Population Study Using Leslie Matrix and Monte Carlo

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Population Based on Several Variables

-Birth Rates
-Survivorship
-Net Immigration
-Fertility Rate

Replicating Research Paper

-Using these variables, make forward-looking population projections

-Fertility Rates are randomly drawn, used to calculate birth rates

-Net Immigration also randomly drawn

How are these variables combined to form projections?....

The Leslie Matrix

$$\begin{bmatrix} N_{1t+1} \\ N_{2t+1} \\ N_{3t+1} \\ \vdots \\ N_{it+1} \\ \vdots \\ N_{kt+1} \end{bmatrix} = \begin{bmatrix} F_{1t} & F_{2t} & \cdots & F_{it} & \cdots \\ P_{1t} & 0 & \cdots & \cdots & \\ 0 & P_{2t} & \cdots & 0 & \cdots \\ & & & & \\ & & & & \\ & & & & \\ 0 & 0 & \cdots & & \end{bmatrix} \cdot \begin{bmatrix} N_{1t} \\ N_{2t} \\ N_{3t} \\ \vdots \\ N_{it} \\ \vdots \\ N_{kt} \end{bmatrix} + \begin{bmatrix} NI_{1t} \\ NI_{2t} \\ NI_{3t} \\ \vdots \\ NI_{it} \\ \vdots \\ NI_{kt} \end{bmatrix}.$$

Implementing Monte Carlo to Leslie Matrix

- 100x

-1000x

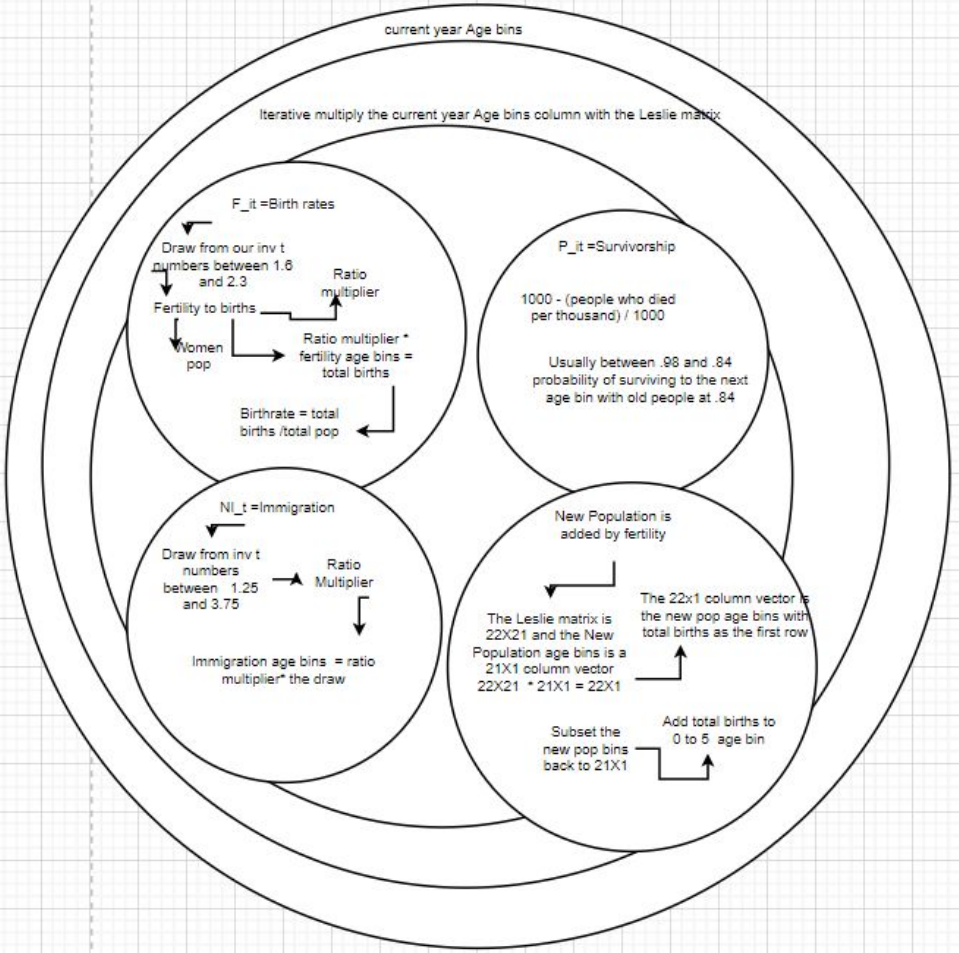
-10000x

Year	Mean
100 trials	
1987	242.0
1992	251.4
1997	259.6
2002	266.6
2012	280.2
2022	291.4
2032	297.8
2042	300.0
2052	300.2
2062	300.9
2072	302.6
2082	303.8
1000 trials	
1987	241.7
1992	251.1
1997	259.2
2002	266.3
2012	279.8
2022	290.9
2032	297.0
2042	298.6
2052	298.4
2062	298.8
2072	300.1
2082	300.6
10000 trials	
1987	241.6
1992	251.0
1997	259.2
2002	266.4
2012	279.8
2022	291.0
2032	297.2
2042	298.9
2052	298.8
2062	299.4
2072	300.8
2082	301.5

Structure of the R Code

Five Main Parts:

- Baseline Population Age Bins
- Birth Rate
- Survivorship
- Net Immigration
- New Population Age Bins



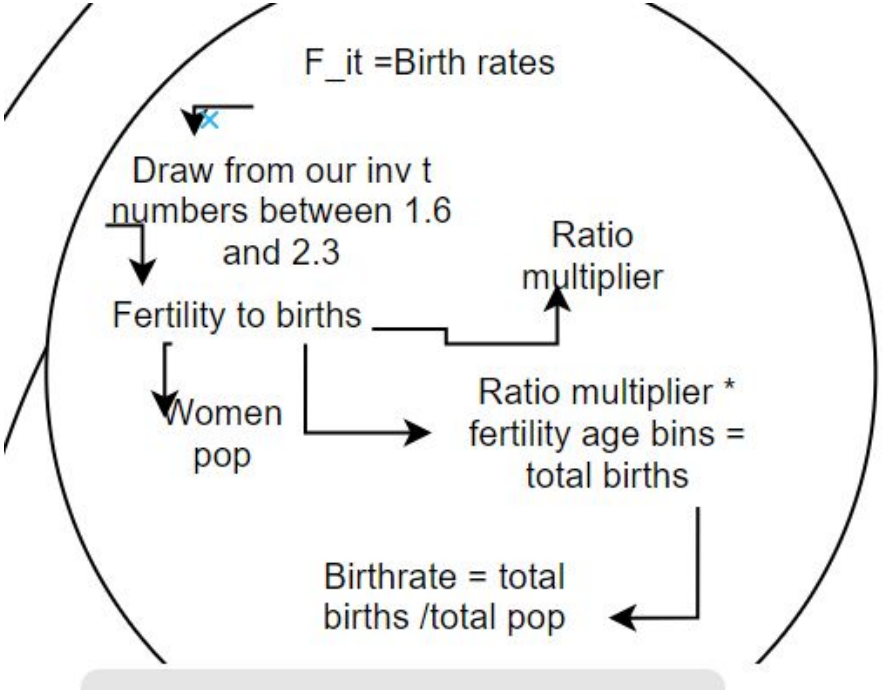
Initial assumptions

Table 1
Principal assumptions.

	Lower bound a_t	Median m_t	Upper bound b_t
Total fertility rate X_t	1.6	1.9	2.3
Quinquennial change Z_t	0.0	0.3	0.5
Life expectancy at birth in 2082 (years)	77.4	81	85.9
Quinquennial net immigration (in millions)	1.25	2.25	3.75

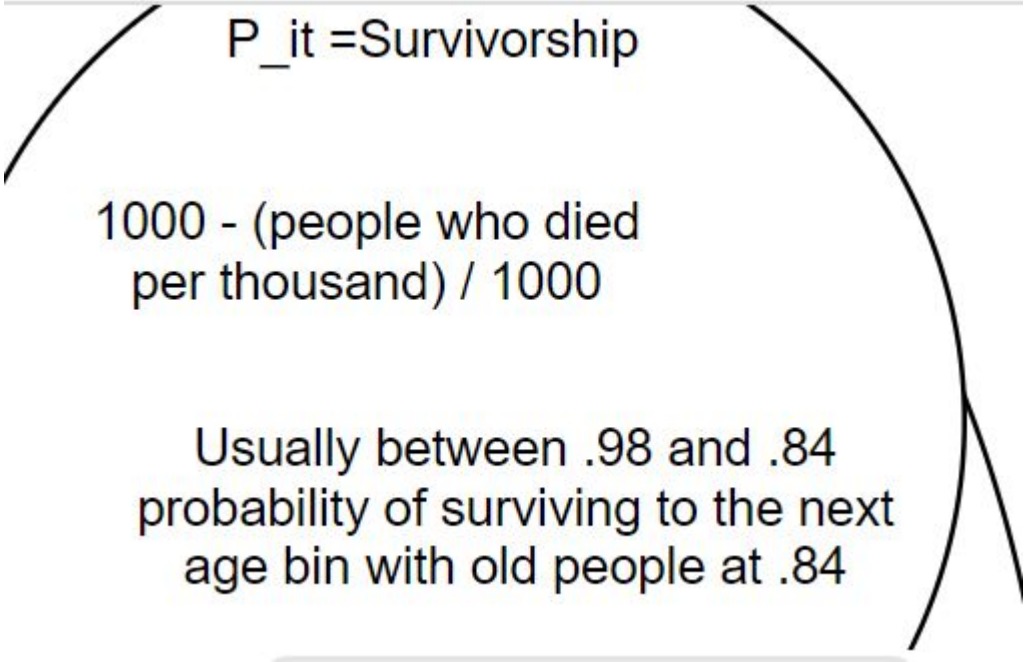
Birth Rate : F_it

- How people enter the population
- Changes stochastically/randomly



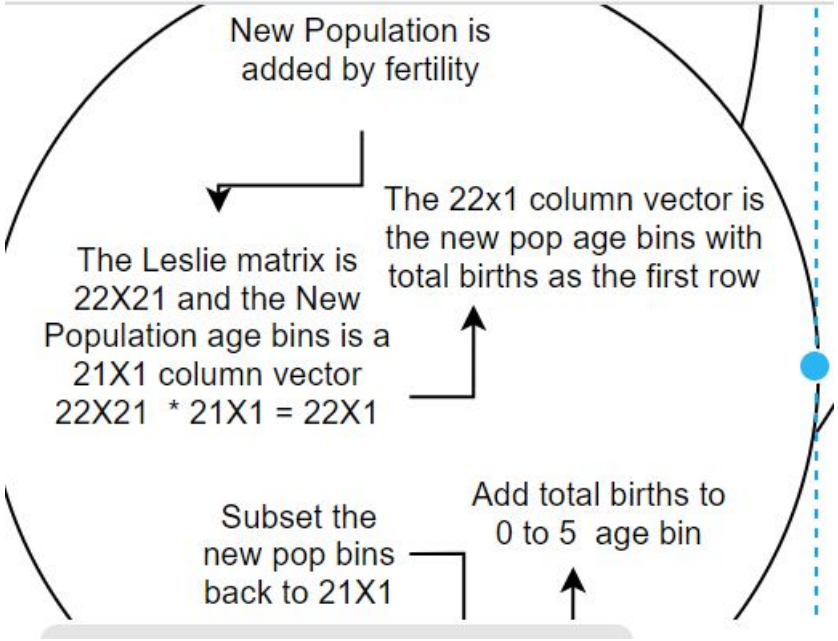
Survivorship: P_it

- How people leave the population
- Does not change randomly in our model



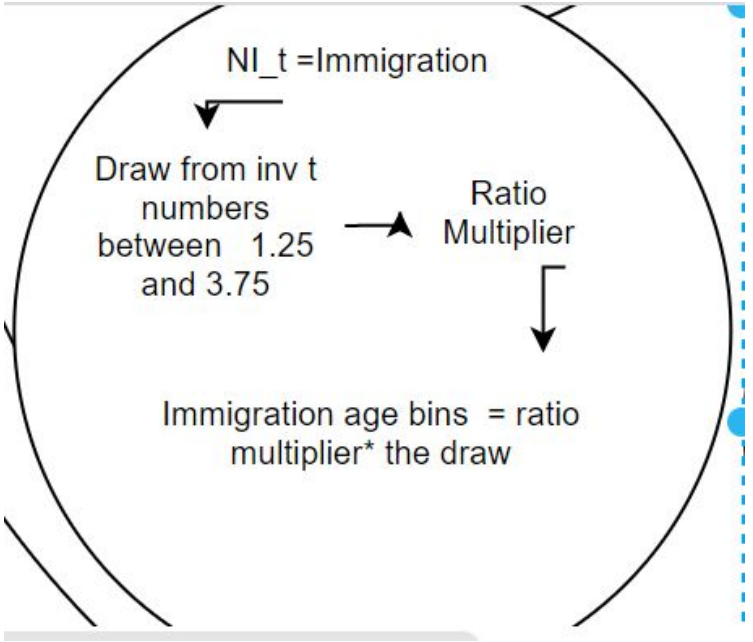
New Population Age Bins : N_it+1

- The goal of the matrix multiplication is to iteratively create new age bins
- Population is the sum or rows in the age bins
- Each row is the population for a particular age group
- T+1 just means a new year



Net Immigration: NI_t

- The secondary way that people are added to the population
- This is random/ stochasticly drawn



Why Randomness

- The Leslie Matrices historically have had poor projections because originally they used deterministic(or non random) variables
- Fertility rates , Birth rates, and Immigration rates have variance and fluctuate.
- Drawing from the inverse transform accounts for this variability
- Consequences of not adding randomness have hurt important projects such as the 1983 social security projections
- These projections where supposed to have funds available until 2058. Funds are running out now!
- According to investopedia, some solutions have been to increase retirement age or lower social security check amounts

Replication Results Using MC:

Years 1982-2021

1982	1983	1984	1985	1986	1987	1988	1989	1990
234.0	245.8	243.3	244.6	246.5	248.6	250.6	253.0	254.7
1991	1992	1993	1994	1995	1996	1997	1998	1999
257.2	259.2	261.4	264.1	266.5	268.6	270.1	273.2	275.4
2000	2001	2002	2003	2004	2005	2006	2007	2008
277.6	279.5	282.1	284.2	286.5	288.7	290.7	293.2	295.3
2009	2010	2011	2012	2013	2014	2015	2016	2017
297.5	299.4	301.6	303.6	305.5	307.7	309.6	311.6	313.6
2018	2019	2020	2021					
315.7	317.9	319.5	321.5					

Comparing Our Estimates and Research Paper's to True U.S Population

Year	Paper's Estimate	Our Estimate	True U.S Population
1987	242.0	248.6	245.0
1992	251.4	259.2	257.0
1997	259.6	270.8	271.7
2002	266.6	279.5	287.2
2012	280.2	301.6	314.0
2022	291.4	321.5	331.0

Expanding Study Using 2010 Census Data

-Monte Carlo & Leslie Matrix methods will be expanded using 2010 Census data as a way to make future projections using more up to date information

United States™ Census 2010

Citations

1982 Census:https://www.sciencedirect.com/science/article/pii/0169207088900155

- Populations, birth tables , fertility tables

1976 Life Table:https://www.cdc.gov/nchs/data/vsus/mort76_2a.pdf

- Survivorship rates

The article:

Actual U.S Population : https://www.worldometers.info/world-population/us-population/

Investopedia:https://www.investopedia.com/ask/answers/071514/why-social-security-running-out-money.asp

- Social security