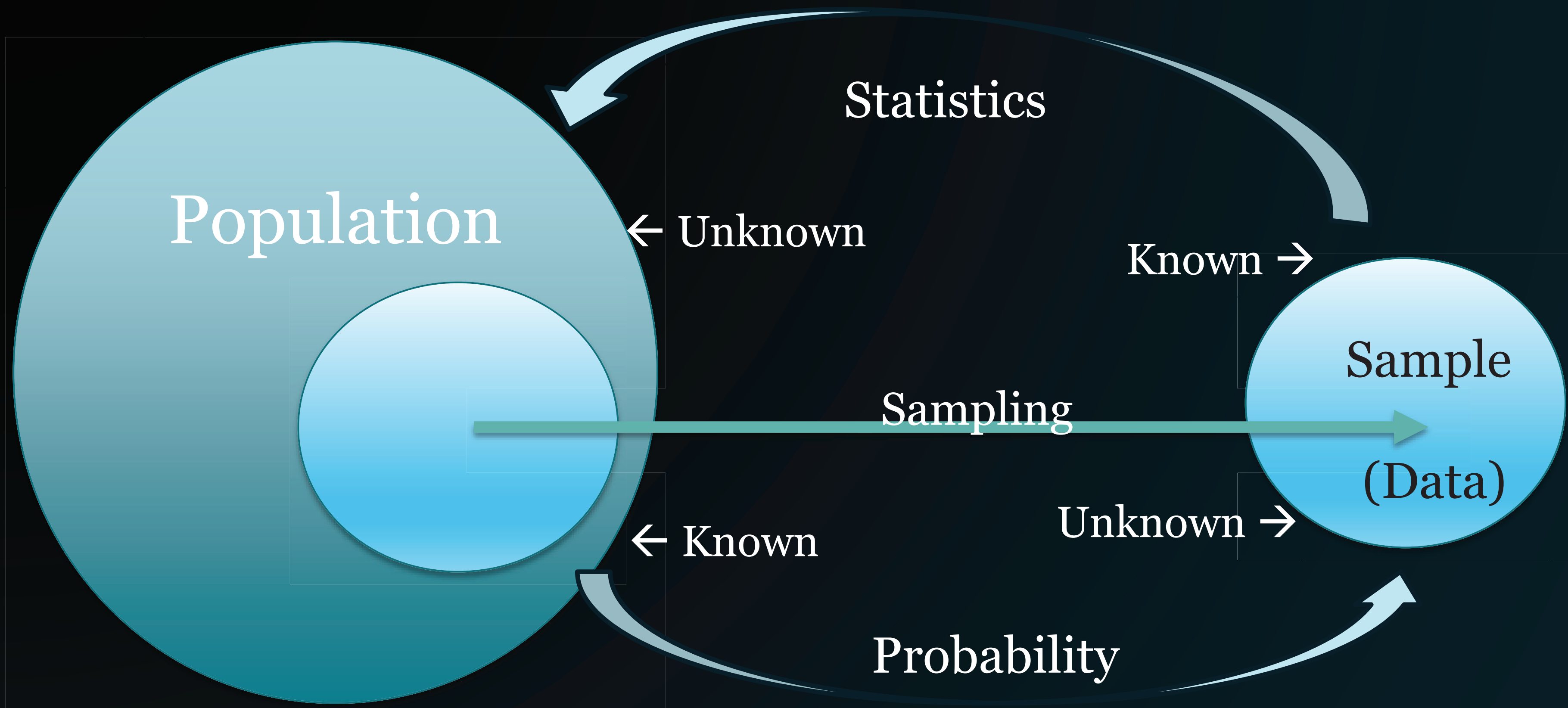
Point Estimation (MLE) (6.4)

Notes

Probability vs Statistics



Some new terms

Sample mean: $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

Sample variance: $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

Sample standard deviation: s = $\sqrt{s^2}$

notes

Point Estimation

Let’s say we are given a distribution (family), and have random samples from this distribution, but don’t know the value of the parameter, θ.

(often, we use θ as the generic term for an unknown parameter).

Definition: The **parameter space, $\boldsymbol{\Omega}$**, is the range of all possible values of θ.

E.g.

X $\sim$ Exp(θ),

X $\sim$ Binom(n, θ)

X $\sim$ Geom(θ)

X $\sim$ N(θ,1)

Point Estimation

Goal: Estimate $\theta \in \boldsymbol{\Omega}$

We will observe n samples, $X_1, X_2, \ldots, X_n$, and estimate θ with n sample values, $x_1, x_2, \ldots, x_n$.

The statistic, $u(X_1, X_2, \ldots X_n)$, is an **estimator** of θ.

Using the values from the observations, we can find an **estimate** of θ, $u(x_1, x_2, \ldots x_n)$.

notes

Simple example of Point Estimate

What was the true mean score on Midterm 2? (μ)

- Population: All students in Math 463/Stat 400
- Right now: only have 30 grades.
- $\bar{x} = \frac{1}{30}\sum_{i=1}^{30} x_i = 85$

estimate of μ

This is a point estimate of μ.

Binomial example

Suppose that I perform an experiment 10 times and define success as 1. I don't know p. We would like to estimate the parameter, p.

- If I get a sample: 1,1,1,1,1,0,1,1,1,1. What is the best estimate for p?

Joint pmf/pdf

- Assuming that X_1, X_2, … X_n are independent and identically distributed, we know that the joint pmf (or pdf) is equal to the product of the pmf/pdfs.

- Bivariate: if X and Y are independent,
  $$f(x, y) = f(x)f(y)$$

Joint pmf/pdf

For multiple independent observations from the same distribution (iid), the **joint distribution** (joint pdf or joint pmf) is the product of the individual pdf/pmfs.

notes

The Likelihood function

The likelihood function looks exactly like the joint pdf (or pmf).

It is obtained through finding the joint distribution.

It is a function of θ, not of x_i.

$$L(\theta) = \prod_{i=1}^{n} f(x_i ; \theta)$$

semicolon

Probability: Know the value of parameters. Calculate probability of observing some data.

Statistics: Know what data look like. Come up with an estimate of parameters.

Binomial Example (MLE)

- Flipping a loaded coin 10 times. It shows heads on 9 of 10 flips.

Let $X \sim Binom(10, \theta)$

$$f(9; \theta) \;=\; \binom{10}{9}\theta^9(1-\theta)^1 \;=\; L(\theta)$$

- Now, given that I have gotten 9 successes, what value of theta makes this expression the largest (most likely)? How can I find that value?

$$f(9;\theta) \;=\; \binom{10}{9}\theta^9(1-\theta)^1 \;=\; L(\theta)$$

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x = seq(from = 0, to = 1, by = 0.01)
y = 10*x^9 * (1-x)
plot(x,y)
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Using Calculus to find MLE

Step 1: Find $L(\theta)$

Step 2: Take the (natural) log of $L(\theta)$, $\log L(\theta)$

Step 3: Take first derivative of $\log L(\theta)$ w.r.t θ.

Step 4: Set expression equal to 0

Step 5: Solve for θ

In case you forgot…

Suppose $f(x)$ is a function of x that is twice differentiable at a stationary point x_0.

1. If $f''(x_0) > 0$, then f has a local minimum at x_0.
2. If $f''(x_0) < 0$, then f has a local maximum at x_0.

notes

MLE Example

Let $X_1, X_2, \ldots X_n$, be iid $\sim f(x;\theta) = \theta^{-2} x e^{-x/\theta}$, $x > 0$, $\theta > 0$

a) Find the MLE of θ, $\hat{\theta}$.

(1)

(2)

MLE Example continued

$$f(x;\theta) = \theta^{-2} x e^{-\frac{x}{\theta}}, \qquad x > 0, \qquad \theta > 0$$

(3,4)

(5)

MLE Example continued

$f(x;\theta) = \theta^{-2} x e^{-x/\theta}$

Find an estimate of θ when

$x_1 = 1,\ x_2 = 0.75,\ x_3 = 2,\ x_4 = 1.5,\ x_5 = 0.75$

MLE Example (for you to practice at home)

Let $X_1, X_2, \ldots X_n, \sim$Bern(p). Find the MLE of p.

$$f(x_i; p) = p^{x_i} (1-p)^{1-x_i}$$

(1) $L(p) = \prod_{i=1}^{n} f(x_i; p)$

$= p^{x_1} (1-p)^{1-x_1} \cdot p^{x_2} (1-p)^{1-x_2} \cdot \ldots \cdot p^{x_n} (1-p)^{1-x_n}$

$=$

(2)

MLE Example (for you to practice at home)

(3,4)

(5)

notes