INTRO TO FACTORIAL EXPERIMENTS

Chapter 6

LEARNING OBJECTIVES

- Describe one factor at a time experiment and why it is a poor design for factorial experiments
- Explain what a simple effect is
- Explain what it means for two factors to interact and for a factor to have a main effect
- Describe and draw design space for given scenario
- Generate and randomize designs

FACTORIAL TREATMENT STRUCTURES

- Focus has been one treatment factor with *t* levels representing different treatment applications
- What makes these treatment applications different?
- Maybe *t* levels arise from a combination of settings across multiple treatment factors
  - Identify the number of factors
  - Identify the number of levels per factor
  - Identify relationship between levels of factor (crossed/nested)
- Say that the treatments have a factorial structure

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- Experiment looking at different paper helicopter models and their flight time
- Each helicopter is manipulated by both
  - Body width (3.25, 3.75, 4.00, 4.25 inches)
  - Wing length (4.00, 4.75, 5.50, 6.00 inches)
- Q: How many treatment factors are there? How many levels per factor? How many total treatment factors?
- Design an experiment that allows us to understand the individual factor effects
- Intuition: investigate one factor at a time
  - Fix the level of the other factor while we explore one of them

ONE FACTOR AT A TIME (OFAT) EXPERIMENTATION

- Wing Length (WL) = 4 when varying Body Width (BW)
- Body Width = 3.25 when varying WL

- Intuitively sounds good, we are simplifying the experiment!

- Are we overlooking something?

- Assumes the effects when changing Body Width are consistent across levels of Wing Length

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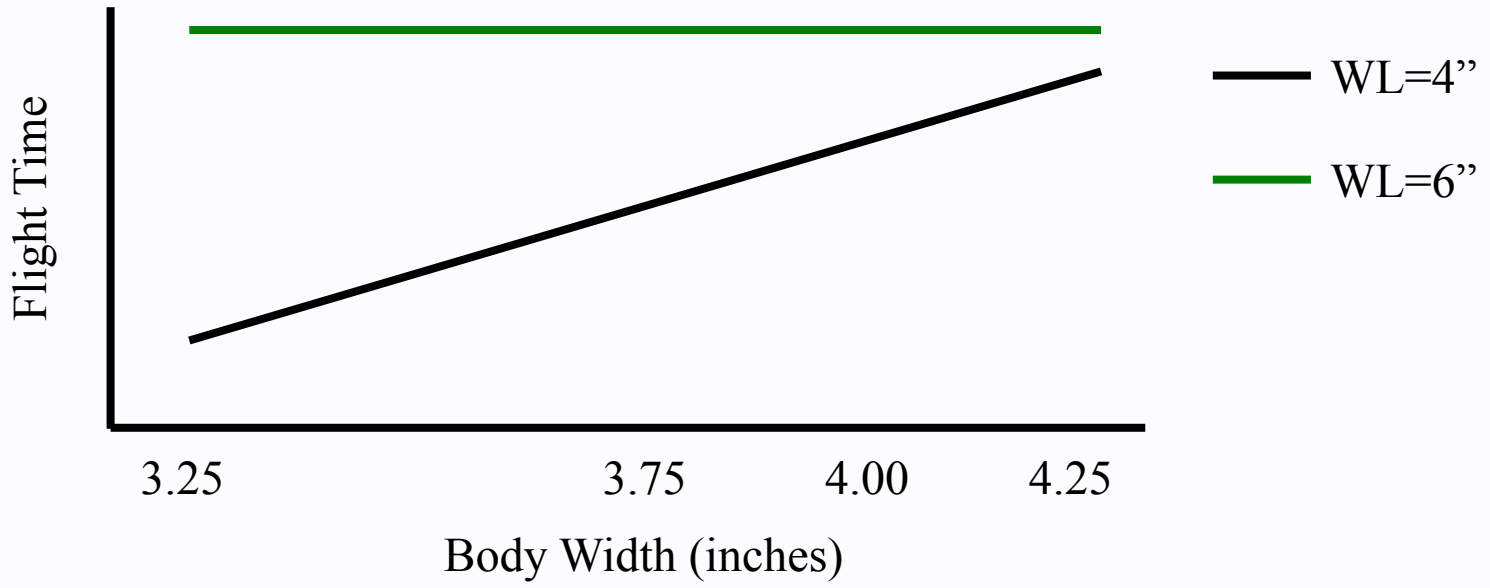
- Potential counter-example:
  - WL = 4”, we see increasing flight time as BW increases
  - WL = 6”, there are no differences between BW
- When a situation like this occurs, we say that the factor effects interact
- Studying factors individually doesn’t make sense
- Want to understand whether effects interact

PAPER HELICOPTER INTERACTION VISUALIZATION

- Visualize the situation just described



- Interpretation: the effect of Body Width on Flight Time depends on the level of Wing Length

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SIMPLE EFFECTS

- Pick BW contrast, temporarily ignoring WL
- To demonstrate, focus on BW(4.25)-BW(3.25)

- Question: when we estimate this contrast, what do we do about the other factor?

- A BW contrast for a fixed level of WL is called a simple effect or simple contrast

- Simple effects are foundation of factorial analyses

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CONTRASTS OF SIMPLE EFFECTS

- Interested in consistency of simple effects as we change levels of other factors

- Step 1: Declare a contrast of interest for BW
  BW(4.25)-BW(3.25)

- Step 2: Estimate BW contrast across every WL level
  - 4 WL levels □ 4 BW(4.25)-BW(3.25) simple effects

- Step 3: Investigate contrasts of simple effects
  - Contrasts of contrasts
  - If simple effects are similar then these contrasts = 0
  - Otherwise the contrasts will be different from 0

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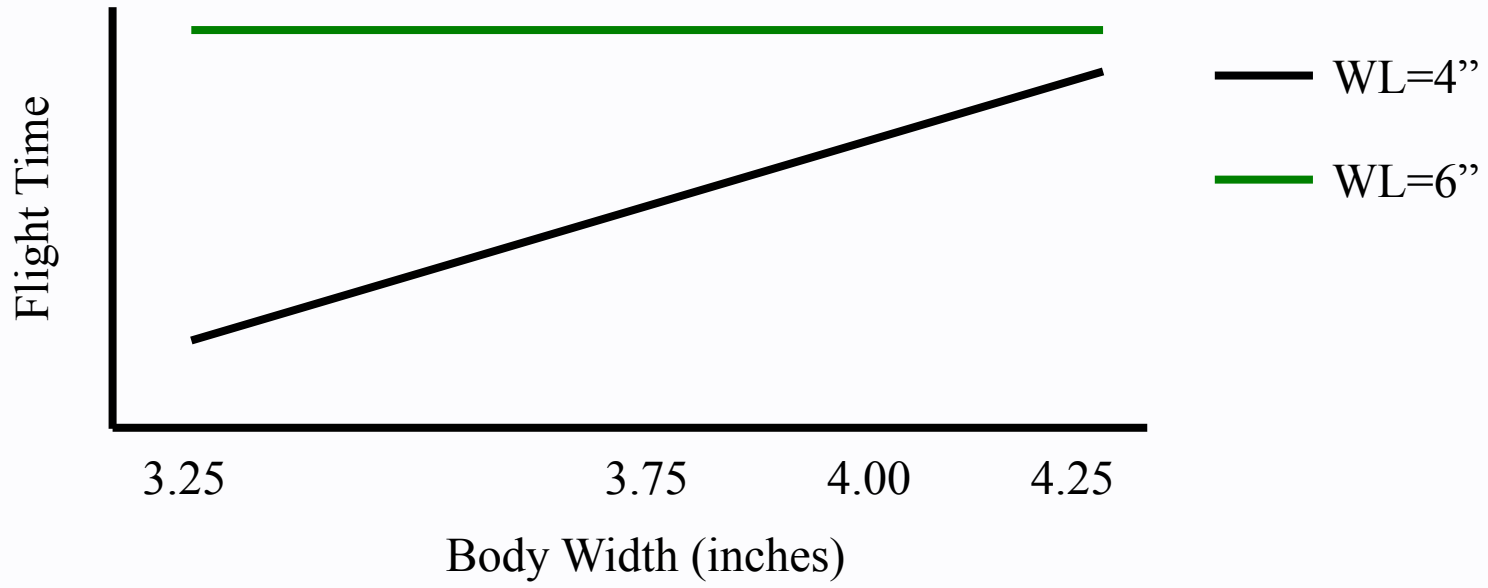
CONTRASTS OF SIMPLE EFFECTS

- Let's just look at 2 levels of WL



- WL=4": BW(4.25) – BW(3.25) = Large, positive #
- WL=6": BW(4.25) - BW(3.25) = 0

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MAIN EFFECTS

- If the simple effects are consistent across the levels of WL, then their average is a useful summary
- A BW contrast's main effect is the average of the simple effects
- Main effects can always be interpreted as the average of simple effects
- Main effects only meaningful in gauging a factor's effect when simple effects are consistent

PAPER HELICOPTER INTERACTION EFFECTS

- If simple effect estimates are inconsistent then that specific BW contrast interacts with levels of WL

- Interaction contrast is a contrast of simple effects

- Goal is to describe the nature of the interaction, which depends on the type of contrast you look at
  - Ex 1: Different for all WL levels
  - Ex 2: Same for levels 1,2 and levels 3,4 but different between these two groups
  - Ex 3: Same for levels 1, 2, and 3, but all different from 4

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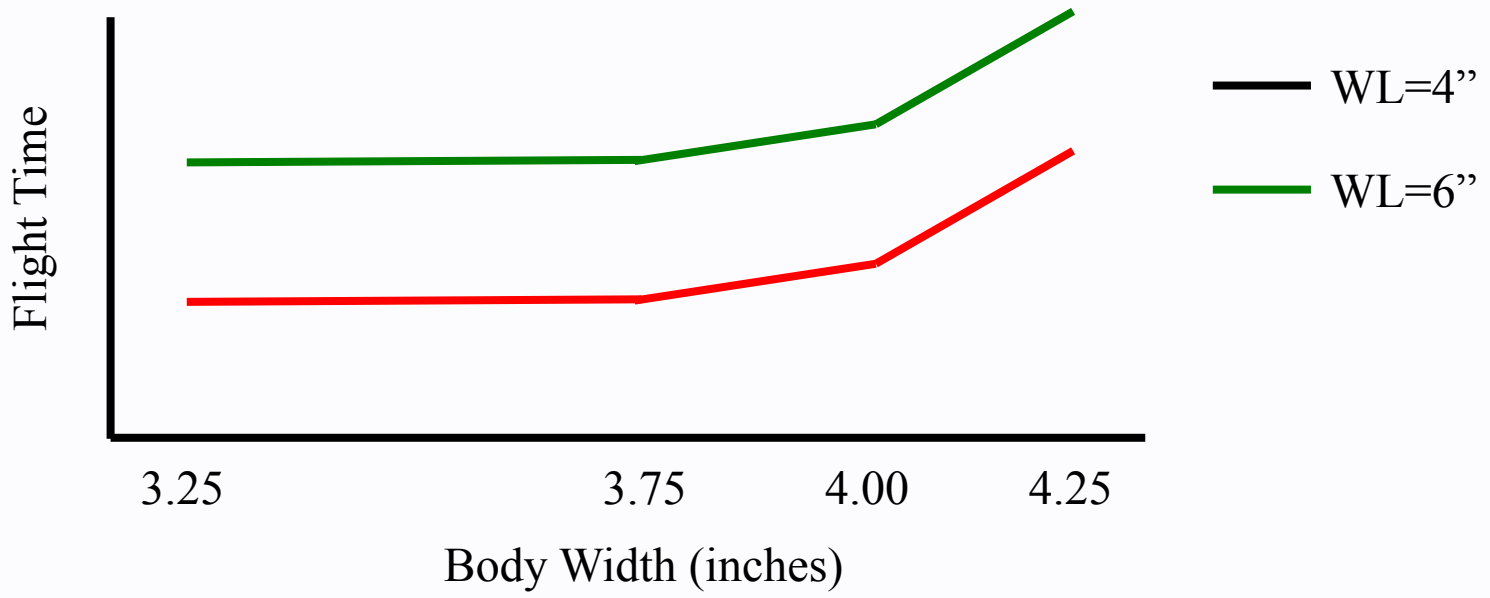
MAIN AND INTERACTION EFFECTS #1

- Visualize the situation just described



- Every BW contrast would give same value no matter the wing length. All BW main effects meaningful!

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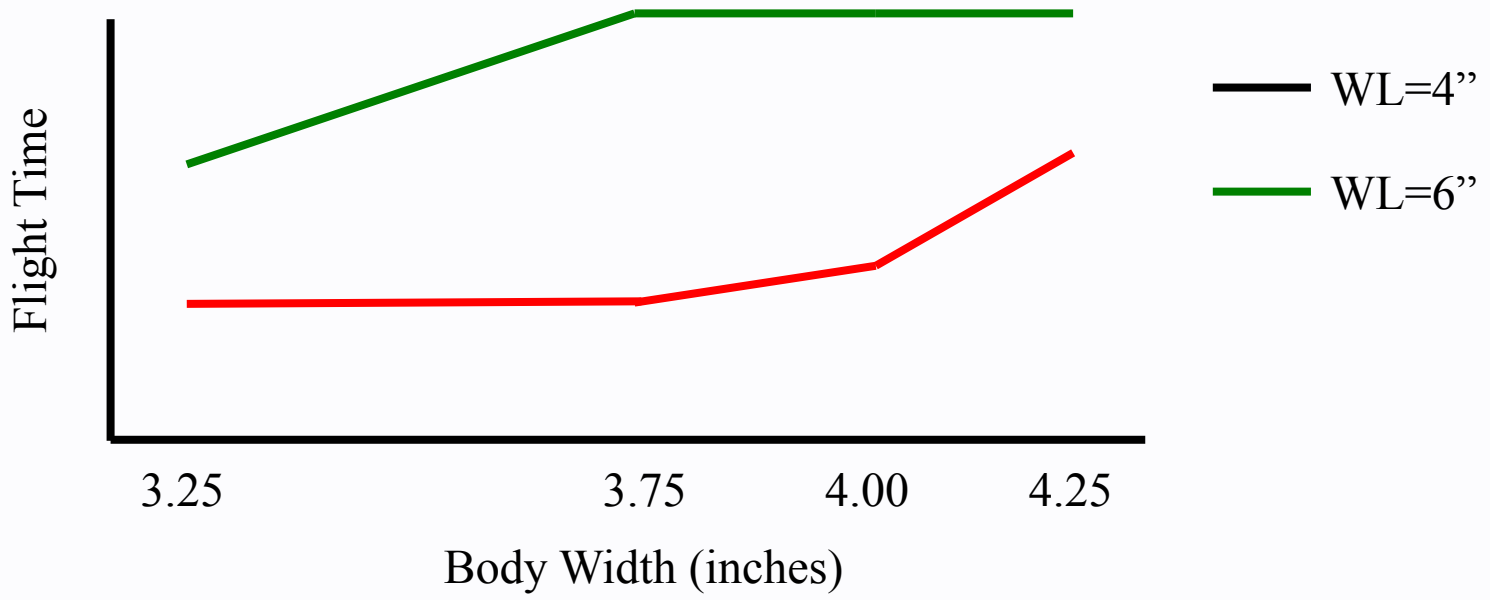
MAIN AND INTERACTION EFFECTS #2

- Visualize the situation just described



- Definitely an interaction HOWEVER the one contrast of BW(4.25) – BW(3.25) is consistent!

REVISITING OFAT DESIGN SPACES

- **Critique 1:** OFAT captures no data that would help us determine the presence of interaction
- Assumes every contrast is a main effect

- Clearly seen by looking at what parts of the **design space** are actually performed

- **Design space:** set of all possible treatment combination settings

- Paper helicopter experiment has $4x4 = 4^2 = 16$ treatment combinations (or design points)

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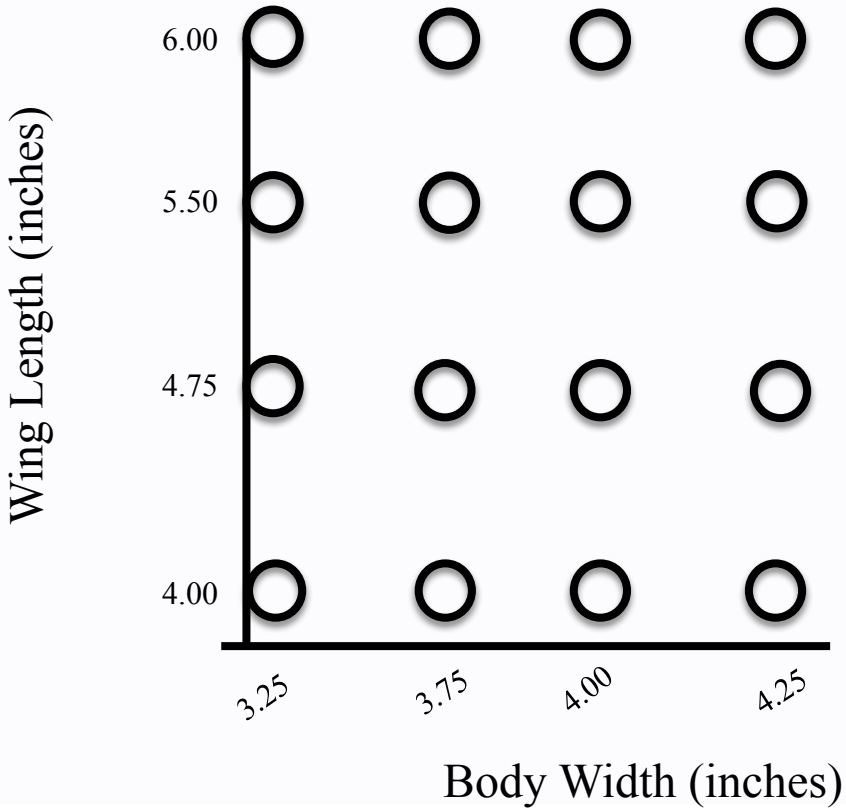
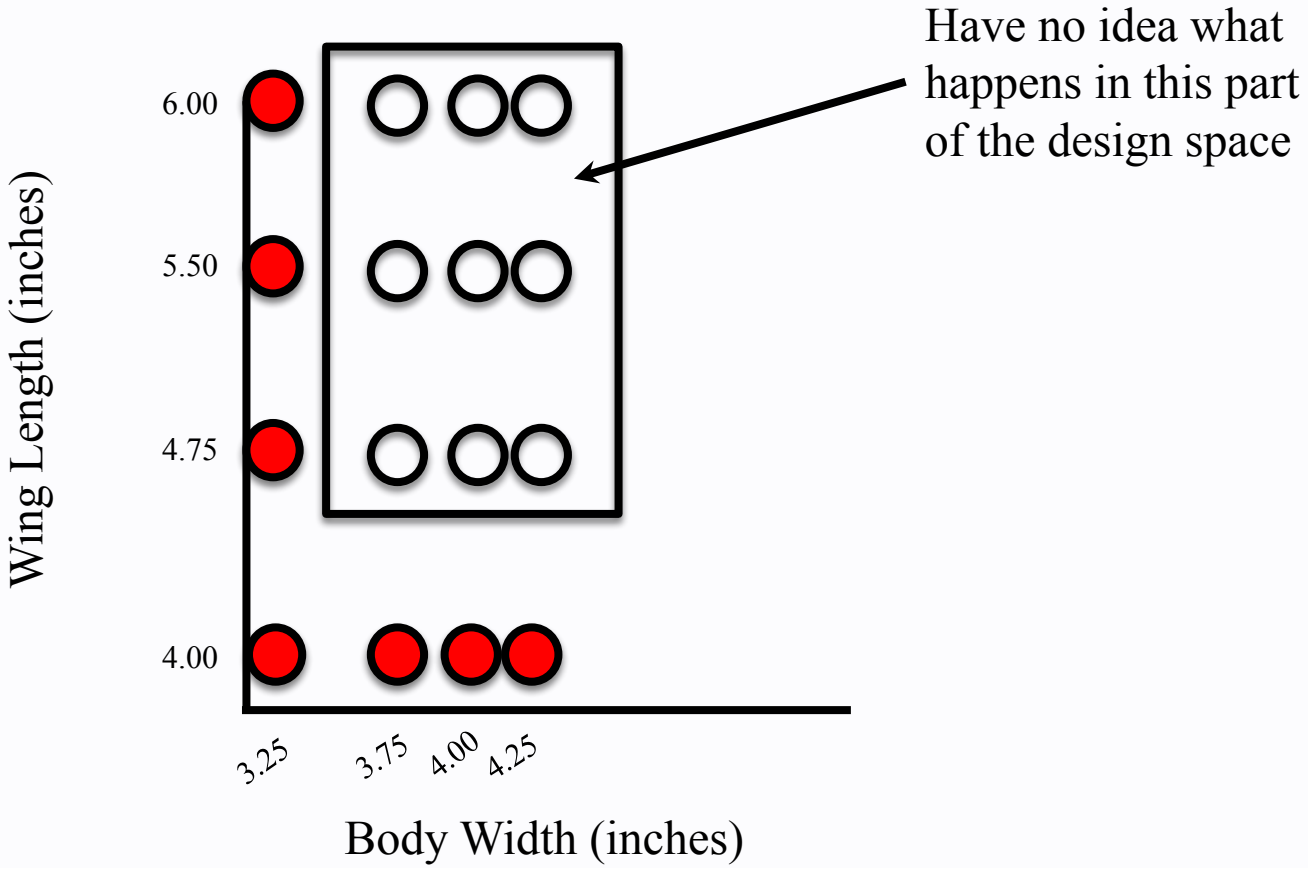
VISUALIZING DESIGN SPACE

- Empty circles are coordinates that represent potential treatment settings



PAPER HELICOPTER OFAT EXPERIMENT

- An OFAT experiment (in red) does a terrible job of exploring the design space



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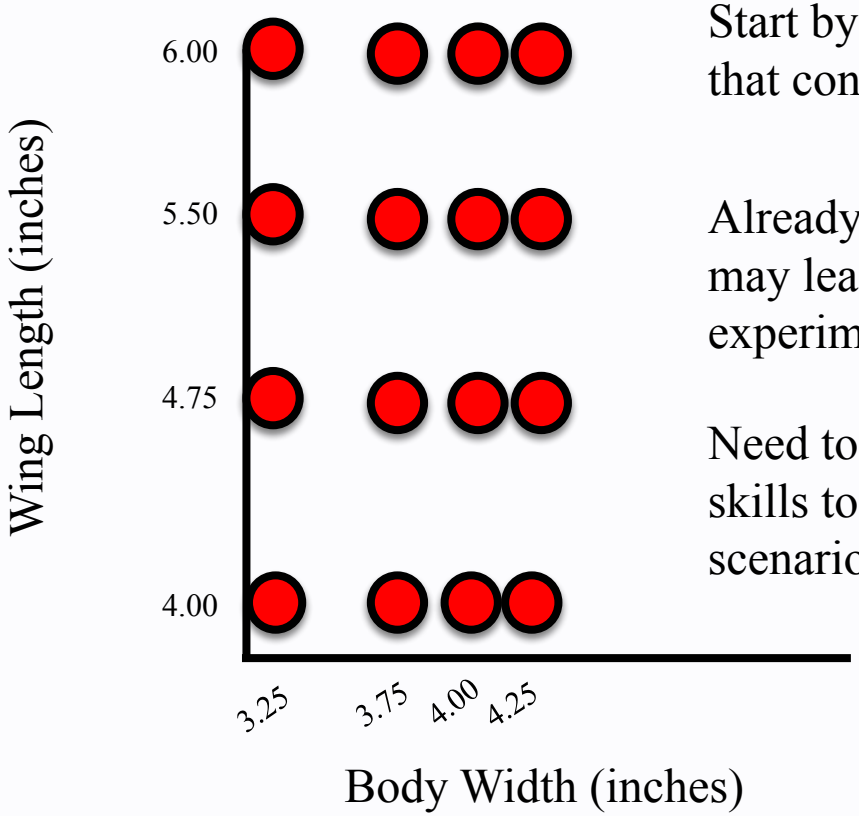
FULL FACTORIAL EXPERIMENT

- **Factorial experiments try to fill as much as the design space as they can**



Start by exploring experiments that consider all possible settings

Already we can see that this may lead to large, expensive experiments

Need to build intuition and analysis skills to deal with more complicated scenarios

REPLICATED FULL FACTORIAL DESIGN PLAN

- Focus on two-factor experiments for now
- Denote factors as *A* and *B*
  - Levels of *A* are 1, 2, …, *a* (similar for *B*)
- Each treatment combination replicated at least twice

- Denote replications by r_{ij}, arrange in replication table

		B			
		1	2	…	*b*
A	1	r_{11}	r_{12}	…	r_{1b}
	2	r_{21}	r_{22}	…	r_{2b}
	…	…	…	…	…
	a	r_{a1}	r_{a2}	…	r_{ab}

RANDOMIZATION AND MODEL

- Factorial designs are randomized just like CRDs
- Assume you have $t=ab$ treatments
- Difference comes from how you analyze the design
- Later look at an analysis using cell-means and factorial effects model
- Model similar to ANCOVA with a categorical covariate except you can't control levels of covariate

LEARNING OBJECTIVES REVIEW

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