Stat 134 Lec 8

Warmup : 1:00-1:10

The probability of being dealt a pair in a poker hand (ranks $aabcd$ where $a \neq b \neq c \neq d$) is:

a $\binom{13}{1}\binom{12}{3}\binom{4}{2}\binom{4}{1}\binom{4}{1}/\binom{52}{5}$

b $\binom{13}{2}\binom{12}{2}\binom{4}{3}\binom{4}{1}\binom{4}{1}/\binom{52}{5}$

c $\binom{13}{1}\binom{12}{3}\binom{4}{2}\binom{4}{1}\binom{4}{1}\binom{4}{1}/\binom{52}{5}$

d none of the above

Answ

$$\frac{\binom{13}{1}\binom{12}{3}\binom{4}{2}\binom{4}{1}\binom{4}{1}\binom{4}{1}}{\binom{52}{5}}$$

aabcd = aabdc = aacbd = aacdb

= aacbd = aacdb

Announcement: Wednesday's Q2 logistics will be released soon.

last time interpretation of $\binom{5}{2}$:

1	1	0	0	0
1	0	1	0	0
1	0	0	1	0
1	0	0	0	1
0	1	1	0	0
0	1	0	1	0
0	1	0	0	1
0	0	1	1	0
0	0	1	0	1
0	0	0	1	1

$$\binom{5}{2} = \frac{5!}{2! \cdot 3!} = 10$$

number of arrangements of two 1s in 5 slots

Sec 2.5 hypergeometric distribution

Suppose a population of size N contains G good and B bad elements ($N = G + B$).
A sample, size n, with g good and b bad elements ($n = g + b$) is chosen at random without replacement.

$$P(g \text{ good and } b \text{ bad}) = \frac{\binom{G}{g}\binom{B}{b}}{\binom{N}{n}}$$

Parameters: N = population size
G = # Good in population
n = sample size.

abrev. $HG(N, G, n)$

ex $P(\text{Poker hand is 4 of a kind})$ rank aaaab $a \neq b$

N = number of cards $= 52$
G = number of rank a cards in deck $= 4$
n = sample size $= 5$

$$\binom{13}{1} \cdot \frac{\overset{a}{\binom{4}{4}}\overset{\text{not } a}{\binom{48}{1}}}{\binom{52}{5}} = \binom{13}{1}\binom{12}{1} \cdot \frac{\binom{4}{4}\overset{b}{\binom{4}{1}}}{\binom{52}{5}}$$

choose 1 rank for a

poker hands

Not $\binom{13}{2}$ since aaaab $\neq$ bbbba

Today

1. review student concept test responses,
2. Sec 2.5 more hypergeometric
3. Sec 2.5 counting strategy
4. Sec 2.5 Binomial approx to hypergeometric

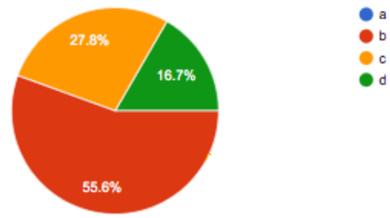
1. The probability of being dealt a three of a kind poker hand (ranks $aaa\underbrace{bc}_{2}$ where $a \neq b \neq c$) is:

a $\binom{4}{3}\binom{4}{1}\binom{44}{1}/\binom{52}{5}$

b $\binom{13}{1}\binom{12}{2}\binom{4}{3}\binom{4}{1}\binom{4}{1}/\binom{52}{5}$

c $\binom{13}{1}\binom{12}{1}\binom{4}{3}\binom{4}{1}\binom{44}{1}/\binom{52}{5}$

d none of the above



d	the correct answer is to add (11 choose 1) to b, because for the single c, there are 11 choices for us to choose from

c	You first have 13 options for the first rank of a. Out of those 4 cards, you choose 3. Then you have 12 options left for the second rank of b. Out of those 4 cards, you choose 1. Lastly, you have 52-8=44 options left for card c, of which you pick 1.

b	The denominator comes from choosing 5 cards out of 52 possible cards. The numerator is (13 1) because you can choose any card for the first triple. It is then multiplied by (12 2) because you have 12 possibilities for each single. Originally I thought it was (12 1) & (11 1) but that would over count because having a poker hand of King & Ace is the same as having a hand of Ace & king.

8 card poker hand

Find Chance of getting aaabbbcd ?

$$\binom{13}{2}\binom{11}{2} \cdot \frac{\binom{4}{3}\binom{4}{3}\binom{4}{1}\binom{4}{1}}{\binom{52}{8}}$$

(2) Practice hypergeometric

You and a friend are playing poker. If each of you are dealt 5 cards from the same deck, what is the chance that you both get a 4 of a kind (ranks aaaab, $a \neq b$)

P(friend and you get 4 of kind)

$= \underbrace{P(\text{friend 4 kind})} \cdot P(\text{you 4 kind} \mid \text{friend 4 kind})$

$$\boxed{\frac{\binom{13}{1}\binom{4}{4}\binom{48}{1}}{\binom{52}{5}} \cdot \frac{\binom{11}{1}\binom{4}{4}\binom{43}{1}}{\binom{47}{5}}}$$

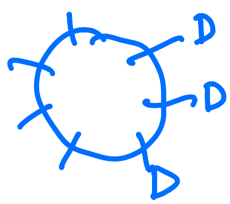
(3) Counting strategy:

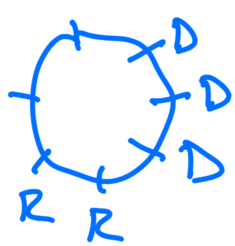
Try and break the problem into a sequence of steps and apply the multiplication rule. Above we find the chance your friend gets 4 of a kind and the probability you get 4 of a kind given that your friend got 4 of a kind.

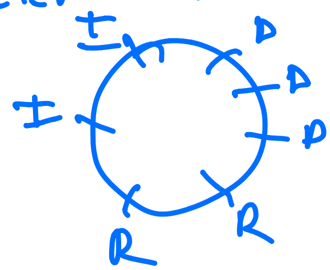
ex There are 3 Democrats, 2 Republicans, and 2 Independents sitting around a table. What is the chance the Dem sit together, the Rep sit together and the Ind sit together?

We break up the Problem

Dem together $\cdot$ Rep together given Dem together $\cdot$ Inden together given D and R together







$$7\cdot\frac{1}{\binom{7}{3}} \cdot 2\cdot\frac{1}{\binom{4}{2}} \cdot 1 = \boxed{\frac{7\cdot 2}{\binom{7}{3}\binom{4}{2}}}$$

7 positions for 1st D

2 positions for 1st R so next to D

Note If DDDRRII are sitting on a bench instead of around a table the answer would be $\dfrac{3!}{\binom{7}{3}\binom{4}{2}}$.

ex In a well shuffled deck, find the probability that J, Q, K appear as 12 consecutive cards and the J are grouped together, Q are grouped together, and K are grouped together?

ex JJJJQQQQKKKK or QQQQJJJJKKKK etc.

We can break the problem into parts:

First find the chance that JJJJQQQQKKKK is the first 12 cards (out of 52) in this order.

This is $\dfrac{1}{\binom{52}{4,4,4,40}} = \dfrac{1}{\binom{52}{4}} \cdot \dfrac{1}{\binom{48}{4}} \cdot \dfrac{1}{\binom{44}{4}}$

Then $\dfrac{3!}{\binom{52}{4,4,4,40}}$ is all ways you can get different orderings JJJJ, QQQQ and KKKK in the 1st 12 cards.

Imagine the 12 cards are glued together as one fat card, so our deck now has 41 cards. There are 41 positions for the fat card. Hence the answer is

$$\boxed{\dfrac{41 \cdot 3!}{\binom{52}{4,4,4,40}}}$$