Spring 2021 Thurs Dis

Thursday, February 4, 2021 12:58 PM

1. Given the series $\sum_{n=1}^{\infty} \frac{2}{2^{n-1}}$

   (a) Identify the type of infinite series and find the value it converges to

$$\sum_{n=1}^{\infty} \frac{2}{2^{n-1}} = 2\underbrace{\sum_{n=1}^{\infty} \frac{1}{2^{n-1}}}_{\text{series}}$$

$$\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} = 1 + \frac{1}{2} + \frac{1}{4}, \quad \text{geometric series: } \frac{a}{1-r}$$

$$\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} = \frac{1}{1-\frac{1}{2}} = 2$$

$$\sum_{n=1}^{\infty} \frac{2}{2^{n-1}} = 2 \cdot \sum_{n=1}^{\infty} \frac{1}{2^{n-1}} = 2 \cdot 2 = \boxed{4}$$

2. Suppose $S = 0, 1, 2, \ldots$, with $P(0) = .08$ and $P(1) = C$, $P(k) = \frac{1}{2^k \cdot k!}$ with $k = 2, 3, 4, \ldots$. Find the constant value of C for which the following is a valid probability distribution. (Hint: What type of series is P(K)?)

$$P(S) = 1, \quad P(0) + P(1) + P(k) = 1$$

$\mathbb{P}(S) = 1$, $\mathbb{P}(0) + \mathbb{P}(1) + \mathbb{P}(k) = 1$

$$0.08 + C + \underbrace{\sum_{k=2}^{\infty} \frac{1}{2^k \cdot k!}} = 1$$

$$\sum_{k=2}^{\infty} \frac{1}{2^k \cdot k!} = \sum_{k=0}^{\infty} \frac{1}{2^k \cdot k!} - \sum_{k=0} \frac{1}{2^k \cdot k!}$$

2, 3, 4, … 0, 1, 2, 3, 4, … 0, 1

$$\boxed{\sum_{k=0}^{\infty} \frac{1}{2^k \cdot k!}} - \left[\frac{3}{2}\right]$$

$$\boxed{e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}}$$

$$\sum_{k=0}^{\infty} \frac{1}{2^k \cdot k!} = e^{1/2}$$

$$\boxed{e^{1/2} - \frac{3}{2}}$$ for the series

$$\mathbb{P}(S) = \underbrace{0.08 + C + e^{1/2} - \frac{3}{2} = 1}$$

$$\boxed{C = 2.42 - e^{1/2}}$$

$$\boxed{C = 2.42 - e^{1/2}}$$

$P(A \cap B) = P(A) \cdot P(B)$

3. It is known that 20% of all the students at Cool College play sports. Suppose that 30% of all the students are females. Among all female students, 30% play sports. (Hint for the last sentence you can say "Given the students are female, 30% play sports")

   (a) What is the probability that a randomly selected student is a female and plays sports?

S = play sports, F = female

$P(S) = 0.20$, $P(F) = 0.30$

$P(S/F) = 0.30$

a.) $P(F \cap S) = ?$

$= P(F) \cdot P(S/F)$

$= 0.30 \cdot 0.30$

$= 0.09$

, $P(A/B) = \frac{\boxed{P(A \cap B)}}{P(B)}$

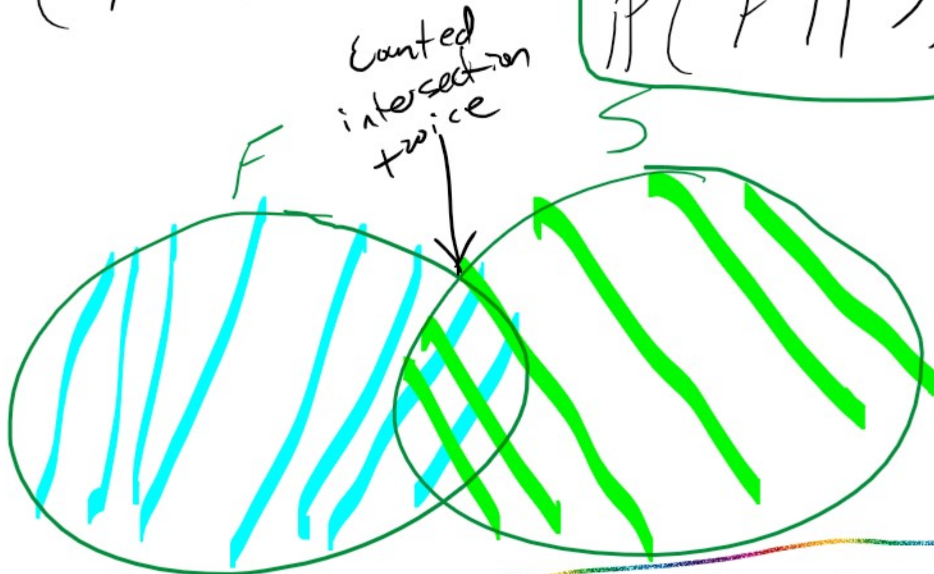
$P(A \cap B) = P(B) \cdot P(A/B)$

   (b) What is the probability that a randomly selected student either is a female or plays sports, or both?

$P(F \cup S) = P(F) + P(S) - P(F \cap S)$

$\mathbb{P}(F \cup S) =$

$\mathbb{P}(F \cap S)$



$$\boxed{\begin{aligned}\mathbb{P}(F\cup S) &= \mathbb{P}(F) + \mathbb{P}(S) - \mathbb{P}(F\cap S)\\ &= 0.30 + 0.20 - 0.09 = 0.41\end{aligned}}$$

(c) Given a student plays sports, what is the probability they are female?

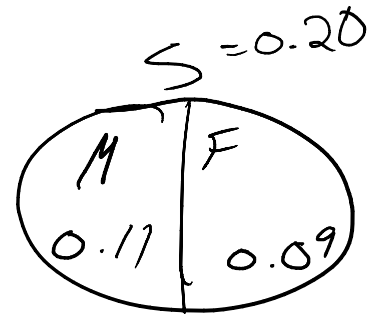
$$\mathbb{P}(F \mid S) = \frac{\mathbb{P}(F \cap S)}{\mathbb{P}(S)} = \frac{0.09}{0.20}$$

$$= \boxed{0.45}$$

(d) Suppose a student is male, what is the probability that they play sports?

$$P(S|M) = \frac{P(S \cap M)}{P(M)}, \qquad P(F) = 0.30$$

$$P(M) = 1 - P(F) = 1 - 0.30 = 0.70$$



$$P(S) = 0.20, \quad P(S \cap F) = 0.09$$

$$P(S \cap M) = ?$$

$$P(S \cap M) = 0.20 - 0.09 = 0.11$$

$$\boxed{P(S|M) = \frac{P(S \cap M)}{P(M)} = \frac{0.11}{0.70} = 0.157143}$$