GENERALIZED RCBDS AND FACTORIAL RCBDS

Chapter 10

LEARNING OBJECTIVES

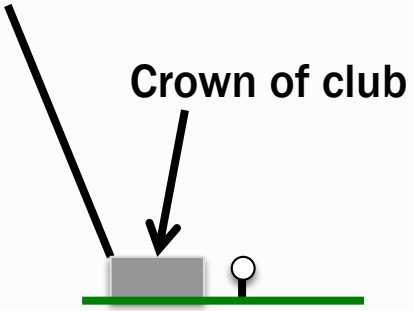
- Define generalized RCBD in terms of block size
- Describe randomization and perform in R
- Write statistical model and compare to RCBD model
- Perform treatment factorial analysis in RCBD
- Explain how some block/treatment interactions could be estimated in this scenario

GOLF TEE HEIGHT

- *Golf Magazine* (2006, June) experimented to determine the ideal tee height for driving a golf ball

- **Purpose:** recommend tee height for any reader

- Treatment levels:
  1. Entire ball below crown
  2. Half ball above crown
  3. Bottom of ball at top of club-face



- Nine golfers chosen that were representative sample of all readers

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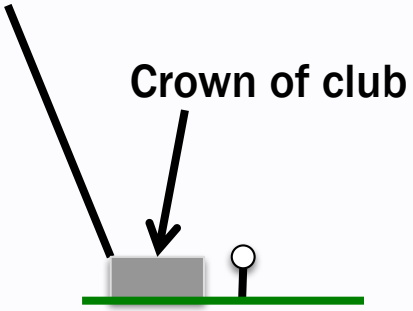

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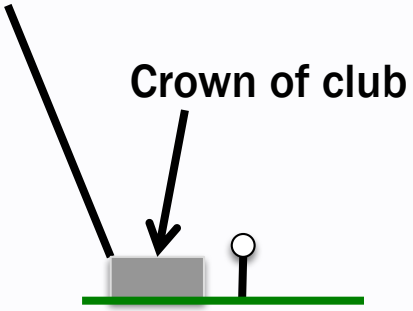

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GOLF TEE HEIGHT

- For this to be an RCBD, each golfer would use each tee height exactly one time
- Most of the experimental error variability comes from golfer-to-golfer variability
- Do not anticipate much variability between drives from same golfer
- Ask each golfer to replicate each treatment 5 times in a randomized order
- Each golfer experiment is a CRD with $r=5$, $t=3$

GENERALIZED RCBD

- This is an example of a generalized RCBD
  - Each block is an equally-replicated CRD with $r > 1$
  - Block size is a multiple of t
- Randomize like RCBD, we just have more replicates
- Only do this if within-block EU variability won't significantly increase by including more EUs
  - Increasing EUs was why we blocked in the first place!
- Gives estimate of pure error and we can separate out block/treatment interaction!

GENERALIZED RCBD STATISTICAL MODEL

- Since we replicate treatments in each block we now have df for error after fitting block/trt interactions

$$Y_{hij} = \mu + \beta_h + \tau_i + \beta\tau_{hi} + E_{hij}$$

- **Group:** write what the index ranges are for *h*, *i*, and *j*

- Pure error estimate based on comparing each block/treatment replicate response to average response

$$\sum_{h,i,j} (Y_{hij} - \bar{Y}_{hi.})^2$$

- Pure error has bt(r-1) degrees of freedom

GENERALIZED RCBD STATISTICAL MODEL

- The F-ratio test for block effects is still invalid
- The F-ratio test for block/treatment interaction though is valid!
  - F = msBT / msE with (b-1)(t-1) numerator df
- If we have a significant interaction, then consider consistency of treatment contrasts across blocks
  - Plots can help guide you on this

GENERALIZED RCBD
GENERAL TREATMENT EFFECTS

- If interaction effect insignificant, remove it from the model and ssBT and its df are pooled with error
- General conclusions about treatment effects in the presence of interaction uses msBT in *F* denominator
  - Strong overall treatment effects should **overcome the block-to-block effect variability**
  - Numerator df = t-1, Denominator df = $(b-1)(t-1)$
  - Mimics test from RCBD
- Only recommended when the blocks are representative sample of a large population
  - Counter-example: experiment with 2 blocks

FACTORIAL TREATMENTS IN RCBD

- Let's revisit the RCBD scenario but with a twist
- What if the treatments have a factorial structure?
  - Single blocking factor
  - Set of treatment factors *A*, *B*, *C*, …
- Called an RCBD factorial experiment
- Randomization is the same, the analysis approach changes

RCBD FACTORIAL ANALYSIS STANDARD MODEL

- Expand treatment effects just like we did before

$$Y_{hij} = \mu + \beta_h + \underbrace{\tau_{ij}}_{\tau_{ij} = \alpha_i + \gamma_j + \alpha\gamma_{ij}} + E_{hij}$$

$$h = 1, \ldots, b$$
$$i = 1, \ldots, a$$
$$j = 1, \ldots, c$$

$$\boxed{Y_{hij} = \mu + \beta_h + \alpha_i + \gamma_j + \alpha\gamma_{ij} + E_{hij}}$$

- This is a 3 factor model that includes:
  - All main effects
  - One two-factor interaction between the last two factors
- We already know how to fit this model!

RCBD FACTORIAL ANALYSIS BLOCK/TREATMENT INTERACTIONS

- For RCBD we cannot estimate the block/treatment interactions because we didn't have enough DF
- Only necessary if we want to consider any treatment contrast can change depending on the block
  - Necessary DF = (b-1)(ac-1)…remember ac=*t*, # treatments
- Block/treatment interactions here expand to three different classes

	$\beta\alpha_{hi}$	$\beta\gamma_{hj}$	$\beta\alpha\gamma_{hij}$
Test DF =	(b-1)(a-1)	(b-1)(c-1)	(b-1)(a-1)(c-1)

- Adding these DF gives total interaction DF (b-1)(ac-1)

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