Stat 400 Homework 7

Spring 2021 -Yu

Due: Tues Mar 23 - 11:59pm

Exercise 1

Yu take a trip to Curtis Orchard are interested in Ambrosia and Winesap apples. Assume the following:

- All apple weights are independent.
- The weight of the Ambrosia apples is normally distributed with a mean of 90 grams and a standard deviation of 4 grams. Let A be the weight of a randomly selected Ambrosia apple.
- The weight of the Winesap apples is normally distributed with a mean of 88 grams and a standard deviation of 6 grams. Let W be the weight of a randomly selected Winesap apple.

a) (0.5 pt) Suppose you pick 5 Winesap apples at random. Assuming independence, what is the probability that that the **average** weight of the 5 applies is less than 89 grams?

b) (0.5 pt) Suppose you pick 5 Ambrosia apples at random. Assuming independence, what is the probability that that the **total** weight of the 5 applies is more than 446 grams?

c) (0.5 pt) Suppose you pick one Ambrosia and One Winesap apple at random. What is the probability that the Ambrosia apple weighs less than the Winesap apple?

d) (1 pt) Suppose you pick 5 Winesap apples and 5 Ambrosia apples. What is the probability that their total weight is less than $900g$?

e) (0.5 pt) Suppose you continue to pick Winesap apples until you get three that weigh over 95g. Let X represent the number of apples you must pick. Find E[X].

Exercise 2

Consider two random variables X and Y, where

- $\sigma_X = 5$
- $\sigma_Y = 2$
- $Var[2X - 3Y] = 80$

(2 pts) Calculate the correlation between X and Y, ρ_{XY}.

Exercise 3

Let $X_1, X_1, ..., X_{100}$ be a i.i.d. random sample of size n=100 from a distribution with probability density function:

$$f(x) = 6x(1-x),\ 0 < x < 1$$

(3 pts) Approximate

$$P(0.45 < \bar{X} < 0.5), \quad where\ \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

Hint: find μ and σ^2.

Exercise 4

You must show a screenshot of your code and output for full credit.

Let `A` ~ N(90,16) and `W` ~ N(88,36).

a) (1 pt) Use R to generate 2 independent, random samples of size 10 from each of these distributions (`A` and `W`).

Comparing by elements, what proportion of `A` are smaller than `W`?

i.e. compare A_1 with W_1, A_2 with W_2, ... A_{10} with W_{10}.

b) (1 pt) Repeat 5(a) with samples of size n=100 and n = 10000. What proportion of `A` are elementwise smaller than `W`? (Compare your answer to 1.c)