Stat 134 lec 8

warmup 9:00 – 9:10

Find the probability that a poker hand has two 2 of a kind

ex K, K, Q, Q, 7

$$\frac{\binom{13}{2}\binom{11}{1}\overset{K}{\binom{4}{2}}\overset{Q}{\binom{4}{2}}\overset{7}{\binom{4}{1}}}{\binom{52}{5}}$$

we have: $\binom{13}{2} = \frac{13\cdot 12}{2}$ instead of $13\cdot 12$

since aabbc = aabbc in a poker hand

so we don't want to double count.

Find the probability that a poker hand has two 2 of a kind and two 1 of a kind

ex K, K, Q, Q, 7, 8

$$\frac{\binom{13}{2}\binom{11}{2}\overset{K}{\binom{4}{2}}\overset{Q}{\binom{4}{2}}\overset{7}{\binom{4}{1}}\binom{4}{1}}{\binom{52}{6}}$$

Last time

independent trials
(draw w/ replacement)
- binomial distribution — 2 outcome trial
- multinomial distribution — K outcome trial

dependent trials
(draw w/o replacement)
- hypergeometric distribution — 2 outcom trial
- multivariate hypergeometric distribution — K outcome trial

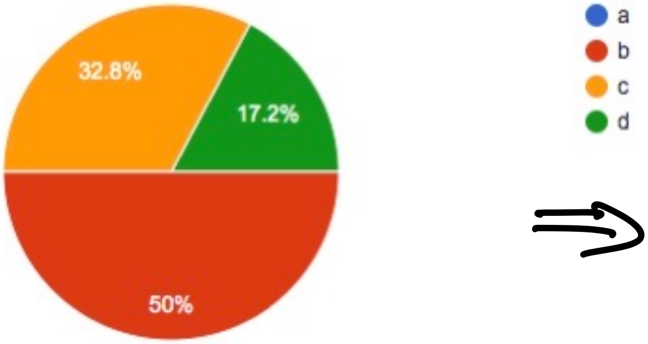
Sec 2.5 hypergeometric distribution

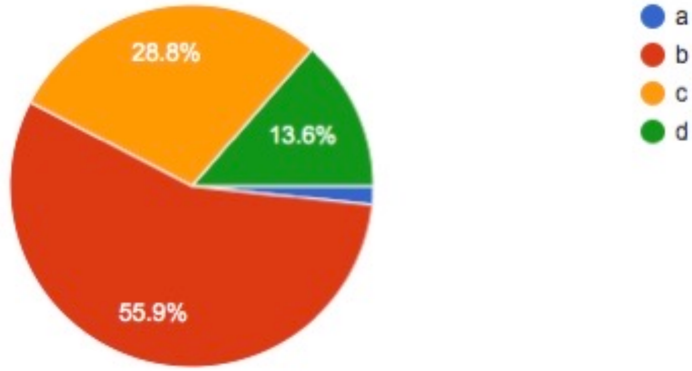
abbrev. $HG(n, N, G)$

Parameters: N = population size
G = # Good in population
n = sample size.

Suppose a population of size N contains G good and B bad elements $(N = G + B)$.
A sample, size n, with g good and b bad elements $(n = g + b)$ is chosen at random without replacement.

$$P(g \text{ good}) = \frac{\binom{G}{g}\binom{B}{b}}{\binom{N}{n}}$$





1. The probability of being dealt a three of a kind poker hand (ranks $aaabc$ where $a \neq b \neq c$) is:

a $\binom{4}{3}\binom{4}{1}\binom{44}{1} / \binom{52}{5}$

b $\binom{13}{1}\binom{12}{2}\binom{4}{3}\binom{4}{1}\binom{4}{1} / \binom{52}{5}$

c $\binom{13}{1}\binom{12}{1}\binom{4}{3}\binom{4}{1}\binom{44}{1} / \binom{52}{5}$

d none of the above

c	First 13 choose 1 to designate the rank of the three of a kind, then 4 choose 3 to get the 3 of a kind, then 12 choose 1 to designate the 2nd rank and 4 choose 1 to get 1 card of that kind, and finally pick 1 from the rest 44 cards
b	Choose a rank out of 13, then choose 3 cards out of that rank, then choose 2 ranks out of the rest 12, each pick 1 card

we have: $\binom{12}{2} = \frac{12\cdot 11}{2}$ instead of $12\cdot 11$
since aaabc = aaacb in a poker hand
so we double count.

today ① sec 2.5 Binomial approx to hypergeometric.

② sec 3.1 – random variables (RV)
– joint distribution of 2 RVs and independence

(1) Sec 2.5 Binomial approx to hypergeometric.

Binomial — independent trials
Hypergeometric — dependent trials.

ex 100 person class with a grade distribution:

A grade : 70 students
B grade : 30 students.

Sample 5 students at random w/o replacement (SRS).

Find $P(3A's, 2B's)$

exact
hypergeometric
$$= \frac{\binom{70}{3}\binom{30}{2}}{\binom{100}{5}} = \binom{5}{3}\frac{70}{100}\frac{69}{99}\frac{68}{98}\frac{30}{97}\frac{29}{96} = .316$$

approx
binomial
$$= \binom{5}{3}(.7)^3(.3)^2 = .309$$

when N is large relative to n, $HG(5, 100, 70) \approx Bin(5, .7)$

why?

$$HG(n, N, G) \approx Bin\left(n, \frac{G}{N}\right)$$

Summary of approximations

$HG(n, N, G)$

approx by binomial
N large, n small
$p = \frac{G}{N}$

binomial (n, p)

approx by Poisson
$p \to 0$, $n \to \infty$, $np \to \mu$

Poisson(μ)

approx by normal
n large
$\mu = np$, $\sigma = \sqrt{npq}$
$0 < \mu \pm 3\sigma < n$
use continuity correction

Normal(μ, σ^2)

② <u>Sec 3.1</u> Intro to Random Variables (RV)

A RV, X, is the outcome of an experiment.

What distribution is the following RV?

X = The number of aces in 5 cards drawn from a standard deck?

$$X \sim HG(5, 52, 4)$$

belongs to

ex flip a prob p coin 2 times

X = # heads

we write $X \sim Bin(2, p)$

More precisely,

outcome space

$$X : \Omega \longrightarrow \mathbb{R} \quad \text{is a function}$$

$$\begin{aligned} HH &\longmapsto 2 \\ HT &\longmapsto 1 \\ TH &\longmapsto 1 \\ TT &\longmapsto 0 \end{aligned}$$

so $X=1$ means $\{HT, TH\} \subseteq \Omega$

$X=1$ is an event

$P(X=1) = \binom{2}{1} p^1 (1-p)^1$ binomial formula

Joint Distribution

Let (X,Y) be the joint outcome of 2 RVs X, Y.

ex X: one draw from [1] [2] [2] [3]

Given $X = x$, Y = number of heads in x coin tosses.

$$P(X=x, Y=y) = P(Y=y \mid X=x) \cdot P(X=x)$$

$$P(X=1, Y=1) = \underbrace{P(Y=1 \mid X=1)}_{= 1/2} \cdot \underbrace{P(X=1)}_{= 1/4} = 1/8$$

Find, what the range of values of X? 1, 2, 3

Y? 0, 1, 2, 3

$$P(\overset{x}{1}, \overset{y}{0}) = \underbrace{P(Y=0 \mid X=1)}_{= 1/2} \underbrace{P(X=1)}_{= 1/4} = 1/8$$

marginal prob of X

$$P(x) = \sum_{y \in Y} P(x,y)$$

	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	
3	0	0	$\frac{1}{32} = \frac{1}{8} \cdot \frac{1}{4}$	$\frac{1}{32}$
2	0	$\frac{1}{8} = \frac{1}{4} \cdot \frac{1}{2}$	$\frac{3}{32} = \frac{3}{8} \cdot \frac{1}{4}$	$\frac{7}{32}$
1	$\frac{1}{8} = \frac{1}{2} \cdot \frac{1}{4}$	$\frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2}$	$\frac{3}{32} = \frac{3}{8} \cdot \frac{1}{4}$	$\frac{15}{32}$
0	$\frac{1}{8} = \frac{1}{2} \cdot \frac{1}{4}$	$\frac{1}{8} = \frac{1}{4} \cdot \frac{1}{2}$	$\frac{1}{32} = \frac{1}{8} \cdot \frac{1}{4}$	$\frac{9}{32}$
Y \ X	1	2	3	

marginal prob of Y

$$P(y) = \sum_{x \in X} P(x,y)$$

Is X, Y dependent?

<u>Def^n</u> Two RVs are <u>independent</u> if

$P(Y=y \mid X=x) = P(Y=y)$ for all $x \in \mathcal{X}$
$y \in \mathcal{Y}$

By the multiplication rule,

if X, Y are indep,

$$P(X=x, Y=y) = \underset{\overset{\shortparallel}{P(Y=y)}}{P(Y=y \mid X=x)}\, P(X=x)$$

so $\boxed{P(X=x, Y=y) = P(X=x)P(Y=y)}$.

tinyurl.com/sept12-2022

try this before the next class you self

stat 134 concept test

September 12 2022

The joint distribution of X and Y is drawn below:

	3/8	1/2	1/8	P(x) / P(y)
1	1/4	1/3	1/12	2/3
0	1/8	1/6	1/24	1/3
Y / X	0	1	2	

a) X and Y are independent

b) If we divide both rows by their marginal probability we get the same answer.

c) $P(X = x|Y = 0) = P(X = x|Y = 1)$

d) All of the above