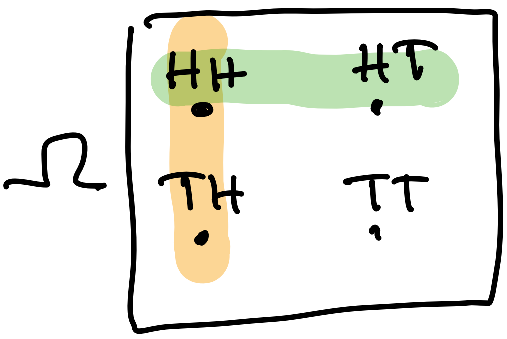
Stat 134 lec 3

warmup 9:00 - 9:10

The outcome space of flipping two coins is drawn below.



Find two indep. events and show them in the outcome space.

A = heads on 1st toss

B = heads on 2nd toss

Notice that if A, B are independent nonempty sets that they must have a nonempty intersection.

mutually exclusive ⇒ not independent

independent ⇒ not mutually exclusive

last time

Sec 1.4 Conditional Probability and independence

we saw last time the multiplication rule

$P(AB) = P(A|B)P(B)$ and $P(AB) = P(B|A)P(A)$

$\Rightarrow P(A|B)P(B) = P(B|A)P(A)$

Hence if $P(A|B) = P(A)$ then

$\overset{= \cancel{P(A)}}{P(A|B)}\, P(B) = P(B|A)\cancel{P(A)}$

$\Rightarrow P(B) = P(B|A)$

So to show independence you can show

$P(A|B) = P(A)$

or

$P(B|A) = P(B)$

or

$P(AB) = P(A)P(B)$

ex

Two separate decks of cards are shuffled. What is the chance that the top card of the ~~first~~ deck is the **king** of spades **or** the bottom card of the ~~second~~ deck is the **king** of spades

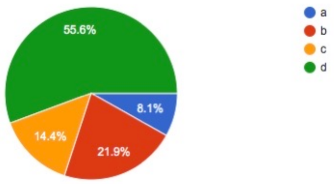
a $\frac{1}{52} + \frac{1}{52} - \frac{1}{52} \times \frac{1}{52}$

b $\frac{1}{52} + \frac{1}{51}$

c $\frac{1}{52} + \frac{1}{52} - \frac{1}{52} \times \frac{1}{51}$

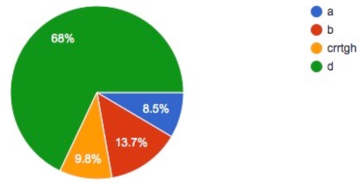
d none of the above

round 1



$\Rightarrow$

round 2



a	The answer is a by the inclusion exclusion rule

d	Two ways: 1. The two are mutually exclusive events so their intersection is 0 so while taking the union, the individual probabilities add up which is 1/52+1/52 or 1/26 which isn't among the options 2. By the complement rule, the probability is 1 - $\underbrace{(51/52)*(50/51)}$ which is 1 - 50/52 which again gives 1/26 which isn't among the options

prob top card and bottom card not king of spades

51/52 is the chance the first card isn't king of spades.

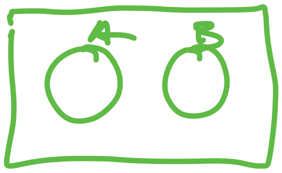
50/51 is the chance the last card isn't king of spades given that first card isn't king of spades.

Today

1. Sec 1.4 Mutually Exclusive versus Independent
2. Sec 1.5 Bayes' Rule

① Sec 1.4 Mutually Exclusive (ME) versus Independent

ME: $P(AB) = 0$

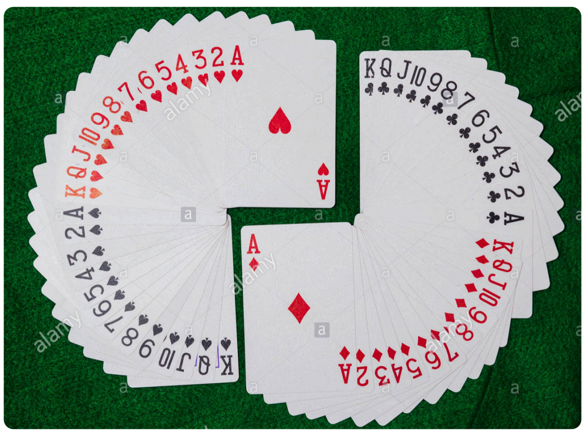


Ind: $P(A|B) = P(A)$

eg Consider different kinds of cards

Is red and Heart ME ✗, Ind ✗?

$\underset{1}{\underset{\shortparallel}{P(R|H)}} \neq \underset{1/2}{\underset{\shortparallel}{P(R)}} \Rightarrow$ dep



eg Is red and Spade ME ✓, Ind ✗?

$\underset{0}{\underset{\shortparallel}{P(R|S)}} \neq \underset{1/2}{\underset{\shortparallel}{P(R)}}$

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Suppose A and B are two events with

$$P(A) = 0.8 \text{ and } P(A \cup B) = 0.8.$$

Is it possible for A and B to be both **mutually exclusive** and **independent**?

a yes

b no

c there isn't enough information to decide

$B = \emptyset$

$$\underset{\substack{\parallel \\ 0}}{P(AB)} = \underset{\substack{\parallel \\ .8}}{P(A)}\,\underset{\substack{\parallel \\ 0}}{P(B)} \checkmark \Rightarrow A, B \text{ indep if } B = \emptyset$$

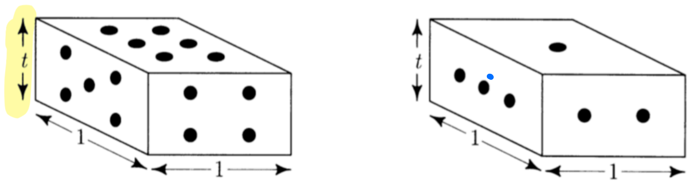
<u>Note</u> better to check for independence using ~~test~~ $P(AB) = P(A)P(B)$ since $P(A|B)$ is hard to think about if $B = \emptyset$.

ex

Sec 1.5 Baye's rule

P(A and B)
P(AB)
P(A,B)
$P(A \cap B)$
all the same

Shapes.
A *shape* is a 6-sided die with faces cut as shown in the following diagram:



A box contains 3 shaped die (see pic above), D_1, D_2, D_3, with probability 1/3, 1/2, 2/3 respectively of landing flat (with 1 or 6 on top).

Note: the numbers 1/3, 1/2, 2/3 dont add up to 1 because they are the chance of landing flat for 3 different die.

forward or likelihood conditional

a) what is $P(\text{get } 6 \mid D_1) = \frac{1}{2}\cdot\frac{1}{3} = \frac{1}{6}$

b) what is $P(\text{get } 6, D_1) = P(\text{get } 6 \mid D_1)P(D_1) = \frac{1}{18}$

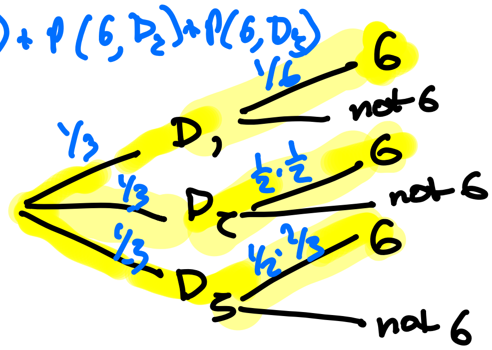
$P(\text{get } 6 \mid D_1) = \frac{1}{6}$, $P(D_1) = \frac{1}{3}$

c) what is $P(\text{get } 6)$

$P(6) = P(6, D_1) + P(6, D_2) + P(6, D_3)$

$= \frac{1}{3}\cdot\frac{1}{6} + \frac{1}{3}\cdot\frac{1}{4} + \frac{1}{3}\cdot\frac{1}{3}$

$= \boxed{\frac{1}{4}}$



d) Find Posterior $P(D_1 \mid 6) = \frac{P(D_1, 6)}{P(6)} = \frac{1/18}{1/4} = \boxed{\frac{2}{9}}$

backwards or Posterior conditional

HHHHT is allowed

Suppose you draw a number from a bag, with equal probabilities across the choices {1, 2, 3}.

Once you draw a number, you toss a coin until you get that many number of heads followed by a tails—so if you draw a 3, you keep tossing until you encounter the sequence {Heads, Heads, Heads, Tails}.

What is the probability of tossing a coin seven times given that you draw the number 2?

$P(7 \mid \text{draw } 2)$ forward conditional

* * * * * * * — 2^7 possibilities.

Cant have HHTH for first four tosses.

HHTT

HHHT so $16-4=12$ possibilities

THHT

$$\boxed{\frac{12}{2^7}}$$