stat 134 lec9

last time sec 2.5 harder hypergeometric and counting problems.

today sec 2.5 Binomial approx to hypergeometric.

sec 3.1 — random variables (RV)
— joint distribution of 2 RVs and independence

sec 2.5 Binomial approx to hypergeometric.

Binomial — independent trials (draw with replacement)
Hypergeometric — dependent trials. (draw w/o replacement).

ex 100 person class with a grade distribution:

A grade : 70 students
B grade : 30 students.

Sample 5 students at random w/o replacement (SRS).

Find $P(3A's, 2Bs)$

exact hypergeometric $= \dfrac{\binom{70}{3}\binom{30}{2}}{\binom{100}{5}} = \binom{5}{3}\dfrac{70}{100}\dfrac{69}{99}\dfrac{68}{98}\dfrac{30}{97}\dfrac{29}{96} = .316$

approx binomial $= \binom{5}{3}(.7)^3(.3)^2 = .309$

When N is large relative to n, $Hyper(100, 70, 5) \approx Bin(5, .7)$

why? w/o replacement = w/ replacement.

$Hyper(N, G, n) \approx Bin(?)$ $n, \frac{G}{N}$

Summary of approximations

hypergeometric (N, G, n)

↓ approx by binomial
N large, n small
$p = \frac{G}{N}$

binomial (n, p)

↙ approx by Poisson
$p \to 0$, $n \to \infty$, $np \to \mu$

Poisson(μ)

↘ approx by normal
n large
$\mu = np$, $\sigma = \sqrt{npq}$
$0 < \mu \pm 3\sigma < n$
use continuity correction

normal(μ, σ^2)

Sec 3.1 Intro to Random Variables (RV)

A RV, X, is the outcome of an experiment.

ex flip a prob p coin 2 times

$X =$ # heads

$X = 1$ is an event

$P(X=1) = \binom{2}{1} p^1 (1-p)^2$ binomial formula

we write $X \sim \text{Bin}(2, p)$

More precisely, outcome space

$X : \Omega \longrightarrow \mathbb{R}$ is a function

$$\begin{aligned} HH &\longmapsto 2 \\ HT &\longmapsto 1 \\ TH &\longmapsto 1 \\ TT &\longmapsto 0 \end{aligned}$$

So $X=1$ means $\{HT, TH\} \subseteq \Omega$

X has a probability distribution

X	0	1	2
$P(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
$g(X)=\lvert X-1\rvert$	1	0	1

$P(HT \text{ or } TH)$

function of a RV

You can find the distribution of $g(X)=|X-1|$ — range? — 0, 1

$\lvert X-1\rvert$	0	1
$P(x)$	$\frac{1}{2}$	$\frac{1}{2}$

What distribution is $|X-1|$? — $\text{Bin}(1, \frac{1}{2})$

Joint Distribution

Let (X, Y) be the joint outcome of 2 RVs X, Y.

The event $(X=x, Y=y)$ is the intersection of events $X=x$ and $Y=y$.

ex

X: one draw from $\boxed{1}\ \boxed{2}\ \boxed{2}\ \boxed{3}$

Given $X=x$, Y = number of heads in x coin tosses.

$$P(X=x, Y=y) = P(Y=y \mid X=x) \cdot P(X=x)$$

$$P(X=1, Y=1) = \underbrace{P(Y=1 \mid X=1)}_{= 1/2} \cdot \underbrace{P(X=1)}_{= 1/4} = 1/8$$

What the range of values of X? — 1, 2, 3

Y? — 0, 1, 2, 3

Find,

$P(1,0) = \frac{1}{2} \cdot \frac{1}{4} = 1/8$

$P(1,1) = \frac{1}{2} \cdot \frac{1}{4} = 1/8$

$P(2,0) = \frac{1}{4} \cdot \frac{1}{2} = 1/8$

$P(2,1) = \frac{1}{2} \cdot \frac{1}{2} = 1/4$

$P(2,2) = \frac{1}{4} \cdot \frac{1}{2} = 1/8$

$P(3,0) = \frac{1}{8} \cdot \frac{1}{4} = 1/32$

$P(3,1) = \frac{3}{8} \cdot \frac{1}{4} = \frac{3}{32}$

$P(3,2) = \frac{3}{8} \cdot \frac{1}{4} = \frac{3}{32}$

$P(3,3) = \frac{1}{8} \cdot \frac{1}{4} = \frac{1}{32}$

marginal prob of X

$P(x) = \sum_{y \in Y} P(x,y)$

Y \ X	1	2	3	
	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	
3	0	0	$\frac{1}{32}$	$\frac{1}{32}$
2	0	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{7}{32}$
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{32}$	$\frac{15}{32}$
0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{9}{32}$

marginal prob of Y

$P(y) = \sum_{x \in X} P(x,y)$

$X-1 \sim Bin(2, \frac{1}{2})$ since

$P(X-1=0) = \binom{2}{0}\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$P(X-1=1) = \binom{2}{1}\left(\frac{1}{2}\right)^2 = \frac{1}{2}$

$P(X-1=2) = \binom{2}{2}\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Y not a named distribution.

Is X, Y dependent? — yes.

ex
$$\left.\begin{array}{l} P(Y=0 \mid X=1) = \frac{1}{2} \\ P(Y=0) = 9/32 \end{array}\right\} \Rightarrow X, Y \text{ dep}$$

Def^n Two RVs are <u>independent</u> if

$$P(Y=y \mid X=x) = P(Y=y) \quad \text{for all } x \in X, \; y \in Y$$

By the multiplication rule,

if X, Y are indep,

$$P(X=x, Y=y) = \underbrace{P(Y=y \mid X=x)}_{= P(Y=y)} P(X=x)$$

so $\boxed{P(X=x, Y=y) = P(X=x)P(Y=y)}$.

Check X, Y dependent from table above.

Stat 134

Chapter 3 Monday February 11 2019

1. The joint distribution of X and Y is drawn below:

Y \ X	0	1	2	P(y)
1	1/4	1/3	1/12	2/3
0	1/8	1/6	1/24	1/3
P(x)	3/8	1/2	1/8	

a X and Y are independent

Chk $(3/8)(2/3) = 6/24 = 1/4$
$(1/2)(2/3) = 1/3$ etc ✓

b $P(X = x|Y = 0) = P(X = x|Y = 1)$, for all x.

$P(X=0|Y=0) = \frac{P(0,0)}{P(Y=0)} = \frac{1/8}{1/3} = 3/8$

$P(X=0|Y=1) = \frac{P(0,1)}{P(Y=1)} = \frac{1/4}{2/3} = 3/8$ ✓

c More than one of the above

d None of the above

(a) $\Rightarrow$ (b).

This is different from $P(x,0) = P(x,1)$

we have $P(x,0) = \frac{1}{2}P(x,1)$

2. A fair coin is tossed twice.
Let $X = \#$ heads on the first toss.
Let $Y = \#$ heads on the first 2 tosses.

	$\frac{1}{2}$	$\frac{1}{2}$	$P(x)$
			$P(y)$
2	impos. 0	HH $\frac{1}{4}$	$\frac{1}{4}$
1	TH $\frac{1}{4}$	HT $\frac{1}{4}$	$\frac{1}{2}$
0	TT $\frac{1}{4}$	impos. 0	$\frac{1}{4}$
Y \ X	0	1	

a The table above is correct — yes ex $P(0,0)$ is chance you get TT

b $Y \sim Bin(2, \frac{1}{2})$

Y	0	1	2
$P(y)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

c More than one of the above

d None of the above