# *Stat 134: Conditional Probabilities, Distributions, & Expectations Review*

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## *Problem 1*

Let $X_1 \sim$ Geom $(p_1)$, $X_2 \sim$ Geom $(p_2)$, $X_1 \perp X_2$, both on $\{1, 2, \ldots\}$. Find:

a. $P(X_1 \leq X_2)$;

b. $P(X_1 = x \mid X_1 \leq X_2)$. Recognize $X_1 | X_1 \leq X_2$ as a named distribution, and state the parameter(s).

## *Problem 2*

Let $Y \sim$ Beta $(r, s)$. Conditioned on $Y = y$, let $X \sim$ Geometric $(y)$ on $\{0, 1, 2, \ldots\}$ For simplicity, assume $r, s > 1$.

a. Find $\mathbb{E}(X)$.

b. P(X = x, Y $\in$ dy)

c. Find $P(X = x)$, for $x \in \{0, 1, 2, \ldots\}$.

## *Problem 3*

Suppose a proportion $p$ of a population has a gene $m$ that makes them predisposed to migraines. Of these people, the number of migraines they experience in a year follows a Poisson process with rate $\mu$ per year, whereas the rest of the population experiences migraines according to a Poisson process with rate $\lambda$.

a. What is the probability that a randomly selected individual experiences no migraines in a given year?

Hint: Condition on whether the individual has gene $m$.

b. Let $N_t$ denote the number of migraines a randomly selected individual experiences in $t$ years. Find $\mathbb{E}(N_t)$.

## *Problem 4*

Let $X, Y$ have joint density $f_{X,Y}(x, y) = 2\lambda^2 e^{-\lambda(x+y)}, 0 < x < y$. It can be shown that $f_X(x) = 2\lambda e^{-2\lambda x}, x > 0$. Find:

a. The conditional density of $Y$, given $X = x$;

b. $\mathbb{E}(Y|X = x)$.