Hypothesis Tests for Two Proportions

8.3 (Tues)

Power of a Statistical Test

8.5 (Thurs)

Tuesday’s topics

Review Midterm 3

2-sample Hypothesis testing review

- (paired t-test example)

Hypothesis test for two proportions

Midterm 3

notes

Paired t-test

For comparing means of two **dependent** samples.

Saliva samples from 10 people were sent to each of two laboratories (Lab 1 and Lab 2) to test for antibody levels. Is there a statistically significant difference in the mean at the 0.01 significance level?

Subject	Lab 1	Lab 2	Difference
1	296	318	-22
2	268	287	-19
…			
10	262	285	-23
Mean	260.6	275	-14.4 s_d =6.77

notes

$H_0 =$
$H_A =$

$t =$ $= -6.73 \sim$

Type I Error: Review

Type I Error: Reject H_0 when H_0 is true

Hypothesis testing:

Set maximum acceptable rate of Type I error:

α (significance level)

Choose a test with the most power to detect H_A.

notes

$p_1 - p_2$

Consider two populations, with "success" proportions p_1 and p_2, respectively. Consider the sample proportions:

$\hat{p}_1 = \frac{x_1}{n_1}$ and $\hat{p}_2 = \frac{x_2}{n_2}$. n_1 and n_2 are sample sizes

x_1 and x_2 are the numbers of "successes" in the two samples from populations 1 and 2.

If n_1 and n_2 are large, $(\hat{p}_1 - \hat{p}_2)$ is approximately distributed

$$\boxed{(\hat{p}_1 - \hat{p}_2) \sim N\left(p_1 - p_2, \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}\right)}$$

CI for $p_1 - p_2$

A 100(1-α)% CI for $p_1 - p_2$ is

$$\boxed{(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}$$

Hypothesis Testing for p_1 - p_2

$H_0: p_1 - p_2 = 0$

$H_1: p_1 - p_2 \neq 0$

$H_1: p_1 - p_2 > 0$

$H_1: p_1 - p_2 < 0$

Test statistic:

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}}}, \quad \text{where} \quad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

Hypothesis Testing - 2 proportion

For developing countries in Africa and the Americas, let p_1 and p_2 be the respective proportions of babies with a low birth weight (below 2500 grams).

Test H_0: $p_1 = p_2$ against H_1: $p_1 > p_2$.

a) Define a critical region that has an $\boldsymbol{\alpha}$ **= 0.05** significance level.

Hypothesis Testing - 2 proportion

For developing countries in Africa and the Americas, let p_1 and p_2 be the respective proportions of babies with a low birth weight (below 2500 grams).

Test H_0: $p_1 = p_2$ against H_1: $p_1 > p_2$.

b) If respective random samples of sizes $\boldsymbol{n_1}$ **= 900** and $\boldsymbol{n_2}$ **= 700** yielded $\boldsymbol{y_1}$ **= 135** and $\boldsymbol{y_2}$ **= 77** babies with a low birth weight, what is your conclusion at $\alpha = 0.05$?

Hypothesis Testing - 2 proportion

For developing countries in Africa and the Americas, let p_1 and p_2 be the respective proportions of babies with a low birth weight (below 2500 grams).

Test H_0: $p_1 = p_2$ against H_1: $p_1 > p_2$.

c) If respective random samples of sizes $\boldsymbol{n_1 = 900}$ and $\boldsymbol{n_2 = 700}$ yielded $\boldsymbol{y_1 = 135}$ and $\boldsymbol{y_2 = 77}$ babies with a low birth weight, what is the rejection region and conclusion at α = 0.01?

Hypothesis Testing - 2 proportion

For developing countries in Africa and the Americas, let p_1 and p_2 be the respective proportions of babies with a low birth weight (below 2500 grams).

If respective random samples of sizes $\boldsymbol{n_1}$ **= 900** and $\boldsymbol{n_2}$ **= 700** yielded $\boldsymbol{y_1}$ **= 135** and $\boldsymbol{y_2}$ **= 77** babies with a low birth weight,

d) Perform the following test (at α = 0.1)

H_0: $p_1 \leq p_2 + 0.2$

H_1: $p_1 > p_2 + 0.2$

Power

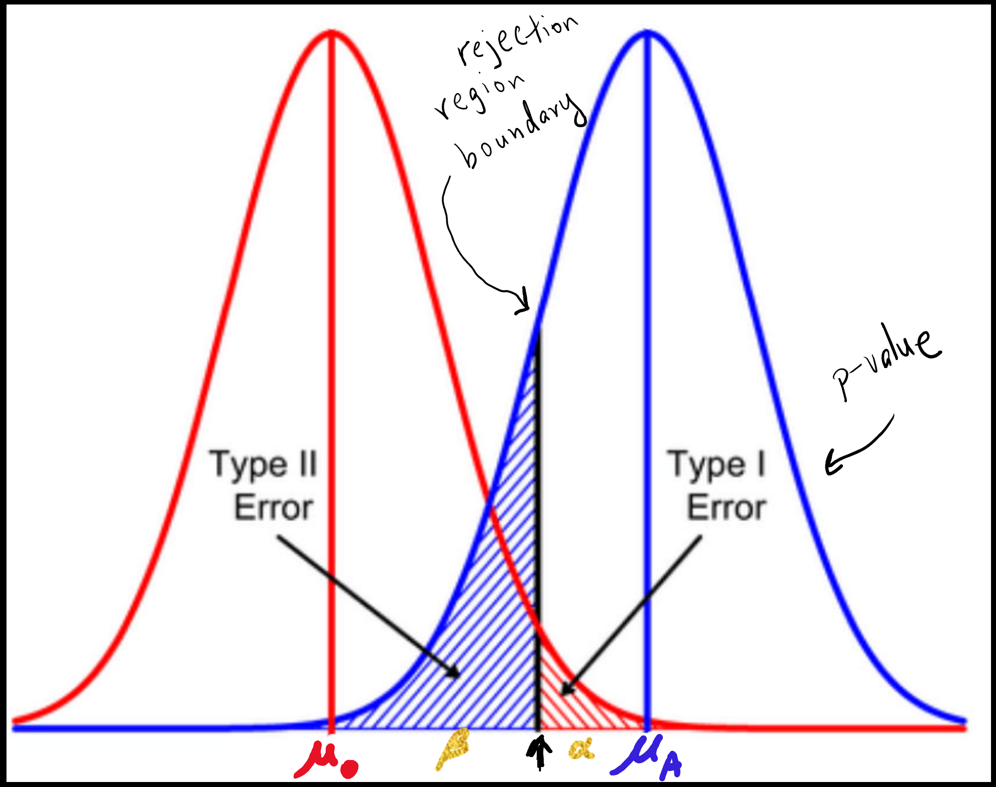
The **power** of a statistical test is related to Type II error.

Power = 1 – P[Type II Error]

Some potentially confusing notation (that I didn't invent)

- β is the probability of Type II error
- Sometimes the power function is written as $\beta(\theta)$.
  - Need to look at context
  - These β's are not the same. In the second case, β is just the name of the function.
  - Your textbook calls $\beta(\theta)$ as K(μ) when dealing with the mean.

Type I and Type II Errors



Example 8.5-2

Let $X_1, X_2, \ldots, X_n$ be a random i.i.d. sample $\sim N(\mu, 100)$. Let's say these are final exam scores of students in a large online stats course.

Suppose a researcher wanted to see if there is a significant difference between Zoom and Twitch. Zoom is the current default method of teaching, and has mean 60)

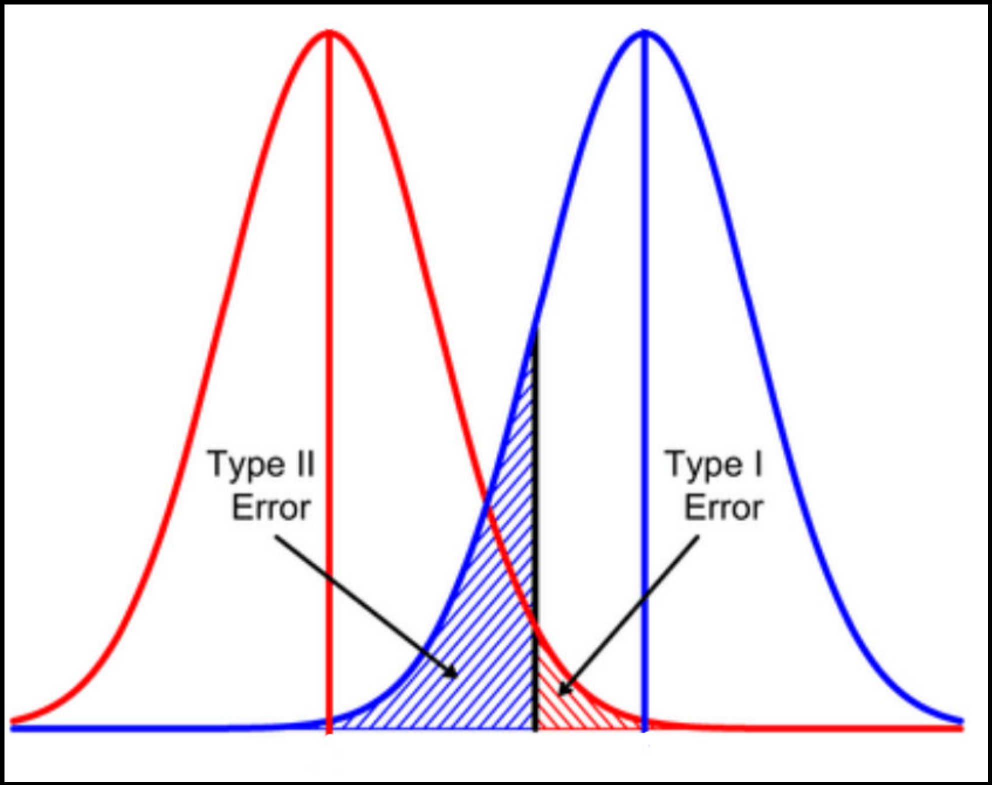
H_0: $\mu = 60$

H_1: $\mu > 60$ (we want to test to see if Twitch is better)

Test statistic: $\bar{X}$ (it is the MLE of μ)

Initially, we use the rule to reject H_0 if and only if $x \geq 62$. Consider a sample of size $n = 25$. What are the consequences of this test?

Type I and Type II Errors



$\beta(\theta)$

here, $K(\mu)$

Power Function

If the true mean under H_A (for Twitch) is μ, then

X ~ N(μ,100).   If n = 25, $\bar{X}$ ~ N(μ, 4).

power function

The probability of rejecting H_0 is given by

$K(\mu) = P[\bar{X} \geq 62 ; \mu]$

$= P\left[\frac{\bar{X}- \mu}{2} \geq \frac{62- \mu}{2} ; \mu\right] \qquad = P\left[Z \geq \frac{62- \mu}{2} ; \mu\right]$

Table 8.5-1 Values of the power function

μ	$K(\mu)$
60	0.1587
61	0.3085
62	0.5000
63	0.6915
64	0.8413
65	0.9332
66	0.9772

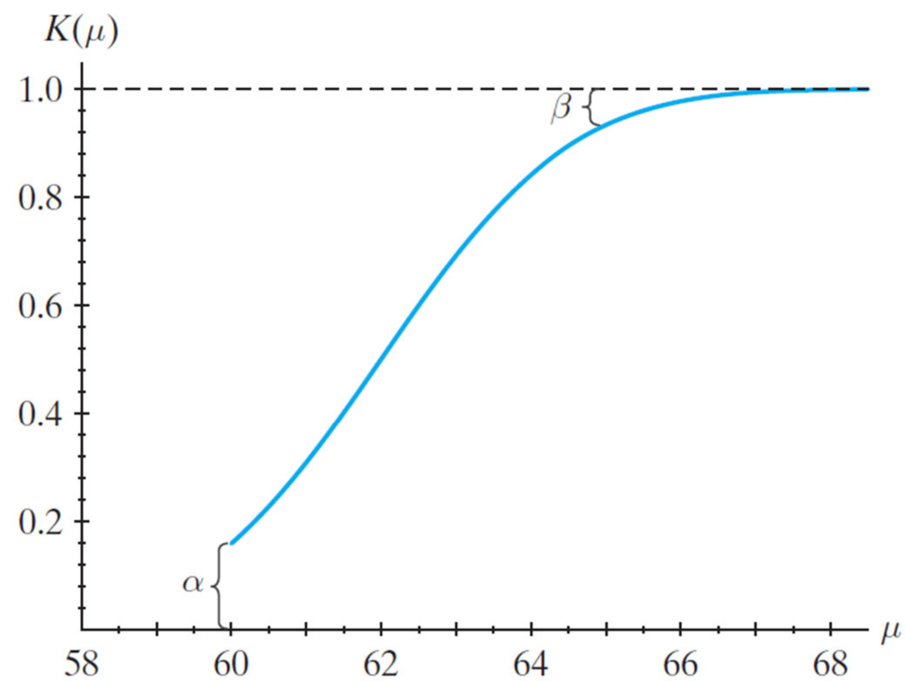


Figure 8.5-2 Power function $K(\mu) = 1 - \Phi([62 - \mu]/2)$

Ideal power function?

- What would an ideal power function look like?

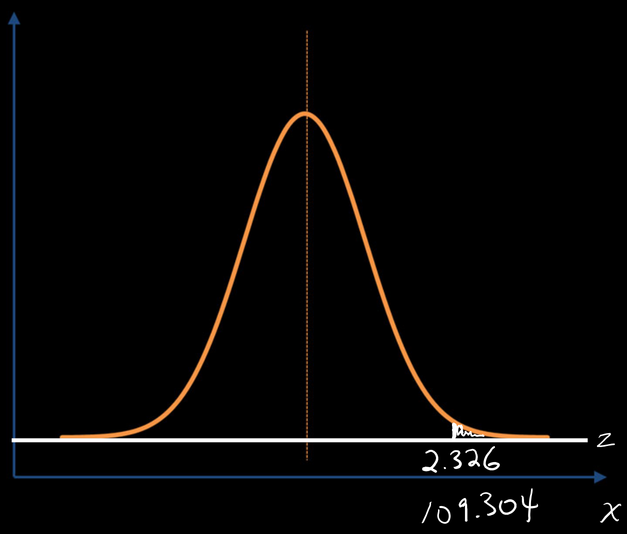
Example 2

Assume that the number of grams of coffee that Albert selects every day follows an approximately normal distribution with unknown mean and standard deviation 16.

Let $n = 16$, $\alpha = 0.01$

Test H_0: $\mu = 100$ vs H_A: $\mu > 100$.

When should I reject H_0?

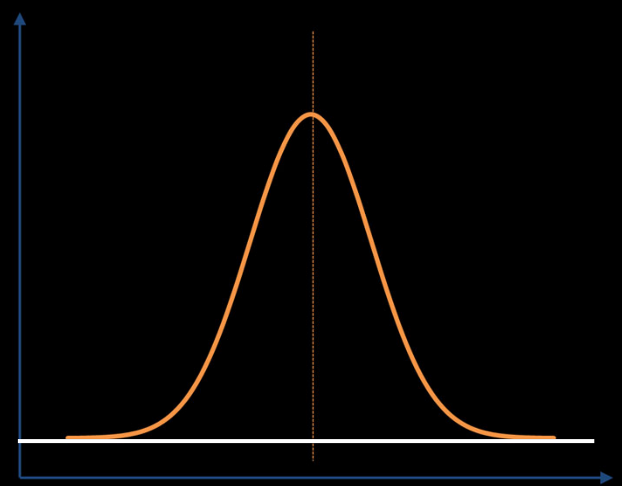


Example 2 – Power at μ =108

Power = P[$\bar{X} \geq 109.304$ | μ = 108]

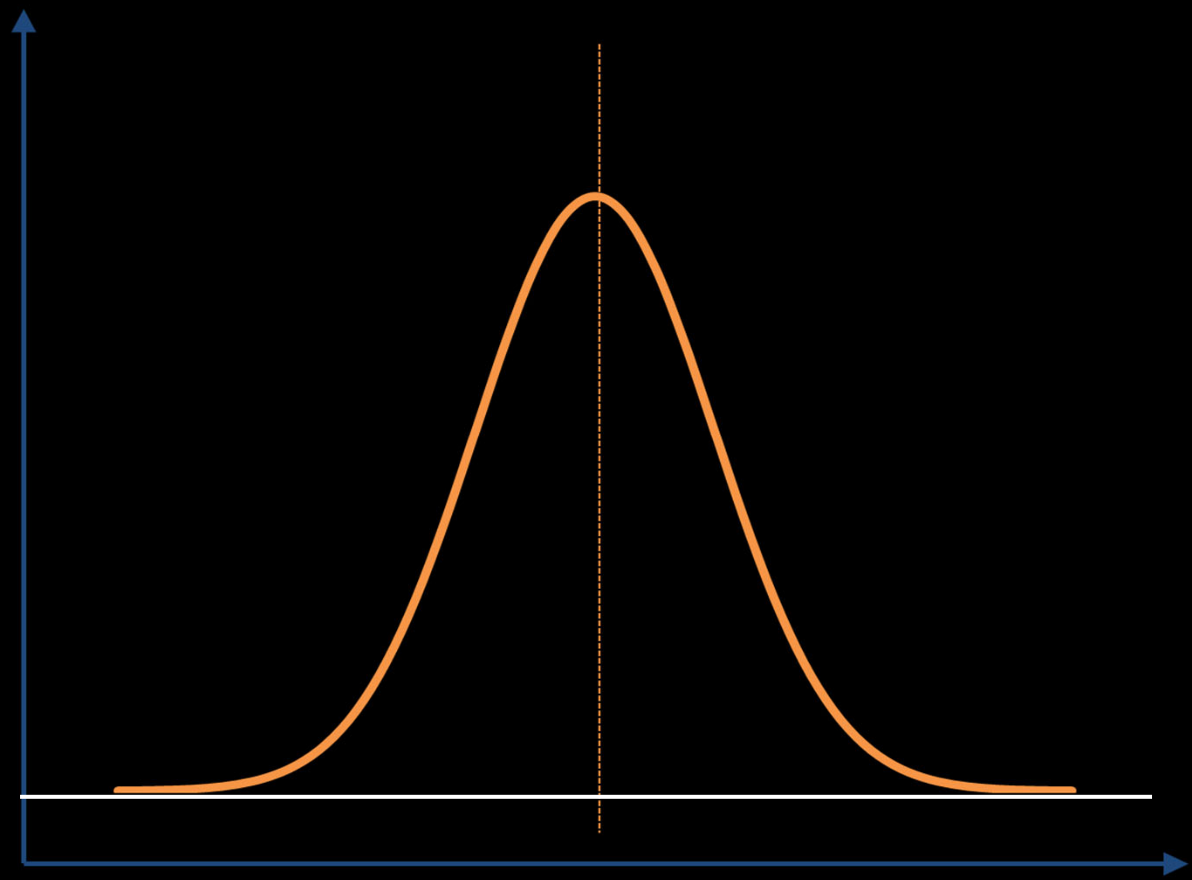
= P[Z $\geq \frac{109.304 - 108}{16/\sqrt{16}}$] = P[Z $\geq$ 0.326]

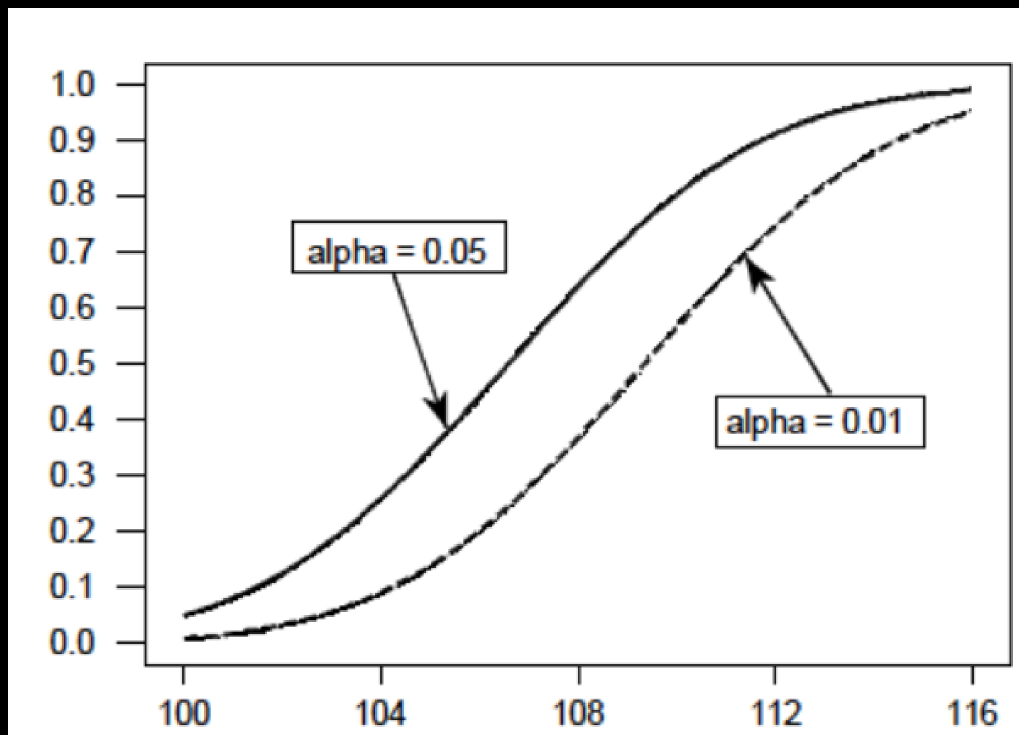
= 0.3722



What if we used α = 0.05?

Cutoff = 106.58, Power = P[$\bar{X} \geq 106.58] = 0.6404$




1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
alpha = 0.05
alpha = 0.01
100
104
108
112
116