Stat 134: Bivariate Normal

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Conceptual Review

1. The definition of the bivariate normal distribution.
2. The construction of the bivariate normal random variable (the decomposition representation).
3. Conditional distributions for the bivariate normal distribution.
4. Linear transformation of the multivariate normal distribution.

Problem 1

Let X and Y have bivariate normal distribution with parameters μ_X, μ_Y, σ_X^2, σ_Y^2, and ρ.

1. Predict Y given $X = x$.
2. Find $\mathbb{P}(Y > y | X = x)$.
3. Find $\mathbb{P}(Y > \mu_Y, X > \mu_X)$.
4. Find $\mathbb{E}(Y | a < X < b)$, where $a < b$.

Problem 2

Let $X \sim N(\mu_X, \sigma_X^2)$ and $Y \sim N(\mu_Y, \sigma_Y^2)$.

1. If X and Y have bivariate normal distribution with correlation ρ, show that $\rho = 0$ if and only if X, Y are independent.
2. Find a counter-example such that X and Y are uncorrelated but they are not independent.