Exercise 1

Spring 2021 STAT400 Homework 11 Solutions

Suppose we have a group of stat majors (population 1) and another group of non-stat majors (population 2). We take a random sample from each group ($n_1 = 46, \quad n_2 = 132$). 30 of the 46 students in population 1 preferred funnels to straws. 80 of 132 in population 2 preferred funnels to straws. (suppose that the only two possible preferences are funnel or straw.)

a) (1.5 points) Conduct a χ^2 test for independence at $\alpha = 0.1$ to see if preference is related to population. State all of the following:

- H_0 and H_A
- value and distribution of the test statistic under H_0
- decision (You may use either a p-value or rejection region)
- conclusion (as a **full sentence** in terms of the problem context. For the conclusion, write a *full sentence on all future questions* from now on, including exercise 2 and the final exam).

b) (1 points) Conduct a 2 sample proportion test to see whether the proportion of people who prefer funnels is equal in both populations. Let p_1 and p_2 be the proportion of stat majors and non-stat majors who prefer funnels, respectively.

a) H_0: Straw vs. funnel preference is independent from whether they are a statistics major.
H_a: The null hypothesis is false: there exists a relationship between straw vs. funnel preference and major.

Observed:

	Statistics (1)	Non-Stat (2)	Total
Funnel	30	80	110
Straw	16	52	68
Total	46	132	178

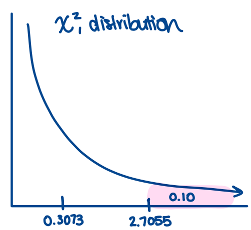
Expected:

	Statistics (1)	Non-Stat (2)	Total
Funnel	28.4270	81.5730	110
Straw	17.5730	50.4270	68
Total	46	132	178

degrees of freedom = (number of rows − 1)(number of columns − 1) = 1

$$\chi^2_1\text{-Statistic} = \sum_{i=1}^{n} \frac{(Obs_i - Exp_i)^2}{Exp_i} = \frac{(30-28.4270)^2}{28.4270} + \frac{(16-17.5730)^2}{17.5730} + \frac{(80-81.5730)^2}{81.5730} + \frac{(52-50.4270)^2}{50.4270}$$

χ^2_1-Statistic $\approx$ 0.3073



P-value = 1 − pchisq(0.3073, 1) = 1 − χ^2cdf(0.3073, 999, 1) $\approx$ 0.5794

R

TI-84

rejection region = 1 − qchisq(1−0.90, 1) $\approx$ 2.7055

Decision: Because p-value > α, or because the χ^2-statistic is less than the boundary value for the rejection region, we fail to reject the null hypothesis.

Conclusion: To a 90% confidence level, there is not enough evidence to suggest there is a dependent relationship between funnel vs. straw preference and major.

b) $H_0: p_1 = p_2$

$H_a: p_1 \neq p_2$

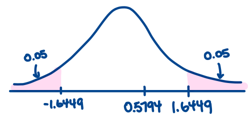
sample values: $\hat{p}_1 = \frac{30}{46}$, $\hat{p}_2 = \frac{80}{132}$ $\rightarrow$ $\hat{p} = \frac{110}{178}$; $n_1 = 46$, $n_2 = 132$

test statistic: $z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p}) \cdot \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \dfrac{35/759}{\sqrt{\frac{110}{178}\left(\frac{68}{178}\right) \cdot \left(\frac{1}{46} + \frac{1}{132}\right)}} \approx 0.55431$

p-value $= 2 \cdot (1 - \text{pnorm}(0.55431)) = 2 \cdot \text{normalcdf}(0.55431, 999, 0, 1) \approx 0.5794$ ← notice the consistency w/ part a)

R   TI-84

rejection region $= \text{qnorm}(1 - 0.10/2) = \text{invNorm}(1 - 0.10/2, 0, 1) = 1.6449$



Decision: Because p-value $> \alpha$, or because the z-statistic is less than the boundary value for the rejection region, we fail to reject the null hypothesis.

Conclusion: To a 90% confidence level, there is not enough evidence to suggest that the proportions of students who prefer funnel to straws is different between the two population groups.

Exercise 2

Chloe breaks into the pantry after Albert goes to work. She finds and opens a family-pack of mixed nuts. After surfing the web, she finds that these packs claim to be 20% almond (A), 30% cashew (C), 10% macadamia (M), and 40% peanut.

Chloe discovers that her bag contains 14 almonds, 28 cashews, 6 macadamias, and 52 peanuts.

a) (1.5 points) Perform an appropriate test to determine whether this claim is true or not at $\alpha = 0.05$. State **all** the relevant steps (listed in exercise 1).

b) (0.5 points) What is the decision and conclusion (write a sentence) at $\alpha = 0.1$?

c) (0.5 points) Chloe is a choosy and sneaky doggie. She likes to find a p-value first and then come up with a significance level, α later so she can reject as many H_os as possible. Is her method acceptable? Yes or no. *Write a short (1 sentence) justification for your answer.*

a) $H_0: p_A = 0.20,\ p_C = 0.30,\ p_M = 0.10,\ p_P = 0.40$

H_a: H_0 is false.

$n = 14 + 28 + 6 + 52 = 100$ total nuts

	Almonts	Cashews	Macadamias	Peanuts
Observed Count	14	28	6	52
Expected Count	20	30	10	40

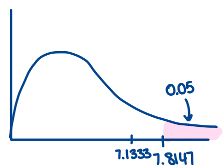
degrees of freedom = number of categories − 1 = 3

test-statistic: $\chi^2_3 = \sum_{i=1}^{4} \frac{(Obs_i - Exp_i)^2}{Exp_i} = \frac{(14-20)^2}{20} + \frac{(28-30)^2}{30} + \frac{(6-10)^2}{10} + \frac{(52-40)^2}{40} = \frac{107}{15} \approx 7.1333$

p-value $= 1 - \text{pchisq}(107/15, 3) = \chi^2\text{cdf}(107/15, 999, 3) \approx 0.067767$

R TI-84

rejection region = qchisq(1−0.05) ≈ 7.8147



Decision: Because p-value > α, or because the χ^2_3-statistic is less than the boundary value for the rejection region, we fail to reject the null hypothesis.

Conclusion: To a 95% confidence level, there is not enough evidence to suggest that the true distribution of mixed nuts is different from what the web claims.

b)

Decision: Because p-value < α, or because the χ^2_3-statistic is greater than the boundary value for the rejection region, we reject the null hypothesis.

Conclusion: To a 95% confidence level, there is enough evidence to suggest that the true distribution of mixed nuts is different from what the web claims.

c) This method is not acceptable, because this increases the rate of committing a Type I Error (P[reject H_0 | H_0 is true])

Exercise 3

Chloe opens another smaller package from the same company, which is also supposedly 20% almonds (A), 30% cashews (C), 10% macadamias (M), and 40% peanuts. Out of 20 nuts, she find that there are 12 peanuts. Chloe does not like peanuts and thinks there are is a **higher** proportion of peanuts than the manufacturer claims.

a) (1 point) Perform a proportion test at $\alpha = 0.05$ to determine whether Chloe's hunch is correct.

b) (1 point) Chloe now wants to calculate the **exact p-value** of this (12 peanuts or more) occuring. i.e. she does not want to do a proportion test. Please perform this test for Chloe, show your work for solving the problem, and calculate the **exact p-value**.

a) $H_o: p = 0.40$; $p \leq 0.40$
$H_a: p > 0.40$

$\hat{p} = \frac{12}{20} = 0.60$

test statistic: $z = \dfrac{0.60 - 0.40}{\sqrt{\dfrac{0.40(0.60)}{20}}} \approx 1.8257$

p-value $= 1 - \text{pnorm}(1.8257) = \text{normalcdf}(1.8257, 999, 0, 1) \approx 0.03394$

R

TI-84

rejection region: $\text{qnorm}(1 - 0.05) = \text{normInv}(1 - 0.05, 0, 1) = 1.6449$

Decision: Because p-value < α, or because the z-statistic is greater than the boundary value for the rejection region, we reject the null hypothesis.

Conclusion: To a 95% confidence level, there is enough evidence to suggest that the true proportion of peanuts is greater than what the manufacturer claims.

b) p-value = P[12 or more peanuts | H_o is true]

Assume a random variable $X \sim \text{Binomial}(n = 20, p = 0.40)$

p-value $= P[X \geq 12] = 1 - P[X < 11]$

R

TI-84

$= 1 - \text{pbinom}(q = 11, \text{size} = 20, \text{prob} = 0.40) = 1 - \text{binomcdf}(20, 0.40, 11)$

≈ 0.05653

Decision: Because p-value > α, we fail to reject the null hypothesis.

Conclusion: To a 95% confidence level, there is not enough evidence to suggest that the true proportion of peanuts is greater than what the manufacturer claims.

Exercise 4

Hulk is testing concrete compressive strength in a particular batch. Under the null hypothesis, the mean strength is 3000psi. Assume that the strengths are normally distributed with population standard deviation = 500 psi.

a) (1 point) Given a sample of size 10, define a rejection region (in terms of mean strength, $\bar{X}$, under a 1-sided lower tailed alternative hypothesis ($\mu < 3000$) at significance level $\alpha = 0.05$. Your answer should look like this: "Reject if ________"

Note: Parts (b-c) are all based on the same rejection region

b) (1 point) Based on a 1-sided lower tailed rejection region from part (a), calculate the power of the test at the following true mean strengths:

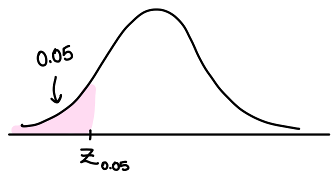
- 2300 psi
- 2600 psi

a) Given: $n=10$, $\mu_0 = 3000$, $\sigma = 500$ psi, $\alpha = 0.05$

$H_0: \mu = 3000$ psi ; $\mu \geq 3000$ psi

$H_a: \mu < 3000$ psi

Because this is a lower-tailed test, we are looking for:



<u>rejection region</u>: qnorm(0.05) = invNorm(0.05, 0, 1) = -1.64485 (R: qnorm; TI-84: invNorm)

reject if z-statistic is < -1.64485

<u>unstandardize to get $\bar{x}$ value</u>: $\dfrac{\bar{x} - 3000}{500/\sqrt{10}} < -1.64485$

$\bar{x} < 2739.9258$ psi

Therefore: Reject if $\bar{x} < 2739.93$ psi

b) Power = P[reject H_0 | H_0 is false]

if $\mu = 2300$ psi: $P[\bar{x} < 2739.93 \mid \mu = 2300 \text{ psi}] = P\left[z < \dfrac{2739.93 - 2300}{500/\sqrt{10}}\right] = P[z < 2.7823] \approx 0.9973$

if $\mu = 2600$ psi: $P[\bar{x} < 2739.93 \mid \mu = 2600 \text{ psi}] = P\left[z < \dfrac{2739.93 - 2600}{500/\sqrt{10}}\right] = P[z < 0.8850] \approx 0.8119$

c) (1 point) Using the information from part (a) and (b), (roughly) sketch a graph by hand of the power curve for this test with mean strength on the horizontal axis, and rejection probability on the vertical axis (connect the dots in a curved fashion). Label/include the following:

- Power at 2300 psi and at 2600 psi.
- Probability of Type II Error at 2300 psi and at 2600 psi.
- Probability of Type I error.