Stat 134 lec 9

warmup 1:00 - 1:10

Adam, Jess and Tom are standing in a group of 12 people. The group is randomly split into two lines of 6 people each. The chance that Adam, Jessica, and Tom are standing next each other in one of these lines is:

a $\dfrac{\binom{3}{3}\binom{9}{3}}{\binom{12}{6}} * \dfrac{\binom{4}{1}}{\binom{6}{3}}$

b $2 * \dfrac{\binom{3}{3}\binom{9}{3}}{\binom{12}{6}} * \dfrac{\binom{4}{1}}{\binom{6}{3}}$

c none of the above

We first find the chance A,J,T are together in line 1 or line 2 but not necessarily consecutive

Out of 12 we take a sample of 6.

A,J,T are good and the other 9 are bad,

we have $\dfrac{\binom{3}{3}\binom{9}{3}}{\binom{12}{6}}$

Have 2 possible half lines so multiply by 2.

Next we find the chance they are consecutive given that they are in the same half line.

Thinking of A J T as one person there are 4 in the line.

$\binom{4}{1}$ ways AJT can be in the line.

The chance they are consecutive is $\dfrac{\binom{4}{1}}{\binom{6}{3}}$.

Now apply mult rule!

$$\boxed{2 \cdot \frac{\binom{3}{3}\binom{9}{3}}{\binom{12}{6}} \cdot \frac{\binom{4}{1}}{\binom{6}{3}}}$$

Last time Harder hypergeometric problems.

today ① sec 2.5 Binomial approx to hypergeometric.

② sec 3.1 – random variables (RV)

joint distribution of 2 RVs and independence

Binomial approx to hypergeometric.

Binomial — independent trials
Hypergeometric — dependent trials.

ex 100 person class with a grade distribution:

A grade : 70 students
B grade : 30 students.

Sample 5 students at random w/o replacement (SRS).

Find $P(3A's, 2Bs)$

exact
hypergeometric
$$= \frac{\binom{70}{3}\binom{30}{2}}{\binom{100}{5}} = \binom{5}{3}\frac{70}{100}\frac{69}{99}\frac{68}{98}\frac{30}{97}\frac{29}{96} = .316$$

approx
binomial
$$= \binom{5}{3}(.7)^3(.3)^2 = .309$$

When N is large relative to n, $HG(\underset{n}{5}, \underset{N}{100}, \underset{G}{70}) \approx Bin(\underset{n}{5}, \underset{p}{.7})$

why? — w/o replacement ≈ w/ replacement when $N >> n$

$$HG(n, N, G) \approx Bin\left(n, \frac{G}{N}\right)$$ — Prob of drawing a good elt from the population.

Summary of approximations

$HG(n, N, G)$

↓ approx by binomial
N large, n small
$p = \frac{G}{N}$

binomial(n, p)

↙ approx by Poisson
$p \to 0$, $n \to \infty$, $np \to \mu$

Poisson(μ)

↘ approx by normal
n large
$\mu = np$, $\sigma = \sqrt{npq}$
$0 < \mu \pm 3\sigma < n$
use continuity correction

normal(μ, σ^2)

Sec 3.1 Intro to Random Variables (RV)

A RV, X, is the outcome of an experiment.

What distribution is the following RV?

X = The number of aces in 5 cards drawn from a standard deck?

$$X \sim HG(n=5\ N=52,\ G=4)$$

belongs to

ex flip a prob p coin 2 times

X = # heads

$X=1$ is an event

$P(X=1) = \binom{2}{1} p^1 (1-p)^2$ binomial formula

we write $X \sim Bin(2,p)$

More precisely, outcome space

$X: \Omega \longrightarrow \mathbb{R}$ is a function

$HH \longmapsto 2$

$HT \longmapsto 1$

$TH \longmapsto 1$

$TT \longmapsto 0$

so $X=1$ means $\{HT, TH\} \subseteq \Omega$

X has a probability distribution

X	0	1	2
$P(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

P(HT or TH)

$P(X=0) = P(TT) = 1/4$

$P(X=1) = P(HT \text{ or } TH) = 1/2$

$P(X=2) = 1/4$

You can find the distribution of $g(X) = |X-1|$? — values 0,1

function of a RV

$g(x)$	0	1
$P(g(x))$	$\frac{1}{2}$	$\frac{1}{2}$

since this happens when $X=1$ with prob $1/2$.

$Ber(1/2) = Bin(1, 1/2)$

Joint Distribution

Let (X,Y) be the joint outcome of 2 RVs X, Y.

The event $(X=x, Y=y)$ is the intersection of events $X=x$ and $Y=y$.

ex

X: one draw from $\boxed{1}\ \boxed{2}\ \boxed{2}\ \boxed{3}$

Given $X=x$, Y = number of heads in x coin tosses.

$$P(X=x, Y=y) = P(Y=y \mid X=x) \cdot P(X=x)$$

$$P(X=1, Y=1) = \underbrace{P(Y=1 \mid X=1)}_{= 1/2} \cdot \underbrace{P(X=1)}_{= 1/4} = 1/8$$

What the range of values of X? — 1, 2, 3

Y? — $0, 1, 2, \dots, x$

Find,

$P(1,0) = \frac{1}{2} \cdot \frac{1}{4} = 1/8$

$P(1,1) = \frac{1}{2} \cdot \frac{1}{4} = 1/8$

$P(2,0) = 1/4 \cdot 1/2 = 1/8$

$P(2,1) = 1/2 \cdot 1/2 = 1/4$

$P(2,2) = \frac{1}{4} \cdot \frac{1}{2} = 1/8$

$P(3,0) = 1/8 \cdot 1/4 = 1/32$

$P(3,1) = \frac{3}{8} \cdot \frac{1}{4} = \frac{3}{32}$

$P(3,2) = \frac{3}{8} \cdot \frac{1}{4} \quad \frac{3}{32}$

$P(3,3) = \frac{1}{8} \cdot \frac{1}{4} \quad \frac{1}{32}$

marginal prob of X

$P(x) = \sum_{y \in Y} P(x,y)$

Y \ X	1	2	3	
	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	
3	0	0	$\frac{1}{32}$	$\frac{1}{32}$
2	0	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{7}{32}$
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{32}$	$\frac{15}{32}$
0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{9}{32}$

$P(X=1, Y=2)$ $\rightarrow$ 0

$\neq$

$P(X=1)P(Y=2)$

marginal prob of Y

$P(y) = \sum_{x \in X} P(x,y)$

$X-1 \sim \text{Bin}(2, \tfrac{1}{2})$

Y not a named distribution.

Is X, Y dependent? — yes.

ex $\left.\begin{array}{l} P(Y=0 \mid X=1) = \frac{1}{2} \\ P(Y=0) = \frac{9}{32} \end{array}\right\} \Rightarrow X, Y$ dep

$\frac{1/8}{1/4} = \frac{1}{2}$

$P(Y=0 \mid X=1) = \frac{P(Y=0, X=1)}{P(X=1)}$ $\; //$ Bayes

<u>Def^n</u> Two RVs are <u>independent</u> if

$$P(Y=y \mid X=x) = P(Y=y) \quad \text{for all } \begin{array}{l} x \in \mathcal{X} \\ y \in \mathcal{Y} \end{array}$$

By the multiplication rule,

if X, Y are indep,

$$P(X=x, Y=y) = \underbrace{P(Y=y \mid X=x)}_{= P(Y=y)} P(X=x)$$

for all x, y

so $\boxed{P(X=x, Y=y) = P(X=x)P(Y=y)}$.

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Stat 134

1. The joint distribution of X and Y is drawn below:

Y \ X	0	1	2	P(y)
1	1/4	1/3	1/12	2/3
0	1/8	1/6	1/24	1/3
P(x)	3/8	1/2	1/8	

a X and Y are independent

$3/8 \cdot 2/3 = 1/4$ ✓

$1/2 \cdot 2/3 = 1/3$ ✓

do all 6 cases

b X and Y are dependent since the two rows $Y = 0$ and $Y = 1$ give different probabilities for each value of X.

False

To check independence show that

$P(X=x, Y=y) = P(X=x)P(Y=y)$ for all $x \in X$, $y \in Y$ or

show $P(X=x \mid Y=y) = P(X=x)$

(b.) is false since just because the rows are different we can still have

$P(X=x \mid Y=0) = P(X=x \mid Y=1)$ (i.e. independence),

If we divide the first row by $P(Y=1) = 2/3$

we get $\frac{1/4}{2/3}, \frac{1/3}{2/3}, \frac{1/12}{2/3} = \left(P(X=0 \mid Y=1), P(X=1 \mid Y=1), P(X=2 \mid Y=1)\right)$

$\quad\quad\; \| \quad\quad \| \quad\quad \|$

$\quad\quad 3/8 \quad 1/2 \quad 1/8$

If we divide the 2^{nd} row by $P(Y=0) = 1/3$

we also get $3/8, 1/2, 1/8$ ✓