COMPLETELY RANDOMIZED DESIGNS

Chapter 3

LEARNING OBJECTIVES

- Explain what complete randomization means
- Perform randomization in R (lab 7)
- State assumptions that make this design appropriate
- Define experimental and observational error
- Give hypothetical examples for sources of variation contributing to the two types of error

COMPLETELY RANDOMIZED DESIGNS (CRD)

- Every experimental design starts with a pre-randomized design plan

- CRD plan is most basic:
  1. List out all possible treatment levels (could come from one or more treatment factors)
  2. Decide number of replicates per treatment level

- Given CRD plan, assignment of treatments to EUs is done completely at random
  - Randomization is valid if it preserves the desired replication

BREAD RISING EXAMPLE RANDOMIZATION

- Treatment factor: time to rise (3 levels)
  - Assign levels 1, 2, and 3 to r_1, r_2, and r_3 EUs ($r_1+r_2+r_3=N$)

- Example: N=12 loaf pans with $r_1 = r_2 = r_3 = 4$

EU	1	2	3	4	5	6	7	8	9	10	11	12
Trt	1	1	1	1	2	2	2	2	3	3	3	3
Trt	1	2	3	3	3	2	2	1	1	3	2	1
Trt	2	2	2	3	1	2	3	1	1	3	1	3

- Randomization procedure is valid is every permutation of treatment labels is equally likely
- NOT THE SAME AS SAYING EVERY EU HAS SAME CHANCE TO BE ASSIGNED A GIVEN TREATMENT

INCORRECT RANDOMIZATION SCHEMES

- Examples of the wrong way to randomize:
  1. For each EU, randomly generate number from {1,2,3}
  2. Start with EU 1 and randomly generate number from {1,2,3}, continue to other EUs until desired replication is met

- Possible designs under (1) and (2)

Randomize Scheme	EU	1	2	3	4	5	6	7	8	9	10	11	12
1	Trt	1	2	2	1	1	3	3	3	2	2	1	1
1	Trt	1	1	1	1	1	1	1	1	1	1	1	1
2	Trt	1	2	3	3	2	1	1	2	3	3	2	1
2	Trt	1	1	1	1	2	2	2	2	3	3	3	3

INCORRECT RANDOMIZATION SCHEMES

- Scheme 2 is invalid because the two assignments have different probabilities of occurring

Randomize Scheme	EU	1	2	3	4	5	6	7	8	9	10	11	12
2	Trt	1	2	3	3	2	1	1	2	3	3	2	1

Each have prob 1/3 Prob 1/2 Prob 1

$$\frac{1^{10}}{3} \times \frac{1}{2} \times 1 = \frac{1}{118098}$$

Randomize Scheme	EU	1	2	3	4	5	6	7	8	9	10	11	12
2	Trt	1	1	1	1	2	2	2	2	3	3	3	3

CORRECT RANDOMIZATION SCHEME

- Many ways to generate correct randomization
- **Brute force:** enumerate all possibilities and then randomly select one (impractical)
- Computer-generated randomization:
  1. Create data set with EUs listed from 1 to *N*
  2. Generate random uniform number for each EU
  3. Sort EUs by random number
  4. Assign first r_1 to treatment 1, next r_2 to treatment 2, etc.
- **Takeaway:** random ordering of EUs is sufficient for valid randomization

COMMENTS ABOUT CRDS

- Treatments do not have to be equally replicated for a design to be called a CRD

- Called a CRD because of randomization procedure

- Recommended when EUs are fairly homogenous

- Nuisance factors are either
  1. Held constant
  2. Measured and adjusted for with analysis of covariance

EXPERIMENTAL ERROR

- Experimental error: variation between responses subjected to the same treatment

- Sources of experimental error:
  - EU variability (present independently of treatment)
  - Treatment application error (inability to apply treatment exactly same)
  - EU state variability (changes in EU that occur during/after treatment separate from the treatment effect)
    - Environmental factors realized through state variability

- Does not have OU variability = observational error

OBSERVATIONAL ERROR

- Observational error: response variation between OUs within same EU
- Sources of observational error:
  - OU variability (present independently of treatment)
  - Measurement error (inability to accurately measure a constant)
  - Inconsistent within-EU treatment application (inability to apply a treatment uniformly across EU)
- Estimate experimental and observational variance separately only with multiple measurements per EU
- Won’t consider these models right now, just average the multiple responses from same EU

VISUALIZING OU VS EU VARIABILITY

- $t = 3, r = 3, m = 2$, m is # of measurements per EU
- Y_{ijk} = ***k*-th measurement of *j*-th replicate of treatment *i***

i	1						2						3					
j	1		2		3		1		2		3		1		2		3	
k	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2

OU variability between measurements from same EU
(variability for fixed values *i* and *j*)

VISUALIZING OU VS EU VARIABILITY

- $t = 3, r = 3, m = 2,$ m **is # of measurements per EU**
- Y_{ijk} **=** ***k*-th measurement of *j*-th replicate of treatment *i***

i	1						2						3					
j	1		2		3		1		2		3		1		2		3	
k	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2

EU variability between average measurements across EU with same treatment (variability for fixed value *i)*

FERTILIZER AND TOMATO PLANTS

- Box et al (1978) describe experiment to determine whether a change in fertilizer mixture would change yield of tomato plants (# tomatoes from plant)

- 11 tomato plants were planted in a single row, fertilizer A or B was randomly assigned to each plant

Container	1	2	3	4	5	6	7	8	9	10	11
Fertilizer	A	A	B	B	A	B	B	B	A	A	B

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