Stat 134: Section 22

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Conceptual Review

a. What are the properties of PDF and CDF?

b. What is the memoryless property of exponential distribution?

c. What is the relation of T_r, W_r and N_t? What are their distributions?

d. What is the distribution of order statistics $X_{(k)}$?

Problem 1

Electrical components of a particular type have exponentially distributed lifetimes with mean 48 hours. In one application the component is replaced by a new one if it fails before 48 hours, and in case it survives 48 hours it is replaced by a new one anyway. Let T represent the potential lifetime of a component in continuous use, and U the time of such a component in use with the above replacement policy. Sketch the graphs of:

i. the c.d.f of T?

ii. the c.d.f of U. Is U discrete, continuous, or neither?

iii. Does the replacement policy serve any good purpose? Explain.

Ex 4.rev.6 in Pitman's Probability

Problem 2

C.d.f. of the beta distribution for integer parameters.

a. Let X_1, X_2, ... , X_n be independent uniform (0, 1) random variables, and let $X_{(k)}$ be the kth order statistic of the X's. Find the c.d.f. of $X_{(k)}$ by expressing the event $X_{(k)} \leq x$ in terms of the number of X_i that are $\leq x$.

b. Use a) to show that for positive integers r and s, the c.d.f. of the beta (r, s) distribution is given by

$$\sum_{i=r}^{r+s-1} \binom{r+s-1}{i} x^i (1-x)^{r+s-i-1} \qquad (0 \leq x \leq 1)$$

Ex 4.6.5 in Pitman's Probability

Problem 3

At the ticket stand outside a Cal football game there are 3 tellers, each of whom has a service time that is distributed $Exp(\lambda)$, independent of the other tellers. Suppose all 3 tellers currently have customers they are servicing, and there are 5 people waiting in a single line. The rules for the line are as follows: the person at the front of the line waits until a teller becomes available and goes to the first available teller. Let T be the time until everyone gets a ticket. Find:

i. $E(T)$.

ii. $Var(T)$.