Stat 134: Section 7

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Conceptual Review

Please discuss these short questions with those around you in section. These problems are intended to highlight concepts from lecture that will be relevant for today's problems.

a. What is $\mathbb{E}(X)$?

b. When do we want to use indicators instead of the weighted sum formula to calculate expectation? What's the rule for choosing indicators?

Problem 1

Suppose the Stat department teaches 15 classes a semester: 2 have 60 students, 1 has 300 students, and 12 have 20 students. Each course is taught by a different professor, and each student only takes one class in the department.

a. For a randomly selected professor, what is the expected size of the class they teach?

b. For a randomly selected student, what is the expected size of the class they are in? How does this compare to part (a)?

Problem 2

In a well-shuffled standard deck of cards, we are interested in the number of adjacent pairs; i.e., cards which are the same rank as the card before or after them in the deck. Calculate the expected number of adjacent pairs.

Hint: consider the probability that a card is the same as the card before it.

Problem 3

Let A and B be independent events, with indicator random variables I_A and I_B.

a. Describe the distribution of $(I_A + I_B)^2$ in terms of $P(A)$ and $P(B)$;

b. What is $\mathbb{E}(I_A + I_B)^2$?

c. Suppose we now have a set of identical but not necessarily independent indicators $I_1, I_2, \ldots, I_n$. Derive a useful formula for $\mathbb{E}(I_1 + I_2 + \ldots + I_n)^2$