

Stat 400 Homework 3

Spring 2021 (Yu)

Due: *Tues Feb 23 @ 11:59pm*

Exercise 1

While wandering around the halls of North Shore High School after hours, Regina George encounters lots of enemies. Suppose enemies appear one-at-a-time. You may assume that these encounters are independent and that there are only 4 types of enemies she may run into:

Enemy	Probability
Freshman	0.5
Sophomore	0.2
Junior	0.2
Senior	0.1

- a) Suppose she encounters 10 enemies. What is the probability that at least 1 is a Junior? (0.4 pt)
- b) What is the probability that the first 8 enemies she encounters are all Freshmen? (0.4 pt)
- c) Suppose she encounters 10 enemies. What is the probability that more than 8 are Freshmen? (0.4 pt)
- d) What is the probability that she sees her first Senior **after** her tenth encounter? (0.4 pt)
- e) What is the probability that she sees her third Senior **on** (exactly) her thirtieth encounter? (0.4 pt)
- f) What is the variance of the number of Sophomore she sees in her first 20 encounters? (0.5 pt)
- g) What is the variance of the number of trials required to find her first Junior? (0.5 pt)

Exercise 2 (Use R to find the following. Show your commands)

(Use the same probabilities shown above for #1) **Must use R** to get credit on these!

- a) Find the probability that it takes fewer than 8 trials to find the first Sophomore. (0.5 pt)
- b) Suppose she encounters 100 enemies. What is the probability that fewer than 18 are a Juniors? (0.5 pt)
- c) Suppose she encounters 100 enemies. What is the probability that at least 65 are Freshmen? (0.5 pt)
- d) What is the probability that she sees her 4th Junior **by** (including) her twentieth encounter? (0.5 pt)

Exercise 3

In a football league, suppose a particular team's offensive line allows their quarterback to be sacked on 40% of their plays if they are not playing against the Bucs. On any given night, there is a 10% chance that they are playing against the Bucs, in which case the team will allow their quarterback to be sacked on 90% of the plays. *Note: Assume that a play either does or does not result in sack. You can only be sacked once in a play.*

- a) Suppose your roommate saw a randomly selected highlight reel (play) where the quarterback got sacked. Based on this information, what is the probability that this team was playing against the Bucs? (1 pt)
- b) Assume that the result of every play is independent. Out of the first 10 plays, your roommate says that the quarterback got sacked 7 times. What is the probability that this team was playing against the Bucs? (1.5 pt)

Exercise 4

While taking Chloe for a bike ride, we encounter squirrels according to a Poisson process with a rate of 4 squirrels per 10 blocks.

- a) What is the probability of not encountering any squirrels in 1 block? (0.5 pt)
- b) What is the probability of not encountering any squirrels in 20 blocks? (0.5 pt)
- c) What is the probability of encountering 4 squirrels in 10 blocks? (0.5 pt)
- d) What is the probability of encountering at least 2 squirrels in 10 blocks? (0.5 pt)
- e) What is the probability that exactly 4 of the 10 blocks contain no squirrels? (0.5 pt)