— MON, JAN 27 —

* MIDTERM: FRI, MAR 6
  → IN CLASS

* HW #1: TUES, FEB 4
  → GRADESCOPE BY 11:59PM

* QUIZ #1: WED, FEB 5
  → IN DISCUSSION SECTIONS
  → COVERS ALL OF §1.

GROUPS A DECK IS SHUFFLED.

$\mathbb{P}$(TOP CARD IS KING SPADES
OR
BOTTOM CARD IS KING SPADES)

= ?

ANS

ADDITION RULE:

$$= P(\text{KS TOP}) + P(\text{KS BOTTOM})$$

$$= \frac{1}{52} + \frac{1}{52}$$

GROUPS 2 DECKS ARE SHUFFLED.

P(TOP OF 1^{st} IS KS

OR

BOTTOM OF 2^{nd} IS KS)

= ?

ANS INCLUSION-EXCLUSION:

$$= P(\text{KS TOP } 1^{st}) + P(\text{KS BOTTOM } 2^{nd}) - P(\text{KS TOP } 1^{st}, \text{KS BOTTOM } 2^{nd})$$

$$= \frac{1}{52} + \frac{1}{52} - \frac{1}{52} \cdot \frac{1}{52}$$

§1.4 CONDITIONAL PROBABILITY & INDEPENDENCE

<u>DEF</u> 2 EVENTS $A, B \subset \Omega$ ARE **INDEPENDENT** IF

$$\mathbb{P}(A \cap B) = \mathbb{P}(A)\,\mathbb{P}(B).$$

[IF NOT, WE SAY THAT A, B ARE **DEPENDENT**.]

CAREFUL:

Independent $\neq$ Disjoint

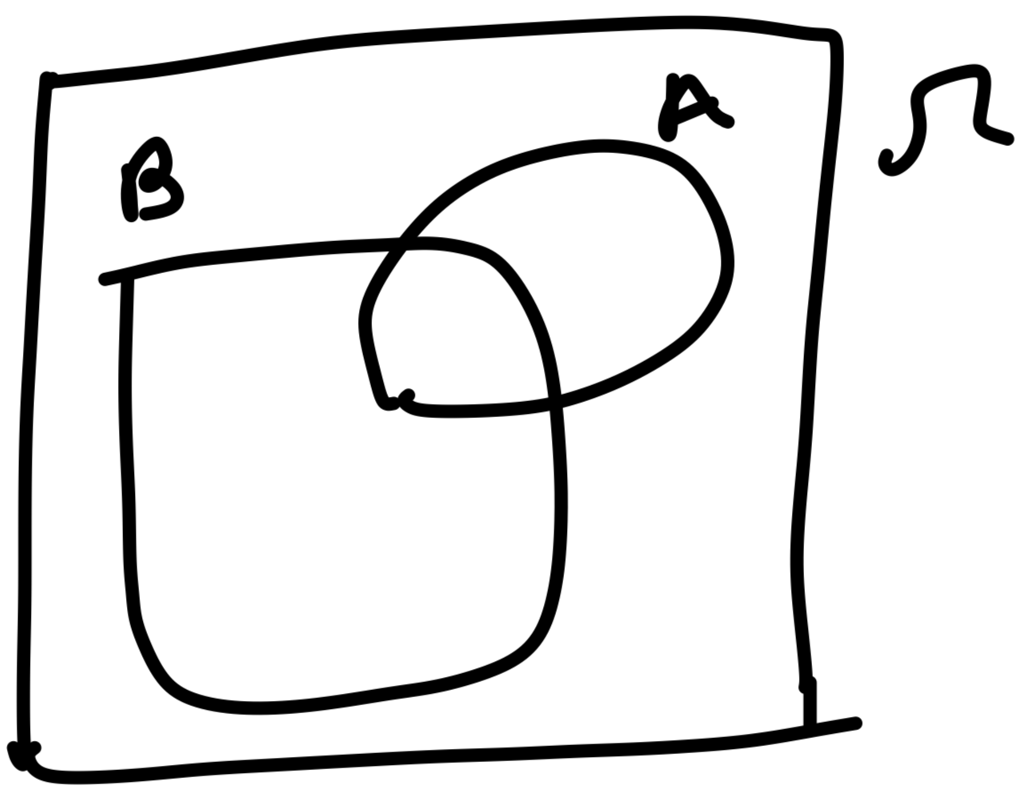
if $A \cap B = \emptyset$, then $\mathbb{P}(A \cap B) = 0$.

In this case, A, B are independent only if $\mathbb{P}(A) = 0$

or $\mathbb{P}(B) = 0$.

DEF THE CONDITIONAL PROBABILITY OF A GIVEN B IS

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \quad [P(B) \neq 0]$$



$$\text{"} \frac{\text{area}(A \cap B)}{\text{area}(B)} \text{"}$$

NOTE: IF A, B INDEPENDENT,

THEN,

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \frac{\mathbb{P}(A)\cancel{\mathbb{P}(B)}}{\cancel{\mathbb{P}(B)}}$$

$$= \mathbb{P}(A).$$

"KNOWING B DOES NOT EFFECT THE PROB. OF A".

THE CONDITIONAL PROBABILITY FORMULA GIVES US:

$$\mathbb{P}(A \cap B) = \mathbb{P}(A \mid B)\,\mathbb{P}(B)$$

<u>MULTIPLICATION RULE</u>

GROUPS SUPPOSE A, B WITH

$\mathbb{P}(A) = 0.5$, $\mathbb{P}(A \cup B) = 0.8$.

IS IT POSSIBLE THAT A, B

ARE INDEPENDENT **AND**

MUTUALLY EXCLUSIVE (DISJOINT) ?

<u>NO</u>.

IF $A \cap B = \emptyset$, THEN

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$$

$$\Rightarrow \quad \mathbb{P}(B) = 0.3$$

BUT THEN,

$$\mathbb{P}(A)\,\mathbb{P}(B) = (0.5)(0.3) \neq 0.$$

SO $\mathbb{P}(A \cap B) \neq \mathbb{P}(A)\,\mathbb{P}(B)$,

SINCE $\mathbb{P}(A \cap B) = \mathbb{P}(\emptyset) = 0$.

GROUPS

$$\mathbb{P}(A) = \mathbb{P}(A \cup B) = 0.8.$$

CAN A, B BE INDEPENDENT AND MUTUALLY EXCLUSIVE?

YES.

FOR EXAMPLE, $B = \emptyset$.

THEN, $\mathbb{P}(A \cup B) = \mathbb{P}(A) = 0.8$.

AND, $\mathbb{P}(A \cap B) = \mathbb{P}(\emptyset) = 0$

$$= \mathbb{P}(A)\,\mathbb{P}(B),$$

SINCE $\mathbb{P}(B) = 0$.

§1.5 BAYES' RULE

$$\boxed{\mathbb{P}(A \mid B) = \frac{\mathbb{P}(B \mid A)\,\mathbb{P}(A)}{\mathbb{P}(B)}}$$

<u>GROUPS</u> : WHY ?

① $\mathbb{P}(A \mid B) = \dfrac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$

$+$

② $\mathbb{P}(A \cap B) = \mathbb{P}(B \mid A)\,\mathbb{P}(A)$.

MORE GENERALLY: SUPPOSE $A \subset \Omega$ AND $B_1, \ldots, B_n$ PARTITIONS Ω.

1. $\mathbb{P}(A) = \sum_{i=1}^{n} \mathbb{P}(B_i \cap A)$ [LOTP]

$= \sum_{i=1}^{n} \mathbb{P}(A \mid B_i)\mathbb{P}(B_i)$ [MR]

2. SO,

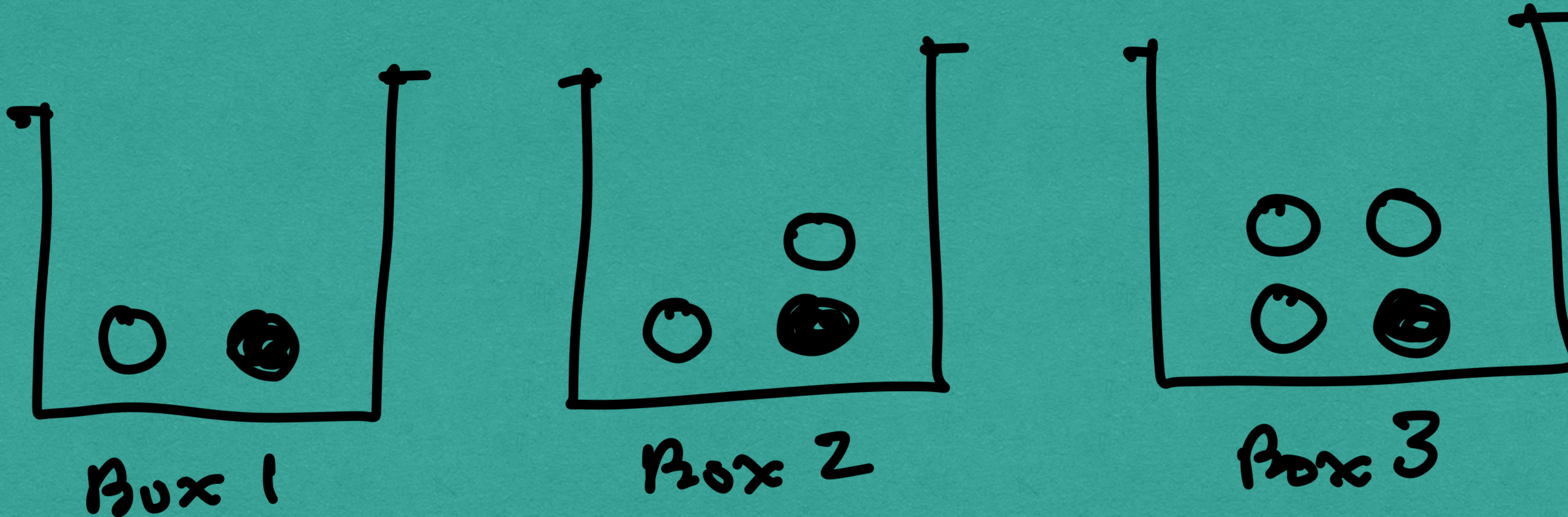
$$\mathbb{P}(B_j \mid A) = \frac{\mathbb{P}(A \mid B_j)\,\mathbb{P}(B_j)}{\sum_{i=1}^{n} \mathbb{P}(A \mid B_i)\,\mathbb{P}(B_i)} \quad (\forall j)$$

BAYES' RULE HELPS YOU THINK ABOUT THINGS "THE OTHER WAY AROUND".

FOR EXAMPLE

$\downarrow$

GROUPS



* PICK A BOX AT RANDOM, AND THEN A BALL FROM THAT BOX RANDOMLY.

* SUPPOSE THAT IT IS WHITE. FIND THE PROB. THAT IT CAME FROM BOX 2.

<u>ANS</u> $\mathbb{P}(1) = \mathbb{P}(2) = \mathbb{P}(3) = \frac{1}{3}$

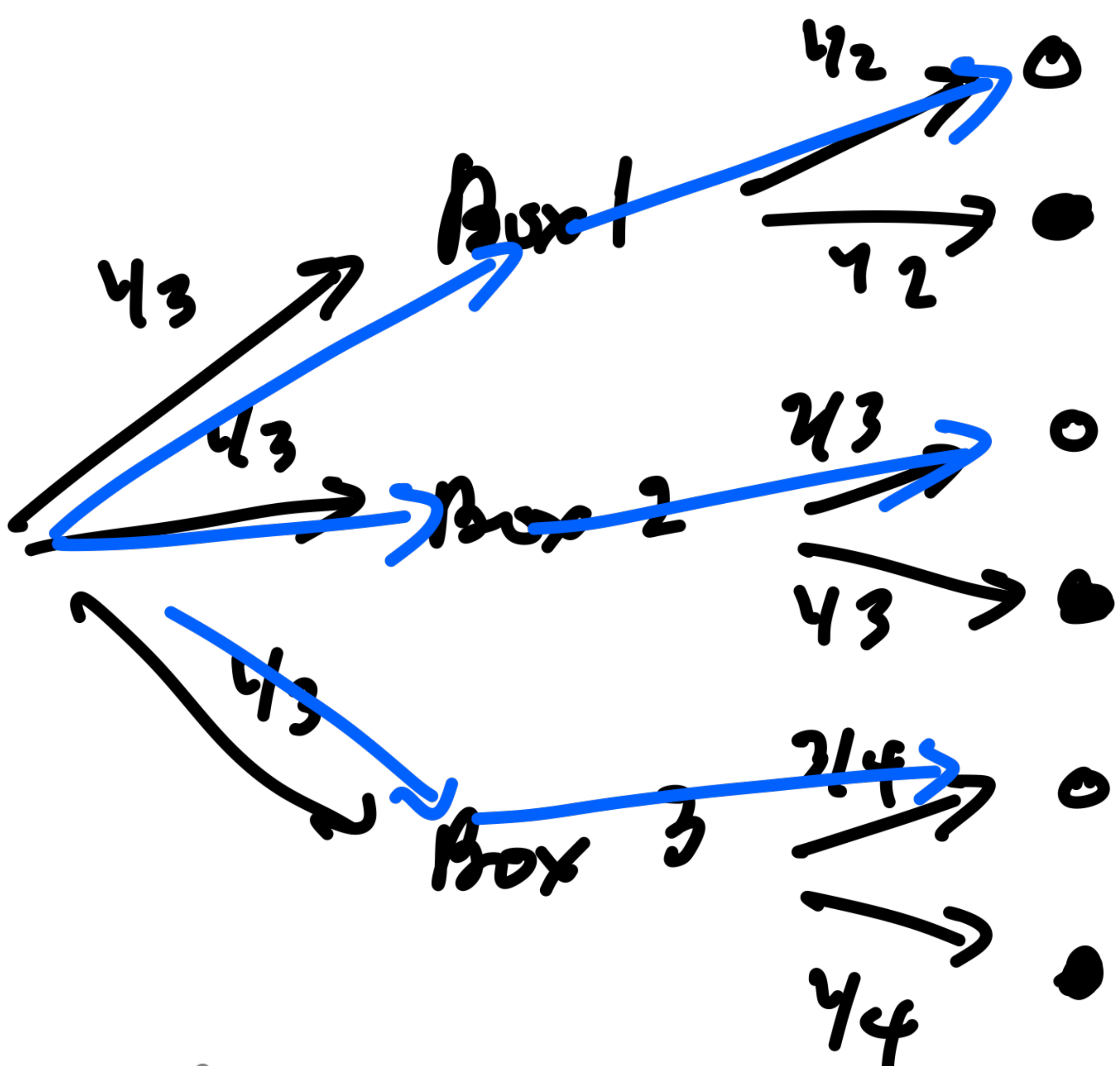
$\mathbb{P}(W|1) = \frac{1}{2}$, $\mathbb{P}(W|2) = \frac{2}{3}$, $\mathbb{P}(W|3) = \frac{3}{4}$.

$$\therefore \mathbb{P}(2|W) = \frac{\mathbb{P}(W|2)\,\cancel{\mathbb{P}(2)}}{\sum_1^3 \mathbb{P}(W|i)\,\cancel{\mathbb{P}(i)}}$$

$$= \frac{\frac{2}{3}}{\frac{1}{2}+\frac{2}{3}+\frac{3}{4}} = \frac{2/3}{23/12} = 8/23$$

$\longrightarrow$ ABOUT A 35% CHANCE.

"TREE DIAGRAM":



$\frac{1}{3} \cdot \frac{1}{2}$

$\frac{1}{3} \cdot \frac{2}{3}$ *

$\frac{1}{3} \cdot \frac{3}{4}$

$P(:)$ $\quad$ $P(\cdot \mid :)$ $\quad$ $P(\cdot \mid :)\,P(:)$