Stat 400 Homework 4

Spring 2021 (Yu)

Due: *Tues Mar 2 @ 11:59pm* via Gradescope.

Please show all work for full credit. When in doubt, just include it :)

Exercise 1

Suppose the moment-generating function of X is

$$M_X(t) = 0.2 + 0.1 \cdot e^t + 0.3 \cdot e^{2t} + 0.4 \cdot e^{3t}$$

a) Calculate $E[X]$. (0.5 pt)

b) Calculate $Var[X]$. (0.5 pt)

Exercise 2

Let X ~ Bernoulli(p).

a) Derive an expression for the moment generating function $M_x(t)$. Note: you can check your answer with the given mgf in the table (textbook). (1 pt)

b) Use the mgf and calculus to derive expressions for E[X] and Var[X]. Show all steps for credit. (1 pt)

Exercise 3

Let random variable X have probability mass function

$$f(x) = \frac{54}{3^x}, \ x = 4, 5, 6, \dots$$

a) Derive and simplify an expression for the moment-generating function of $X, M_X(t)$. (1 pt)

b) Calculate E[X] using the definition of E[X]. (The expected value formula) (0.5 pt)

c) Calculate $E[X]$ using the MGF method. (0.5 pt)

Exercise 4

Students often worry about the time it takes to complete the final exam. Suppose students begin taking the exam at 8:00 am. Suppose that the completion time in hours, T, for a STAT 400 final exam follows a distribution with density

$$f(t) = \frac{2}{9} \cdot (3t - t^2), \ 0 \le t \le 3.$$

a) What is the probability that a randomly chosen student finishes the exam **beween** 9:00 am and 11:00am? (0.5 pt)

b) What is the probability that a randomly chosen student finishes the exam **during** the first hour of the exam? (0.5 pt)

c) Calculate the expected value of the time it take to finish the final, $E[T]$. (0.5 pt)

d) Calculate the 20th percentile of the time it takes to finish this exam. (0.5 pt)

e) **Use R:** Suppose 600 students take the final exam. Find the probability that **at least** 450 finish the exam between 9:00 and 11:00 am. **Show your code and output**. For partial credit, feel free to write comments explaining what you did. (1 pt)

Exercise 5

Suppose the scores on a certain midterm were not very high and their probability density function was

$$f(s) = \frac{1}{14480} \cdot (3s + 1), \ 20 \leq s \leq 100.$$

a) What is the expected value of the scores, $E[S]$? (0.5 pt)

b) What is the standard deviation of the scores, $SD[S]$? (1 pt)

c) What score is in the 90th percentile? (0.5 pt)