Stat 134: Section 3

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Problem 1

The fraction of persons in a population who have a certain disease is 0.01. A diagnostic test is available to test for the disease. But for a healthy person the chance of being falsely diagnosed as having the disease is 0.05, while for someone with the disease the chance of being falsely diagnosed as healthy is 0.2. Suppose the test is performed on a person selected at random from the population.

a. What is the probability that the test shows a positive result (meaning the person is diagnosed as diseased, perhaps correctly, perhaps not)?

Drawing a diagram to illustrate the problem more clearly would probably help.

b. What is the probability that the person selected at random is one who has the disease but is diagnosed healthy?

c. What is the probability that the person is correctly diagnosed and is healthy?

d. Suppose the test shows a positive result. What is the probability that the person tested actually has the disease?

Ex 1.5.5 in Pitman's Probability

Problem 2

Suppose that the birthday of each of three people is equally likely to be anyone of the 365 days of the year, independently of others. Let B_{ij} denote the event that person i has the same birthday as person j, where the labels i and j may be 1, 2, or 3.

a. Are the events B_{12} and B_{23} independent?

b. Are the events B_{12} , B_{23} , and B_{13} independent?

c. Are the events B_{12} , B_{23} , and B_{13} pairwise independent?

Ex 1.6.8 in Pitman's Probability

To answer the question "Are A and B independent?", first ask yourself this simpler question: If I know something about A, do I know *anything* about B? If the answer is yes, then A and B are **not** independent.

Problem 3

The Birthday Problem

Suppose there are n students in a class. What is the probability that at least two students in the class have the same birthday?

Bonus Question: Give an expression, that is numerically simple to evaluate, to estimate this probability. (i.e. For any value of n, you should be able to plug this expression into a simple calculator and get an answer that is *close* to the true probability.)

Try solving the problem for small values of n (e.g. $n = 3, 4, 5, \ldots$) to build some intuition about the problem.

See if you can spot a pattern in your solutions for these smaller problems and then try to extend that pattern to arrive at a solution for the general problem

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