Stat 134: Section 20

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Conceptual Review

a. For strictly positive variables X, Y, write out the convolution formula for the density of $Z = Y/X$.

b. If $X \sim \text{Exp}\,(\mu)$, what is the distribution of aX (for $a > 0$)?

Problem 1

A system consists of two components. Suppose each component is subject to failure at constant rate λ, independently of the other, up to when the first one fails. After that moment the remaining component is subject to additional load and failure at constant rate 2λ.

a. Find the distribution of time until both components have failed.

b. What are the mean and variance of this distribution?

Ex 5.4.4 in Pitman's Probability

Problem 2: Competing Exponentials

Suppose $X \sim \text{Exp}(\lambda_X)$, $Y \sim \text{Exp}(\lambda_Y)$, and X, Y are independent.

a. Find $P(X < Y)$.

b. Now suppose $\lambda_X = \lambda_Y = \lambda$. Using part (a), find the density of $Z = X/Y$. (Hint: look at the CDF of Z.)

c. By a similar process as in (b), find the density of $W = \frac{X}{X+Y}$.

Problem 3

Suppose I toss three coins. Some of them land heads and some land tails. Those that land tails I toss again. Let X be the number of heads showing after the first tossing, Y the total number showing after the second tossing, including the X heads appearing on the first tossing. So X and Y are random variables such that $0 \leq X \leq Y \leq 3$ no matter how the coins land. Write out distribution tables and sketch histograms for each of the following distributions:

a. the distribution of X;

b. the conditional distribution of Y given $X = x$ for $x = 0, 1, 2,$;

c. the joint distribution of X and Y;

d. the distribution of Y;

e. the conditional distribution of X given $Y = y$ for $y = 0, 1, 2, 3$.

Ex 6.1.1 in Pitman's Probability

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