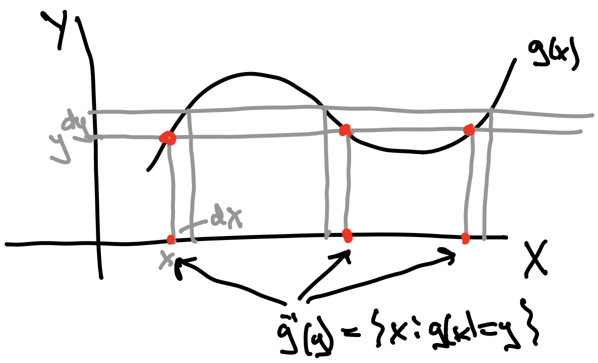
Stat 134 lec 24

last time

sec 4.4 change of variable formula for density of $Y = g(X)$

many to one g:



addition rule

$$P(Y \in dy) \overset{\text{addition rule}}{=} \sum_{x \in g^{-1}(y)} P(X \in dx) = \sum_{x \in g^{-1}(y)} f_X(x)|dx|$$

$$\| \quad f_Y(y)|dy|$$

$$\boxed{\Rightarrow f_Y(y) = \sum_{x \in g^{-1}(y)} f_X(x)\left|\frac{dx}{dy}\right| = \sum_{x \in g^{-1}(y)} f_X(x) \cdot \frac{1}{\left|\frac{dy}{dx}\right|} \quad \left|\frac{dy}{dx}\right| = |g'(x)| \quad \text{where } x \in g^{-1}(y).}$$

Today

1. Review student explanations of concept test.
2. sec 4.5 Cummulative density functions (CDF)

use describe distribution

1

1. Let V be a standard normal RV. The distribution of $X = V^2$ is?

   a Gamma
   b Uniform
   c Normal
   d none of the above

$$f(x) \propto \left. 2 \cdot \frac{e^{-\frac{1}{2}v^2}}{|2v|} \right|_{v = \pm\sqrt{x}}$$

$$= x^{\overbrace{-1/2}^{r-1}} e^{-\overbrace{\tfrac{1}{2}}^{\lambda} x}$$

Concept test

a	a	V=\sqrt{x}, we get f_X(x) \propto e^{-\frac{x}{2}}
a	a	This is a change of variable from V=Norm(0,1) to X=V^2. Plugging in you see that the variable part consists of e^(-x/2)/sqrrootX. This matches gamma.

c	c	
d	d	You get standard normal divided by V
c	c	
d	d	
d	d	
c	c	The distribution of a normal variable squared is still normal probably I hope
c	c	
d		
c	c	V is normal so is the X
d		
c	c	
d	d	
a	d	It is like a gamma distribution and gaussian distribution mixed.
b	c	
d	d	
c	c	
c	c	
c	c	Plug in to change of variable formula to get variable part of e^(-y/2) => normal
a	c	
d		density is incosistent with a-c
c	c	After changing scale y still follow normal distribution
d	d	X to the -1/2 times e to the -x squared/2
b	b	E(Y)=E(X^2)=1, Var(Y)=0
d		
a		Without constants f(x) = x^(-1/2)*e^-(2x) which is the game distribution
c	c	
c	c	
c	c	
c	c	
b	b	
c	c	If we have a normal distribution and square our RV then we will see that it stays normal

② Sec 4.5 Cumulative Distribution Function (CDF)

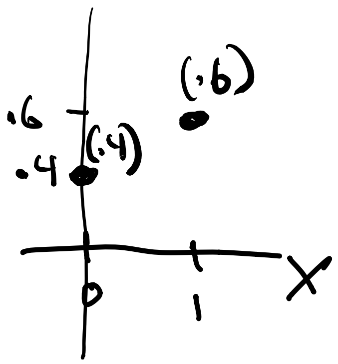
defn X RV

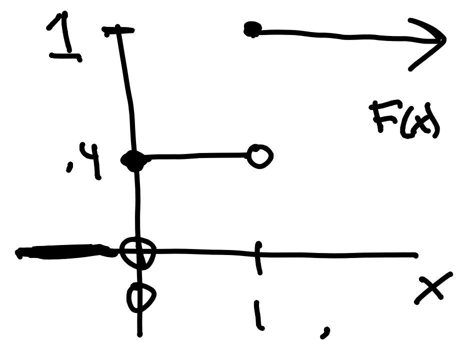
$$F_X(x) = P(X \le x)$$

Use Describes a distribution

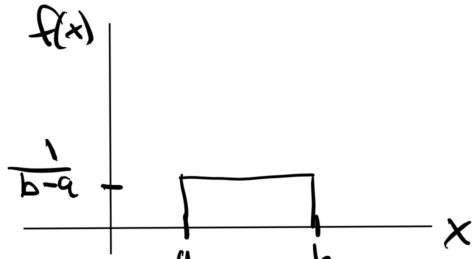
(equivalent to a density or probability mass function)

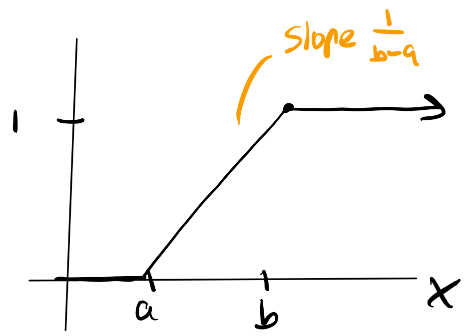
ex $X \sim \text{Bernoulli}(p=.6)$



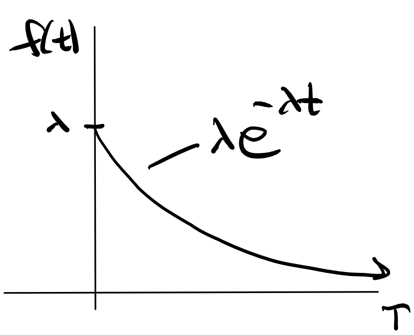


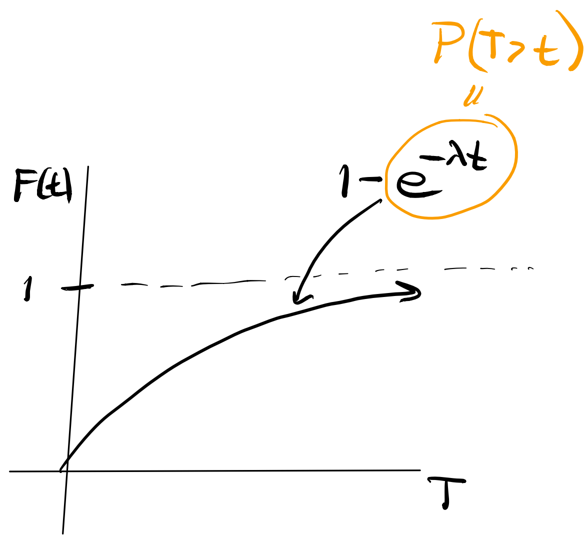
ex $X \sim \text{Unif}(a,b)$





ex $T \sim \text{Exp}(\lambda)$





ex (Mixed distribution)

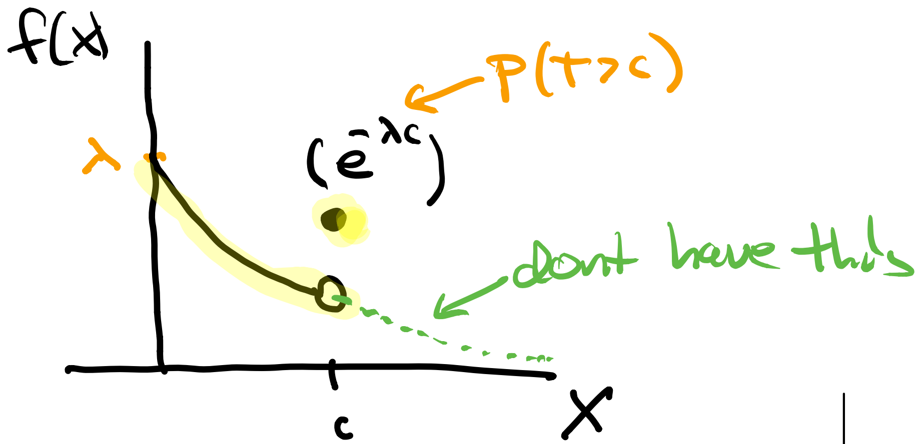
$T \sim Exp(\lambda)$

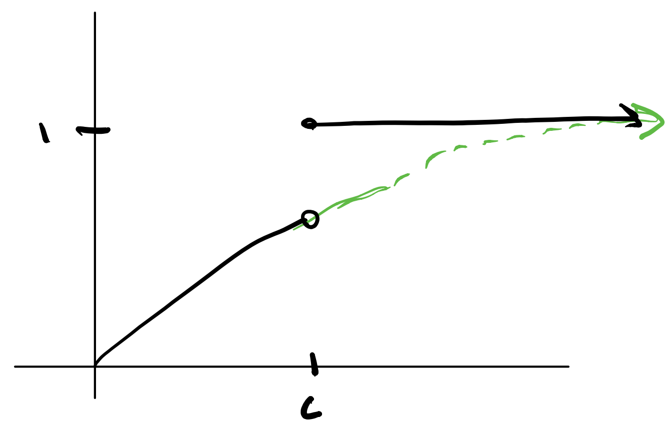
$c > 0$

$$X = \begin{cases} T & \text{if } x < c \\ c & \text{if } x = c \end{cases},$$

$X = \min(T, c)$

"T killed by c"

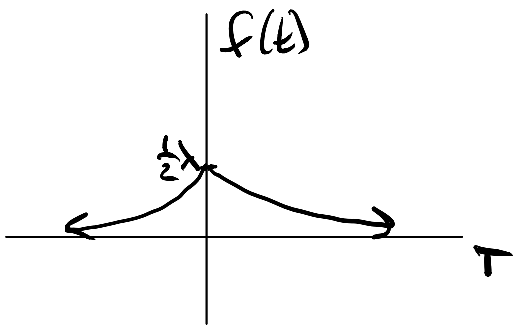




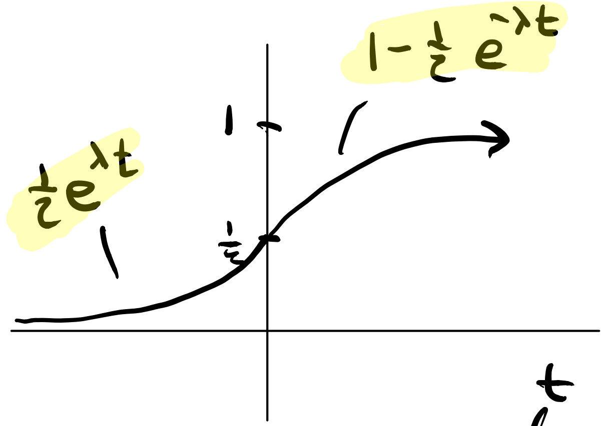
ex

Find the cdf of T where $f(t) = \frac{1}{2}\lambda e^{-\lambda|t|}$

double exponential



You should be able to draw the rough shape of $F(t)$.



$$t<0: \; P(T \le t) = \int_{-\infty}^{t} \tfrac{1}{2}\lambda e^{\lambda t}\, dt = \tfrac{1}{2} e^{\lambda t}$$

$$t>0: \; P(T \le t) = \frac{1}{2} + \int_{0}^{t} \tfrac{1}{2} e^{-\lambda t}\, dt = \tfrac{1}{2} + \tfrac{1}{2}\left(1 - e^{-\lambda t}\right)$$

$$= 1 - \tfrac{1}{2} e^{-\lambda t}$$

Note

$$\lim_{t \to 0^-} F'(t) = \lim_{t \to 0} \frac{\lambda}{2} e^{\lambda t} = \frac{\lambda}{2}$$

$$\lim_{t \to 0^+} F'(t) = \lim_{t \to 0} \frac{\lambda}{2} e^{-\lambda t} = \frac{\lambda}{2}$$

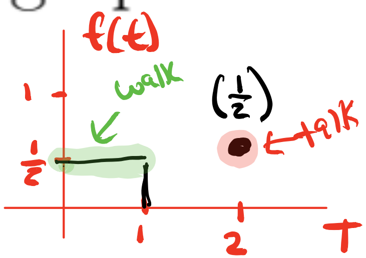
So there is no "kink" in the CDF at $t=0$.

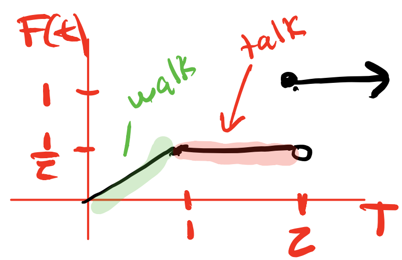
Solution

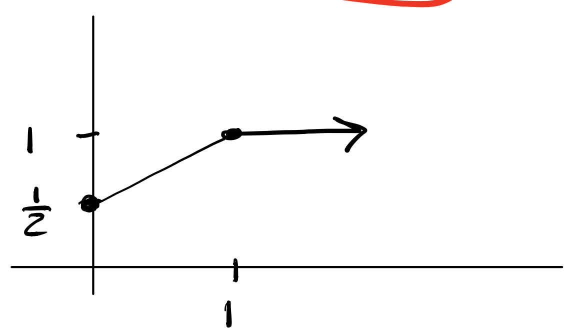
1

Concept Test

Suppose you are trying to discretly leave a party. Your time to leave is uniform from 0 to 2 minutes. However, if your walk to the exit takes more than 1 minute, you run into a friend at the door and and must spend the full 2 minutes to leave. Let T represent the time it takes you to leave. True or false, the graph of the cdf of T is:







Soln

Suppose stop lights at an intersection alternately show green for one minute, and red for one minute (no yellow). Suppose a car arrives at the lights at a time distributed uniformly from 0 to 2 minutes. Let X be the delay of the car at the lights (assuming there is only one car on the road). True or false, the graph of the cdf of X is: