Stat 134 lec 7

Warmup 9:00 – 9:10

p large

large

ex 97.8% of approx 30 million poor families in the U.S. have a fridge. If you randomly sample 100 of these families roughly what is the chance 98 or more have a fridge?

<u>Defn</u> Poisson(μ)

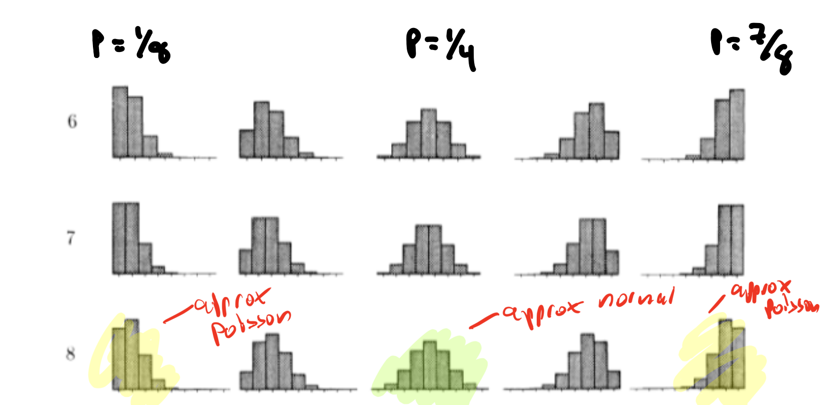
$$P(k) = \frac{e^{-\mu}\mu^k}{k!} \text{ for } k = 0, 1, 2, \ldots$$

P(2 or less dont have a fridge)

$= P(0) + P(1) + P(2)$

use Poisson approx with $q = 1 - p = .022$

$$\approx e^{-2.2} + \frac{e^{-2.2}(2.2)^1}{1!} + \frac{e^{-2.2}(2.2)^2}{2!}$$



Last time

Announcement: Quiz 1 covers sec 1.1 – 1.6 and 2.1
3 or 4 questions in 45 minutes.

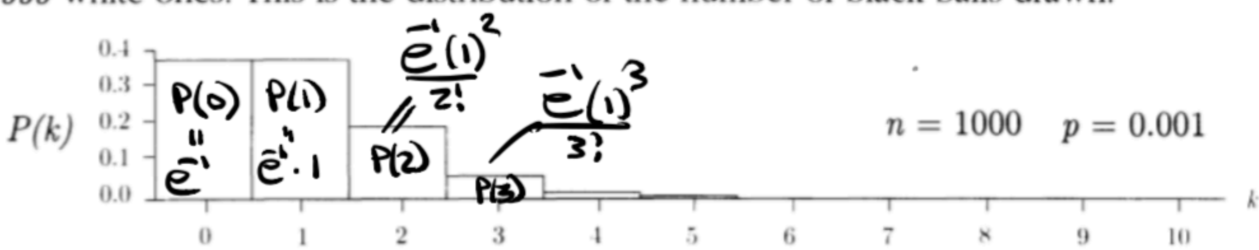
sec 2.4 Poisson Distribution

$$P(k) = \frac{e^{-\mu}\mu^{k}}{k!}, \quad k = 0, 1, 2, \ldots$$

We saw that Pois(μ) is a limit of binomials for $n \to \infty$, $p \to 0$ and $np \to \mu$

The binomial (1000, 1/1000) distribution.

Now take 1000 random draws with replacement from a box with 1 black ball and 999 white ones. This is the distribution of the number of black balls drawn:

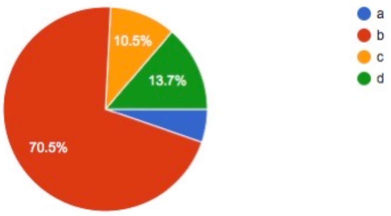


mode of Bin(n,p):
$m = \lfloor np+p \rfloor$

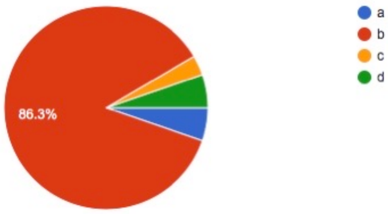
$$\text{mode} = \begin{cases} m & \text{if } np+p \notin \mathbb{Z} \\ m-1, m & \text{if } np+p \in \mathbb{Z} \end{cases}$$

mode of Pois(μ):
$m = \lfloor \mu \rfloor$ since $np + p \to \mu + p \approx \mu$

$$\text{mode} = \begin{cases} m & \text{if } \mu \notin \mathbb{Z} \\ m-1, m & \text{if } \mu \in \mathbb{Z} \end{cases}$$



$\Rightarrow$



stat 134 concept test

September 7 2022

Southwest overbooks its flights, knowing some people will not show up. Suppose for a plane that seats 350 people, 360 tickets are sold. Based on previous data the airline claims that each passenger has a 90% chance of showing up. **Approximately**, what is the chance that at least one empty seat remains? (There are no assigned seats)

a) $P(Z < \frac{350.5-\mu}{\sigma})$

b) $P(Z < \frac{349.5-\mu}{\sigma})$

c) $P(Z < \frac{360.5-\mu}{\sigma})$

d) none of the above

Probability no empty seats

$1 - P(Z > \frac{349.5-\mu}{\sigma})$

this is also a valid approximation

d	we need to apply the compliment rule

b	we want the area to the left of 350 (not including 350 itself, since we want at least one open seat). the formula in (b) thus gives us P(X < 350).

Today

① sec 2.5 Random sampling

independent trials (draw w/ replacement)
- binomial distribution — 2 outcome trial
- multinomial distribution — k outcome trial

dependent trials (draw w/o replacement)
- hypergeometric distribution — 2 outcome trial
- multivariate hypergeometric distribution — k outcome trial

1. Sec 2.5

Random sampling with replacement

ex Class 100 students

grade distribution:

A 50 students

B 30 students

C 15 students

D 5 students

You sample 10 students with replacement.

a) What is the chance you get

AAAAABBBCCD ?

$(.5)^4(.3)^3(.15)^2(.05)$

AABAABB

$(.5)^2(.3)(.5)^2(.3)^2$

b) Find P(4 A's, 3B's, 2C's, 1D)

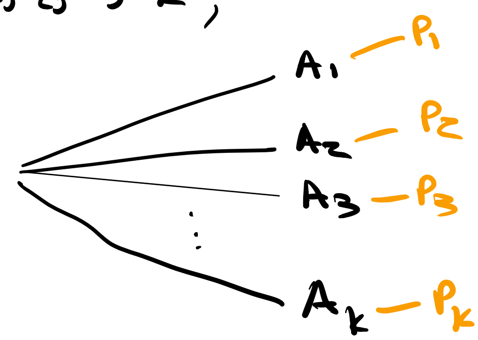
$$\binom{10}{4,3,2,1}(.5)^4(.3)^3(.15)^2(.05)$$

$$\binom{10}{4,3,2,1} = \frac{10!}{4!3!2!1!}$$

$$\text{or } = \binom{10}{4}\binom{6}{3}\binom{3}{2}\binom{1}{1} = \binom{10}{4,3,2,1}$$

Def^n <u>Multinomial Distribution</u>   written $\mathrm{Multi}(n, p_1, \ldots, p_k)$

If you have n independent trials, where each trial has k possible outcomes, $A_1, A_2, \ldots, A_k$ with probabilities $p_1, p_2, \ldots, p_k$,



then the probability you get n_1 outcome A_1, n_2 outcome A_2, ..., n_k outcome A_k is

$$P(n_1, n_2, \ldots, n_k) = \binom{n}{n_1, n_2, \ldots, n_k} p_1^{n_1} p_2^{n_2} \cdots p_k^{n_k}$$

$$\frac{n!}{n_1! n_2! \cdots n_k!}$$

<u>Note</u> Binomial distribution is a special case with $k=2$.

independent trials (draw w/ replacement) $\Big\langle$ binomial distribution — 2 outcome trial; multinomial distribution — k outcome trial

random sample without replacement

ex In a very student friendly class with 100 students the grade distribution is:

A 70 students
B 30 students

You sample 5 students at random without replacement (called a simple random sample (SRS))

a) Find the chance you get

A A A B B

$$\frac{70}{100}\,\frac{69}{99}\,\frac{68}{98}\,\frac{30}{97}\,\frac{29}{96}$$

b) Find P(3A's, 2B's).

$$\binom{5}{3,2}\frac{70}{100}\,\frac{69}{99}\,\frac{68}{98}\,\frac{30}{97}\,\frac{29}{96} = \frac{\frac{70\cdot 69\cdot 68}{3!}\cdot\frac{30\cdot 29}{2!}}{\frac{100\cdot 99\cdot 98\cdot 97\cdot 96}{5!}}$$

$$\binom{5}{3,2} = \frac{5!}{3!\,2!}$$

$$= \frac{\overset{A}{\binom{70}{3}}\overset{B}{\binom{30}{2}}}{\binom{100}{5}}$$

hypergeometric formula

written

Def^n <u>hypergeometric distribution</u> $HG(n, N, G)$

Suppose a population of size N contains G good and B bad elements $(N = G + B)$.

A sample, size n, with g good and b bad elements $(n = g + b)$ is chosen at random <u>without replacement</u>.

$$P(g \text{ good and } b \text{ bad}) = \frac{\binom{G}{g}\binom{B}{b}}{\binom{N}{n}}$$

this generalizes to the <u>multivariate hypergeometric distribution</u>

Now instead of 2 types of elements we have K with sizes $G_1, G_2, \ldots, G_K$ $(N = G_1 + \cdots + G_K)$ and in our sample we have $n = g_1 + \cdots + g_K$.

$$P(g_1, g_2, \ldots, g_K) = \frac{\binom{G_1}{g_1}\binom{G_2}{g_2}\cdots\binom{G_K}{g_K}}{\binom{N}{n}}$$

<u>ex</u> Class 100 students
grade distribution:

A	50 students
B	30 students
C	15 students
D	5 students

You sample 10 students without replacement (SRS)

Find $P(4\text{ A's}, 3\text{ B's}, 2\text{ C's}, 1\text{ D})$

$$\frac{\binom{50}{4}\binom{30}{3}\binom{15}{2}\binom{5}{1}}{\binom{100}{10}}$$

ex A 5 card <u>poker hand</u> consists of a SRS of 5 cards from a 52 card deck. There are $\binom{52}{5}$ poker hands.

a) Find P(poker hand has 4 aces and a king)

$$\frac{\overset{\text{aces}}{\binom{4}{4}}\overset{\text{kings}}{\binom{4}{1}}}{\binom{52}{5}} \quad \binom{44}{0} = 1$$

n

b) Find P(poker hand has 4 aces).

$$\frac{\binom{4}{4}\binom{48}{1}}{\binom{52}{5}} \quad \text{or} \quad \frac{\binom{12}{1}\binom{4}{4}\binom{4}{1}}{\binom{52}{5}}$$

c) Find P(poker hand has 4 of a kind)

← pick your quad ← pick your single

$$\frac{\binom{13}{1}\binom{4}{4}\binom{12}{1}\binom{4}{1}}{\binom{52}{5}}$$

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Stat 134

Chapter 2 Wednesd[illegible] February 5

1. The probability of being dealt a three of a kind poker hand (ranks $aaabc$ where $a \neq b \neq c$) is:

   a $\binom{4}{3}\binom{4}{1}\binom{44}{1}/\binom{52}{5}$

   b $\binom{13}{1}\binom{12}{2}\binom{4}{3}\binom{4}{1}\binom{4}{1}/\binom{52}{5}$

   c $\binom{13}{1}\binom{12}{1}\binom{4}{3}\binom{4}{1}\binom{44}{1}/\binom{52}{5}$

   d none of the above

Try your best. We will go over next time