Stat 134 Lec 8

warmup: 11:00-11:10

The probability of being dealt a pair in a poker hand (ranks $aabcd$ where $a \neq b \neq c \neq d$) is:

a $\binom{13}{1}\binom{12}{3}\binom{4}{2}\binom{4}{1}\binom{4}{1} / \binom{52}{5}$

b $\binom{13}{2}\binom{12}{2}\binom{4}{3}\binom{4}{1}\binom{4}{1} / \binom{52}{5}$

c $\binom{13}{1}\binom{12}{3}\binom{4}{2}\binom{4}{1}\binom{4}{1}\binom{4}{1} / \binom{52}{5}$

d none of the above

ex 8 card poker hand

Find chance of getting aaabbbcd $a \neq b \neq c \neq d$

$$\frac{\binom{13}{2}\binom{4}{3}\binom{4}{3}\binom{11}{2}\binom{4}{1}\binom{4}{1}}{\binom{52}{8}}$$

Last time

independent trials (draw w/ replacement)
- binomial distribution — 2 outcome trial
- multinomial distribution — K outcome trial

dependent trials (draw w/o replacement)
- hypergeometric distribution — 2 outcome trial
- multivariate hypergeometric distribution — K outcome trial

Sec 2.5 hypergeometric distribution

abbrev. $HG(n, N, G)$ Parameters: N = population size
G = # Good in population
n = sample size.

Suppose a population of size N contains G good and B bad elements $(N = G + B)$.
A sample, size n, with g good and b bad elements $(n = g + b)$ is chosen at random without replacement.

$$P(g \text{ good}) = \frac{\binom{G}{g}\binom{B}{b}}{\binom{N}{n}}$$

Today

① Sec 2.5 harder hypergeometric and counting strategies

① harder hypergeometric and counting strategies

Counting strategies

① Try and break the problem into a sequence of steps and apply the multiplication rule.

$P(AB) = P(A)P(B|A)$

② Find the count for a simple special case and see how to generalize.

Flip coin 5 times. Find prob get 2 heads.

Simpler: What is $P(HHTTT)$

ex

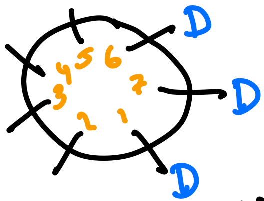
You and a friend are playing poker. If each of you are dealt 5 cards from the same deck, what is the chance that you both get a 4 of a kind (ranks aaaab, $a \neq b$)

$$P(\text{friend and you both get 4 of a kind})$$

$$= P(\text{friend 4 kind}) \cdot P(\text{you get 4 of kind} \mid \text{friend 4 of kind})$$

$$\frac{\binom{13}{1}\binom{4}{4}\binom{12}{1}\binom{4}{1}}{\binom{52}{5}} \cdot \frac{\binom{11}{1}\binom{4}{4}\left[\binom{10}{1}\binom{4}{1} + \binom{3}{1}\right]}{\binom{47}{5}}$$

ex There is an empty round table with 7 seats.



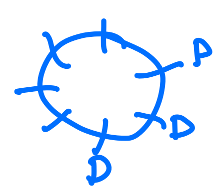
3 Democrats randomly sit at the table. What is the chance they sit next to each other?

$$\frac{7}{\binom{7}{3}}$$

ex There are 3 Democrats, 2 Republicans, and 2 Independents sitting around a table. What is the chance the Dem sit together, the Rep sit together and the Ind sit together?

$$P\left(\begin{matrix}\text{Dem}\\\text{together}\end{matrix}\right) \cdot P\left(\begin{matrix}\text{Rep together}\\\text{and Indep together}\\\text{given Dem together}\end{matrix}\right)$$

$$\frac{7}{\binom{7}{3}} \cdot \frac{2}{\binom{4}{2}}$$



ex In a well shuffled deck, find the probability that JJJJQQQQKKKK are the first 12 cards?

$$P\left(\begin{matrix}\text{JJJJ first}\\ \text{4 cards}\end{matrix}\right)\cdot P\left(\begin{matrix}\text{QQQQ}\\ \text{next 4}\end{matrix}\,\middle|\,\begin{matrix}\text{JJJJ}\\ \text{first 4}\end{matrix}\right)$$

$$\cdot P\left(\begin{matrix}\text{KKKK}\\ \text{next 4}\end{matrix}\,\middle|\,\begin{matrix}\text{JJJJQQQQ}\\ \text{first 8}\end{matrix}\right)$$

$$= \frac{1}{\binom{52}{4}}\cdot\frac{1}{\binom{48}{4}}\cdot\frac{1}{\binom{44}{4}}$$

$$= \frac{1}{\binom{52}{4,4,4,40}} \quad \text{or} \quad \frac{4!\,4!\,4!}{52\cdot 51\cdots 41}$$

$$\frac{1}{\frac{52\cdot 51\cdot 50\cdot 49}{4!}\cdot\frac{48\cdots 45}{4!}\cdot\frac{44\cdots 41}{4!}}$$

ex Continuing the previous problem, what is the chance the J are together, Q are together and K are together in the first 12 cards but in any order, for example QQQQKKKKJJJJ.

$$\frac{3!}{\binom{52}{4}\binom{48}{4}\binom{44}{4}}$$

ex Continuing the previous problem, what if the J, Q, K quartets can be together anywhere in the deck as 12 consecutive cards (for example cards 3-15)

$$\frac{41 \cdot 3!}{\binom{52}{4,4,4,40}}$$

Imagine the 12 cards are glued together as one fat card, so our deck now has 41 cards, there are 41 positions for the fat card.