RANDOMIZED COMPLETE BLOCK DESIGNS (RCBDS)

Chapter 10

LEARNING OBJECTIVES

- Identify blocking factor and explain how it is different from a treatment factor
- Define a complete block design; explain how it should be randomized and perform it in R
- Write statistical model for RCBD and explain where estimate of error comes from
- Compare model to main effect factorial model and ANCOVA model and explain their differences
- Define and calculate relative efficiency for determining impact of blocking

REPLICATION AND EXPERIMENTAL ERROR

- Increasing replication only increases precision of treatment contrasts if experimental error stays same
- As we increase the # of EUs in our design, more likely to observe greater experimental error…
- …but the study becomes more representative
- "Reduce" experimental error by identifying EU sources of variation and accounting for them in the (1) randomization procedure and (2) analysis

FLOWER LIFE EXAMPLE

- Investigate different watering techniques for extending the life of cut flowers put in a vase
- **Treatment factor:** type of liquid put in vase
  1. Tap water
  2. Tap + spoonful sugar
  3. Tap + cup of carbonated water
  4. Tap + cup of 7-up
- **EUs:** single flowers of comparable age
- **Response:** Time in days until flower wilted

- **Group:** What are some sources of variation for EUs?

FLOWER LIFE EXAMPLE

- Focus on type of flower as an important source of variation
  1. Rose
  2. Carnation
  3. Daisy
  4. Tulip

- Groups: critique the following experiments
  1. Use 16 roses and completely randomize 4 treatments (4 reps)
  2. Use 4 of each type of flower and completely randomize 4 treatments (4 reps)

BLOCK WHAT YOU CAN; RANDOMIZE WHAT YOU CAN'T

- Complete randomization of treatments to EU's reduces impact of bias caused by EU-to-EU variability
  - Example: Unlikely we will confound treatments and flowers
- Idea: Group EUs into homogeneous sets and perform separate CRD in each group (call a block)

- A design has been blocked when:
  - EU's divided into *b* blocks of some size (≥ 2)
  - Different treatments assigned to EUs in each block
  - Separate randomization performed in each block

- ANCOVA with categorical covariates does not randomize this way

BLOCKING FACTORS AND LEVELS

- First identify EUs and their experimental conditions
- Blocking factor: source of variation that potentially explains large amount of EU variation
  - Levels of this factor = EU groups/blocks
- Choose levels so that EUs in each blocks are homogenous but different from EUs in other blocks

- Different from treatment factor because we do not randomly assign these levels to the EUs
  - Usually an inherent characteristic of the EU or conditions the EU is put under that we do not control
  - Example: Flower type is a blocking factor

ADVANTAGES OF BLOCKING

- Reduce experimental error by comparing treatments WITHIN each block, then "pool" the effects
- Able to use heterogenous EUs
- Increase replication without increasing experimental error
- Increase representativeness of treatment effects

DISADVANTAGES OF BLOCKING

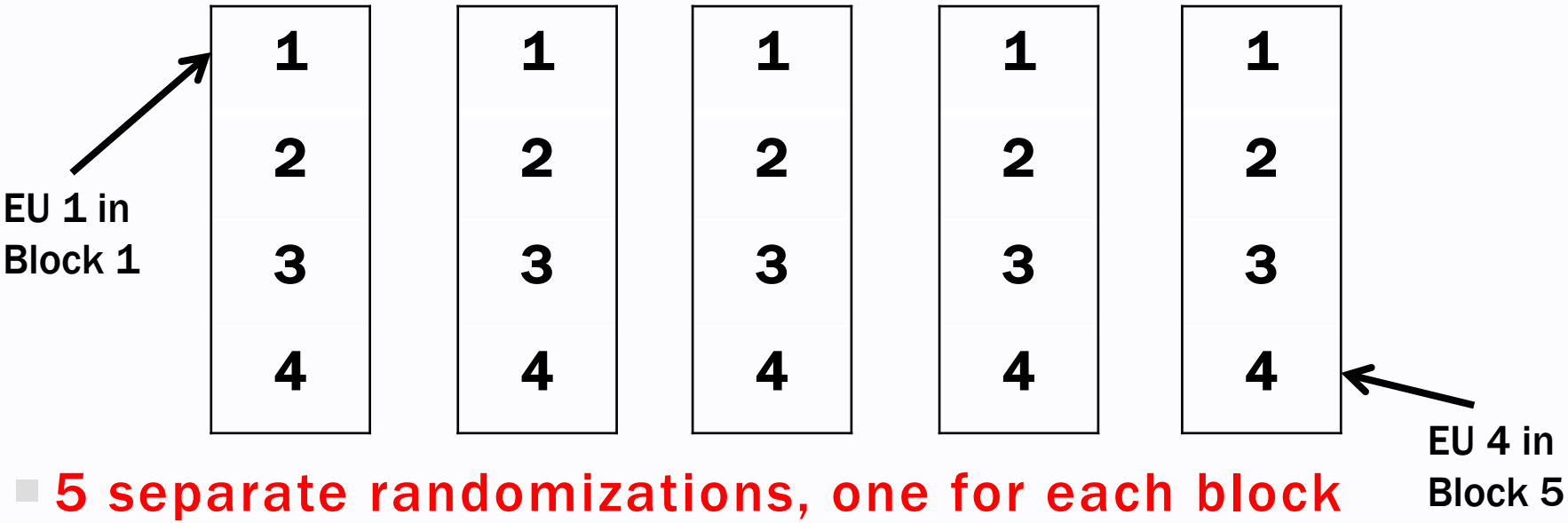
- Increases “complexity” of analysis (but no more than factorial experiments)
- Complicated block designs more difficult to conduct, often done incorrectly
  - Critical you can explain how to randomize experiment
- Reduces df for error more than if we had done a CRD without blocking
- Hope the reduction in df is accompanied by a significant reduction in ssE

TERMINOLOGY OF BLOCK DESIGNS

- **Block size** = number of EU's in a block
  - Dictates how many different treatments we can assign
- **Proper block design:** each block has the same size
- **Complete block design with *t* treatments** has
  - *b* blocks each of size *t*
  - Each treatment appears in each block **once**
- Only call a group of EUs a block when **we do separate randomization within each group**

RANDOMIZATION OF COMPLETE BLOCK DESIGN

- Visualize a complete block design with t=4 treatments and b=5 blocks
- Design plan shows which treatments go to which block (pre-randomization)



- 5 separate randomizations, one for each block

RANDOMIZATION OF COMPLETE BLOCK DESIGN

- Visualize a complete block design with t=4 treatments and b=5 blocks
- Randomized plan might look something like this
- Still have each treatment once in each block

1	2	1	4	3
4	4	3	1	1
3	3	4	2	4
2	1	2	3	2

- Like 5 CRDs each with one replicate

COMPLETE BLOCK DESIGN STATISTICAL MODEL

- RCBD model is a main effect factorial model

$$Y_{hi} = \mu + \beta_h + \tau_i + E_{hi} \qquad \begin{matrix} h = 1, \ldots, b \\ i = 1, \ldots, t \end{matrix}$$

- No need for an index of replicates

- Exactly like a main effect factorial model!
  - Perform analysis just like before*
  - Interaction effects are the only way to estimate error!

CONTROVERSY OF TESTING FOR BLOCK EFFECTS

- **Randomization-based models** explicitly use the randomization procedure to generate model
- **Result:** Model looks exactly like the previous slide

- **Problem:** The usual F-ratio approach we would use to "test for block differences" is an invalid test

- Ignore the p-values for testing for block, assess impact using **relative efficiency**
- NEVER DROP BLOCK EFFECTS FROM MODEL
  - Those *b*-1 degrees of freedom can never be put back into error

TESTING FOR TREATMENT EFFECTS

- Remember, blocking is primarily considered to be a variance reduction tool
- We care more about testing for treatment effects

- **Result:** the usual F-ratio approach for testing treatment effects is still valid!

- Summary of analysis of RCBD
  - Always includes block effects
  - Focus analysis on treatment effects
  - Post-hoc analysis only done for treatment effects

RELATIVE EFFICIENCY OF RCBD

- Informally assess how effective blocking was by comparing MSB and MSE (look at F-value)
  - If MSB >> MSE then blocking was worthwhile
  - If MSB < MSE we reduced power…why?
- Block impact assessed by the anticipated reduction in variance if we had used a CRD
- Motivated by the idea of a uniformity trial which is a design with pseudo-treatments

UNIFORMITY TRIAL

- **Uniformity trial:** RCBD with dummy treatments would have an ANOVA table like this

Source	df	SS	MS
Block	b-1	ssB(U)	msB(U)
Within Blocks (Error)	b(t-1)	ssE(U)	msE(U)
Total	bt-1	ssTot(U)	

$$\hat{\sigma}^2_{RBCD} = msE(U)$$

$$\hat{\sigma}^2_{CRD} = \frac{ssTot(U)}{bt-1}$$

Relative efficiency compares variances: $$RE = \frac{\hat{\sigma}^2_{CRD}}{\hat{\sigma}^2_{RCBD}}$$

ESTIMATED RELATIVE EFFICIENCY

- Rarely perform a uniformity trial, we actually apply real treatments
- Use $\hat{\sigma}^2_{RCBD} = \dfrac{ssE}{(b-1)(t-1)} = msE$

- Need estimates for ssTot(U) from our data

$$\widehat{ssTot(U)} = (b-1)msB + b(t-1)msE$$

$$\hat{\sigma}^2_{CRD} = \frac{(b-1)msB + b(t-1)msE}{bt-1}$$

$$ERE = \frac{(b-1)msB + b(t-1)msE}{(bt-1)msE}$$

LEARNING OBJECTIVES REVIEW

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- Write statistical model for RCBD and explain where estimate of error comes from
- Compare the model to main effect factorial model and explain the implicit difference between them
- Define and calculate relative efficiency for determining impact of blocking