STAT 400 Discussion 10

1. Target conducted a survey of 100 customers as they entered a store in Wisconsin. The survey asked which item category each customer was looking for (household, toy/games, clothing, food) and whether the customer had visited the store already that week. The survey found that of those individuals who had been to the store before that week, 16 were shopping for household items, 7 for toys/games, 4 for clothing, and 20 for food. Among those who had not been to the store that week, 8 shopped for household, 9 for toys/games, 7 for clothing and 29 were shopping for food.

   Using the .02 level of significance, can we reject the null that there is independence between the item category a person is shopping for and whether the customer had been to the store before that week?

Sol: we use the χ^2-test of independence

	Household		Toys/Games		Clothing		Food		Total
visited	16	11.28	7	7.52	4	5.17	20	23.03	47
not visited	8	12.72	9	8.48	7	5.83	29	25.97	53
total	24		16		11		49		100

H_0: no association between columns and rows

H_A: $\sim H_0$

Now, we use red to write the eight expected values above.

THEN let $T = \sum \frac{(O_i - E_i)^2}{E_i}$ where O_i is ith observed value, E_i is ith expected value. Under H_0, $T \sim \chi^2_{(\text{row}-1)\cdot(\text{columns}-1)}$

Since $T = 5.05 < \chi^2_{.98,3} = 9.84$, we fail to reject.

2. A snack mix company claims that half of it's best selling snack mix is made out of pretzel pieces, thirty percent is made of rye chips and the rest of the mix is made of peanuts. Suppose you purchase a bag of the snack mix and count the number of each piece present in the bag. You find 64 pretzel pieces, 18 rye chips and 20 peanuts. Test to see whether the company is making an invalid claim. Use the .05 level of significance.

H_0:

$P_P = \frac{1}{2}$

$P_{rye} = 3/10$

$P_{peanut} = 1/5$

$H_A: \sim H_0$

Again, a table!

	Pretzel	Rye	Peanut	Total
observed	64	18	20	102
expected proportion	.5	.3	.2	1
expected count	51	30.6	20.4	102

Let $M = \frac{(64-51)^2}{51} + \frac{(18-30.6)^2}{30.6} + \frac{(20-20.4)^2}{20.4}$

Under H_0, $M \sim \chi^2_{(2-1)(3-1), .95} = \chi^2_{2, .95} = 5.99$

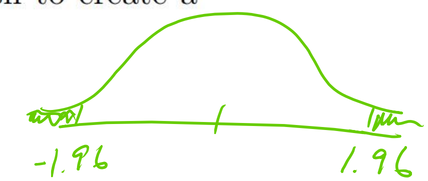
SINCE $M = 8.5$, we reject the null.

3. Suppose we are given $\sigma = 5$, and n=25. Using $\bar{X}$, we wish to create a 2-sided z-test at $\alpha = 0.05$.

Type I error

$H_0 : \mu = 75$

$H_A : \mu \neq 75$



$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{\bar{X} - 75}{5/\sqrt{25}}$$

What is the power of this test at

- $\mu = 72$?
- $\mu = 77$?

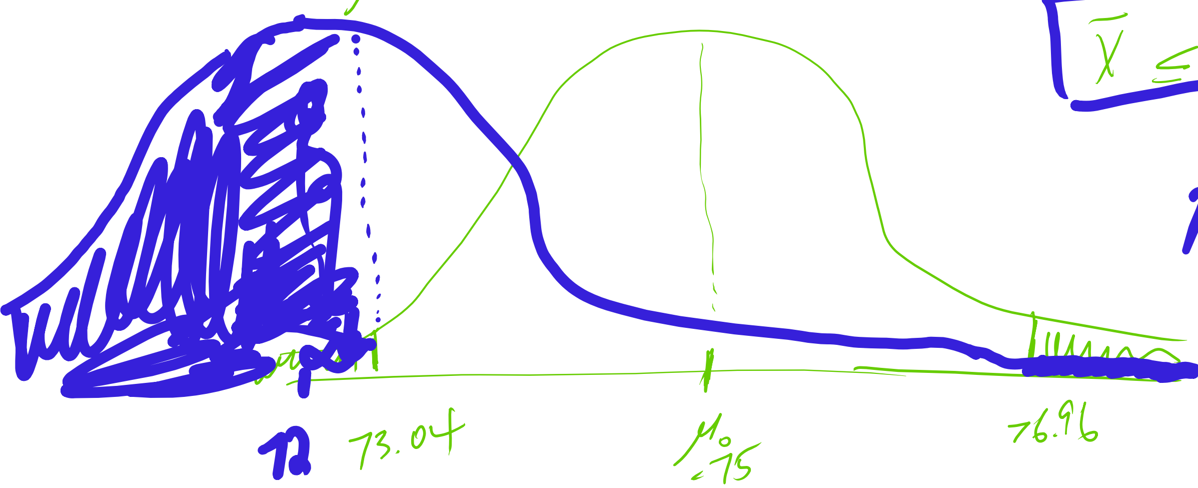
Can you sketch a power curve?

RR: $\frac{\bar{X} - 75}{5/\sqrt{25}} < -1.96$ or $\frac{\bar{X} - 75}{5/\sqrt{25}} > 1.96$

$\boxed{\bar{X} \leq 73.04} \longleftrightarrow \boxed{\bar{X} \geq 76.96}$

or

if $\mu = 72$



$P(\bar{X} \leq 73.04 \mid \mu = 72) +$

$P(\bar{X} \geq 76.96 \mid \mu = 72)$

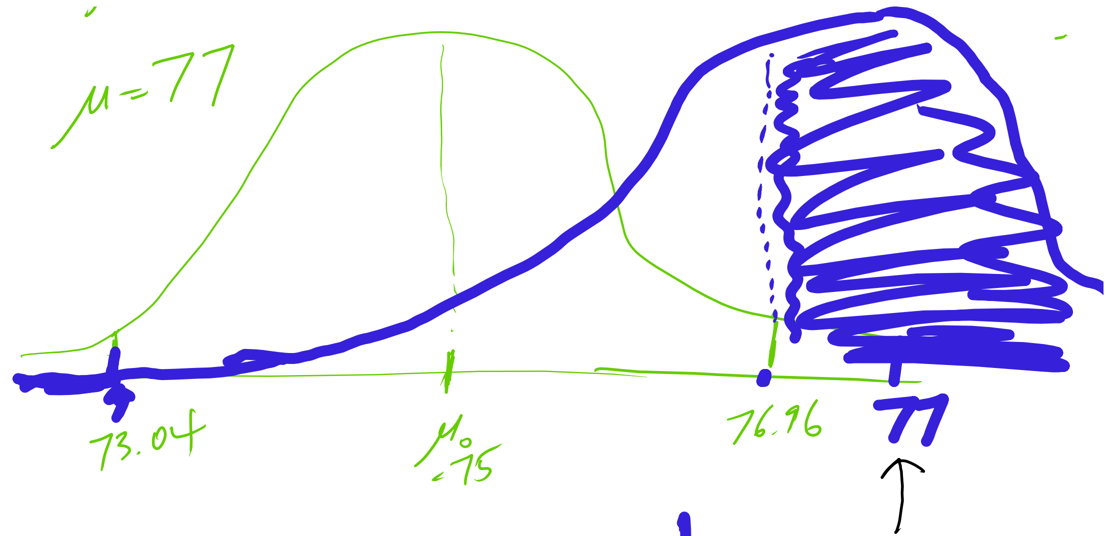
$P(\bar{X} \le 73.04 \mid \mu = 72) +$

$P(\bar{X} \ge 76.96 \mid \mu = 72)$

$P\left(Z \le \frac{73.04 - 72}{5/\sqrt{25}}\right) + P\left(Z \ge \frac{76.96 - 72}{5/\sqrt{25}}\right)$

$= 0.846 + \cancel{3.5 \times 10^{-7}}$

0.00000035

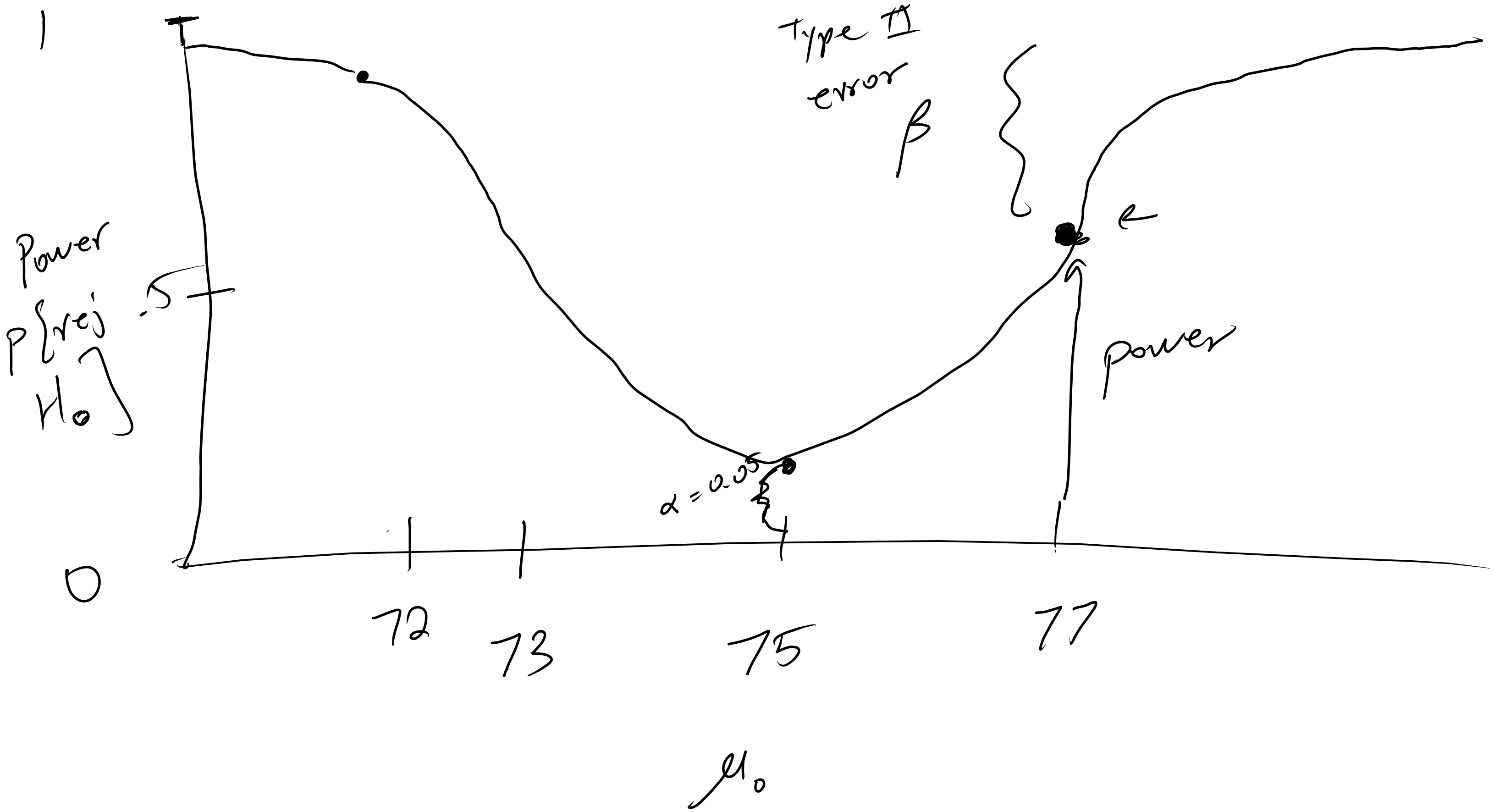


Not to scale

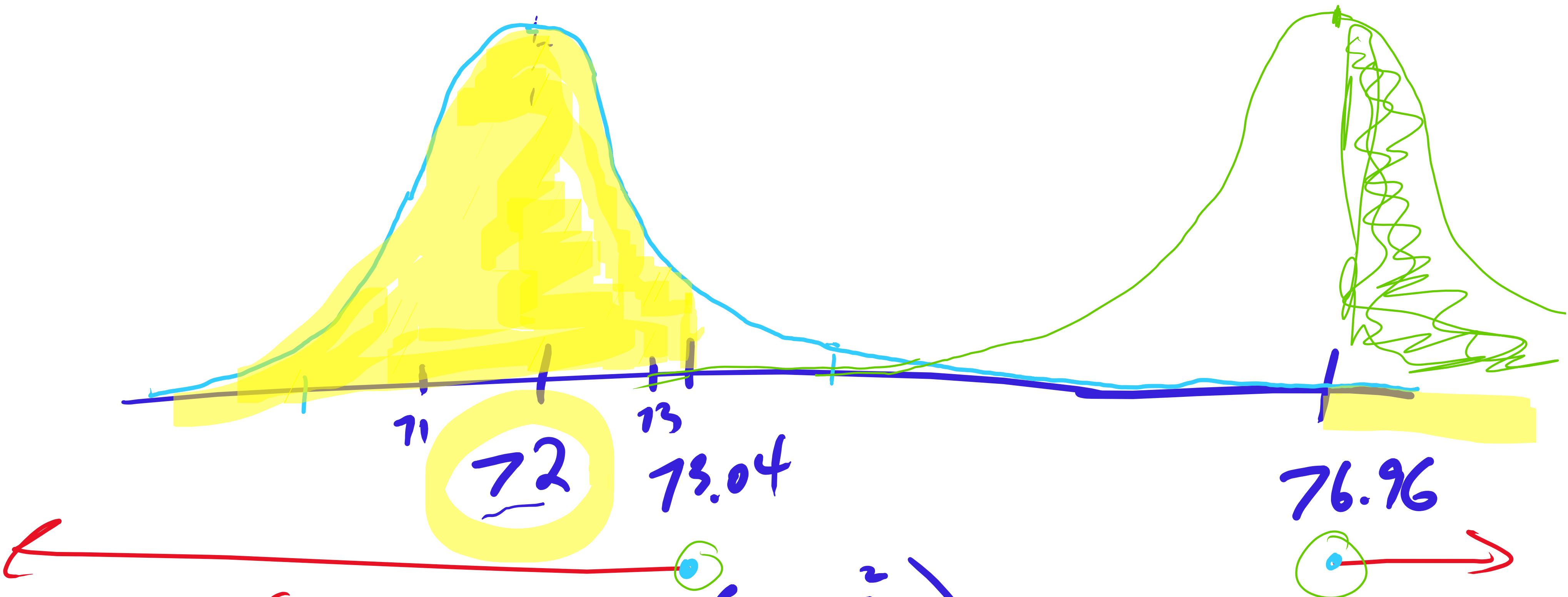
$P(\bar{X} \le 73.04 \mid \mu = 77) + P[\bar{X} \ge 76.96 \mid \mu = 77]$

$= 3 \times 10^{-5} + 0.516 \approx 0.516$

0.00003



1
Power
P{rej
H0}
.5
0
Type II
error
β
power
α = 0.05
72
73
75
77
μ0



reject if

$\bar{X} \sim N\left(72, \frac{\sigma^2}{n}\right)$

25

25

$\sigma = 1$