STAT 400 Discussion 3

1. Your friend Justin claims that he is an MLG and wants to 1v1 you at checkers where you always play first. Let's assume that you know you only have a chance of winning any given game equal to .43

   (a) If you play 5 games, what is the probability that you have more wins than Justin?

   Solution: Since you always go first and know that your probability of winning is .43 we have a binomial distribution. Let X be the number of wins, then

   $$P(X > 2) = \sum_{x=3}^{5} \binom{5}{x} .(43)^x (.57)^{(5-x)} = .37$$

   (b) After the previous 5 warm up games, Justin wants to play a best of 5. What is the probability that he defeats you in the best of 5 series on the 5th game.

   Solution: In this case, order does not matter for the first four games. For him to defeat you on the fifth, you must win two of the first four and then lose on the fifth.

   $$P(\text{Lose on 5th game}) = P(\text{Win two of 4})P(\text{lose a game}) =$$

   $$[\binom{4}{2} .(43)^2 (.57)^{(2)}] * [.57] = .205$$

2. We would like to review some properties of expectation and variance. Let X be a discrete random variable with expected value 4 and variance 3. Also, let Y be an R.V. with expectation 2 and variance 1. Find the following

   (a) $E(2X + 1)$

Solution:

$$E(2X+1) = E(2X) + E(1) = 2E(X) + 1 = 9$$

(b) $E(3X - 1 - 2Y + 2)$

Solution:

$$E(3X - 1 - 2Y + 2) = E(3X) - E(2Y) + 1 =$$

$$3E(X) - 2E(Y) + 1 = 12 - 4 + 1 = 9$$

(c) $Var(X + 15)$

Solution:

$$Var(X + 15) = Var(X) = 3$$

(d) $Var(5X - 2)$

Solution:

$$Var(5X - 2) = 25Var(X) = 25 * 3 = 75$$

(e) Assume that we have some function of our random variable X, call it g(X). Using the definition, write out the both the expectation variance for g(X).

Solution: Here we will just be reminding the definition for generic functions of random variables. If you let g(X) be the identify function such that g(x)=x then these formulas should look as you have seen already.

$$E(g(X)) = \sum_{\text{All x}} g(x)P(x)$$

$$Var(g(X)) = E([g(X)]^2 - E(g(X))^2 =$$

thus we have

$$\sum_{\text{All x}} [g(x)]^2 P(x) - [\sum_{\text{All x}} g(x)P(x)]^2$$

3. You find an urn which contains N white and M black balls. You randomly select balls one at a time until you get a black one. If we assume sampling with replacement, what is the probability it takes at least k draws?

Solution:

$$P(X \geq k) = \frac{M}{M+N} \sum_{n=k}^{\infty} (\frac{N}{M+N})^{n-1} = (\frac{N}{M+N})^{k-1}$$

4. Lets assume you find a second similar urn with 100 total balls of many different colors, where 20 of them are cyan. If you draw 30 balls from the urn what is the probability that you select half of the cyan balls?

Solution: From the given information we can see that we have a hypergeometric distribution, thus we can write the probability as

$$P(X = 10) = \frac{\binom{20}{10} * \binom{80}{20}}{\binom{100}{30}} = .0222$$

Note: Rather than calculating this by hand we can use the R command dhyper(10, 20, 80, 30)