

Stat 134: Section 25

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Conceptual Review

- a. What are covariance and correlation?
- b. Let X, Y be two i.i.d Gaussians and α, β real numbers what is the distribution of $\alpha X + \beta Y$
- c. What does standard bivariate normal distribution mean?

Problem 1: Exchangeability

Let X_1, \dots, X_n exchangeable random variables, meaning that their joint distribution is the same no matter what order they are presented. Let $S_n = \sum_{k=1}^n X_k$ and show that

$$\text{Var}(S_n) = n\text{Var}(X_1) + n(n-1)\text{Cov}(X_1, X_2)$$

Ex 6.4.18 in Pitman's Probability

Problem 2: Covariance and normal distribution

Let X_1, X_2 two i.i.d normal random variables. Define $Y_1 = X_1 + X_2$ and $Y_2 = \alpha X_1 + 2X_2$ where α is a real number such that $\text{Cov}(Y_1, Y_2) = 0$. Find the density of Y_2 and $\text{Cov}(X_2, Y_2)$.

Ex 6.5.8 in Pitman's Probability

Problem 3: Normal marginals do not mean bivariate normal

As in problem 2 of section 24, suppose that X is an $N(0, 1)$ random variable and let Y be a random variable independent of X such that $P(Y = 1) = P(Y = -1) = \frac{1}{2}$. Consider the variable $Z = YX$. Are X, Z bivariate normal?