STAT 491: Midterm Exam
Name:

Short Answer Questions (20 points)

For questions in this section, keep your answers concise. You are welcome to use a combination of prose, math, and pseudocode, but your responses should be well thought out and defended.

1. (4 points)

Describe the differences in marginal, joint, and conditional probabilities.

2. (4 points)

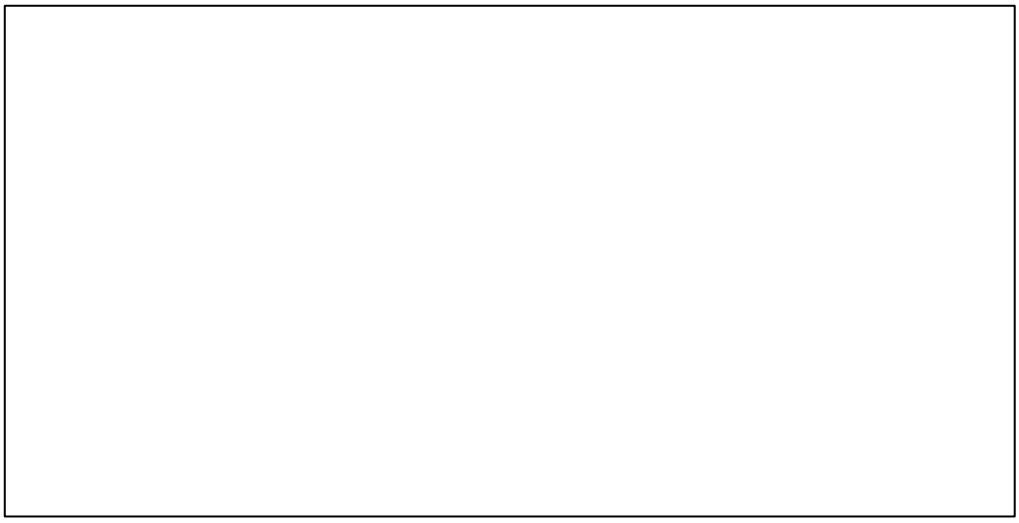
Why is a model, and the underlying probability distribution with parameters, essential to statistical analysis?

3. (4 points)

What is a prior distribution and why is it necessary for Bayesian data analysis?

4. (4 points)

Sketch and defend a prior that incorporates your belief about the average test score on the 491 midterm (this exam). Make sure to label your axes.



5. (4 points)

Describe the four components of the following equation:

$$p(\theta|X) = \frac{p(X|\theta)p(\theta)}{p(X)}$$

- $p(\theta|X)$

- $p(X|\theta)$

- $p(\theta)$

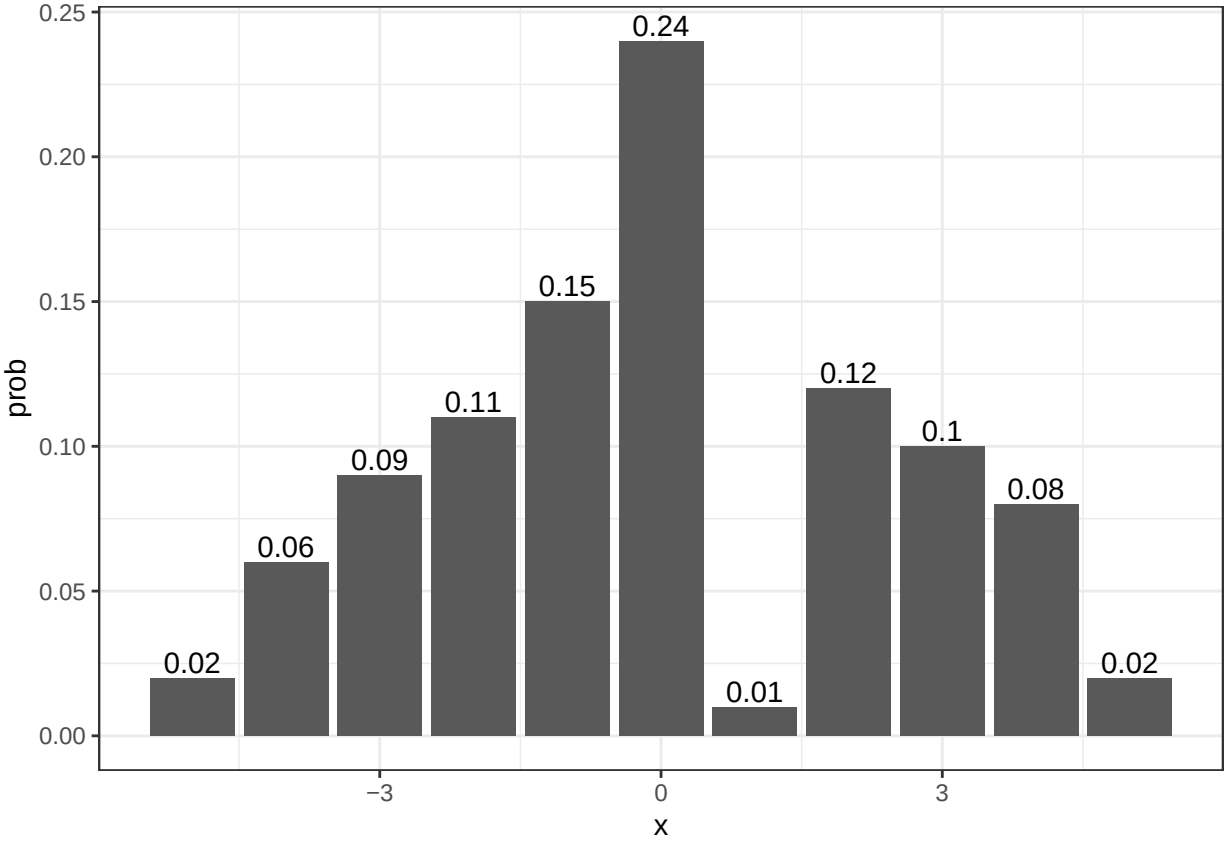
- $p(X)$

6. (4 points)

What is a posterior distribution and how can be it be considered a compromise between the prior distribution and the data?

7. (4 points)

For the following distribution, identify the shortest 95 % region.



8. (4 points)

Write out the statistical model and prior distribution implied by the following `JAGS` code.

```
model{
  # Likelihood
  z ~ dbinom(theta, N)

  # Prior
  theta ~ dbeta(1, 1)
}
```

9. (4 points)

Suppose that you are estimating the probability of catching a leprechaun in a trap (θ). Interpret the results below, using the code from Q8 based on 2 unsuccessful attempts and a uniform prior on θ.

```
##
## Iterations = 6001:16000
## Thinning interval = 1
## Number of chains = 1
## Sample size per chain = 10000
##
## 1. Empirical mean and standard deviation for each variable,
##    plus standard error of the mean:
##
##           Mean             SD       Naive SE Time-series SE
##        0.24834        0.19404        0.00194        0.00335
##
## 2. Quantiles for each variable:
##
##     2.5%      25%      50%      75%    97.5%
## 0.007841 0.089248 0.203820 0.369838 0.713388
```

10. (4 points)

Critique and/or defend using a Beta(1,1) prior for θ - the probability of catching a leprechaun - in Q9.