

Stat 400 Homework 8

Spring 2021 -Yu

Due: Tues Mar 30 - 11:59pm

Exercise 1

Let $X_1, X_2, \dots, X_5 \sim \text{Poisson}(\lambda)$

- a) (1.5 point) Find an expression for the MLE of $\lambda, \hat{\lambda}$.
- b) (1.5 point) Find an expression for the MOM of $\lambda, \tilde{\lambda}$.
- c) (0.5 point) Find an estimate of λ when $x_1 = 2, x_2 = 7, x_3 = 3, x_4 = 1, x_5 = 2$
- d) (1.5) Using the 5 data points in 1(c) and the likelihood from 1(a), use R to plot the Likelihood as a function of θ . Must show code and plot output from R!

Plot θ on the x-axis: $0 < \theta < 10$. The y-axis is Likelihood, $L(\theta)$.

Exercise 2

Let

$$f(x) = \frac{x}{\theta} e^{-\frac{x^2}{2\theta}}, \quad x > 0, \quad \theta > 0.$$

(2 points) Find an estimator of θ using the MLE method.

Exercise 3

Suppose X_1, X_2, \dots, X_n are iid with mean θ and variance θ^2 .

Suppose super stats sluths Peralta, Boyle, Santiago, and Diaz propose the following estimators:

$$\hat{\theta}_{Peralta} = X_1, \quad \hat{\theta}_{Boyle} = \frac{X_1 + 3X_2}{6}, \quad \hat{\theta}_{Santiago} = \bar{X}, \quad \hat{\theta}_{Diaz} = \frac{X_1 + X_2 + X_3}{3}.$$

- a) (1 point) Find the Bias of each estimator (please provide 4 answers, 1 for each detective).
- b) (2 points) Find the Variance of each estimator (4 answers).