Stat 134: Section 17

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Conceptual Review

Please discuss these short questions with those around you in section. These problems are intended to highlight concepts from lecture that will be relevant for today's problems.

a. X, Y are jointly distributed on the region $(x, y) : 1 < x < y < 3$. True or false: X, Y could be independent.

b. Set up a double integral that would yield the CDF of Y in terms of the joint density $f_{X,Y}(x, y)$.

c. If W, Z are jointly uniformly distributed over a region, why can we use areas instead of volumes to calculate probabilities?

Problem 1

A metal rod is ℓ inches long. Measurements made using this rod are distributed uniformly from $\ell - 0.1$ to $\ell + 0.1$ inches, accounting for random error. Assume measurements are independent of each other.

a. Find the chance that a measurement is within 0.01 inches of ℓ.

b. Find the chance that two measurements are within 0.01 inches of each other.

Draw a picture to help visualize this event.

Ex 5.1.2 in Pitman's Probability

Problem 2

Suppose that (X, Y) is uniformly distributed over the region $\{(x, y) : 0 < |y| < x < 1\}$. Find:

As before, draw a picture of the region. This will help you to set bounds for integration, and may provide a hint for part (d).

a. The joint density of (X, Y)

b. The marginal densities $f_X(x)$ and $f_Y(y)$

c. Are X and Y independent?

d. Find $\mathbb{E}(X)$ and $\mathbb{E}(Y)$.

Ex 5.2.1 in Pitman's Probability

Problem 3

Minimum and maximum of two independent exponentials. Suppose S and T are i.i.d. Exponential (λ) random variables. Define $X = \min\{S, T\}$, $Y = \max\{S, T\}$, and $Z = Y - X$.

Consider $P(X \in dx, Y \in dy)$. What are the possible ways this could happen?

a. Find the joint density of X and Y. Are X, Y independent?

b. Find the joint density of X and Z. Are X, Z independent?

c. Identify the marginal distributions of X and Z.

Ex 5.2.9 in Pitman's Probability

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