Stat 134: Section 18

Adam Lucas

November 18th, 2019

Conceptual Review

a. Suppose $X, Y \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0,1)$. Identify the distribution of:

(i) X^2

(ii) $X^2 + Y^2$

(iii) $\sqrt{X^2 + Y^2}$

(iv) $4X + 3Y + 5$

b. Let X, Y have joint density $f_{X,Y}$. How do we find the density of $Z = X + Y$? (Write out the formula.)

Problem 1

X, Y are i.i.d. standard Normal variables. Find (without integration):

a. $P(X > 3Y + 2)$;

b. $P(0 < X < Y)$;

c. $P(|\min\{X, Y\}| < 1)$

For (b) and (c), it may be helpful to draw the relevant region in the plane.

Adapted from Ex 5.3.3, 5.3.6 in Pitman's Probability

Problem 2: Competing Exponentials

Suppose $X \sim \text{Exp}\,(\lambda_X)$, $Y \sim \text{Exp}\,(\lambda_Y)$, and X, Y are independent.

a. Find $P(X < Y)$.

b. Now suppose $\lambda_X = \lambda_Y = \lambda$. Using part (a), find the density of $Z = X/Y$. (Hint: look at the CDF of Z.)

c. By a similar process as in (b), find the distribution of $W = \frac{X}{X+Y}$. (Simplify F_W, and you should recognize W as one of our famous distributions.)

Problem 3

Suppose $X \sim \text{Unif}\,(0, 2)$, and $Y \sim \text{Unif}\,(0, 1)$. Find the density of $Z = X + Y$, using:

a. the convolution formula (draw a picture as well);

b. the C.D.F. of Z.

Prepared by Neil Sharma and Yiming Shi