Spring 2021 Dis 1 Wed

Wednesday, February 3, 2021 12:56 PM

1. Given the series $\sum_{n=1}^{\infty} \frac{2}{2^{n-1}}$

   (a) Identify the type of infinite series and find the value it converges to.

$$\sum_{n=1}^{\infty} \frac{2}{2^{n-1}} = 2 \sum_{n=1}^{\infty} \frac{1}{2^{n-1}}$$

$$\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} = 1 + \frac{1}{2} + \frac{1}{4} + \cdots$$

Geometric series: $\frac{a}{1-r}$

$$\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} = \frac{1}{1 - \frac{1}{2}} = 2$$

$$\sum_{n=1}^{\infty} \frac{2}{2^{n-1}} = 2 \cdot \sum_{n=1}^{\infty} \frac{1}{2^{n-1}} = 2 \cdot 2 = 4$$

2. Suppose $S = 0, 1, 2, \ldots$, with $P(0) = .08$ and $P(1) = C$, $P(k) = \frac{1}{2^k \cdot k!}$ with $k = 2, 3, 4, \ldots$. Find the constant value of C for which the following is a valid probability distribution. (Hint: What type of series is P(K)?)

series

$$\sum_{k=2}^{\infty} \frac{1}{2^k \cdot k!} = \sum_{k=0}^{\infty} \frac{1}{2^k \cdot k!} - \sum_{k=0}^{1} \frac{1}{2^k \cdot k!}$$

steps of the series

$k=2$

show steps of the series

$2, 3, 4, \ldots, \infty = 0, 1, 2, 3, 4 \ldots, \infty - 0, 1$

$$\sum_{k=0}^{\infty} \frac{1}{2^k \cdot k!} - \sum_{k=0}^{1} \frac{1}{2^k \cdot k!}$$

$$\sum_{k=0}^{\infty} \frac{1}{2^k \cdot k!} - \left[\frac{1}{2^{(0)} \cdot (0)!} + \frac{1}{2^{(1)} \cdot (1)!} \right]$$

$$\sum_{k=0}^{\infty} \frac{1}{2^k \cdot k!} - \frac{3}{2} = e^{1/2} - \frac{3}{2} = P(k)$$

$$\star \sum_{k=0}^{\infty} \frac{1}{2^k \cdot k!} = e^{1/2} \star$$

$$1 = P(0) + P(1) + P(k)$$

$$1 = 0.08 + C + e^{1/2} - \frac{3}{2}$$

$$C = 2.42 - e^{1/2}$$

3. It is known that 20% of all the students at Cool College play sports. Suppose that 30% of all the students are females. Among all female students, 30% play sports. (Hint for the last sentence you can say "Given the students are female, 30% play sports")

(a) What is the probability that a randomly selected student is a female and plays sports?

F = female, S = plays sports

$P(F \cap S) = P(F) \cdot P(S|F) =$

$0.3 \cdot 0.3 = 0.09$

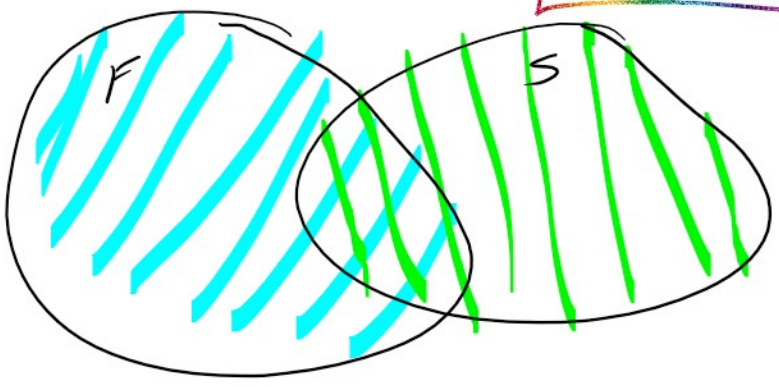
$P(A|B) = \frac{P(A \cap B)}{P(B)}$) $P(A|B) \cdot P(B) = P(A \cap B)$

(b) What is the probability that a randomly selected student either is a female or plays sports, or both?

$P(F \cup S) = P(F) + P(S) - P(F \cap S) =$

$0.30 + 0.2 - 0.09 =$

$F \cup S = 0.41$



(c) Given a student plays sports, what is the probability they are female?

$P(F \cap S)$

(c) Given a student plays sports, what is the probability they are female?

$$P(F|S) = \frac{P(F \cap S)}{P(S)}$$

$$= \frac{0.09}{0.20} = 0.45$$

(d) Suppose a student is male, what is the probability that they play sports?

$$P(S|M) = \frac{P(S \cap M)}{P(M)}$$

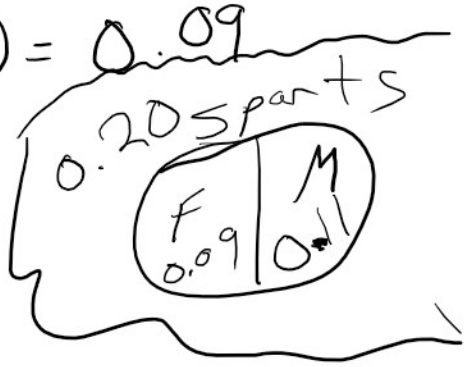
$P(F) = 0.30$

$P(M) = 1 - P(F)$

$P(M) = 0.70$

$P(S \cap M) = ?$ We know $P(S) = 0.20$

$P(F \cap S) = 0.09$



$P(S \cap M) = P(S) - P(F \cap S)$

$P(S \cap M) = 0.20 - 0.09 = 0.11$

$$P(S|M) = \frac{0.11}{} = 0.157143$$

$$\frac{P(S|M)}{P(M)} = \frac{0.11}{0.70} = [0.1\ldots]$$