Stat 400 Homework 2

Fall 2023

Due: *Fri Sep 15 @ 11:59pm* via Gradescope

Exercise 1 (4 pts)

Suppose $X = 1, 2, 3, 4, 5, ...$ and for any element of S,

$$P[X = k] = c\frac{3^k}{k!}$$

a) (1 pt) Find the value of c that makes this a valid probability distribution.

b) (0.5 pt) Find $P(X > 2)$. (Hint: you do not need to calculate an infinite sum).

c) (1 pt) Find $E(X)$.

d) (1 pt) Find $Var(X)$, and write it as an expression in terms of μ_X. (Hint: $E(X^2) = E(X(X-1)) + E(X)$, it is easier to figure out $E(X(X-1))$).

e) (0.5 pt) Let $Y = 2X + 3$. Denote $E(X)$ as μ_X and $Var(X)$ as σ_X^2. What are $E(Y)$ and $Var(X)$ in terms of μ_X and σ_X^2?

Exercises 2 and 3 refer to the following information:

Chancellor Palpatine wants to work on his tan. Let **X** denote the number of times that Palpatine goes to the beach in a given week. Suppose X has the following probability distribution:

x	f(x)
0	0.2
1	0.3
2	0.5

Exercise 2 (3.5 pts)

a) (0.25 pt) Find the expected number of times that Palpatine will go to the beach in a given week.

b) (0.5 pt) Evaluate $Var(X)$.

c) (0.5 pt) Anakin tells Palpatine that he is at high risk of skin cancer and should avoid going to the beach more than 2 times total in any 2 week period, but Palpatine doesn't care. Denote the total number of visits in two weeks as Y. Does $Y = 2X$? If not, explain why. (Assume the frequency of beach visits each week is independent)

d) (1 pt) What is the PMF, $g(y)$, for $y = 0, 1, 2, .., 4$?

e) (0.25 pt) What is the probability that Palpatine will go to the beach more than 2 times total in any 2 week period?

f) (1 pt) At the end of the week, "Ninki Minjaj" says she knows Palpatine has been to the beach at least twice. What is the probability that Palpatine went to the beach at least 3 times that week? (Hint: we are finding the probability of Y given that $Y \geq 2$).

Exercise 3 (R / Rmarkdown) (1.5 pts)

Must use R or RMarkdown for this question. Code and markdown output must both be shown to receive credit. (If the rest of your assignment is handwritten, you may take a screenshot of the pdf file generated by markdown or a picture of your code on the computer)

a) (0.5) Using the probability distribution given above, use R to draw 1000 samples from this distribution, and store the results in a vector called `beach`. *(In other words, create a random vector of length 100 using the given probability distribution. You may use any pre-built functions to do this)*

b) (0.5) Give the sample mean and sample variance of your vector from 4(a).

c) (0.5) Plot a histogram of your sample.

Exercise 4 (Conceptual Random Variable question) (1 pt)

Denote X as the random variable as S as the outcome space. You can use examples you learned in our lecture or your own example to answer the following questions. If your answer is yes, explain in your own words If your answer is no, provide a counter example.

a) (0.5 pt) Recall the relation between X and S in the definition of a random variable. Is it necessary for S to be the same as the space of X?

b) (0.5 pt) Consider a discrete random variable X. Is X's space countable? Is S always countable?