Stat 400 / Math 463
Spring 2021

1.3 Conditional Probability

Notes

Conditional Probability

	Early (E)	Late (L)	Totals
Red (R)	5	8	13
Yellow (Y)	3	4	7
Totals	8	12	20

What is the probability of selecting a red bulb? (marginal) 13/20

What is the probability of selecting a red bulb if you know the flower will bloom early? (conditional)

5/8

A **conditional probability** is a probability that is updated to take into account the (known) occurrence of another event.

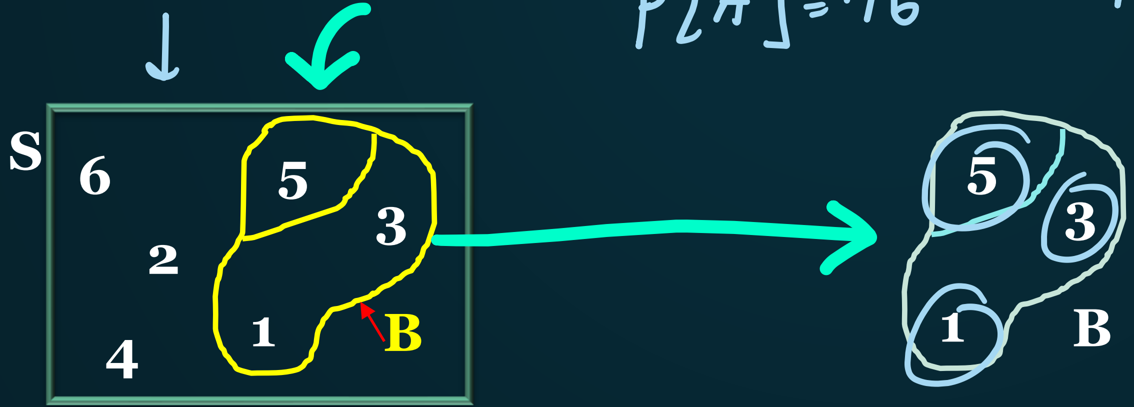
Conditional Probability Example

$$P[A|B] = \frac{P[A \cap B]}{P[B]} = \frac{1/6}{1/2} = 1/3$$

- With a fair die being rolled once, define **A = {5}**
- Then, P[A] = 1/6

What if someone rolls the die and doesn't tell us the number showing. Tells us only that event **B = {odd number}** occurs?

$P[A] = 1/6$ $\qquad$ $P[B] = 1/2$



$1/3$

$$P[B|A] = \frac{P[B \cap A]}{P[A]}$$

Conditional Probability

Definition 1.3-1

The **conditional probability** of an event A, given that event B has occurred, is defined by

"given"

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)},$$

"Bayes Rule"

provided that $P(B) > 0$.

Multiplication Rule

Definition 1.3-2

The probability that two events, A and B, both occur is given by the **multiplication rule**,

umb rain

$$P(A \cap B) = P(A)P(B \mid A),$$

provided $P(A) > 0$ or by

$$P(A \cap B) = P(B)P(A \mid B)$$

$$\frac{P(A \cap B)}{P(B)} = P(A \mid B)$$

provided $P(B) > 0$.

1.3 Conditional Probability

Examples

For a randomly selected off-campus student at UIUC on any given day, assume:

- P[Bikes to campus] = 0.4,
- P[Rides bus to campus] = 0.3,
- P[Does both] = 0.04.

	Bus	Bus^c	
Bike	0.04	.36	0.4
$Bike^c$	.26	.34	0.6
	0.3	0.7	1

1) What is the probability that a student bikes to campus, given that they ride the bus? $P[Bike \mid Bus] = \frac{P[Bike \cap Bus]}{P[Bus]} = \frac{0.04}{0.3} = \boxed{0.1\overline{3}}$

2) What is the probability that a student bikes to campus, given that they don't ride the bus?

$$P[Bike \mid Bus^c] = \frac{P[Bike \cap Bus^c]}{P[Bus^c]} = \boxed{\frac{0.36}{0.7}} =$$

continued

P[Bikes to campus] = 0.4,
P[Rides bus to camps] = 0.3,
P[Does both] = 0.04.

3) Suppose you know that a student does not bike to campus. Find the probability that this student does not take the bus.

	Bus	Bus^c	
Bike	0.04	.36	0.4
$Bike^c$	.26	.34	0.6
	0.3	0.7	1

$$P[Bus^c \mid Bike^c]$$

$$= \frac{P[Bus^c \cap Bike^c]}{P[Bike^c]} = \boxed{\frac{0.34}{0.6}}$$

While running from Shia LaBeouf, you stumble upon a group of 20 kittens. 8 are going to explode. You decide to grab 2 of them anyway.

$\left(\frac{12}{20} \cdot \frac{11}{19}\right)$ ← both don't

- 4) Find the probability that both will explode.

$P[\text{Both Explode}] = P[\text{1st Explode} \cap \text{2nd Explode}]$

$= P[\text{1st Ex}] \cdot P[\text{2nd Ex} \mid \text{1st Ex}] = \frac{8}{20} \cdot \frac{7}{19} = \underline{0.147}$

- 5) Find the probability that at least one of the kittens will explode.

$\{\underbrace{\underline{EE}, \underline{EE^c}, \underline{E^cE}}_{A}, \underbrace{E^cE^c}_{A^c}\}$

A = at least 1 E

A^c complement

$\underline{P[A] = 1 - P[E^cE^c]} = 1 - \left(\frac{12}{20} \cdot \frac{11}{19}\right) = \boxed{0.653}$

Two fair 6-sided dice are rolled.

36

6) What is the probability that the number on the first die was at least as large as **4** given that the sum of the two dice was **8**?

6^2

$$P[\text{1st} \geq 4 \mid \text{sum} = 8] = \frac{P[\text{1st} \geq 4 \wedge \text{sum} = 8]}{P[\text{sum} = 8]} = \frac{P\{(4,4), (5,3), (6,2)\}}{P\{(2,6), (3,5), (4,4), (5,3), (6,2)\}}$$

$$= \frac{3/36}{5/36} = \boxed{3/5}$$

- Blue 4 Red 4
- Red 4 Blue 4 <- same

- Blue 2 Red 6
- Red 2 Blue 6 ← diff