Exercise 1 Spring 2021 STAT400 Homework 10 Solutions

King Joffrey's mom has a million muffins and thinks that at least half of them are poisoned. She obtains a random sample of a sample of 200 muffins and finds that only 80 of them are poisoned. She now thinks it's possible that fewer than half are poisoned. Let p represent the true proportion of poisoned muffins.

a) (1.5 points) Perform an appropriate test at significance level $\alpha = 0.01$ to test her hypothesis.

b) (1 point) Change the Null Hypothesis from 1(a) to $p \geq 0.4$, and perform a test at significance level $\alpha = 0.05$.

c) (0.5 point) Create a 95% confidence interval for p.

d) (0.5 point) Create a 95% confidence upper bound for p.

a) $H_0: p \geq 0.50$

$H_a: p < 0.50$

$\hat{p} = \frac{80}{200} = 0.40$; standard error : $\sqrt{\frac{0.50(1-0.50)}{200}} \approx 0.035355$

test statistic : $z = \frac{0.40 - 0.50}{0.035355} \approx -2.8284 \rightarrow$ p-value ≈ 0.0023389

Because p-value $< \alpha$, we reject the null hypothesis. To a 99% confidence level, we have sufficient evidence to conclude the true proportion of poisonous muffins is less than 0.50.

b) $H_0: p \geq 0.40$

$H_a: p < 0.40$

$\hat{p} = 0.40$; test statistic : $z = \frac{0.40 - 0.40}{\sqrt{\frac{0.40(1-0.60)}{200}}} = 0 \rightarrow$ p-value $= 0.50$

Because p-value $> \alpha$, we fail to reject the null hypothesis. To a 95% confidence level, we do not have sufficient evidence to conclude the true proportion is less than 0.40.

c) $\hat{p} = 0.40$

standard error $= \sqrt{\frac{0.40(1-0.40)}{200}} \approx 0.03464$; $z_{\alpha/2} \approx 1.95996$

$(0.3321, 0.4679)$

d) $z_\alpha \approx 1.64485$

$(0, 0.45698)$

Exercise 2

Vision is at the new Infinity Stone kiosk at Marketplace Mall. His current Infinity Stone allows him to fire lasers with high accuracy. If he is shooting at targets a mile away, his laser blasts are currently on target $\mu_{current} = 0$, but have a (population) standard deviation of $\sigma_{current} = 2$cm.

With the new infinity stone, he tries shooting the same target, and misses by the following amounts:

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new.laser = c(1.3, -0.8,  2.3,  3.3,  2.7,  5.8,  0.6,  0.2,  2.0,  0.0)
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*Notes:

- 0 means the shot was on target, negative means it missed to the left, and positive means it missed to the right.
- Assume we don't care about missing up or down.
- You may use R to calculate s^2

a) (1.5 point) Perform a hypothesis test at $\alpha = 0.05$ to test whether there is sufficient evidence to suggest that the new Infinity Stone gives his laser blasts a smaller standard deviation, σ_{new} than the current one.

b) (1 point) Vision is noticing that these measurements have more positive numbers than negative ones but Wanda is not around to interpret Stats things for him. Perform an appropriate test at $\alpha = 0.05$ to see whether the new stone still keeps his lasers centered on target (e.g. whether $\mu_{new} = 0$ for the new stone).

c) (0.5 points) Create a 95% Confidence interval for σ_{new}.

d) (0.5 points) Create a 95% Confidence interval for μ_{new}.

a) $H_0: \sigma_{new} \geq 2.0$ cm

$H_a: \sigma_{current} < 2.0$ cm

test statistic : $\chi^2_9 = \frac{(10-1)\cdot 3.7293}{(2.0)^2} = 8.3910 \rightarrow$ p-value ≈ 0.5047

Because p-value > α, we fail to reject the null hypothesis. To a 95% confidence level, we do not have sufficient evidence to conclude the new Infinity Stone is more precise than the current.

b) $H_0: \mu_{new} = 0$

$H_a: \mu_{new} \neq 0$

$\bar{x} = 1.74$; $s \approx 1.931148$; $SE = \frac{1.931148}{\sqrt{10}}$ $\rightarrow$ test statistic : $t_9 = \frac{1.74}{0.6106827} \approx 2.84927$

p-value ≈ 0.019112 Because p-value < α, we reject the null hypothesis. To a 95% confidence level, we have significant evidence to conclude the new lasers are not centered.

c) $\chi^2_{0.025,9} = 2.700389$; $\chi^2_{0.975,9} = 19.02277$

95% Confidence Interval for σ^2_{new} : $\left(\frac{9\cdot 3.7293}{19.02277}, \frac{9.3723}{2.700389}\right) = (1.764412, 12.42932)$

95% Confidence Interval for σ_{new} : $(1.328312, 3.525524)$

d) $t_{\alpha/2,9} = 2.262157$; standard error $= \frac{1.931148}{\sqrt{10}} \approx 0.6106827$; $\bar{x} = 1.74$

95% Confidence Interval for $\mu_{new} = (1.74 - 2.26\cdot 0.6107, 1.74 + 2.26\cdot 0.6107)$

95% Confidence Interval for $\mu_{new} = (0.3585, 3.12146)$

Exercise 3

Albert collects two samples of exam scores for the same midterm: students who are are reading the textbook (Group 1), and students who **are not** reading the textbook (Group 2), and notices that exams scores seem lower in group 2.

Test whether this difference is significant or not. Here are some summary statistics:

Group 1: $n_1 = 30$, $\bar{x} = 92.2$. $s_X = 2.5$,

Group 2: $n_2 = 20$, $\bar{y} = 83.1$. $s_Y = 4.0$

a) (1.5 points) Perform the appropriate test to determine if this difference is significant at $\alpha = 0.05$, using Welch's formula for degrees of freedom. Please show all work. Identify your test statistic, calculate its value and distribution under H_0 (show your work for the df calculation), p-value, and decision.

b) (1 point) Perform a similar appropriate test at $\alpha = 0.05$, but now assume that there is equal variance between the two groups (use pooled variance)

c) (0.5 points) How many degrees of freedom would there be if we used the conservative method instead of Welch's t? Would you expect the p-value to be larger or smaller than using Welch's?

a) $H_0: X \leq Y \rightarrow X - Y \leq 0$

$H_a: X > Y \rightarrow X - Y > 0$

$$\text{Welch's df} = \left\lfloor \frac{\left(\frac{2.5^2}{30} + \frac{4.0^2}{20}\right)^2}{\frac{1}{29}\left(\frac{2.5^2}{30}\right)^2 + \frac{1}{19}\left(\frac{4.0^2}{20}\right)^2} \right\rfloor = 28 \;;\; \text{Standard error} = \sqrt{\frac{2.5^2}{30} + \frac{4.0^2}{20}} \approx 1.004158$$

$$\text{test statistic}: t = \frac{92.2 - 83.1}{1.004158} \approx 9.062319 \rightarrow \text{p-value} = 0$$

Because p-value < α, we reject the null hypothesis. To a 95% confidence level, we have sufficient evidence to conclude that students who read the textbook score better.

b) $H_0: X \leq Y \rightarrow X - Y \leq 0$

$H_a: X > Y \rightarrow X - Y > 0$

$$S_{pooled} = \sqrt{\frac{29 \cdot 2.5^2 + 19 \cdot 4.0^2}{30 + 20 - 2}} \approx 3.1795$$

$$\text{t-statistic} = \frac{92.2 - 83.1}{3.1795\sqrt{\frac{1}{30} + \frac{1}{20}}} \approx 9.91448 \;,\; df = 30 + 20 - 2 = 48 \rightarrow \text{p-value} = 0$$

Because p-value < α, we reject the null hypothesis. To a 95% confidence level, we have sufficient evidence to conclude that students who read the textbook score better.

c) conservative df $= \min(30, 20) - 1 = 19$

I would expect the p-value from the conservative df method to be greater than using Welch's.