2.3 Discrete Random Variables

Review

Variance

- One way to characterize a random variable is by its location (**mean, median**).
- Another way is to describe how spread out it is (**variance**).

For a random variable, X, we can say $Var[X]$, σ^2, or $\sigma^2{}_X$

Variance

$$\boxed{\sigma^2 = E[(X-\mu)^2] = \sum_{all\ x}(x-\mu)^2 f(x)}$$

Also,

$$\begin{aligned}\sigma^2 &= E[(X-\mu)^2] = E[X^2 - 2\mu X + \mu^2] \\ &= E(X^2) - 2\mu E(X) + \mu^2 \\ &= E(X^2) - \mu^2.\end{aligned}$$

, or $\boxed{\sigma^2 = E[X^2] - (E[X])2}$

Linear Transformation of a Random Variable – Basic Properties

- $E[aX + b] = a \cdot E[X] + b$
- $Var[aX + b] = a^2 \cdot Var[X]$
- $SD[aX + b] = |a| \cdot SD[X]$

2.1 – 2.3

Examples

1) A pocket contains 5 billiard balls numbered 1 to 5. Jake reaches in and pulls out two of them randomly.

- a) How many different subsets of 2 billiards are there in this pocket?
- b) Let X by the larger of the two numbers drawn. What is the pmf of X?
- c) What is E[X]?
- d) What is Var[X]?

Outcome	X
1,2	2
1,3	3
1,4	4
1,5	5
2,3	3
2,4	4
2,5	5
3,4	4
3,5	5
4,5	5

2) Suppose a fair die is tossed 3 times. Let X be the largest number that shows up.

a) Find an **expression** for F(x).

b) Find an expression for f(x).

Note: the following applies to (discrete) p.m.f.'s

$$\boxed{f(x) = P[X = x] = P[X \leq x] - P[X \leq (x-1)]}$$

3) A fair coin is tossed three times. Let X be $\# \; of \; heads \; - \; \# \; of \; tails$ in the three tosses.

a) What is the space of X?

b) What is the pmf of X?

c) What is $E[X]$?

d) What is $Var[X]$?

3) A fair coin is tossed three times. Let X be $\# \; of \; heads \; - \; \# \; of \; tails$ in the three tosses.

c) What is $E[X]$?

d) What is $Var[X]$?

Suppose $E(X) = 20,\ SD(X) = 2$

$Let\ Y = 3X + 1.$

- Find $E[Y]$ and $Var[Y]$

$Let\ Z = 3 - X$

- Find $E[Z]$ and $SD[Z]$

Additional Examples