Hypothesis Tests for Variances and for two means

8.2

Today's topics

Review:

- Hypothesis test steps

New:

- Hypothesis test for variances
- Hypothesis test for two means

Note for 8.1, 8.3

In all the examples from Tuesday's lecture, we are assuming that these samples are coming from distributions that are **approximately normal** for these methods to work.

(Otherwise, with small sample sizes, the CLT would not hold)

Hypothesis Testing (Steps)

1. Formulate H_0 and H_A (based on the scenario)
2. Identify a test statistic to use and its distribution under H_0
3. Evaluate the test statistic
4. Calculate a p-value, compare to α.
   RU#lghqwli|#d#uhmhfwlrq#uhjlrq
5. Make a decision
   - if $p < \alpha$, reject H_0. Otherwise, (if $p > \alpha$), do not reject H_0.
6. State conclusion **in the context of the original question.**
   - "There is/isn't enough evidence to show that…"

General Form of CI for mean (review)

Estimate $\pm$ (Critical Value * SE of estimate)

e.g. if σ is known:

$$\bar{x} \;\pm\; z_{\alpha/2} * \frac{\sigma}{\sqrt{n}}$$

if σ is unknown:

$$\bar{x} \;\pm\; t_{n-1,\alpha/2} * \frac{s}{\sqrt{n}}$$

notes

Confidence Interval for σ^2 (review)

Confidence Interval for σ^2 :

$$\left(\frac{(n-1)\cdot s^2}{\chi^2_{\alpha/2}}, \frac{(n-1)\cdot s^2}{\chi^2_{1-\alpha/2}} \right)$$

Hypothesis Tests for Variance (or SDs)

Test statistic:

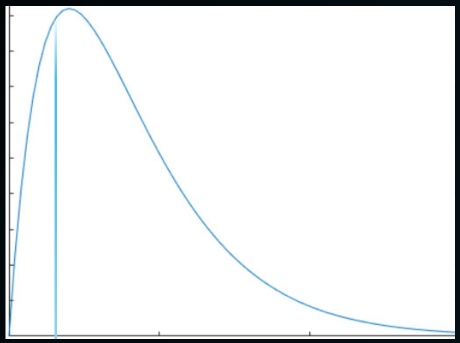
$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{(n-1)}$$

Rejection Region:

Left-tailed:

$H_0: \sigma^2 \geq \sigma_0^2$ vs $H_1: \sigma^2 < \sigma_0^2$

$H_0: \sigma \geq \sigma_0$ vs $H_1: \sigma < \sigma_0$

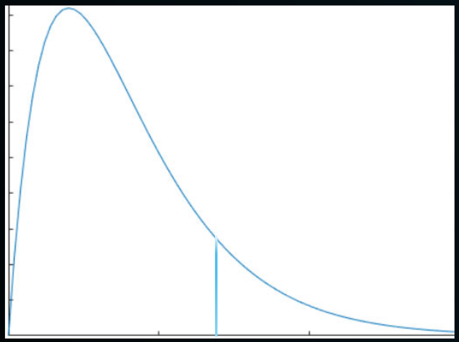


Rejection Region:

Right-tailed:

$H_0: \sigma^2 \leq \sigma_0^2$ vs $H_1: \sigma^2 > \sigma_0^2$

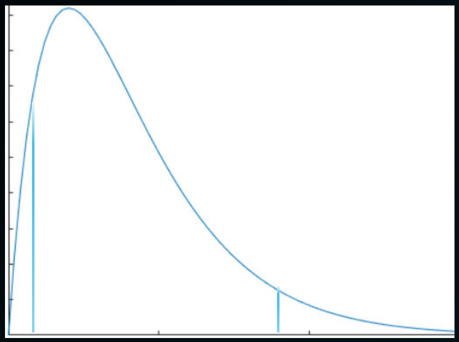
$H_0: \sigma \leq \sigma_0$ vs $H_1: \sigma > \sigma_0$



Two-tailed:

$H_0: \sigma^2 = \sigma_0^2$ vs $H_1: \sigma^2 \neq \sigma_0^2$

$H_0: \sigma = \sigma_0$ vs $H_1: \sigma \neq \sigma_0$



notes

Hypothesis Tests for Variance (or SDs)

Left-tailed:

$H_0: \sigma^2 \geq \sigma_0^2$ vs $H_1: \sigma^2 < \sigma_0^2$

$H_0: \sigma \geq \sigma_0$ vs $H_1: \sigma < \sigma_0$

Right-tailed:

$H_0: \sigma^2 \leq \sigma_0^2$ vs $H_1: \sigma^2 > \sigma_0^2$

$H_0: \sigma \leq \sigma_0$ vs $H_1: \sigma > \sigma_0$

Two-tailed:

$H_0: \sigma^2 = \sigma_0^2$ vs $H_1: \sigma^2 \neq \sigma_0^2$

$H_0: \sigma = \sigma_0$ vs $H_1: \sigma \neq \sigma_0$

Example

A sample of 12 random exam scores from the final exam was collected: 78, 81, 82 ,77, 86, 79, 84, 87, 86, 91, 88, 88. (s = 4.48). $\text{Last year's final had } \sigma = 7$. Is there enough evidence to suggest that the population standard deviation for this year is lower than 7?

Two Sample tests for means

Say we have two populations: with mean μ_1, variance σ_1^2, and mean μ_2, variance σ_2^2.

Let $X_1, X_2, \ldots, X_{n1}$ and $Y_1, Y_2, \ldots, Y_{n2}$ be two independent samples from each of these populations.

If n_1 and n_2 are large, or these populations are both approximately normal then a confidence interval for μ_1 - μ_2 is:

$$\boxed{(\bar{X} - \bar{Y}) \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Review – Functions of Normal Distributions

Assuming i.i.d:

If each $X_i \sim N(\mu, \sigma^2)$, what is the distribution of $\bar{X}$?

If each X_i has mean μ_1, and variance σ_1^2, $\bar{X} \sim$?

If each Y_i has mean μ_2, and variance σ_2^2, $\bar{Y} \sim$?

$\bar{X} - \bar{Y} \sim$?

General form of CI for mean

Estimate $\pm$ Critical Value * SE(estimate)

$$(\bar{X} - \bar{Y}) \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

2-sample confidence interval

$$\boxed{(\bar{X} - \bar{Y}) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Two approaches for df:

1. Conservative

   df = min(n_1, n_2) -1

2. Welch's T:

$$\text{df} = \left\lfloor \frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{s_1^2}{n_1}\right)^2+\frac{1}{n_2-1}\left(\frac{s_2^2}{n_2}\right)^2} \right\rfloor$$

Two sample means – pooled variance

If we can assume that population 1 and population 2 standard deviations are equal, we can use s_{pooled} as an estimator for both.

$$\left(\overline{X}-\overline{Y}\right) \pm t_{\alpha/2} \cdot s_{\text{pooled}} \cdot \sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

where

$$s_{\text{pooled}}^2 = \frac{(n_1-1)\cdot s_1^2 + (n_2-1)\cdot s_2^2}{n_1+n_2-2}$$

and df = $n_1 + n_2 - 2$

Two sample means – Matched pairs

Assume that the differences $D_i = X_i - Y_i$ are a random sample from normal distribution with mean δ_D and standard deviation σ_D.

Pair			Difference
1	X_1	Y_1	$D_1 = X_1 - Y_1$
2	X_2	Y_2	$D_2 = X_2 - Y_2$
.	.	.	.
.	.	.	.
.	.	.	.
n	X_n	Y_n	$D_n = X_n - Y_n$

A confidence interval for δ_D is

$$\bar{D} \pm t_{\alpha/2} \frac{s_D}{\sqrt{n}}. \qquad \text{df} = n-1$$

The test statistic for testing H_0: $\delta = \delta_0$ is

$$T = \frac{\bar{D} - \delta_0}{s_D/\sqrt{n}}$$

Hypothesis test for 2 means

Steps

- Determine which statistic to use t/z
- Are the data paired?
- Can the variances be pooled?
- Follow the usual procedures for hypothesis testing:
  - Come up with H_0 and $H_{A,}$
  - Create and evaluate test statistic, identify the null distribution
  - Find p-value (or rejection region)
  - Make decision and conclusion.

Example

Among the data collected for the World Health Organization air quality monitoring project is a measure of suspended particles, in μg/m^3. Let X and Y equal the concentration of suspended particles in μg/m^3 in the city centers of Melbourne and Houston, respectively. Using n = 13 observations of X and m = 16 observations of Y, we obtain $\bar{x}$=72.9, s_x = 25.6, $\bar{y}$=81.7, s_Y = 28.3. Perform a test to see if Houston has worse air quality at 0.05 significance and state the result.

H_0: $\mu_X = \mu_Y$

H_1: $\mu_X < \mu_Y$.

Continued:

$\bar{x}$=72.9, s_x = 25.6, $\bar{y}$=81.7, s_Y = 28.3. Perform a test to see if Houston has worse air quality at 0.05 significance and state the result.

H_0: $\mu_X = \mu_Y$ H_1: $\mu_X < \mu_Y$.

$$\left(\overline{X}-\overline{Y}\right) \pm t_{\alpha/2} \cdot s_{\text{pooled}} \cdot \sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$s_{\text{pooled}}^2 = \frac{(n_1-1)\cdot s_1^2+(n_2-1)\cdot s_2^2}{n_1+n_2-2}$$

What if we want to pool the variance?

$\bar{x}$=72.9, s_x = 25.6, $\bar{y}$=81.7, s_Y = 28.3. Perform a test to see if Houston has worse air quality at 0.05 significance and state the result.

H_0: $\mu_X = \mu_Y$

H_1: $\mu_X < \mu_Y$.