

August 28th, 2019

Please discuss these short questions with those around you in section. These problems are intended to highlight concepts that will be relevant for today's problems.

- What is an outcome space (typically notated as  $\Omega$ )?
- What are the three axioms of probability?
- Find a convenient formula for  $\sum_{k=1}^n k$ .

Consider an outcome space  $\Omega$ , and two subsets  $A, B \subset \Omega$ . In each of the parts below, fill in the blanks with either sets, or  $\cup$  or  $\cap$  symbols so that the equalities hold. (It may help to draw Venn diagrams.)

- Partitioning:  $A = (A \text{ — } B) \text{ — } (A \text{ — } B^c)$
- DeMorgan's Law I:  $(A \cap B)^c = A^c \text{ — } B^c$
- DeMorgan's Law II:  $(A \cup B)^c = A^c \text{ — } B^c$
- Suppose  $B \subset A$ . Then  $A \cap B = \underline{\hspace{2cm}}$ .

## *The Birthday Problem*

CLASS ACTIVITY: In your discussion section, how many students do you think have the same birthday? As time permits, your GSI will go around the room and have students say their birthdays.

Suppose you are in a classroom of  $n$  students ( $n \leq 365$ ). In the following calculations, ignore leap days and assume that students' birthdays are independent and distributed uniformly at random across the year. Find:

- a. the chance that at least one other student shares *your* birthday.
- b. the chance that at least two students share the same birthday.
- c. (continued from part b): Using your answer from part (b), derive a useful approximation for this expression, using the approximation  $\log(1 + x) \approx x$  for small  $x$ .

How are these assumptions violated in reality? How does this affect the true probability of these events?

*From Section 1.6, Example 5 (pg 62) in Pitman's Probability*