Stat 134 Old Finals

Here are 3 practice finals. Each was originally intended to be roughly 110 minutes long, though Final #2 is on the short side. Your final length will be 8-10 problems with multiple parts, roughly 3 midterms in length but since you get more than 3 midterms of time to do it there shouldn't be much time pressure. Roughly 3 problems will be on Ch 1-3. The finals are all my problems (except some from text).

Stat 134 Final #1

1) Consider a standard deck of 52 cards, with 13 each of the four suits: spades, hearts, diamonds, and clubs. Suppose one draws 13 cards without replacement and let X be the number of hearts in the hand. Find:

a) $E(X)$ *10 pts* b) $P(X = 3)$ *10 pts*

2) 10 cards numbered 1 to 10 are shuffled and dealt one by one. Say a "record" occurs if the present card is higher than all the previous cards. Thus if the cards come out in the order 1,2,3,4,5,6,7,8,9,10 then 10 records are set; if the 10 comes out first then only one record can be set. What is the expected number of records? *10 pts*

3) A biased coin is flipped; the probability of heads on each flip is .4.

a) What is the distribution of the number of heads in 100 flips? *5 pts*

b) Give an approximation for $P(X = 45)$. *10 pts*

c) What is the distribution for the number Z of flips required to produce 4 heads? *10 pts*

d) What is $E(Z)$? *5 pts*

4) Let X have density function $f(x) = cx^2$ for $0 < x < 1$, and $f(x) = 0$ otherwise. Find:

a) the value of c. *10 pts* b) $E(X)$. *10 pts* c) $SD(X)$. *10 pts*

d) the cdf of X. *10 pts* e) the density function for $Y = \sqrt{X}$. *10 pts*

5) Let X, Y be independent exponentials with parameters λ_1 and λ_2. Let $T = X + Y$. **For parts b), c), d), e) assume that** $\lambda_1 = \lambda_2 = 2$. Note that this problem is long, but in general the parts do not build from each other, so if you can't do one part, still try to do the later parts.

a) Calculate $P(X \geq Y)$. *10 pts* b) Calculate $P(\frac{1}{2} < \frac{X}{Y} < 2)$. *10 pts*

c) Find $P(T < 3)$. *10 pts* d) Calculate $P(X < x | X < Y)$. *10 pts*

e) Find the joint density of X and T. *10 pts*

6) I roll a random number of dice. The number of dice has Poisson(6) distribution; let X = total number of spots showing on the dice. Find (and justify your answers):

a) $E(X)$ *10 pts* b) $SD(X)$ *10 pts*

Stat 134 Fi

1) Consider a standard deck of 52 cards with 4 aces. 5 cards are drawn from the deck, and let X be the number of aces in the 5 cards.

a) $E(X)$ b) $P(X = 3)$ c) Var(X)

2) 10 dice are shaken and rolled. Any dice which show sixes are set aside, and the remaining dice are rolled again. This is repeated until all the dice show sixes. As an example, say after the first roll there are 4 sixes, so then the remaining 6 dice are rolled again. After this there are 2 non-sixes remaining, and finally after the 3rd roll all the dice are sixes. Let N be the number of times the dice are shaken and rolled; $N = 3$ in the example. Let T be the number of individual die rolls; $T = 10 + 6 + 2 = 18$.

a) $P(N \leq 3)$ b) $E(T)$

3) Let X have density function $f(x) = ce^{-|x|}$ for $-\infty < x < \infty$ Find:

a) The value of c. b) The density function of X^2.

c) Var(X). d) A function g such that $g(X)$ is uniform (0,1).

4) A particle counter records two types of particles, types 1 and 2. Type 1 particles arrive at an average rate of 1 per minute, while type 2 particles arrive at an average rate of 2 per minute.

a) Find P(1st particle to arrive is of type 1).

b) Let T_3 be the time that the 3rd particle of any kind arrives. Find $P(1 < T_3 < 2)$.

5) Annie Oakley and Butch Cassidy each shoot at the center of a target. Annie hits the point (X_1, Y_1) where X_1 and Y_1 are independent normal random variables with mean 0 and SD 1 inch. Butch hits the point (X_2, Y_2) where X_2 and Y_2 are independent normals with mean 0 and SD 2 inches. Find:

a) P(Annie's shot is within 1 inch of the center)

b) P(Annie's shot is closer to the center than Butch's shot)

6) Let X be a fair die roll; Y is the number of heads resulting from a number of coin tosses equal to X.

a) $P(Y = 2|X = 4)$ b) $P(X = 4|Y = 2)$

Stat 134 Final #3

1) Bus lines A and B service a particular spot. Beavis and Daria are waiting at the bus stop. Beavis is waiting for an A bus, and Daria doesn't care which bus she gets on as long as Beavis isn't on it. Thus if the first bus is an A, then Beavis will get on and Daria will wait for the next bus of any kind. If the first bus is a B then Daria will get on and Beavis will wait for the next A. The buses arrival times come from independent Poisson processes: on average 6 A buses and 4 B buses come per hour. Let T be the amount of time that Beavis waits, D is the amount of time that Daria waits. Find:

a) $P(T > t)$

b) $P(D > t)$

c) $E(D)$

d) P(3rd bus of any kind comes after at least 30 minutes)

2) A box contains four tickets marked 0, 1, 1, and 2. Let S_n be the sum of the numbers obtained from n draws with replacement from the box.

a) Give the distribution of S_2 in a distribution table.

b) Find $P(S_{50} = 50)$ approximately.

c) Find an exact formula for $P(S_n = k)$.

3) X and Y are two random variables with joint density $f(x, y) = 2\lambda^2 e^{-2\lambda x - \lambda y}$ for $x > 0, y > 0; 0$ elsewhere. Let $Z = min(X, Y)$. *Hint: if you can see what the distribution of X and Y are from a), you don't need to integrate anything for c), d), and e).*

a) Find the marginal densities $f_X(x)$ and $f_Y(y)$.

b) Are X and Y independent? Why? c) Find $P(X + Y > 2)$

d) Find $P(X < Y)$ e) Find the cdf for Z.

4) Roll a die repeatedly. Let X be the number of sixes in the first four die rolls and let T be the time of the first six (could be more than four rolls if X is 0).

a) What are the unconditional distributions of X and T?

b) Write out a conditional distribution table for $T|X = 1$.

c) Write out a conditional distribution table for $T|X = 2$.

5) A gambler makes bets at a casino game which has $P(win) = \frac{1}{100}$ on each bet; all bets are independent. Let X be the number of wins in 80 bets.

a) find $P(X = 1)$ exactly. b) find $P(X \geq 2)$ approximately.

6) A chemistry class has 200 students; 80 are male and 120 are female. Lab partners are assigned at random. Let X be the number of male students who have female lab partners.

a) Find E(X). b) Find var(X).