Stat 400 / Math 463
Spring 2021

1.3 Conditional Probability

Notes

Conditional Probability

	Early (E)	Late (L)	Totals
Red (R)	5	8	13
Yellow (Y)	3	4	7
Totals	8	12	20

What is the probability of selecting a red bulb? (marginal)

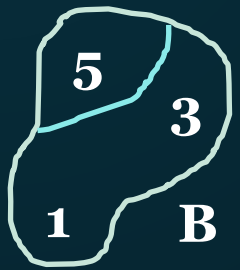
What is the probability of selecting a red bulb if you know the flower will bloom early? (conditional)

A **conditional probability** is a probability that is updated to take into account the (known) occurrence of another event.

Conditional Probability Example

- With a fair die being rolled once, define **A = {5}**
- Then, P[A] = 1/6

What if someone rolls the die and doesn't tell us the number showing. Tells us only that event **B = {odd number}** occurs?



Conditional Probability

Definition 1.3-1

The **conditional probability** of an event A, given that event B has occurred, is defined by

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)},$$

provided that $P(B) > 0$.

Multiplication Rule

Definition 1.3-2

The probability that two events, A and B, both occur is given by the **multiplication rule**,

$$P(A \cap B) = P(A)P(B \mid A),$$

provided $P(A) > 0$ or by

$$P(A \cap B) = P(B)P(A \mid B)$$

provided $P(B) > 0$.

1.3 Conditional Probability

Examples

For a randomly selected off-campus student at UIUC on any given day, assume:

P[Bikes to campus] = 0.4,
P[Rides bus to camps] = 0.3,
P[Does both] = 0.04.

1) What is the probability that a student bikes to campus, given that they ride the bus?

2) What is the probability that a student bikes to campus, given that they don't ride the bus?

continued

P[Bikes to campus] = 0.4,
P[Rides bus to camps] = 0.3,
P[Does both] = 0.04.

3) Suppose you know that a student does not bike to campus. Find the probability that this student does not take the bus.

While running from Shia LaBeouf, you stumble upon a group of 20 kittens. 8 are going to explode. You decide to grab 2 of them anyway.

- 4) Find the probability that both will explode.

- 5) Find the probability that at least one of the kittens will explode.

Two fair 6-sided dice are rolled.

6) What is the probability that the number on the first die was at least as large as **4** given that the sum of the two dice was **8**?