

# Stat 400 Homework 7

Spring 2021 -Yu

Due: Tues Mar 23 - 11:59pm

## Exercise 1

You take a trip to Curtis Orchard are interested in Ambrosia and Winesap apples. Assume the following:

- All apple weights are independent.
  - The weight of the Ambrosia apples is normally distributed with a mean of 90 grams and a standard deviation of 4 grams. Let  $A$  be the weight of a randomly selected Ambrosia apple.
  - The weight of the Winesap apples is normally distributed with a mean of 88 grams and a standard deviation of 6 grams. Let  $W$  be the weight of a randomly selected Winesap apple.
- a) Suppose you pick 5 Winesap apples at random. Assuming independence, what is the probability that that the **average** weight of the 5 apples is less than 89 grams?
- b) Suppose you pick 5 Ambrosia apples at random. Assuming independence, what is the probability that that the **total** weight of the 5 apples is more than 446 grams?
- c) Suppose you pick one Ambrosia and One Winesap apple at random. What is the probability that the Ambrosia apple weighs less than the Winesap apple?
- d) Suppose you pick 5 Winesap apples and 5 Ambrosia apples. What is the probability that their total weight is less than 900g?
- e) Suppose you pick Winesap apples until you get three that weigh over 95g. Let  $X$  represent the number of apples you must pick. Find  $E[X]$ .

## Exercise 2

Consider two random variables  $X$  and  $Y$ , where

- $\sigma_X = 5$
- $\sigma_Y = 2$
- $Var[2X - 3Y] = 80$

Calculate the correlation between  $X$  and  $Y$ ,  $\rho_{XY}$ .

## Exercise 3

Let  $X_1, X_2, \dots, X_{100}$  be a i.i.d. random sample of size  $n=100$  from a distribution with probability density function:

$$f(x) = 6x(1-x), 0 < x < 1$$

Approximate

$$P(0.45 < \bar{X} < 0.5), \quad \text{where } \bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$$

Hint: find  $\mu$  and  $\sigma^2$ .

#### Exercise 4

Let  $\mathbf{A} \sim N(90,16)$  and  $\mathbf{W} \sim N(88,36)$ .

- a) Use R to generate 2 independent, random samples of size 10 from each of these distributions ( $\mathbf{A}$  and  $\mathbf{W}$ ).

Comparing by elements, what proportion of  $\mathbf{A}$  are smaller than  $\mathbf{W}$ ?

i.e. compare  $A_1$  with  $W_1$ ,  $A_2$  with  $W_2$ ,  $\dots$   $A_{10}$  with  $W_{10}$ .

- b) Repeat 5(a) with samples of size  $n=100$  and  $n = 10000$ . What proportion of  $\mathbf{A}$  are elementwise smaller than  $\mathbf{W}$ ? (Compare your answer to 1.c )