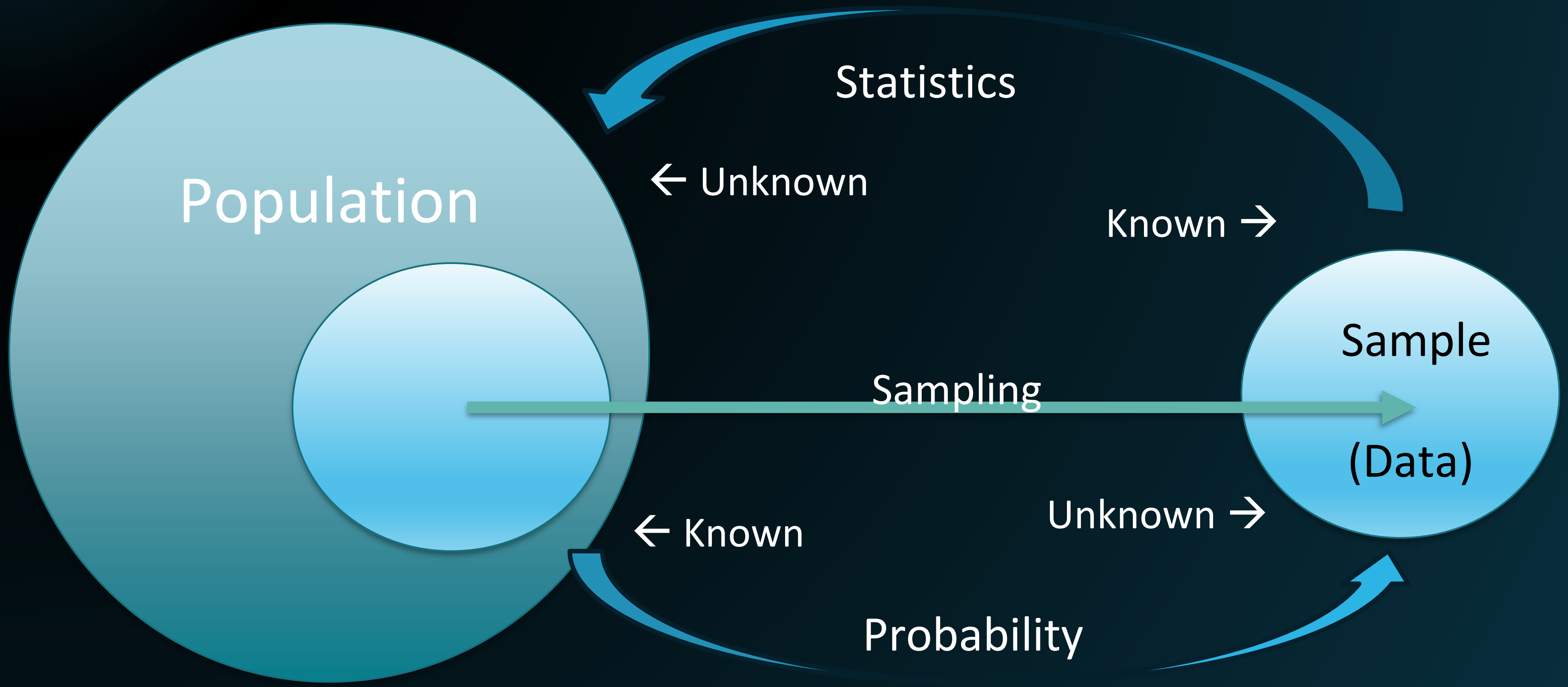
1. Probability

1.1 Properties of Probability
+ Infinite Series

What is Statistics?

- What is a statistic? A function of **data**
- **Statistics**: study of the collection, analysis, interpretation, presentation, and organization of **data.**


Statistics
← Unknown
Known →
Population
Sample
Sampling
(Data)
Unknown →
← Known
Probability

1. Probability

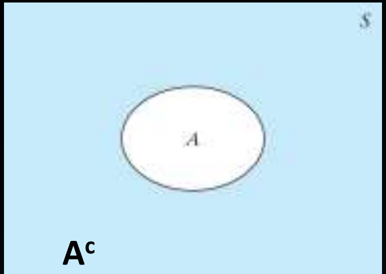
1.1 Properties of Probability

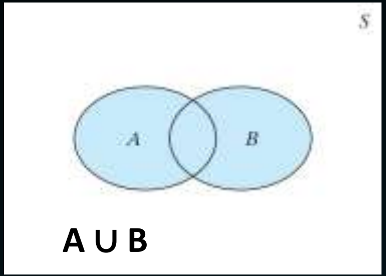
Random Experiments

In Statistics, we consider experiments where the outcome can not be predicted with certainty.

- **Outcome space** or **Sample space**, *S* – collection of all possible outcomes
- An **Event** is a collection of outcomes in *S*.
- If a random experiment is performed and the outcome of the experiment is in A, we say **event *A* has occurred.**

Set notation and operations





Notation	Meaning
$\emptyset$, {}	**Null** or empty set
$x \in A$	x is an element of A
$A \cup B$	the union of A and B
$A \cap B$	the intersection of A and B
$A \subseteq B$	A is a subset of B
$A \subset B$	A is a proper subset of B
A', A^c	the complement of A

Probability is a real-valued set function P that assigns, to each event A in the sample space S, a number $P(A)$, called the probability of the event A, such that the following properties are satisfied:

(a) $P(A) \geq 0$;

(b) $P(S) = 1$;

(c) if $A_1, A_2, A_3, \ldots$ are events and $A_i \cap A_j = \emptyset$, $i \neq j$, then

$$P(A_1 \cup A_2 \cup \cdots \cup A_k) = P(A_1) + P(A_2) + \cdots + P(A_k)$$

for each positive integer k, and

$$P(A_1 \cup A_2 \cup A_3 \cup \cdots) = P(A_1) + P(A_2) + P(A_3) + \cdots$$

for an infinite, but countable, number of events.

Theorem 1.1-1 For each event A,

$$P(A) = 1 - P(A').$$

Proof [See Figure 1.1-1(a).] We have

$$S = A \cup A' \qquad \text{and} \qquad A \cap A' = \emptyset.$$

Thus, from properties (b) and (c), it follows that

$$1 = P(A) + P(A').$$

Hence

$$P(A) = 1 - P(A'). \qquad \square$$

Probability Theorems

- **Theorem 1**
  - $P[A'] = 1 - P[A]$
- **Theorem 2**
  - $P[\emptyset]=0$
- **Theorem3**
  - If $A \subset B$, then $P[A] \leq P[B]$.

Probability Theorems

- **Theorem 4**
  - For any event A, $P[A] \leq 1$
- **Theorem 5**
  - If A and B are any two events, then

$P[A \cup B] = P[A] + P[B] - P[A \cap B]$

- **Theorem 6**
  - $P[A \cup B \cup C] = P[A] + P[B] + P[C] - P[A \cap B] -$

$P[A \cap C] - P[B \cap C] + P[A \cap B \cap C]$

1. Suppose a 6-sided die is rolled:

- **Let event A = {The outcome is even}**
- **Let event B = {The outcome is greater than 3}**
  - a) What are the outcomes in $[A \cap B]$?

    ii) *What is $P[A \cap B]$?*

1. Suppose a 6-sided die is rolled:

- **Let event A = {The outcome is even}**
- **Let event B = {The outcome is greater than 3}**
  - b) What are the outcomes in [A U B]?

    ii) *What is P[A U B]?*

1... Suppose the die is loaded so that the probability of an outcome is proportional to the outcome, i.e.

P[1] = *p*, P[2] = *2p*, P[3] = *3p*, P[4] = *4p*, P[5] = *5p*, P[6] = *6p*,

- c) Find the value of *p* that would make this a valid probability model
- d) Find the following probabilities:
  - i) P[A], ii) P[A’], iii) P[A U B]

2. The probability that a randomly selected student at Zoom University owns a bicycle is 0.55, the probability that a student owns a car is 0.30, and the probability that a student owns both is 0.10.

- A) What is the probability that a student selected at random does not own a bicycle?
- B) What is the probability that a selected student at random owns either a car or a bicycle (or both)?

2…

The probability that a randomly selected student at Zoom University owns a bicycle is 0.55, the probability that a student owns a car is 0.30, and the probability that a student owns both is 0.10.

- C) What is the probability that a student selected at random neither has a car nor a bicycle?

3. Let $a > 2$. Suppose $S = \{0, 1, 2, 3, \ldots.\}$ and

$P[0] = c$, $P[k] = \frac{1}{a^k}$, $k = 1, 2, 3\ldots$

A) Find the value of c that will make this a valid probability distribution.

B) Find the probability of an odd outcome

4. Suppose S = {0, 1, 2, 3, …. }, P[0] = p, and P[k] = $\frac{1}{2^k k!}$, k = 1, 2, 3…

Find the value of p that will make this a valid probability distribution.