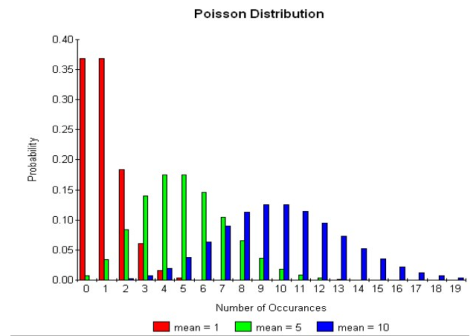
Last time   Stat 134   lec 7

sec 2.4   Poisson Distribution

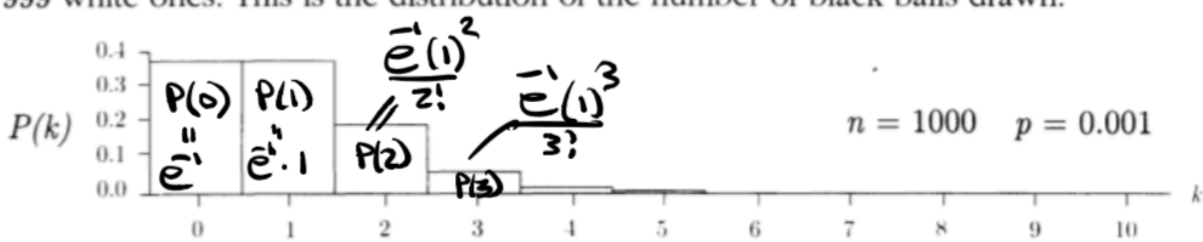
$$P(k) = \frac{e^{-\mu}\mu^k}{k!}, \quad k = 0, 1, 2, \ldots$$



We saw that Pois(μ) is a limit of binomials for $n \to \infty$, $p \to 0$ and $np \to \mu$.

The binomial (1000, 1/1000) distribution.

Now take 1000 random draws with replacement from a box with 1 black ball and 999 white ones. This is the distribution of the number of black balls drawn:



mode of Bin(n,p):

$m = \lfloor np + p \rfloor$

$$\text{mode} = \begin{cases} m & \text{if } np+p \notin \mathbb{Z} \\ m-1, m & \text{if } np+p \in \mathbb{Z} \end{cases}$$

mode of Pois(μ):

$m = \lfloor \mu \rfloor$ since $np + p \to \mu$

$$\text{mode} = \begin{cases} m & \text{if } \mu \notin \mathbb{Z} \\ m-1, m & \text{if } \mu \in \mathbb{Z} \end{cases}$$

Today   sec 2.5   Random sampling

independent trials (draw w/ replacement)
- binomial distribution — 2 outcome trial
- multinomial distribution — k outcome trial

dependent trials (draw w/o replacement)
- hypergeometric distribution — 2 outcom trial
- multivariate hypergeometric distribution — k outcome trial

Sec 2.5

Random sampling with replacement

ex Class 100 students

grade distribution:

A	50 students
B	30 students
C	15 students
D	5 students

You sample 10 students with replacement.

a) What is the chance you get

AAAABBBCCD ?

$(.5)^4(.3)^3(.15)^2(.05)^1$

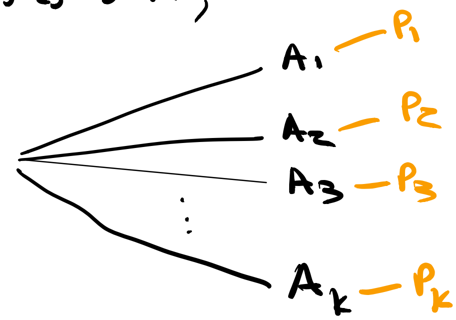
b) Find $P(4\,A's, 3\,B's, 2\,C's, 1\,D)$

$$\frac{10!}{4!3!2!1!}(.5)^4(.3)^3(.15)^2(.05)^1$$

$$\binom{10}{4,3,2,1} = \binom{10}{4}\binom{6}{3}\binom{3}{2}\binom{1}{1}$$

Def^n <u>Multinomial Distribution</u>

If you have n independent trials, where each trial has k possible outcomes, $A_1, A_2, \dots, A_k$ with probabilities $p_1, p_2, \dots, p_k$,



then the probability you get n_1 outcome A_1, n_2 outcome A_2, …, n_k outcome A_k is

$$P(n_1, n_2, \dots, n_k) = \binom{n}{n_1, n_2, \dots, n_k} p_1^{n_1} p_2^{n_2} \cdots p_k^{n_k}$$

$$\frac{n!}{n_1!\, n_2! \cdots n_k!}$$

<u>Note</u> Binomial distribution is a special case with $k=2$.

independent trials (draw w/ replacement) < binomial distribution — 2 outcome trial
multinomial distribution — k outcome trial

random sample without replacement

ex In a very student friendly class with 100 students the grade distribution is:

A 70 students
B 30 students

You sample 5 students at random without replacement (called a simple random sample (SRS))

a) Find the chance you get

A A A B B

$$\frac{70}{100}\frac{69}{99}\frac{68}{98}\frac{30}{97}\frac{29}{96}$$

b) Find P(3A's, 2B's).

$$\binom{5}{2,3}\frac{70}{100}\frac{69}{99}\frac{68}{98}\frac{30}{97}\frac{29}{96}$$

$$\frac{5!}{2!3!}$$

(if we drew with replacement the answer would be $\binom{5}{2,3}(.7)^3(.3)^2$)

$$\frac{5!}{3!2!}\frac{70}{100}\frac{69}{99}\frac{68}{98}\frac{30}{97}\frac{29}{96} = \frac{\frac{70\cdot 69\cdot 68}{3!}\cdot\frac{30\cdot 29}{2!}}{\frac{100\cdot 99\cdot 98\cdot 97\cdot 96}{5!}}$$

$$= \frac{\binom{70}{3}\binom{30}{2}}{\binom{100}{5}}$$

hypergeometric formula

Def^n <u>hypergeometric distribution</u>

Suppose a population of size N contains G good and B bad elements $(N=G+B)$.
A sample, size n, with g good and b bad elements $(n=g+b)$ is chosen at random <u>without replacement</u>.

$$P(g \text{ good and } b \text{ bad}) = \frac{\binom{G}{g}\binom{B}{b}}{\binom{N}{n}}$$

this generalizes to the <u>multivariate hypergeometric distribution</u>

Now instead of 2 types of elements we have K with sizes $G_1, G_2, \dots, G_K$ $(N=G_1+\dots+G_K)$ and in our sample we have $n=g_1+\dots+g_K$.

$$P(g_1, g_2, \dots, g_K) = \frac{\binom{G_1}{g_1}\binom{G_2}{g_2}\cdots\binom{G_K}{g_K}}{\binom{N}{n}}$$

<u>ex</u> Class 100 students

grade distribution:

A 50 students

B 30 students

C 15 students

D 5 students

You sample 10 students with without replacement (SRS)

Find $P(4\text{ A's}, 3\text{ B's}, 2\text{ C's}, 1\text{ D})$ $\quad \frac{\binom{50}{4}\binom{30}{3}\binom{15}{2}\binom{5}{1}}{\binom{100}{10}}$

ex A 5 card <u>poker hand</u> consists of a SRS of 5 cards from a 52 card deck. There are $\binom{52}{5}$ poker hands.

a) Find P(poker hand has 4 aces and a king)

$$\frac{\binom{4}{4}\binom{4}{1}\binom{44}{0}}{\binom{52}{5}}$$

b) Find P(poker hand has 4 aces)

$$\frac{\binom{4}{4}\binom{48}{1}}{\binom{52}{5}} = \frac{\binom{12}{1}\binom{4}{4}\cdot\binom{4}{1}}{\binom{52}{5}}$$

c) Find P(poker hand has $\underbrace{\text{4 of a kind}}_{\text{rank (aaaab)}}$)

here a and b are different.

$$\frac{\underbrace{\binom{13}{1}}_{\text{13 choices for a}}\overset{a}{\binom{4}{4}}\overset{\text{not a}}{\binom{48}{1}}}{\binom{52}{5}}$$

or

$$\frac{\overset{\text{13 choices for a}}{\binom{13}{1}}\overset{\text{12 choices for b}}{\binom{12}{1}}\overset{a}{\binom{4}{4}}\overset{b}{\binom{4}{1}}}{\binom{52}{5}}$$

tinyurl.com/sept13-pt1

Stat 134

Chapter 2 Wednesday February 5 2019

1. The probability of being dealt a three of a kind poker hand (ranks $aaabc$ where $a \neq b \neq c$) is:

   a $\binom{4}{3}\binom{4}{1}\binom{44}{1} / \binom{52}{5}$

   b $\binom{13}{1}\binom{12}{2}\binom{4}{3}\binom{4}{1}\binom{4}{1} / \binom{52}{5}$

   c $\binom{13}{1}\binom{12}{1}\binom{4}{3}\binom{4}{1}\binom{44}{1} / \binom{52}{5}$

   d none of the above

(b) Choose a rank for three of a kind, then choose 2 ranks for the singles. We have $\binom{12}{2} = \frac{12 \cdot 11}{2}$ instead of $12 \cdot 11$ since $aaabc = aaacb$ in a poker hand so we double count.

Note Not $\binom{13}{3}$ since this treats the 3 of a kind and singles as equivalent ranks. The rank assigned to a 3-of a kind is different then the rank assigned to a single.
$aaabc \neq bbbac$

ex if have 8 card poker hand with two 3 of a kind and 2 singles you would have $\binom{13}{2}\binom{11}{2}$ in front, since the two 3 of a kind are equivalent.

2. The probability of being dealt a pair in a poker hand (ranks $aabcd$ where $a \neq b \neq c \neq d$) is:

a $\binom{13}{1}\binom{12}{3}\binom{4}{2}\binom{4}{1}\binom{4}{1}/\binom{52}{5}$

b $\binom{13}{2}\binom{12}{2}\binom{4}{3}\binom{4}{1}\binom{4}{1}/\binom{52}{5}$

c $\binom{13}{1}\binom{12}{3}\binom{4}{2}\binom{4}{1}\binom{4}{1}\binom{4}{1}/\binom{52}{5}$

d none of the above

choose rank of pair

choose 3 ranks of single where order doesn't matter,

Answ

$$\frac{\binom{13}{1}\cdot\binom{12}{3}\cdot\binom{4}{2}\binom{4}{1}\binom{4}{1}\binom{4}{1}}{\binom{52}{5}}$$

because

$aabcd = aabdc = aacbd = aacdb$

$= aacbd = aacdb$

in your poker hand.