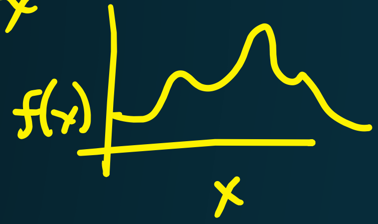
X, Y

Continuous Bivariate Distributions

(4.4)

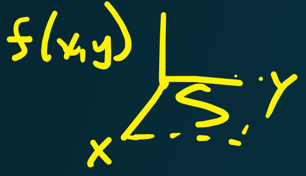
X pdf $f(x)$ density $\int_A f(x)\,dx$



Continuous Bivariate Distributions

4.4
4.2

If X and Y are two continuous random variables, their **joint probability density function,** $f(x, y)$ represents the density at the point (x, y).



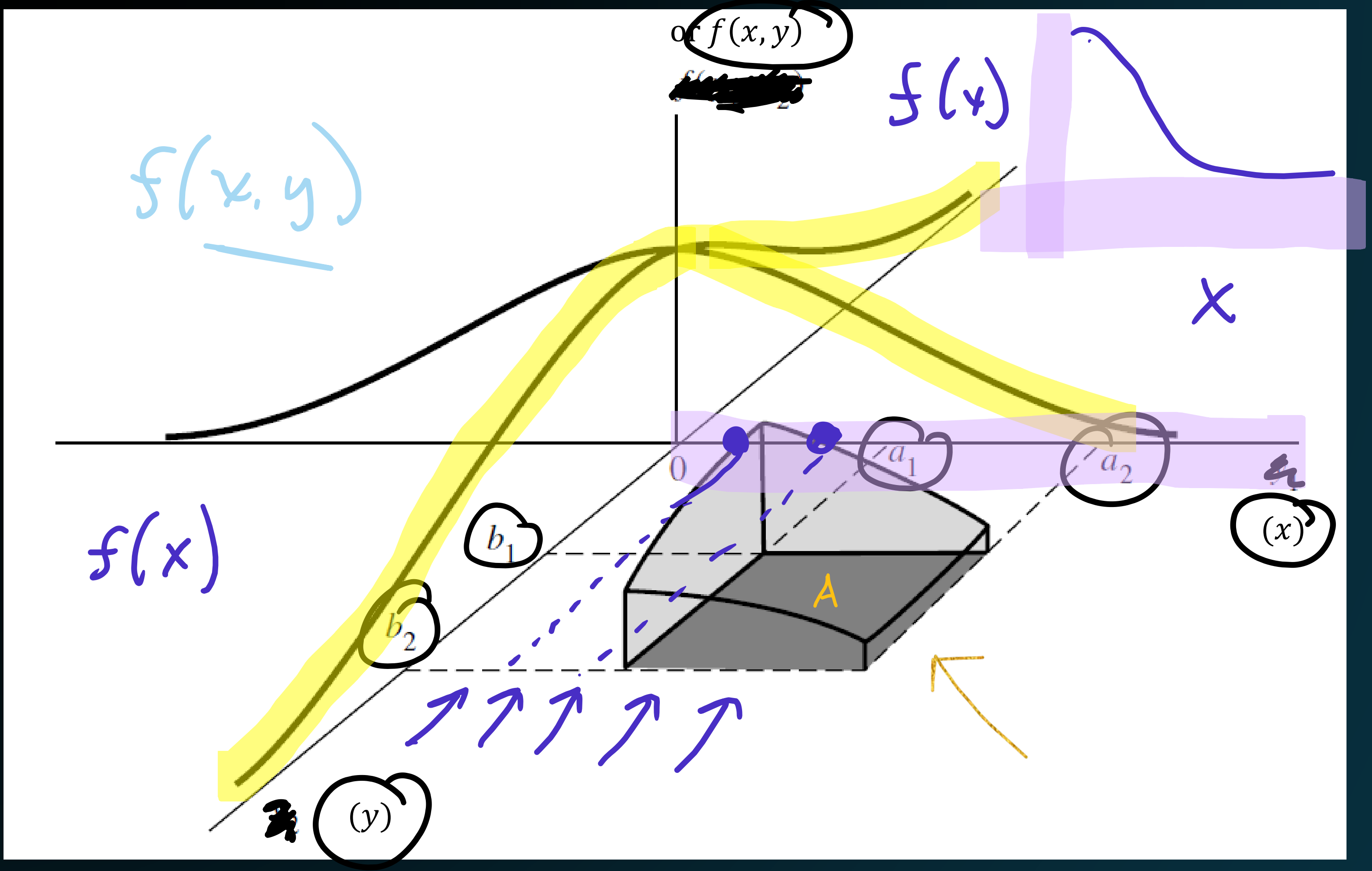
The joint pdf satisfies 3 properties:

(a) $f(x, y) \geq 0$, where $f(x, y) = 0$ when (x, y) is not in the support (space) S of X and Y.

(b) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\, dx\, dy = 1$. $\iint f(x,y)\,dx\,dy = 1$

$y = 4x$

(c) $P[(X, Y) \in A] = \iint_A f(x, y)\, dx\, dy$, where $\{(X, Y) \in A\}$ is an event defined in the plane.



Source: Wackerly, Mendenhall, Scheaffer – Mathematical Statistics

Marginal pdf

$\boxed{f(x)}$

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy, \qquad x \in S_X,$$

integrate over the range of Y

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx, \qquad y \in S_Y,$$

integrate over the range of X

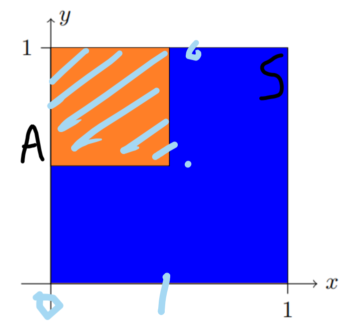
$f_X(x) = \int_y 4xy\,dy = \int_0^1 4xy\,dy = 2xy^2\Big|_0^1 = 2x,\ 0 \le x \le 1$

support

Calculating Probability for joint pdfs:

$$P[(X,Y) \in A] = \iint_A f(x,y)\,dx\,dy$$

- Suppose X and Y both have support [0,1] with joint pdf $f(x,y) = 4xy$.
  - Find P[X < 0.5, Y > 0.5].   3/16
  - Find $f_X(x)$. $f_X(x) = 2x, 0 \le x \le 1$



$\int_{0.5}^{1}\int_{0}^{0.5} 4xy\,dx\,dy =$ ___

Notes

Independence

X and Y are independent **iff:**

- $f(x, y) = f_X(x) f_Y(y), \; x \in S_X, \; y \in S_Y$

joint pdf = product of marginals

Examples

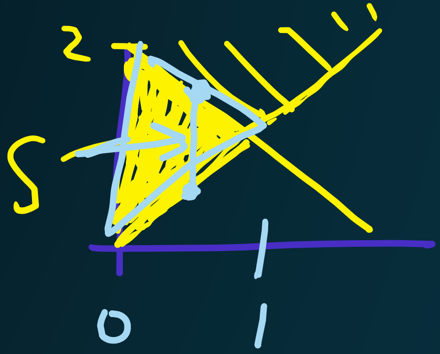
Bivariate Discrete

$1,2$ $\quad y \leq 2-x$

1 Suppose that the random variables X and Y have joint pdf,

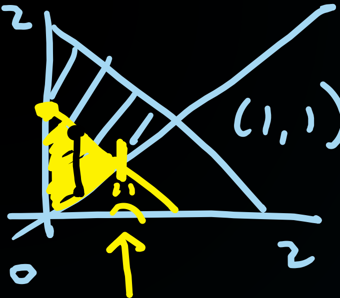
$$f(x,y) = \begin{cases} 6x^2y, & 0 \leq x \leq y, \quad x+y \leq 2 \\ 0, & elsewhere \end{cases}$$

A) Verify that this is a valid joint pdf.

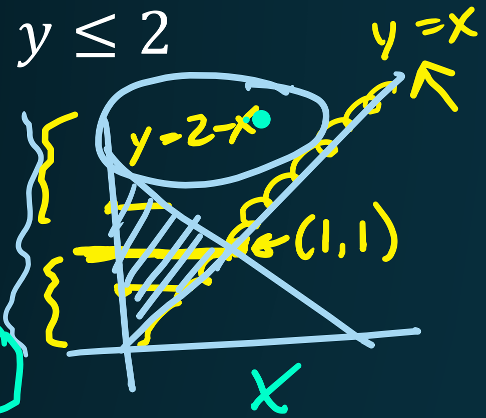


$$\iint 6x^2y\,dy\,dx = 1$$

$$\boxed{\int_0^1 \int_x^{2-x} 6x^2y\,dy\,dx = 1}$$



$$f(x,y) = \begin{cases} 6x^2y, 0 \le x \le y, & x+y \le 2 \\ 0, & elsewhere \end{cases}$$



B) Find $f_Y(y)$.

marg. $y = \int 6x^2y\,dx$

$$f(y) = \begin{cases} \int_0^y 6x^2y\,dx = 2x^3y \Big|_0^y = 2y^4, & 0 \le y \le 1 \\ \int_0^{2-y} 6x^2y\,dx = 2x^3y \Big|_0^{2-y} = 2y(2-y)^3, & 1 < y \le 2 \end{cases}$$

$x = 2-x$

$x = 1, \quad y = 1$

$y = 2-x$

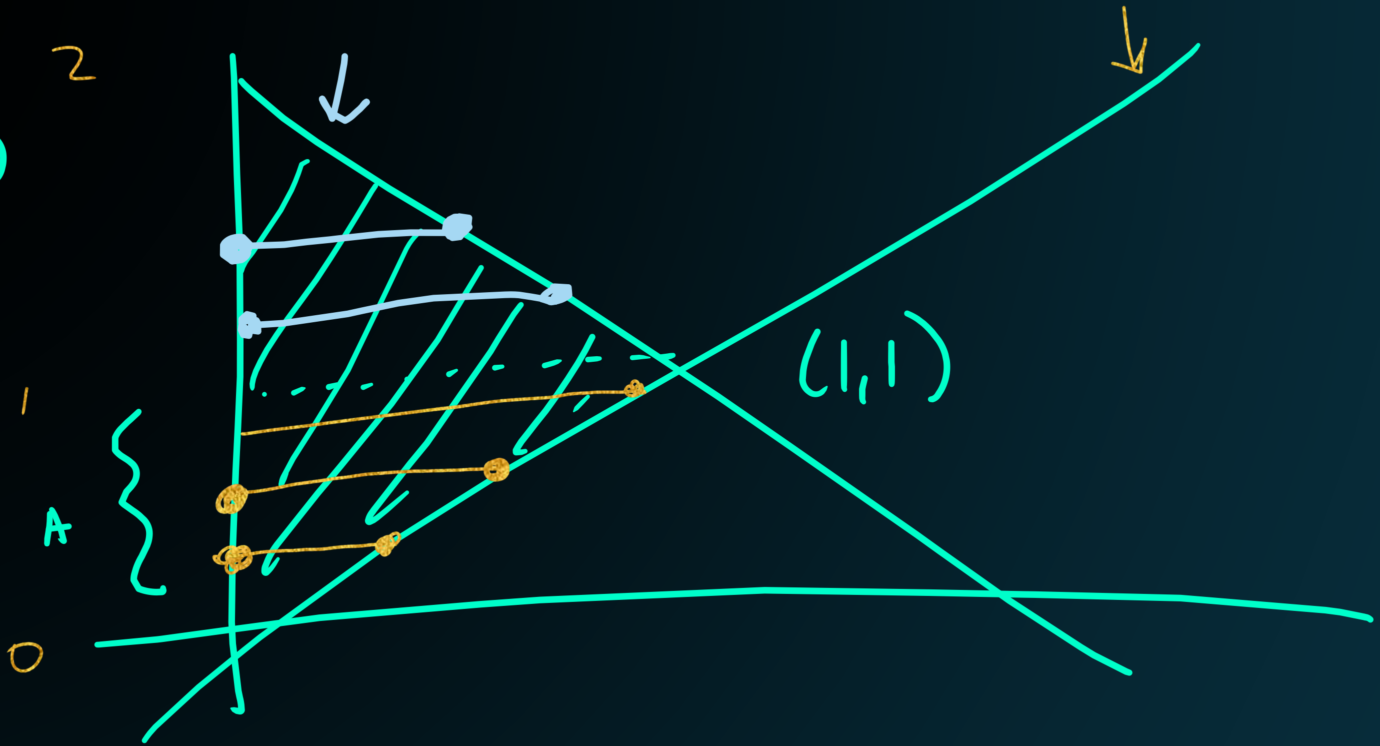
$x = 2-y$

C) What is P[X+Y < 1]?

$P[y < 1-x]$

$$\int_0^{0.5}\int_x^{1-x} 6x^2y\,dy\,dx =$$

Notes



$0 < y < 1$

$$f(x,y)=\begin{cases}6x^2y, 0\le x\le y, & x+y\le 2\\ 0, & elsewhere\end{cases}$$

$E[Y] = \int y \cdot$

$\int_y y \cdot f(y)\,dy$

D) Find (set-up) an expression for E[X].

$E[X] = \int_x x \cdot f(x)\,dx$

$\int_x \int_y x \cdot f(x,y)\,dy\,dx$

OR

$\int_y \int_x x \cdot f(x,y)\,dx\,dy$

Notes