sided die with faces cut as shown in the following diagram:



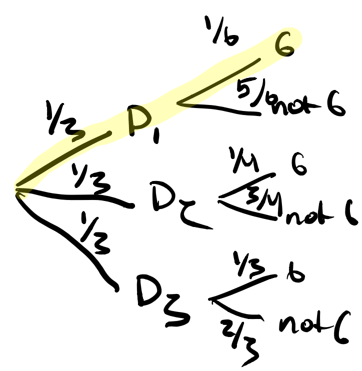
ex — 1.5.9

A box contains 3 shaped die, D_1, D_2, D_3 with probabilities $1/3$, $1/2$, $2/3$ respectively of landing flat (with 1 or 6 on top).

fair die ($t=1$)

a) One of the 3 shapes will be drawn at random and rolled. What is the chance the number rolled is 6?

Can you make a tree?



a

$$P(D_1) = P(D_2) = P(D_3) = 1/3$$

$$P(\text{roll } 6 \mid D_1) = \frac{1}{2}\cdot\frac{1}{3} = 1/6$$

$$P(\text{roll } 6 \mid D_2) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$$

$$P(\text{roll } 6 \mid D_3) = \frac{1}{2}\cdot\frac{2}{3} = 1/3$$

$$P(\text{roll } 6) = P(\text{roll } 6, D_1) + P(\text{roll } 6, D_2) + P(\text{roll } 6, D_3)$$

$$= P(D_1)P(6 \mid D_1) + P(D_2)P(6 \mid D_2) + P(D_3)P(6 \mid D_3)$$

$$= 1/3 \cdot 1/6 + 1/3 \cdot 1/4 + \frac{1}{3}\cdot 1/3 = \boxed{.25}$$

b) Given that 6 is rolled, what is the chance the fair die was chosen?

<u>i.e</u> find the posterior $P(D_1|6)$

$$P(D_1|6) = \frac{1/3 \cdot 1/6}{.25} = \boxed{.222}$$

$$\| $$

$$\frac{P(D_1,6)}{P(6)}$$