Spring 2021 Thursday Discussion 4

Thursday, March 4, 2021 1:00 PM

discrete

Exercise 1

Let the random variable X have PMF $P_X(x) = \frac{\lambda^x e^{-\lambda}}{x!}$. Find the moment-generating function (MGF) of X.

$$\mathbb{E}[e^{tx}] = \sum_{x=0}^{\infty} e^{tx} \cdot \frac{\lambda^x \cdot e^{-\lambda}}{x!}$$

$$= e^{-\lambda} \cdot \sum_{x=0}^{\infty} \frac{(e^t \cdot \lambda)^x}{x!}$$

$$, e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$= e^{-\lambda} \cdot e^{(e^t \cdot \lambda)} = \boxed{e^{\lambda(e^t - 1)}}$$

Exercise 2

Suppose we are trying to model, Y, the winning percentage of the UIUC basketball team. Then Y is a random variable between 0 and 1. Let

$$f_Y(y) = \theta y^{\theta-1}, \theta > 0, 0 \leq y \leq 1.$$

continuous

Find the expected value of Y, i.e. $\mathbf{E}[Y]$.

$$\mathbb{E}[Y] = \int_0^1 y \cdot \theta \cdot y^{\theta-1} \, dy = \int_0^1 \theta y^{\theta} \, dy$$

$$= \theta \frac{y^{\theta+1}}{\theta+1} \Bigg|_0^1 = \theta \left[\frac{1}{\theta+1} - 0\right]$$

$$= \boxed{\frac{\theta}{\theta+1}}$$

$= \boxed{\theta+1}$

occurrences! $\Gamma(n) = (n-1)!$

Exercise 3

Suppose X is a random observation from a Gamma(shape $= \alpha$, rate $= \beta$) distribution. The parameters have the following conditions: $\alpha > 0$ and $\beta > 0$. The pdf of X is.

$f_X(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$ for $x \in (0, \infty)$ B

a) Let $Y \sim \text{Gamma}(1, \lambda)$. Find the pdf of Y. Can you identify an equivalent distribution for Y by looking at its pdf? $\leftarrow \alpha$

$$f_y(y) = \frac{\lambda^1}{\Gamma(1)} x^{1-1} \cdot e^{-\lambda x} = \frac{\lambda}{1} \cdot e^{-\lambda x}$$

$$= \boxed{\lambda e^{-\lambda x} \sim \exp(\lambda)}$$

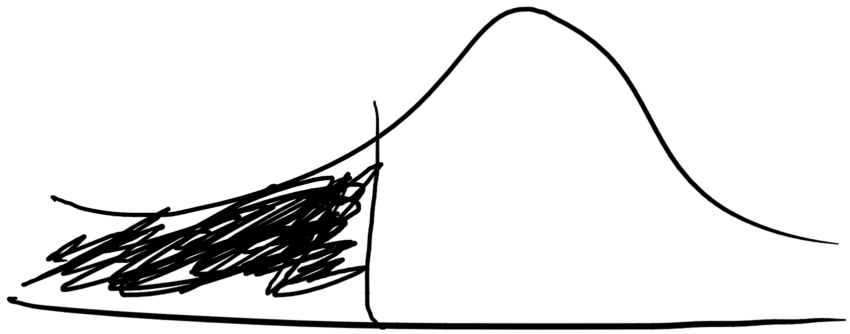
b) Traffic accidents at a particular intersection follow an Exponential distribution with an average rate of 1.4 per week. What is the probability that the fourth accident will occur after the end of the seventh day?

Rate $= 1.4$ per week $) \ \frac{1.4}{7} = 0.2$ per day

$$T \sim \text{Gamma}(4, 0.2)$$

$$P(T > 7) = \int_7^\infty \frac{(0.2)^4}{\Gamma(4)} t^{4-1} e^{-0.2t} \, dt$$

$$\approx \boxed{0.9463}$$



$$P(Z < -1.5)$$

$$= 0.0668$$

$$P(Z < -2.28)$$

$$= 0.0113$$