INTRO TO MULTIPLE BLOCKING FACTORS

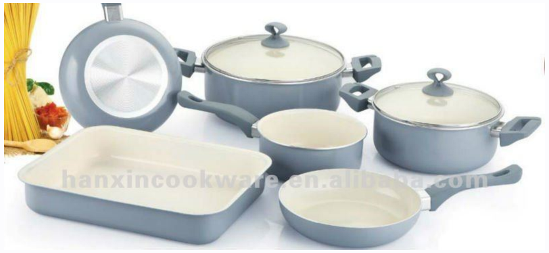
Chapter 12

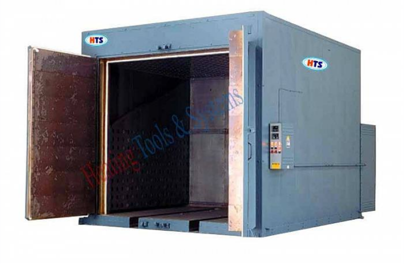
LEARNING OBJECTIVES

- Identify crossed/nested blocking factors
- Describe Latin square design and relate it to an RCBD
- Describe how to properly randomize a LSD
- Write and explain statistical model for a LSD and perform analysis in R

PRODUCTION OF CERAMIC COOKWARE

- Compare different production methods for ceramic cookware
- Anticipate two nuisance sources of variation:
  - Batch of raw material to make cookware
  - Different ovens used for baking cookware





- These make up two blocking factors
- We have c batches of material and r different ovens
  - Every batch type is used with every oven type
- Idea: Think of this as block design with $c*r$ blocks

CROSSED BLOCKING FACTORS

- Have enough raw material in a batch/oven combo to do **each production method one time**

		Batch			
		1	2	…	c
Oven	1				
	2				
	⋮				
	r				

Each cell represents a block

- RCBD with ***b*=*cr* blocks** and t treatments
- **Group:** What are the EUs? How many EUs would this experiment have?

CROSSED BLOCKING FACTORS – STATISTICAL MODEL

- Index blocking factors using *i*=1,…,r and *j*=1,…,c
- Usual block model would be:

$$Y_{ijk} = \mu + \underbrace{\beta_{ij}}_{\beta_{ij} = \theta_i + \gamma_j + \theta\gamma_{ij}} + \tau_k + E_{ijk}$$

$$\boxed{Y_{ijk} = \mu + \theta_i + \gamma_j + \theta\gamma_{ij} + \tau_k + E_{ijk}}$$

- Some block/treatment interaction effects could also be considered

NESTED BLOCKING FACTORS – STATISTICAL MODEL

- Same label "Batch 1" across ovens but Batch 1 in Oven 1 doesn't equal Batch 1 in Oven 2
- Batch "main effect" no longer makes sense, comparisons of Batch must be done Oven-by-oven

- Call batch a nested blocking factor (within oven)
- Call oven a nesting blocking factor

- Let i=1,...,r be nesting factor; j=1,...,c be nested factor

$$\beta_{ij} = \theta_i + \gamma_{j(i)}$$

- Won't worry about these examples in this class

ONE EU PER COMBINATION

- Designs shown earlier require too many EUs
- What if we have one EU per combination?
  - Another example of an unreplicated factorial experiment!
- Assume that $r=c=t$
- If row or column factor dropped, we'd want a RCBD
- Goal: Find design that is RCBD with respect to both rows and columns

t=3

1	2	3
2	3	1
3	1	2

t=4

1	2	3	4
2	3	4	1
3	4	1	2
4	1	2	3

How can we randomize these to maintain this structure?

How do we construct these designs?

LATIN SQUARE DESIGNS
CYCLIC CONSTRUCTION

- Fortunately, these types of designs have been studied for a long time
- Such designs are called Latin square designs (LSD)
  - A t x t array of t symbols such that each symbol appears exactly once in each row and column
- Construct an LSD for any t using cyclic pattern

1	2	3	4
2	3	4	1
3	4	1	2
4	1	2	3

- This is a design plan, we still need to randomize

RANDOMIZING LSD'S

- Properly randomized when all possible LSD's are equally likely to be chosen
- Generally difficult to guarantee this property, requires concept of transformation sets
- In practice, usually start with the cyclic design plan

- Fact 1: If we permute the rows we will maintain this structure (same with columns)

- Fact 2: If we permute treatment labels we will also maintain this structure

RANDOMIZING LSD'S

- Step 1: Randomly permute labels of *t* treatments
- Step 2: Randomly permute rows
- Step 3: Randomly permute columns

1	2	3	4
2	3	4	1
3	4	1	2
4	1	2	3

<u>Label Permutation</u>

1 → 2
2 → 3
3 → 1
4 → 4

RANDOMIZING LSD'S

- **Step 1:** Randomly permute labels of *t* treatments
- **Step 2:** Randomly permute rows
- **Step 3:** Randomly permute columns

2	3	1	4
3	1	4	2
1	4	2	3
4	2	3	1

<u>Row Permutation</u>

1 → 1
2 → 4
3 → 2
4 → 3

RANDOMIZING LSD'S

- **Step 1:** Randomly permute labels of t treatments
- **Step 2:** Randomly permute rows
- **Step 3:** Randomly permute columns

2	3	1	4
1	4	2	3
4	2	3	1
3	1	4	2

<u>Column Permutation</u>

1 → 4
2 → 2
3 → 3
4 → 1

RANDOMIZING LSD'S

- **Step 1:** Randomly permute labels of *t* treatments
- **Step 2:** Randomly permute rows
- **Step 3:** Randomly permute columns

4	3	1	2
3	4	2	1
1	2	3	4
2	1	4	3

All done!

LATIN SQUARE DESIGNS – STATISTICAL MODEL

- Just like with RCBD, we fit main effect model
- Gives us the following additive model:

$$Y_{ijk} = \mu + \theta_i + \gamma_j + \tau_k + E_{ijk}$$

$$\left.\begin{array}{r} i = 1, \ldots, t \\ j = 1, \ldots, t \\ k = 1, \ldots, t \end{array}\right\} \text{Wait, there's only } t^2 \text{ EUs}$$

- For a given $(i\ ,\ j)$ we only have a single k

- How can we write the model that reflects this dependency?

LATIN SQUARE DESIGNS – STATISTICAL MODEL

- Let $d[i, j]$ be a function of *i* and *j* that produces the treatment label assigned to that cell

4	3	1	2
3	4	2	1
1	2	3	4
2	1	4	3

d[1,1]=4
d[1,4]=2
d[3,2]=2
d[4,2]=
d[4,4]=

- Change model to only use indices *i* and *j*

$$Y_{ij} = \mu + \theta_i + \gamma_j + \tau_{d[i,j]} + E_{ij}$$

- Fortunately, R doesn't care about this

LATIN SQUARES – ADVANTAGES AND DISADVANTAGES

- Advantages
  - Accounts for two sources of heterogeneity simultaneously (just like any multiple block factor design)
  - Minimal resources compared to usual crossed block design since only 1 EU per cell
  - Straightforward analysis

- Disadvantages
  - Requires number of row and column blocks to equal *t*
  - If *t* is small there aren't many degrees-of-freedom left for error
  - For small *t* we'd rather have more EU's

LEARNING OBJECTIVES REVIEW

- Identify crossed/nested blocking factors
- Describe Latin square design and relate it to an RCBD
- Describe how to properly randomize a LSD
- Write and explain statistical model for a LSD and perform analysis in R