

# Homework 11

Spring 2021 - Yu

Due: Thurs May 6 – 11:59pm

## Exercise 1

Suppose we have a group of stat majors (population 1) and another group of non-stat majors (population 2). We take a random sample from each group ( $n_1 = 46$ ,  $n_2 = 132$ ). 30 of the 46 students in population 1 preferred funnels to straws. 80 of 132 in population 2 preferred funnels to straws. (suppose that the only two possible preferences are funnel or straw.)

- a) (1.5 points) Conduct a  $\chi^2$  test for independence at  $\alpha = 0.1$  to see if preference is related to population. State all of the following:
- $H_0$  and  $H_A$
  - value and distribution of the test statistic under  $H_0$
  - decision (You may use either a p-value or rejection region)
  - conclusion (as a **full sentence** in terms of the problem context. For the conclusion, write a *full sentence on all future questions* from now on, including exercise 2 and the final exam).
- b) (1 points) Conduct a 2 sample proportion test to see whether the proportion of people who prefer funnels is equal in both populations. Let  $p_1$  and  $p_2$  be the proportion of stat majors and non-stat majors who prefer funnels, respectively.

## Exercise 2

Chloe breaks into the pantry after Albert goes to work. She finds and opens a family-pack of mixed nuts. After surfing the web, she finds that these packs claim to be 20% almond (A), 30% cashew (C), 10% macadamia (M), and 40% peanut.

Chloe discovers that her bag contains 14 almonds, 28 cashews, 6 macadamias, and 52 peanuts.

- a) (1.5 points) Perform an appropriate test to determine whether this claim is true or not at  $\alpha = 0.05$ . State **all** the relevant steps (listed in exercise 1).
- b) (0.5 points) What is the decision and conclusion (write a sentence) at  $\alpha = 0.1$ ?
- c) (0.5 points) Chloe is a choosy and sneaky doggie. She likes to find a p-value first and then come up with a significance level,  $\alpha$  later so she can reject as many  $H_0$ s as possible. Is her method acceptable? Yes or no. *Write a short (1 sentence) justification for your answer.*

## Exercise 3

Chloe opens another smaller package from the same company, which is also supposedly 20% almonds (A), 30% cashews (C), 10% macadamias (M), and 40% peanuts. Out of 20 nuts, she find that there are 12 peanuts. Chloe does not like peanuts and thinks there are is a **higher** proportion of peanuts than the manufacturer claims.

- a) (1 point) Perform a proportion test at  $\alpha = 0.05$  to determine whether Chloe's hunch is correct.
- b) (1 point) Chloe now wants to calculate the **exact p-value** of this (12 peanuts or more) occurring. i.e. she does not want to do a proportion test. Please perform this test for Chloe, show your work for solving the problem, and calculate the **exact p-value**.

## Exercise 4

Hulk is testing concrete compressive strength in a particular batch. Under the null hypothesis, the mean strength is 3000psi. Assume that the strengths are normally distributed with population standard deviation = 500 psi.

- a) (1 point) Given a sample of size 10, define a rejection region (in terms of mean strength,  $\bar{X}$ , under a 1-sided lower tailed alternative hypothesis ( $\mu < 3000$ ) at significance level  $\alpha = 0.05$ . Your answer should look like this: “Reject if \_\_\_\_\_”

Note: Parts (b-c) are all based on the same rejection region

- b) (1 point) Based on a 1-sided lower tailed rejection region from part (a), calculate the power of the test at the following true mean strengths:
- 2300 psi
  - 2600 psi
- c) (1 point) Using the information from part (a) and (b), (roughly) sketch a graph by hand of the power curve for this test with mean strength on the horizontal axis, and rejection probability on the vertical axis (connect the dots in a curved fashion). Label/include the following:
- Power at 2300 psi and at 2600 psi.
  - Probability of Type II Error at 2300 psi and at 2600 psi.
  - Probability of Type I error.