warmup 1:00 - 1:10

Two separate decks of cards are shuffled. What is the chance that the top card of the first deck is the **king** of spades **or** the bottom card of the second deck is the **king** of spades

P(KS top 1^{st} deck)

P(KS bot 2^{nd} deck)

a $\frac{1}{52}+\frac{1}{52}-\underbrace{\frac{1}{52}\times\frac{1}{52}}$ KS top 1^{st} deck and KS bot 2^{nd} deck.

b $\frac{1}{52}+\frac{1}{51}$

c $\frac{1}{52}+\frac{1}{52}-\frac{1}{52}\times\frac{1}{51}$

d none of the above

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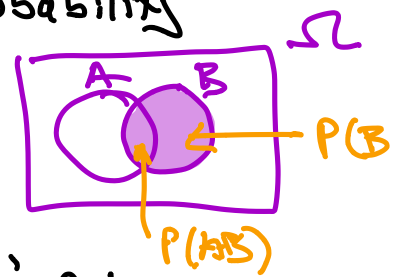
last time

Sec 1.4 Conditional Probability

$A|B$ "A given B"



New sample space is B.

$P(A|B) = \frac{P(AB)}{P(B)}$ Baye's Rule.

Or $P(AB) = P(A|B)P(B)$ multiplication Rule

$= P(A)P(B)$ (if A and B are independent).

Inclusion Exclusion Rule: unconditional prob

$P(A \text{ or } B) = P(A) + P(B) - P(AB)$

ex $P(1KS \text{ or } 2QS) = P(1KS) + P(2QS) - P(1KS \text{ and } 2QS)$

$\qquad\qquad\qquad\qquad\quad \text{"} \quad 1/52 \qquad \text{"} \quad 1/52 \qquad\qquad \text{"} \quad 1/52 \cdot 1/51$

Today

1. Student responses to Concept test
2. Sec 1.4 Mutually Exclusive versus Independent
3. Sec 1.5 Baye's Rule

1. A deck of cards is shuffled. What is the chance that the top card is the **king** of spades **or** the bottom card is the **king** of spades

P(1KS and 52^{nd} KS) = 0

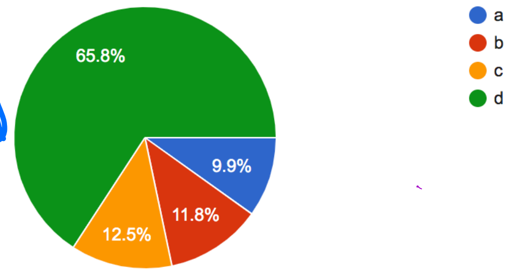
a $\frac{1}{52}+\frac{1}{52}-\overbrace{\frac{1}{52}\times\frac{1}{52}}$

b $\frac{1}{52}+\frac{1}{51}$ — $\frac{1}{52}$

c $\frac{1}{52}+\frac{1}{52}-\frac{1}{52}\times\frac{1}{51}$

d none of the above

correct if K of spades and Q of spades.



a	inclusion exclusion rule

b	When you take the top card, there is a 1/52 chance of picking the K ♠. After taking the top, you would add the probability of taking the second card which is conditional that the first has already been taken (1/51). The two instances are mutually exclusive so there shouldn't be any subtraction. Since it's an "or" problem, add the two probabilities

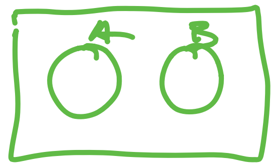
c	no one spoke a word

c	The probability of the top card being king is 1/52. The probability of the bottom card being king is 1/52 as well. The probability of both cards being kings is 1/52 * 1/51. So by the inclusion exclusion principle, c is the answer

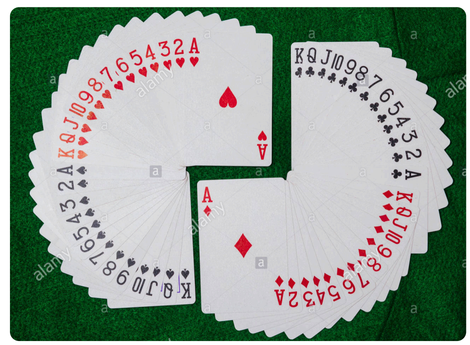
d	These events are disjoint so it's simply the sum of their individual probability

② Sec 1.4 Mutually Exclusive (ME) versus Independent

ME: $P(AB) = 0$



Ind: $P(A|B) = P(A)$



eg Consider different kinds of cards

Is red and Heart ME ✗ , Ind ✗ ?

hearts can be red.

$P(R|H) \neq P(R)$

$= 1 \qquad = 1/2$

or $P(H|R) \neq P(H)$

$= 1/2 \qquad = 1/4$

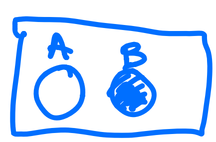
Is red and Spade ME ✓ , Ind ✗ ?

$P(R|S) \neq P(S)$

$= 0 \qquad = 1/4$

If A, B are nonempty sets

A, B ME $\Rightarrow$ A, B Dependent.



if $AB = \emptyset$, $P(A|B) = 0$ and $P(A) \neq 0$

not equal so dependence.

Sec 1.5 Bayes's rule

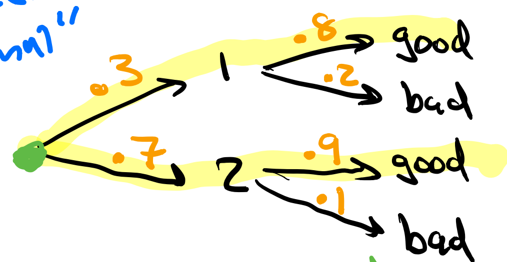
ex A factory produces 2 models of cell phones.

Given $P(1) = .3$

forward conditional prob or "likelihood conditional" → $P(\text{good} \mid 1) = .8$

$P(\text{good} \mid 2) = .9$

$P(A \text{ and } B)$, $P(AB)$, $P(A,B)$, $P(A \cap B)$ — all the same



Find $P(1, \text{good}) = P(1)P(\text{good} \mid 1) = (.3)(.8) = .24$

and

$P(\text{good}) = P(1,\text{good}) + P(2,\text{good})$

$= (.3)(.8) + (.7)(.9) = .87$

$P(1 \mid \text{good}) = \dfrac{P(1,\text{good})}{P(\text{good})} = \dfrac{.24}{.87} = .28$

needs Bayes rule "backwards conditioning" or posterior conditional prob,

$P(1 \mid \text{good}) = \dfrac{P(1,\text{good})}{P(\text{good})} = \dfrac{1}{P(\text{good} \mid 1)\cdot P(1) + P(\text{good} \mid 2)P(2)}$

$P(\text{good} \mid 1)\cdot P(1)$

Proportionality constant

Proportional

$P(1 \mid \text{good}) = .28 \;\propto\; P(1) = .3 \cdot P(\text{good} \mid 1) = .8$

posterior conditional probability

prior

likelihood conditional probability

Sec 1.5

Recall from section 1.3 the example of different shaped die. We assume that it is equally likely to get a 1 or a 6.

Example 3. **Shapes.**

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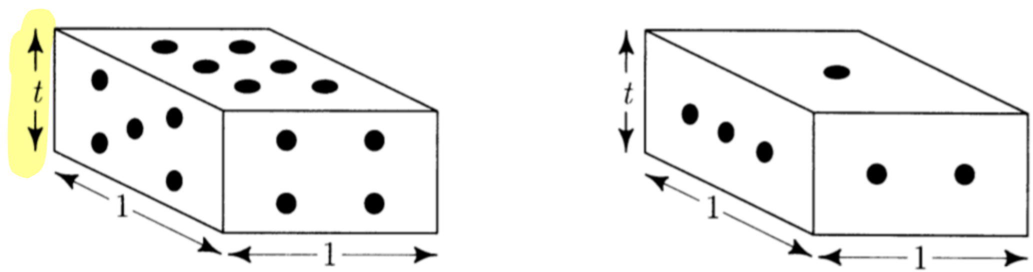
A *shape* is a 6-sided die with faces cut as shown in the following diagram:



ex — 1.5.9 A box contains 3 shaped die, D_1, D_2, D_3 with probabilities $1/3$, $1/2$, $2/3$ respectively of landing flat (with 1 or 6 on top).

↖ fair die (t=1)

Note the numbers $1/3, 1/2, 2/3$ dont add up to 1 because they are the chance of landing flat for 3 different die.

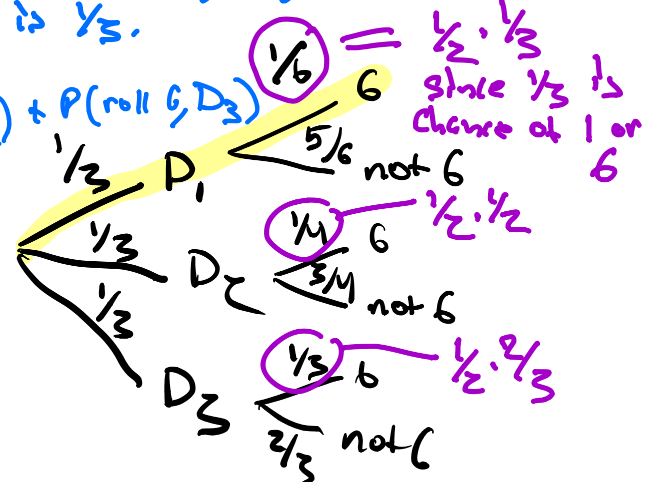
a) One of the 3 shapes will be drawn at random and rolled. What is the chance the number rolled is 6?

There are 3 different shaped die, D_1, D_2, D_3 in a hat. The chance we choose one is $1/3$.

$$P(\text{roll } 6) = P(\text{roll } 6, D_1) + P(\text{roll } 6, D_2) + P(\text{roll } 6, D_3)$$

$$= \tfrac{1}{3}\cdot\tfrac{1}{6} + \tfrac{1}{3}\cdot\tfrac{1}{4} + \tfrac{1}{3}\cdot\tfrac{1}{3}$$

$$= \boxed{1/4}$$



b) Given that 6 is rolled, what is the chance the fair die was chosen?

i.e find the posterior $P(D_1 | 6)$

$$P(D_1|6) = \frac{P(D_1, 6)}{P(6)} = \frac{\tfrac{1}{3}\cdot\tfrac{1}{6}}{1/4} = \boxed{.222}$$

$$= \frac{P(6|D_1)\cdot P(D_1)}{P(6)}$$