STAT 400 Discussion Week 1

1. Given the series $\sum_{n=1}^{\infty} \frac{2}{2^{n-1}}$
   (a) Identify the type of infinite series and find the value it converges to.

   Solution:
   $$\sum_{n=1}^{\infty} \frac{2}{2^{n-1}} = 2\sum_{n=1}^{\infty} \frac{1}{2^{n-1}}$$
   Since we have a geometric series, we need to determine out base value and rate.
   $$\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} = 1 + \frac{1}{2} + \frac{1}{4} + ...$$
   From here we can see that our base case must be 1 with a rate of $\frac{1}{2}$. Using our geometric series formula $\frac{a}{1-r}$ we have
   $$\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} = \frac{1}{1-\frac{1}{2}} = \frac{2}{1}$$
   thus,
   $$2\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} = 4$$

2. Suppose $S = 0, 1, 2, ...,$ with $P(0) = .08$ and $P(1) = C$, $P(k) = \frac{1}{2^k \cdot k!}$ with $k = 2, 3, 4, ...$ Find the constant value of C for which the following is a valid probability distribution. (Hint: What type of series is P(K)?)

Solution: We know that

$$\sum_{k=2}^{\infty} \frac{1}{2^k \cdot k!} = \sum_{k=0}^{\infty} \frac{1}{2^k \cdot k!} - \sum_{k=0}^{1} \frac{1}{2^k \cdot k!} = e^{1/2} - \frac{3}{2}$$

From here we know that

$$1 = .08 + C + e^{1/2} - \frac{3}{2}$$

leaving us with

$$C = 2.42 - e^{1/2}$$

3. It is known that 20% of all the students at Cool College play sports. Suppose that 30% of all the students are females. Among all female students, 30% play sports. (Hint for the last sentence you can say "Given the students are female, 30% play sports")

   (a) What is the probability that a randomly selected student is a female and plays sports?

   Solution: $P(F \cap S) = P(F) * P(S|F)) = 0.30 * 0.30 = 0.09$

   (b) What is the probability that a randomly selected student either is a female or plays sports, or both?

   Solution: $P(F \cup S) = P(F) + P(S) - P(F \cap S)$
   $= 0.30 + 0.20 - 0.09 = 0.41$

   (c) Given a student plays sports, what is the probability they are female?

   Solution: $P(F|S) = P(F \cap S)/P(S)$
   $= 0.09/0.20 = 0.45$

(d) Suppose a student is male, what is the probability that they play sports?

Solution: $P(S|M) = P(S \cap M)/P(M)$
$= (P(S) - P(S \cap F))/P(M)$
$= (0.20 - 0.09)/(1 - 0.30) = 0.157143$