Stat 134: Section 22

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Conceptual Review

Please discuss these short questions with those around you in section. These problems are intended to highlight concepts that will be relevant for today's problems.

a. What is conditional expectation? Can you give a concrete example?

b. Why is the expectation equal to the expectation of conditional expectation? Try to give an intuitive explanation and then a rigorous proof.

Problem 1

Let X_1, X_2 be independent uniform variables on $\{1, \cdots, n\}$. Let U, V be the minimum and maximum of X_1, X_2, respectively. Compute $\mathbb{E}[V|U = u]$ and $\mathbb{E}[U|V = v]$. What if X_1, X_2 are draws without replacement but still independent?
Ex 6.2.2 and 6.2.3 in Pitman's Probability

Problem 2

A deck of cards is cut into two halves of 26 cards each. As it turns out, the top half contains 3 aces and the bottom half just one ace. The top half is shuffled, then cut into two halves of 13 cards each. One of these packs of 13 cards is shuffled into the bottom half of 26 cards, and from this pack of 39 cards, 5 cards are dealt. What is the expected number of aces among these 5 cards?
Ex 6.2.11 in Pitman's Probability

Problem 3

Define the conditional variance of Y on $X = x$ as

$$\mathrm{Var}[Y|X = x] = \mathbb{E}[Y^2|X = x] - (\mathbb{E}[Y|X = x])^2.$$

Now, show that:

$$\mathrm{Var}(Y) = \mathbb{E}[\mathrm{Var}(Y|X)] + \mathrm{Var}[\mathbb{E}(Y|X)].$$

Ex 6.2.18 in Pitman's Probability

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