Stat 134 lec 1

Today:
0) announcements
1) ice breaker
2) sec 1.1 Equally likely outcomes
3) sec 1.2 Interpretations
4) Sec 1.3 Distributions.

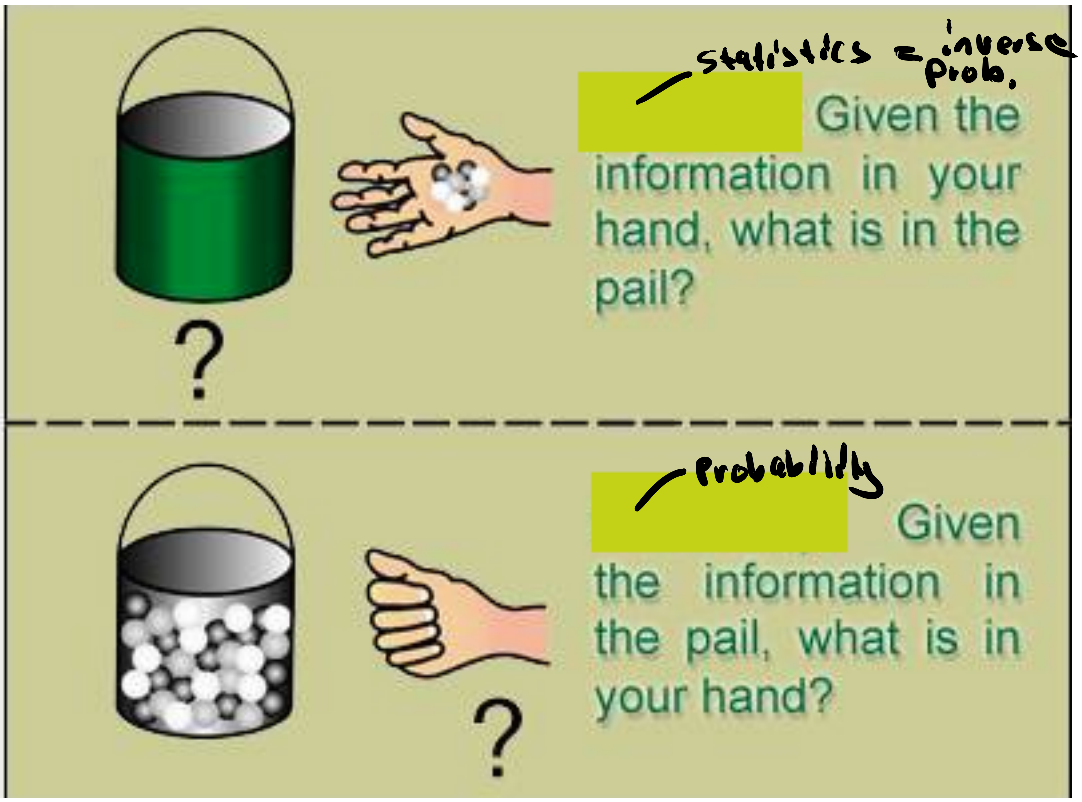
0) Announcements:

- Lab today !!
- prelecture notes are on b-courses/pages
- Class website www.stat134.org
- see schedule on website.
- My OH is before class in SLC.
- The adjunct class 198 started yesterday. Contact Mike Leong mleong@berkeley.edu
- some student comments

Probability requires a different way of thinking compared to most other computational mathematics/statistics courses that younger students are used to, so a lot of people see much lower marks when they just enter the course. As with anything, this requires practice, practice and practice.
Ready to study hard
keep up
Prepare to engage a lot of time
Go to every section and lecture. Doing that gives you an immediate advantage over half the class.
Don't take this class if this class is not your major requirement.
The beginning of the course can be a little scary in terms of breadth and difficulty of the material. If you keep up with the work, you will end up fine. One of the more interesting courses I've had at Cal.

1) ice breaker

One of these pictures describes probability, which one?



2) <u>Sec 1.1</u> Equally likely outcomes

We call the set of all outcomes of an experiment, Ω, the <u>outcome space</u> or the <u>Sample space</u>

ex Imagine you have a 3 card deck J, Q, K.

$$\Omega = \left\{ \begin{matrix} (J,Q,K) \\ (J\ K\ Q) \\ (Q,J\ K) \\ (Q,K,J) \\ (K,Q\ J) \\ (K,J,Q) \end{matrix} \right\}$$

all outcomes are equally likely

Let A be the event that the 2^{nd} card is Q.

$$A = \left\{ \begin{matrix} (J,Q,K) \\ (K,Q,J) \end{matrix} \right\}$$

What is the probability that the 2^{nd} card is Q?

<u>Answ</u> Since all outcomes are equally likely

$$\boxed{P(A) = \frac{\# A}{\# \Omega}}$$

$$= \frac{2}{6} = \frac{1}{3}$$

Deck of cards : 4 suits H, C, D, S

13 ranks Ace, 2-10, J, Q, K

52 cards

ex — #5e p10

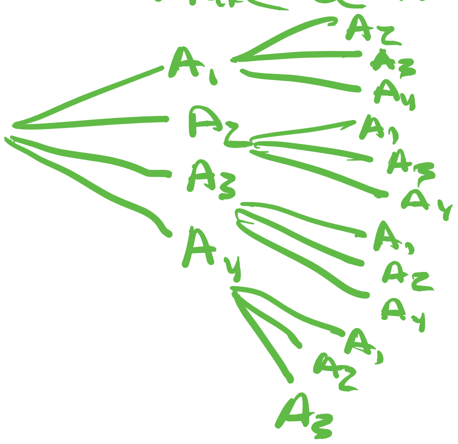
Suppose a deck of cards is shuffled and the top 2 cards are dealt. What is chance you get at least one ace among the 2 cards?

Soln

$$A = \{ \underbrace{(ace, ace)}_{4\cdot 3}, \underbrace{(ace, nonace)}_{4\cdot 48}, \underbrace{(nonace, ace)}_{48\cdot 4} \}$$

$$\Omega = \{ \underbrace{(*, *)}_{52\cdot 51} \}$$

To see why the chance of getting (ace, ace) is $\frac{4\cdot 3}{52\cdot 51}$ you can make a tree diagram:



There are 12 branches corresponding to each pair of aces, so the event (ace, ace) has size 12. There are $52\cdot 51$ pairs of possible cards.

$$\#A = 4(3+48+48) = 4\cdot 99$$

$$\#\Omega = 52\cdot 51$$

$$P(A) = \frac{4\cdot 99}{52\cdot 51} = .149$$

or

$$1 - P(A^c) = 1 - \frac{48\cdot 47}{52\cdot 51} = .149$$

ex Two draws are made at random with replacement from the box

$$\boxed{1}\ \boxed{2}\ \boxed{3}\ \boxed{4}\ \boxed{5}\ \boxed{6}\ \boxed{7}\ \boxed{8}\ \boxed{9}\ \boxed{10}$$

Find the chance the second number is bigger than twice the first.

soln

Ω = all pairs of numbers $\qquad \#\Omega = 10 \cdot 10$

$$A = \left\{ \begin{array}{ll} (1, >2) & 8 \\ (2, >4) & 6 \\ (3, >6) & 4 \\ (4, >8) & 2 \\ (5, >10) & 0 \end{array} \right.$$

$\# A = 8 + 6 + 4 + 2$

$$\Rightarrow P(A) = \frac{\# A}{\# \Omega} = \frac{20}{100} = .2$$

3) Sec 1.2 Interpretations.

Probability has 2 interpretations.

a) frequency interpretation.

- you repeat an experiment many times and count the frequency some event occurs.

ex You flip a fair coin 100 times. You won't always get exactly 50 heads. But if you repeat experiment many times on average you will get 50 heads. This is the law of averages.

b) subjective (Bayesian) interpretation — will discuss in sec 1.5

Probability based on opinion.

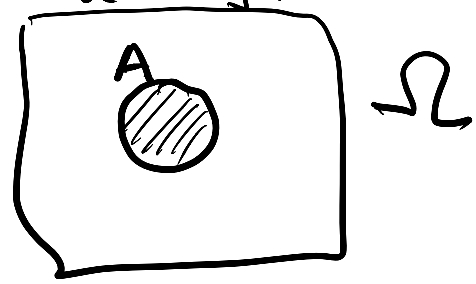
ex The probability of a particular patient surviving an operation.

Your opinion may change over time as you acquire new data. This will change the value of your probability.

Sec 1.3 Distributions

To define probability we start with an <u>outcome space</u>, Ω, and assign to each element a nonneg number and require that all numbers add up to 1.

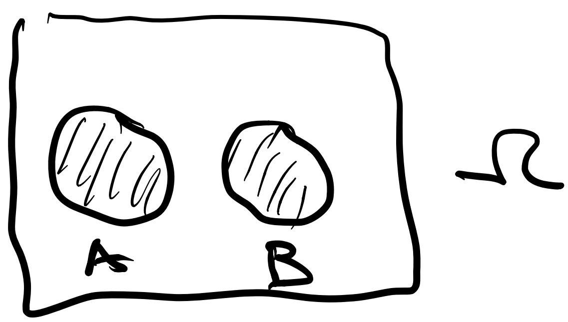
Venn diagram



Axioms

1) $P(A) \geq 0$ for all $A \subseteq \Omega$
2) $P(\Omega) = 1$
3) If A and B are mutually exclusive sets then $P(A \cup B) = P(A) + P(B)$

(addition rule)



ex

Example 3. **Shapes.**

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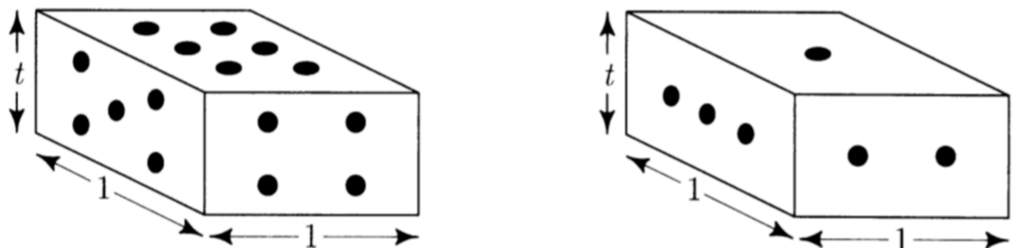
A *shape* is a 6-sided die with faces cut as shown in the following diagram:

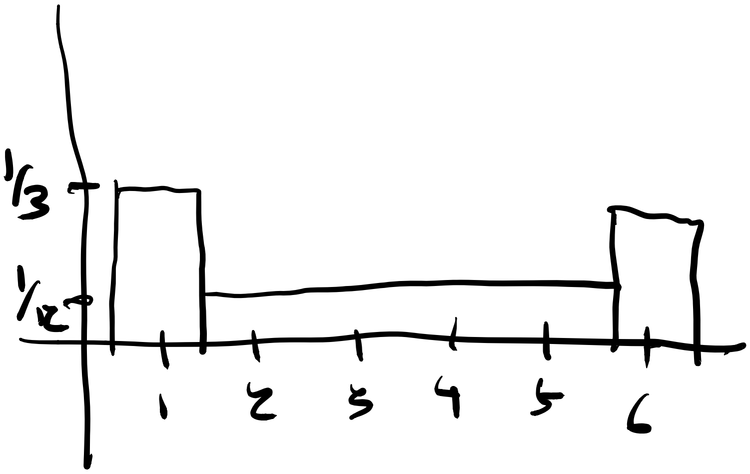


Suppose the thickness of the die, t, is such that the chance of landing flat (1 or 6) is $2/3$.

Find the probability distribution of the shape. Draw a histogram.

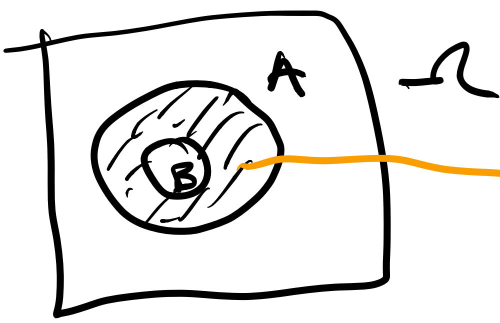
$$\frac{2}{3} + 4x = 1$$
$$\Rightarrow 4x = \frac{1}{3} \Rightarrow x = \frac{1}{12}$$

$P(1) = 1/3$
$P(2) = 1/12$
$P(3) = 1/12$
$P(4) = 1/12$
$P(5) = 1/12$
$P(6) = 1/3$



Difference rule

Suppose $B \subseteq A$



$A \setminus B$ or AB^c

Give formula for $P(A\setminus B)$.

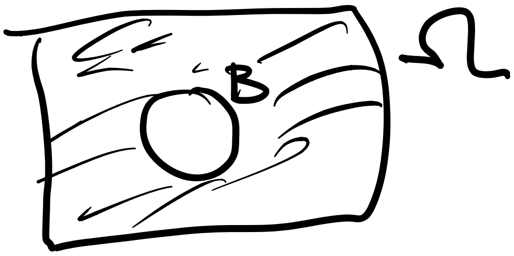
Soln

$A = B \cup (A \setminus B)$ disjoint union

$P(A) = P(B) + P(A\setminus B)$ addn rule.

$\Rightarrow \quad P(A\setminus B) = P(A) - P(B)$

Complement rule



$$P(B^c) = 1 - P(B)$$

Prove the complement rule using the difference rule $P(A \setminus B) = P(A) - P(B)$.

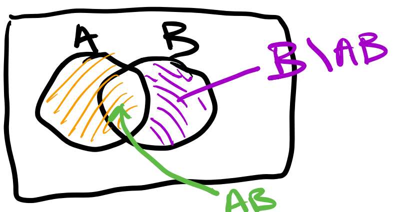
soln

let $A = \Omega$ apply diff formula

$B^c = \Omega \setminus B$

$$\Rightarrow P(B^c) = \underbrace{P(\Omega)}_{= 1} - P(B)$$

Inclusion Exclusion Rule



$$P(A \cup B) = P(A) + P(B) - P(AB)$$

soln

$A \cup B = A \cup (B \setminus AB)$ disjoint

use addn rule,

$$P(A \cup B) = P(A) + P(B \setminus AB) = P(A) + \overbrace{P(B) - P(AB)}^{P(B \setminus AB)}$$

by difference rule.