Stat 134: Section 24

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Conceptual Review

a. What is the covariance of two random variables X, Y.

b. Let X, Y, Z be random variables with finite second moment and $\alpha \in \mathbb{R}$. Check the following basic properties of the covariance:

- $Cov(X, Y) = Cov(Y, X)$
- $Cov(X + \alpha Y, Z) = Cov(X, Z) + \alpha Cov(Y, Z)$
- $Cov(X, X) = Var(X) \geq 0$

c. What is the correlation of two random variables and what is its interpretation?

Problem 1: Conditional distribution

Let $A = (0, 1)$, $B = (1, 0)$ and $C = (-1, 0)$ be three points in the plane. Suppose that (X, Y) is uniform on the triangle ABC. For $x \in (-1, 1)$ find:

a. $P(Y \geq \frac{1}{2} | X = x)$

b. $P(Y < \frac{1}{2} | X = x)$

c. $E[Y | X = x]$

d. $Var(Y | X = x)$

Ex 6.3.5 in Pitman's Probability

Problem 2: Uncorrelated does NOT mean indepedent

Show that if X, Y are independent random variables with finite second moment then $Cov(X, Y) = 0$. Show that the converse of this statement is false as follows:
Suppose that X is an $N(0, 1)$ random variable and let Y be a random variable independent of X such that $P(Y = 1) = P(Y = -1) = \frac{1}{2}$. Let $Z = YX$. Compute $Cov(X, Z)$. Are X, Z indepdendent (justify your answer)?

Problem 3

Suppose that n cards numbered $1, 2, \ldots, n$ are shuffled uniformly randomly and that k cards are dealt. Let S_k be the sum of numbers on the k dealt cards. Compute $E[S_k]$ and $Var(S_k)$ in terms of n and k. (Hint: Use indicators.)

Ex 6.4.9 in Pitman's Probability