Stat 134: Section 21

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Conceptual Review

Please discuss these short questions with those around you in section. These problems are intended to highlight concepts from lecture that will be relevant for today?s problems.

a. Suppose a random variable X depends on an event A which occurs with probability p. Write out a formula to find $E(X)$ by conditioning on A. (Hint: your answer should be a sum of two terms.)

b. How do we find the variance of a random variable X by conditioning on Y?

Problem 1

Suppose that N is a Poisson (λ) R.V., and that given $(N = k)$, for $k > 1$, there are defined random variables $X_1, \ldots, X_k$ such that

$$E(X_j|N = k) = \mu(1 \leq j \leq k)$$

Define a random variable S_N by

$$\begin{cases} X_1 + X_2 + \ldots + X_k & \text{if}(N = k), k \geq 1 \\ 0 & \text{if}(N = 0) \end{cases}$$

Show that $E(S_N) = \mu E(N) = \mu\lambda$.
Ex 6.2.7 in Pitman's Probability

Problem 2

Suppose you have a coin which lands heads with probability p. Let X denote the number of tosses required to observe both heads and tails.

a. Find $E(X)$;

b. Find $Var(X)$.

Problem 3: The Beta-Binomial

Let $S_n = \sum_{i=1}^{n} X_i$ be the number of successes in a sequence of $i = 1$ Bernoulli (Π) trials, where $\Pi \sim$ Beta (r, s). That is, given $\Pi = p$, $S_n \sim$ Binomial (n, p). This arises as a natural model in Bayesian inference when we are uncertain about the true value of p.

a. Given $S_n = k$, show that the posterior distribution of Π is Beta $(r + k, s + n + k)$;

b. Use the fact that the total integral of the beta $(r + k, s + n + k)$ density is 1 to find a formula for the unconditional probability $P(S_n = k)$;

c. Find $E(\Pi|Sn = k)$ and $Var(\Pi|Sn = k)$. (Note that these facts can be used to show as $n \to \infty, \Pi \to \frac{S_n}{n}$, the observed sample proportion of successes, regardless of the values of r, s.)

Ex 6.3.15 in Pitman's Probability

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