Week 12 Activity: T-tests

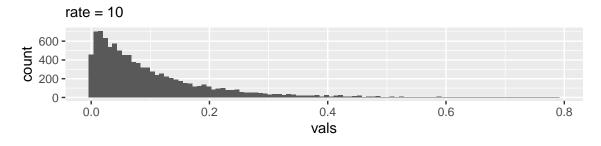
Bayesian modeling with t-distribution

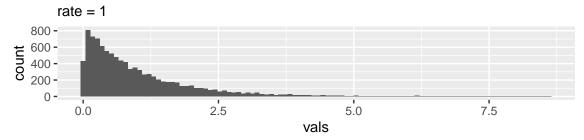
	Sampling	model	$u \sim t$	(11 0	2 $^{\prime\prime}$
•	Samping	moder	$y \sim \iota$	(μ, ι)	ν , ν

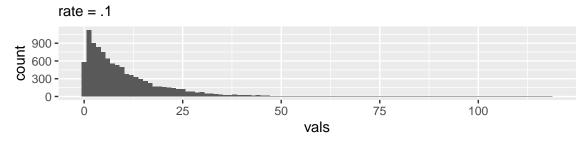
- This requires a prior distribution on:
 - μ : Similiar to the normal sampling model case, we can use a normal distribution with $p(\mu) \sim N(M, S^2)$

 $-\sigma^2$: The variance term also has a similar interpretation, so we can use a uniform or inverse-gamma distribution for a prior.

 $-\nu$: The term ν is often called the degrees of freedom, and this controls the tail behavior of the distribution. The restriction is that the degrees of freedom has to be larger than one. A common prior is to use a shifted exponential distribution.



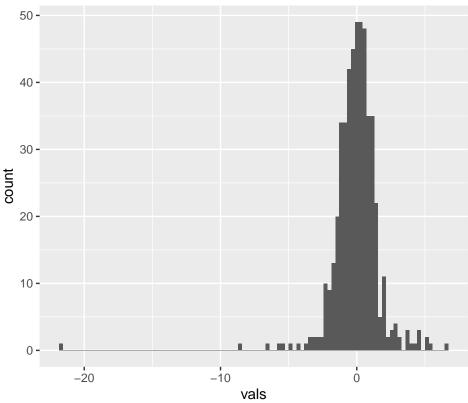




```
t.samples <- data.frame(rt(500, df = 3))
colnames(t.samples) <- 'vals'
ggplot(data=t.samples, aes(vals)) + geom_histogram(bins = 100) +
labs(subtitle = "samples from t(3) distribution")</pre>
```

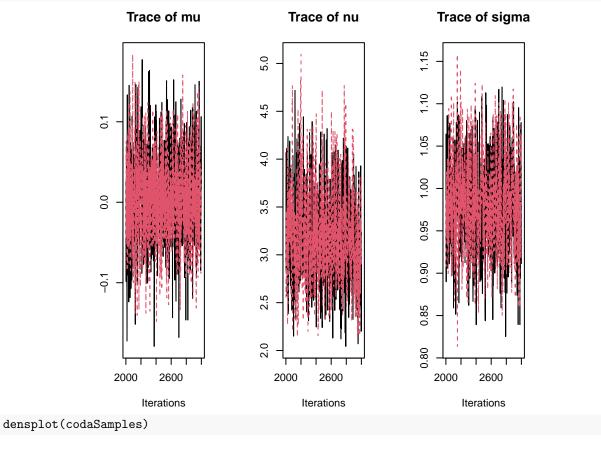
JAGS code

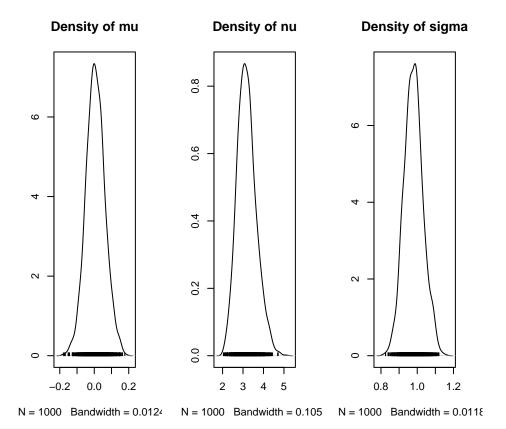
samples from t(3) distribution



```
#Prior parameters
M <- 0
S <- 100
C <- 10
rate <- .1
# Store data
dataList = list(y = t.samples$vals, Ntotal = nrow(t.samples), M = M, S = S, C = C, rate = rate)
# Model String
modelString = "model {
  for ( i in 1:Ntotal ) {
   y[i] ~ dt(mu, 1/sigma^2, nu) # sampling model
 }
 mu ~ dnorm(M,1/S^2)
  sigma ~ dunif(0,C)
 nu <- nuMinusOne + 1 # transform to guarantee n >= 1
 nuMinusOne ~ dexp(rate)
} "
writeLines( modelString, con='Tmodel.txt')
# initialization
initsList <- function(){</pre>
  # function for initializing starting place of theta
  # RETURNS: list with random start point for theta
 return(list(mu = rnorm(1, mean = M, sd = S), sigma = runif(1,0,C),
              nuMinusOne = rexp(1, rate=rate) ))
```

```
}
# Runs JAGS Model
jagsT <- jags.model( file = "Tmodel.txt", data = dataList, inits =initsList,</pre>
                      n.chains = 2, n.adapt = 1000)
## Compiling model graph
##
      Resolving undeclared variables
##
      Allocating nodes
## Graph information:
      Observed stochastic nodes: 500
##
##
      Unobserved stochastic nodes: 3
##
      Total graph size: 516
##
## Initializing model
update(jagsT, n.iter = 1000)
num.mcmc <- 1000
codaSamples <- coda.samples( jagsT, variable.names = c('mu', 'sigma','nu'), n.iter = num.mcmc)</pre>
par(mfcol=c(1,3))
traceplot(codaSamples)
```





HPDinterval(codaSamples)

```
## [[1]]
##
               lower
                         upper
## mu
         -0.09692971 0.1281821
## nu
          2.32465545 4.0713572
## sigma 0.88593432 1.0872612
## attr(,"Probability")
## [1] 0.95
##
## [[2]]
##
               lower
                        upper
         -0.08945252 0.114856
## mu
          2.30738031 4.175330
## nu
## sigma 0.89361504 1.095085
## attr(,"Probability")
## [1] 0.95
```

1.

Simulate 100 responses from a Cauchy distribution, t distribution with $\mu = 1$, $\sigma^2 = 1$ and $\nu = 1$, and describe this data with a plot and brief description of the data.

2.

Use JAGS to fit a normal sampling model and the following priors for this data.

- $\begin{array}{ll} \bullet & p(\mu) \sim N(0, 10^2) \\ \bullet & p(\sigma) \sim U(0, 1000) \end{array}$

Discuss the posterior HDIs for μ and σ .

3.

Use JAGS to fit a t sampling model and the following priors for this data.

- $p(\mu) \sim N(0, 10^2)$
- $p(\sigma) \sim U(0, 1000)$
- $p(\nu) \sim E_{+}(.1)$, where $E_{+}(.1)$ is a shifted exponential with rate = .1.

Discuss the posterior HDIs for μ , σ , and ν .

4.

Create a data visualization to show posterior predictive distributions for Q2 and Q3. Remember this can be done using your posterior samples and combining them with your sampling model. Compare the data and the posterior predictive model curves with posterior predictive models. Note this is the final step in Bayesian data analysis: verifying that our model / prior selection is an accurate representation of the data.

T-test

For this question, we will use classical t-tests (and then see Bayesian analogues on Thursday).

Use the OK Cupid dataset and test the following claim, the mean height OK Cupid respondents reporting their body type as athletic is different than 70 inches.

```
okc <- read.csv('http://www.math.montana.edu/ahoegh/teaching/stat408/datasets/OKCupid_profiles_clean.cs
okc.athletic <- okc %>% filter(body_type == 'athletic')
```

Now consider whether there is a height difference between OK Cupid respondents self-reporting their body type as "athletic" and those self-reporting their body type as "fit"

```
okc.fit <- okc %>% filter(body_type == 'fit')
```