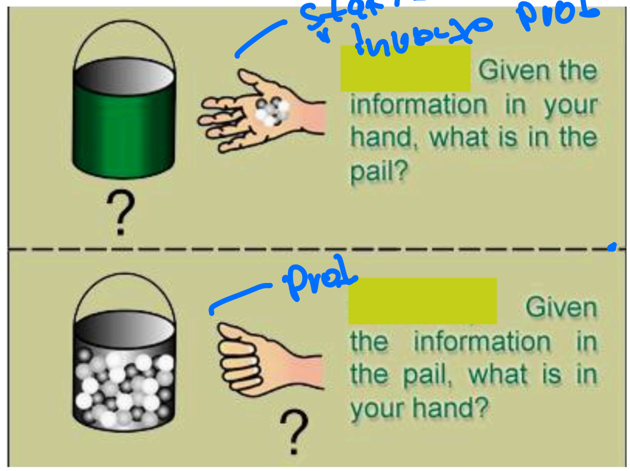
Stat 134 lec 1

warm up 11:00–11:10

Answer T/F

① The top picture is probability and bottom picture is statistics – F



statistics "inverse prob"

prob

② – T Section starts today

③ – T lectures will be recorded and can be found on Stat134.org

④ – T You should download the prelecture notes on bCourses/Pages before class.

⑤ – T I am optimistic I will be able to take people off the WL.

⑥ – T My OH is in SLC after class

Today

① sections 1.1 – 1.3

① **Sec 1.1** **Equally likely outcomes**

We call the set of all outcomes of an experiment Ω, the <u>outcome space</u>, or the <u>Sample space</u>.

let $A \subseteq \Omega$

$$P(A) = \frac{\# A}{\# \Omega}$$

Deck of cards: 4 suits H, C, D, S
13 ranks Ace, 2–10, J, Q, K
52 Cards

#5e p10

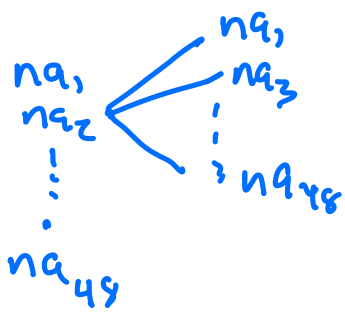
ex Suppose a deck of cards is shuffled and the top 2 cards are dealt. What is the chance you get at least one ace among the 2 cards

$A = \{(ace, ace), (ace, nonace), (nonace, ace)\}$

$A^c = \{nonace, nonace\}$

$P(A) = 1 - P(A^c)$

$\Omega = \{(\overset{52}{*}, \overset{51}{*})\}$ $\quad \#\Omega = 52 \cdot 51$



$\# A^c = 48 \cdot 47$

$$\boxed{P(A) = 1 - \frac{48 \cdot 47}{52 \cdot 51} = .149}$$

ex Two draws are made at random ==with replacement== from the box

$$\boxed{1}\,\boxed{2}\,\boxed{3}\,\boxed{4}\,\boxed{5}\,\boxed{6}\,\boxed{7}\,\boxed{8}\,\boxed{9}\,\boxed{10}$$

Find the chance the 2^{nd} number is bigger than twice the first.

Ω = all pairs of numbers $(\#\Omega = 10 \cdot 10)$

$$A = \left\{ \begin{array}{l} (1, >2) - 8 \\ (2, >4) - 6 \\ (3, >6) - 4 \\ (4, >8) - 2 \end{array} \right. \qquad \#A = 20$$

$$\rightarrow P(A) = \frac{20}{100} = \boxed{1/5}$$

Sec 1.2 Interpretations

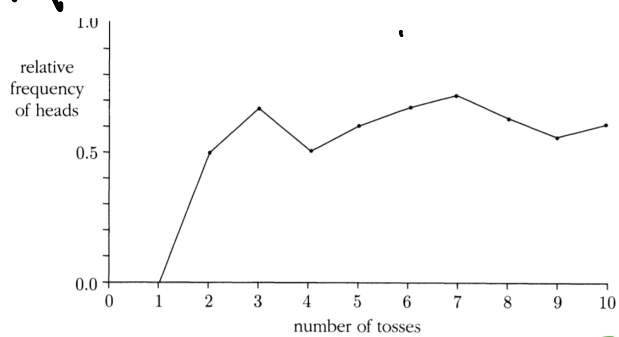
Probability has 2 interpretations

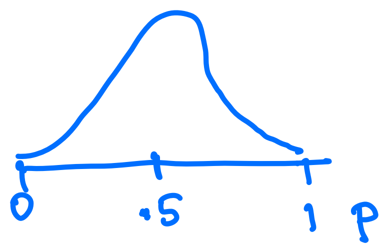
a) frequency interpretation.

eg what is the probability a particular coin lands heads.

There is a correct answer. The answer isn't a random variable.

Make an experiment, Law of averages.





will discuss in section 1.5

p as a RV has a histogram

b) Bayesian interpretation.

eg what is the probability a particular coin lands heads.

There is no correct answer. The probability is a random variable

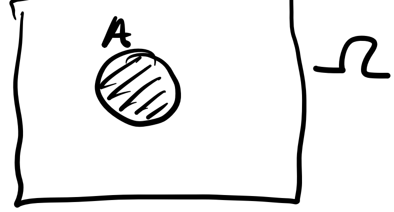
Your opinion may change over time as you acquire new data. This will change the value of your probability.

Sec 1.3 Distributions

To define probability we start with an **outcome space**, Ω, and assign to each element a nonnegative number and require that all numbers add up to 1.

Axioms

Venn Diagram

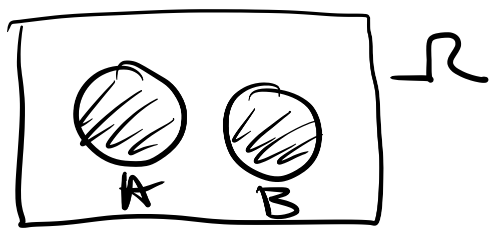


1) $P(A) \geq 0$ for all $A \subseteq \Omega$

2) $P(\Omega) = 1$

3) If A and B are mutually exclusive sets then $P(A \cup B) = P(A) + P(B)$

(addition rule)

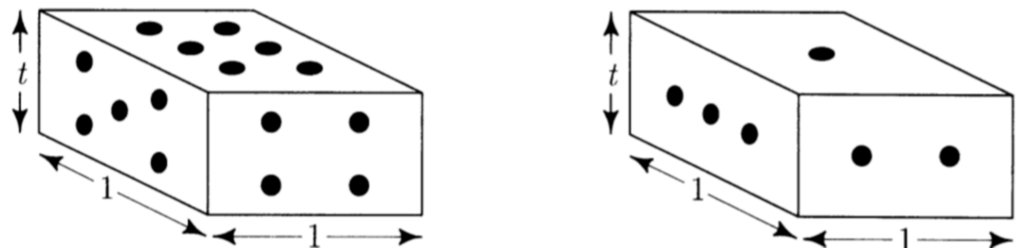


ex

Example 3. **Shapes.**

P 24

A *shape* is a 6-sided die with faces cut as shown in the following diagram:



Suppose the thickness of the die, t, is such that the chance of landing flat (1 or 6) is $2/3$.

Find the probability distribution of the shape. Draw a histogram.

$P(1) = 1/3$

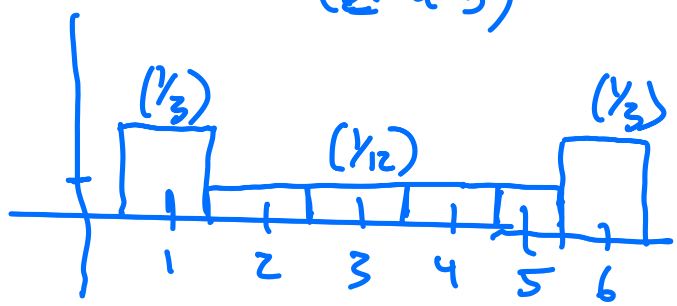
$\left.\begin{array}{l} P(2) \\ P(3) \\ P(4) \\ P(5) \end{array}\right\} = 1/12$

$P(6) = 1/3$

$1 - 2/3 = 4 \cdot x$

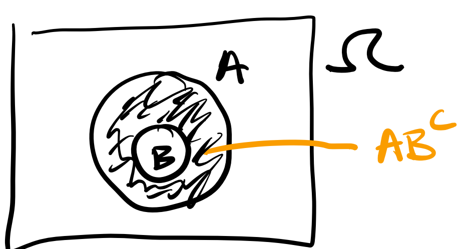
$x = 1/12$

chance of getting a non flat side (ex a 5)



Difference rule

Suppose $B \subseteq A$



Prove a formula for $P(AB^c)$ in terms of $P(A)$ and $P(B)$.

$A \cap B^c$

$$P(AB^c) = P(A) - P(B)$$

$A = B \cup AB^c$ disjoint union

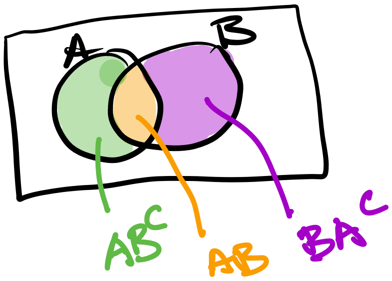
$\Rightarrow P(A) = P(B) + P(AB^c)$ by addition rule. $\square$

Inclusion exclusion rule

$$P(A\cup B) = P(A) + P(B) - P(AB)$$

$A\cap B$

Proof/



$$A\cup B = AB^c \cup AB \cup BA^c$$ disjoint union

$$P(A\cup B) = \underbrace{P(AB^c)}_{= P(A) - P(AB)} + P(AB) + \underbrace{P(BA^c)}_{= P(B) - P(AB)}$$

$$\boxed{P(A\cup B) = P(A) + P(B) - P(AB)}$$