Continuous Bivariate Distributions

(4.4)

Continuous Bivariate Distributions

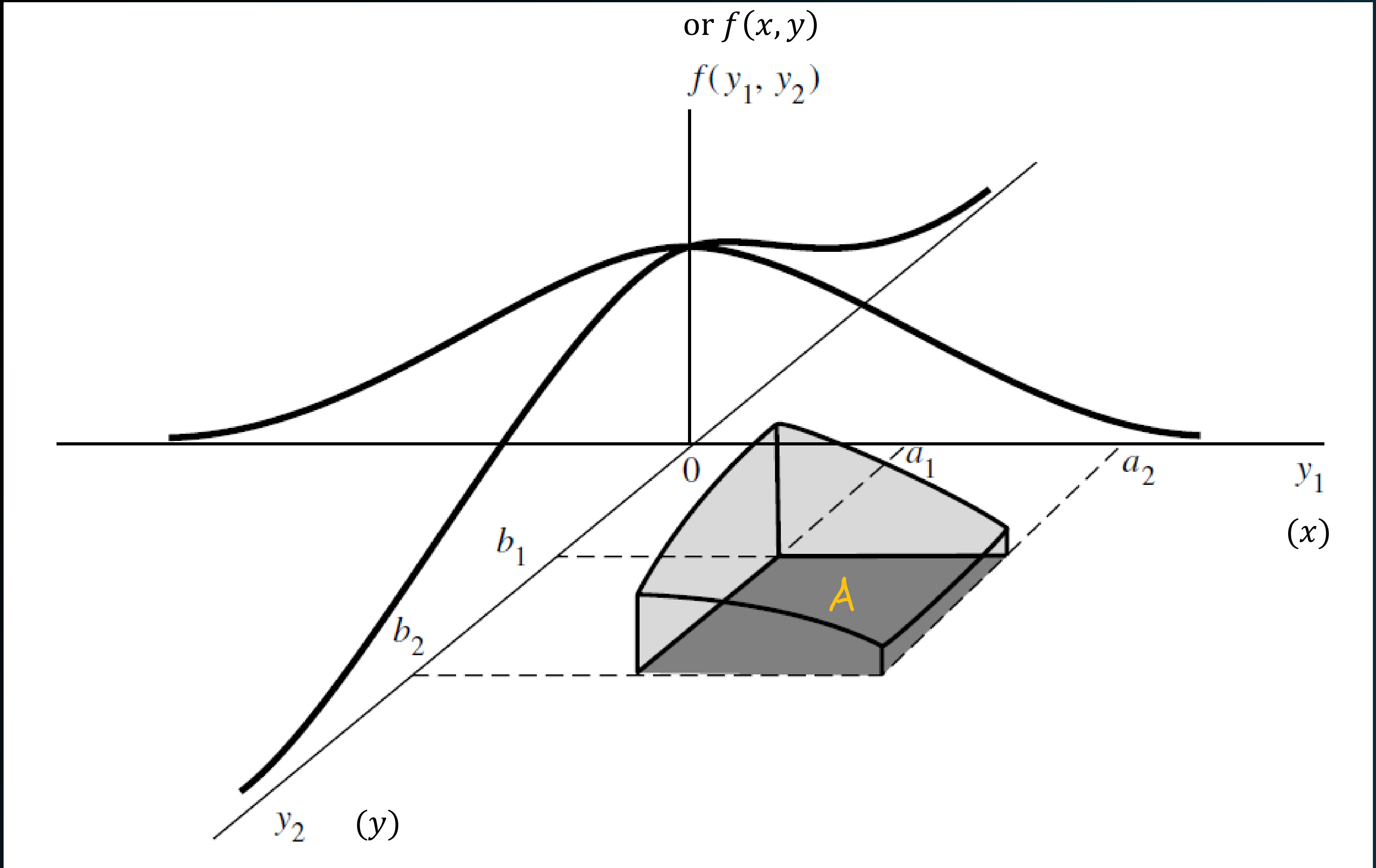
If X and Y are two continuous random variables, their **joint probability density function,** $f(x, y)$ represents the density at the point (x, y).

The joint pdf satisfies 3 properties:

(a) $f(x, y) \geq 0$, where $f(x, y) = 0$ when (x, y) is not in the support (space) S of X and Y.

(b) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)\, dx\, dy = 1$.

(c) $P[(X, Y) \in A] = \iint_A f(x, y)\, dx\, dy$, where $\{(X, Y) \in A\}$ is an event defined in the plane.



Source: Wackerly, Mendenhall, Scheaffer – Mathematical Statistics

Marginal pdf

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy, \qquad x \in S_X,$$

integrate over the range of Y

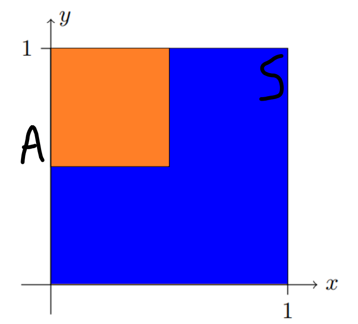
$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx, \qquad y \in S_Y,$$

integrate over the range of X

Calculating Probability for joint pdfs:

$$P[(X,Y) \in A] = \iint\limits_A f(x,y)\,dx\,dy$$

Suppose X and Y both have support [0,1] with joint pdf $f(x,y) = 4xy$.



- Find P[X < 0.5, Y > 0.5]. 3/16
- Find $f_X(x)$. $f_X(x) = 2x,\ 0 \le x \le 1$

Notes

Independence

X and Y are independent **iff:**

- $f(x,y) = f_X(x)f_Y(y),\ x \in S_X,\ y \in S_Y$

Examples

Bivariate Discrete

1 Suppose that the random variables X and Y have joint pdf,

$$f(x,y) = \begin{cases} 6x^2y, 0 \leq x \leq y, \ \ x + y \leq 2 \\ 0, \ \ elsewhere \end{cases}$$

A) Verify that this is a valid joint pdf.

$$f(x,y) = \begin{cases} 6x^2y, 0 \le x \le y, \quad\quad x + y \le 2 \\ 0, \quad\quad elsewhere \end{cases}$$

B) Find $f_Y(y)$.

C) What is P[X+Y < 1]?

Notes

$$f(x,y) = \begin{cases} 6x^2y, 0 \leq x \leq y, & x + y \leq 2 \\ 0, & elsewhere \end{cases}$$

D) Find (set-up) an expression for E[X].

Notes