STAT 456 - Intro Activity

Bayesian Thought Experiment

There are two foundational elements in a Bayesian Analysis:

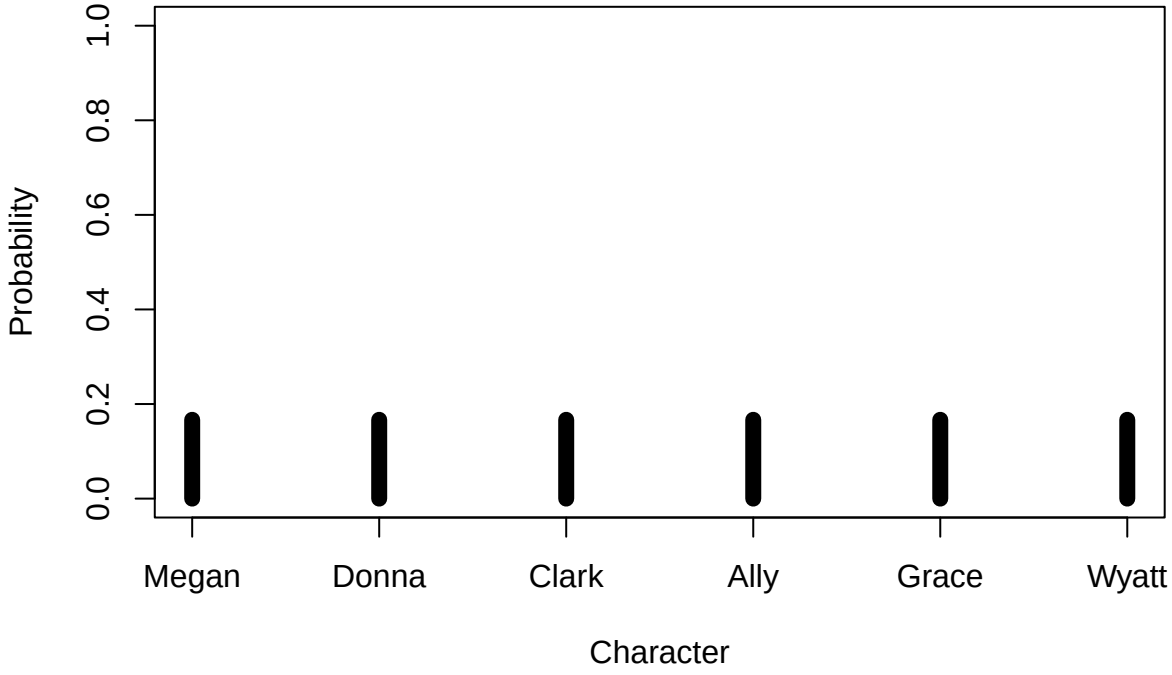
1. Bayesian inference is a re-allocation of credibility across possibilities
2. These possibilities are parameter values in meaningful mathematical models

Guess Who Exercise

Consider the game Guess Who, where the goal is to ask questions to identify an opposing player's character.

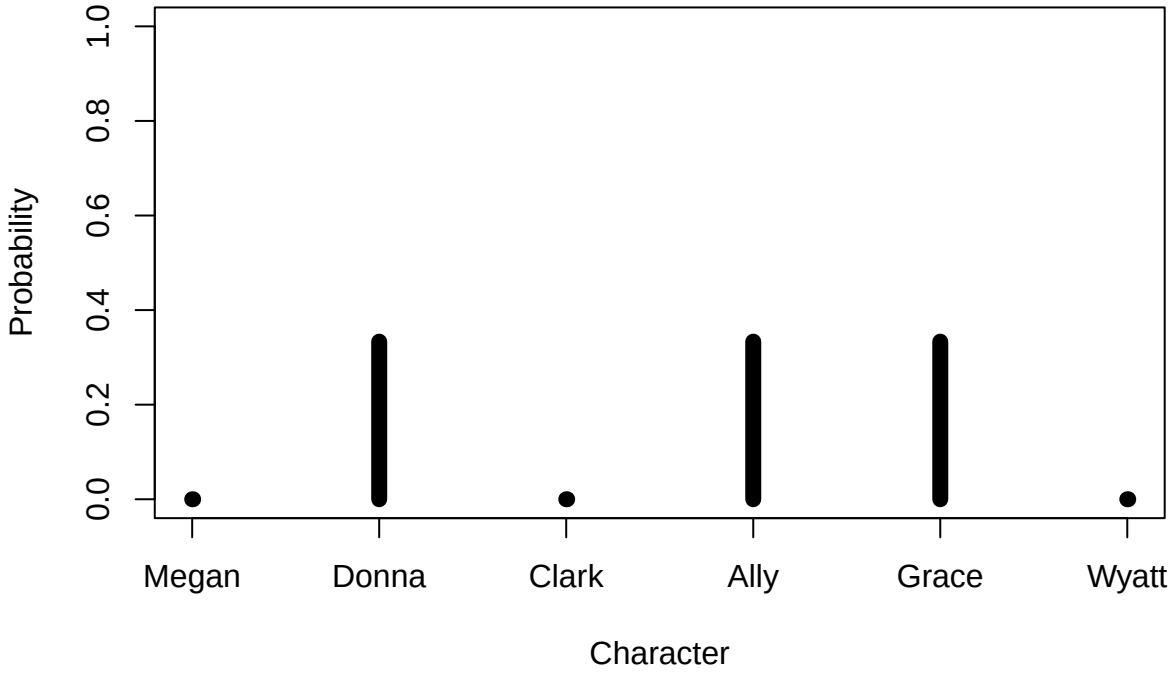
Figure 1: Guess Who Faces

- Given the line up of suspects above, draw you belief (as a probability) for each character. Note these should sum to one and constitute the first foundation element in Bayesian Analysis.
- The first set of probabilities are known as *prior* probabilities.



Follow the conversation and update your probabilities accordingly

- **You**: Does your character have a hat
- **Your adversary**: Yes

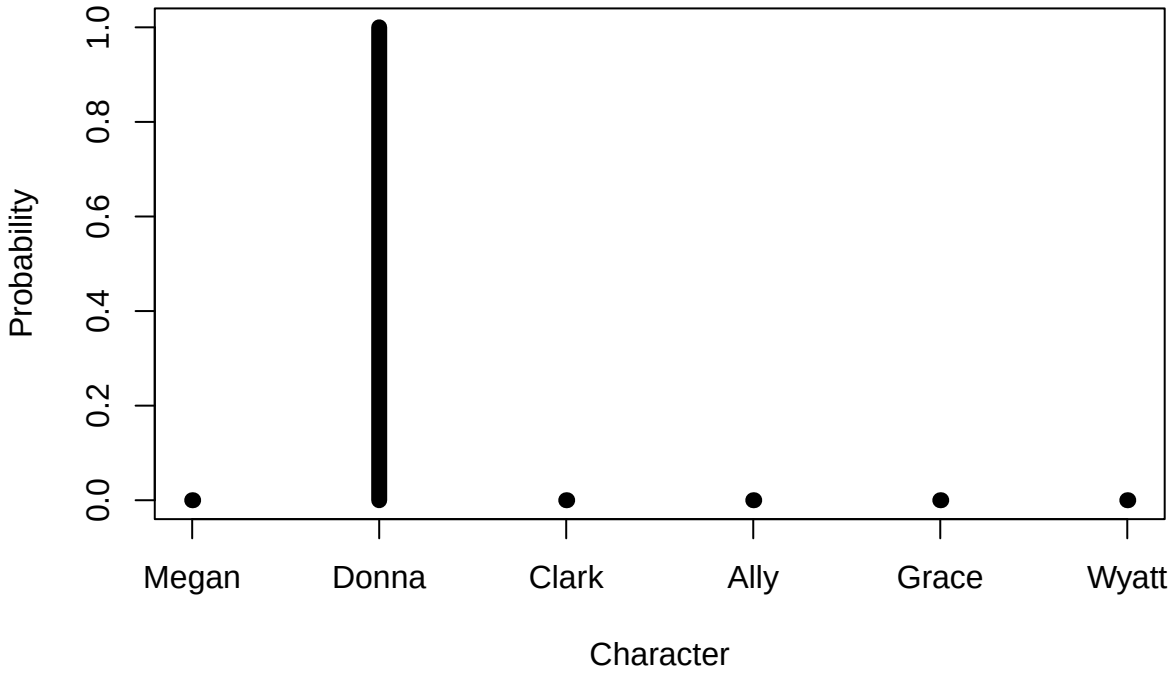


Follow the conversation and update your probabilities accordingly, again

- **You**: Is your character wearing glasses
- **Your adversary**: Yes

Follow the conversation and update your probabilities accordingly, again

- **You**: Is your character wearing purple glasses
- **Your adversary**: No

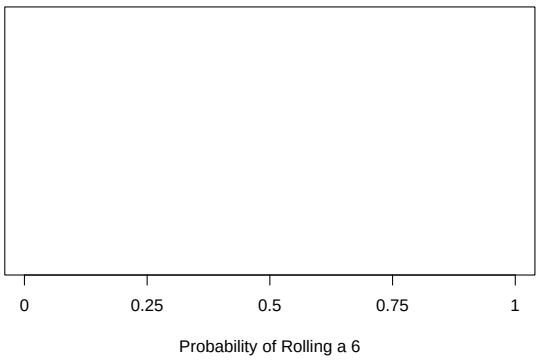


Ex. Rolling a Die

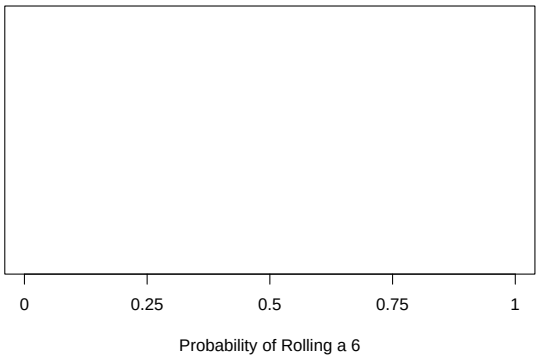
Now consider a similar example using a die. Suppose our goal is to determine the probability of the die landing on 6. Now constuct your prior belief for this die.

Note this is different from the previous example.

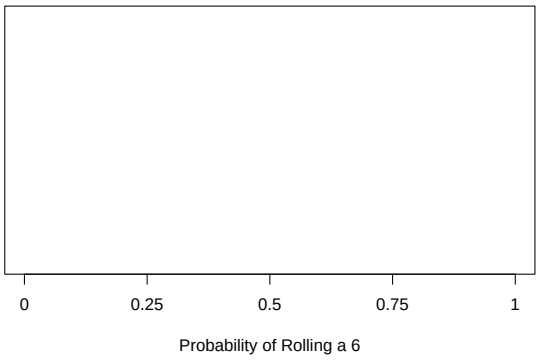
- We are now interested in estimating the probability of rolling a 6, rather than if the character is one of 6 possibilities.
- We are still thinking in a distribution, so the total probability (or credibility) should be allocated across the set of possible values.



Suppose we observe three rolls, now update your beliefs.



Finally suppose we observe 100 more rolls, now update your beliefs.



Key Points From This Activity

- *Thinking with distributions:* A key philosophical idea of Bayesian statistics focuses on uncertainty and thinking about the complete set of possibilities and their associated probabilities. This idea of thinking with a distribution explicitly be seen in the Guess Who example. Furthermore, this idea will enable probabilistic computations such as "What is the probability that the opponents character is wearing a hat?"
- *Specifying a prior distribution:* The starting point for any Bayesian analysis is a prior distribution. Some general principles for prior distributions are to put positive probability on any plausible set and to avoid imparting too much information with a prior distribution.
- *Update distribution with additional data or evidence:* Given a prior distribution and additional information (such as the opponent's character is wearing a hat), we can update our prior distribution. This updated distribution is known as a posterior distribution. This process can continue sequentially as more and more information is included, the posterior distribution updates accordingly.