Infinite Series

Geometric Series

The sum of a geometric series with ratio, r, and first term, a, is $\frac{a}{1-r}$.

e.g. If $|x| < 1$, $a = 1$, $r = x$:

$$\underbrace{1 + x + x^2 + x^3 + \ldots}_{\sum_{k=0}^{\infty} x^k} = \frac{1}{1-x}$$

Also, $\underbrace{x^i + x^{i+1} + x^{i+2} + x^{i+3} + \ldots}_{\sum_{k=i}^{\infty} x^k} = \frac{x^i}{1-x}$

Euler's Constant, e

$$e = \sum_{k=0}^{\infty} \frac{1}{k!} = 1 + 1 + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \ldots$$

Exponentials with e

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

Other stuff

Binomial Expansion

$$(a+b)^n = \sum_{x=0}^{n} \binom{n}{x} a^{n-x} b^x$$

Sums of integers:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$