Stat 134: Section 23

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Conceptual Review

a. What is Φ^{-1}?

b. If X, Y are both linear combinations of independent normals, then what is the joint distribution of X and Y?

c. If X, Y are standard bivariate normal with correlation ρ, where $Y = \rho X + \sqrt{1-\rho^2}Z$, what is the conditional distribution of Y given $X = x$? What about the conditional distribution of X given $Y = y$?

d. If X, Y are independent standard normals, then how do we find P(X > 0, Y > X)?

Problem 1

Suppose PSAT scores have a mean of 1200 and an SD of 100 and SAT scores have a mean of 1000 and an SD of 90. Additonally, PSAT scores and SAT scores are approximately bivariate normal in distribution with correlation 0.6

a. Of the students who scored 1000 on the PSAT, about what percentage scored above average on the SAT?

b. Of the students who scored below average on the PSAT, about what percentage scored above average on the SAT?

c. About what percentage of students got at least 50 points more on the SAT than on the PSAT?

Ex 6.5.1 in Pitman's Probability

Problem 2

Let X and Y be independent standard normal variables.

a. For a constant k, find $\mathbb{P}(X > kY)$.

b. If $U = \sqrt{3}X + Y$, and $V = X - \sqrt{3}Y$, find $\mathbb{P}(U^2 + V^2 < 1)$.

c. Find the conditional distribution of X given $V = v$.

d. What is the joint distribution of U, V?

Ex 6.5.6 in Pitman's Probability