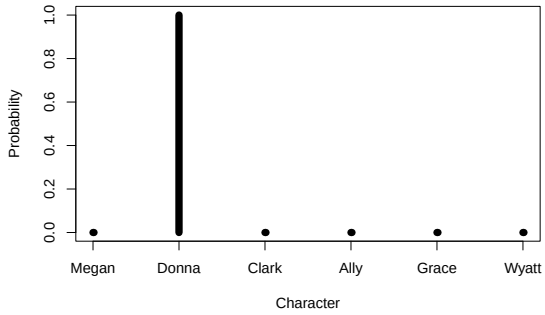
STAT 456 - Week 2

Ch.2 Introduction: Credibility, Models, and Parameters

Reallocation of probabilities

Recall the Guess Who setting, where the goal was to identify the opponents character.

In this setting, after determining that the character wore a hat and non-purple glasses we ended up with the following probability.



In this situation we were able to deduce that Donna was our opponents character.

Similarly, we also considered an example using a die. With the goal to determine the probability of the die landing on 6.

Possibilities are parameter values in descriptive models

Data resulting from the roll of dice can be characterized by a Multinomial distribution. We can formally characterize this using the probability mass function (pmf) of the random variable (more details in later sections.)

$$Pr[\text{die } = i]\frac{n!}{x_1!x_2!x_3!x_4!x_5!x_6!}p_1^{x_1}\ p_2^{x_2}\ p_3^{x_3}\ p_4^{x_4}\ p_5^{x_5}\ p_6^{x_6},$$

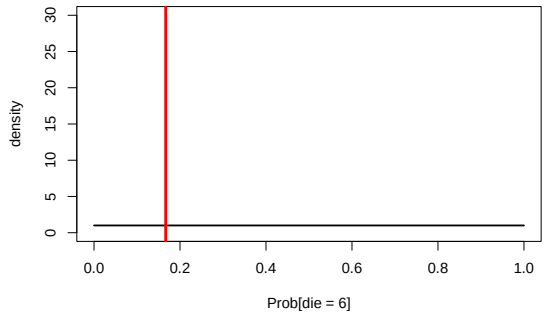
where p_i is the probability of rolling an i, x_i is the total count of $i's$ rolled in n total rolls.

Q: Why do we need a statistical model?

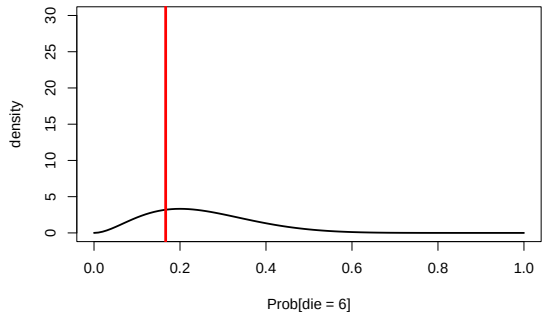
Given that our goal is to only estimate, $P[die = 6]$, we can simplify the pmf above and consider only two cases: 6 and not 6.

Mathematical Notation for Binary Example

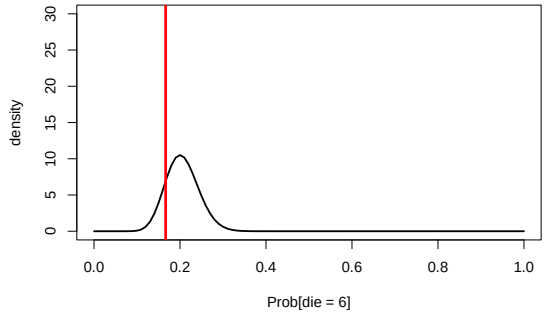
Suppose our initial prior is a uniform prior over the range of values from 0 to 1.



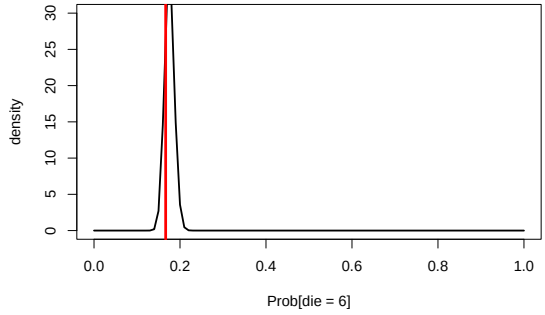
Now assume we are using a fair die and update the probabilities after the following ten rolls: 4, 4, 1, 5, 5, 1, 6, 1, 4, 6. Our distribution can be updated as



Now we observe 100 more rolls of the die, which results in 20 rolls of 1.

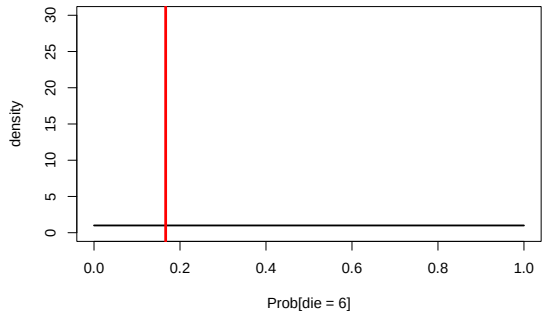


After 1000 more rolls of the die, which results in 172 rolls of 6.

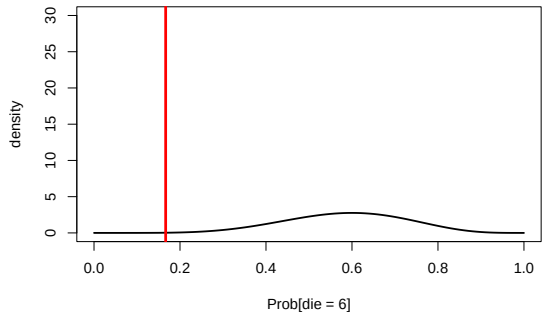


Unfair Coin

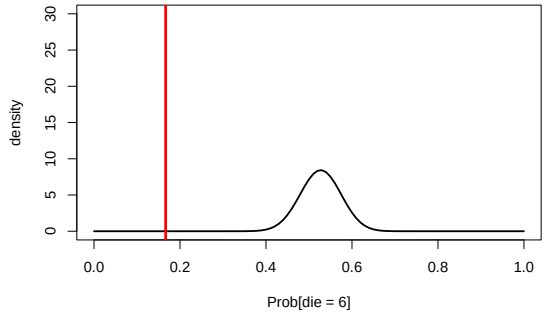
Suppose our initial prior is a uniform prior over the range of values from 0 to 1.



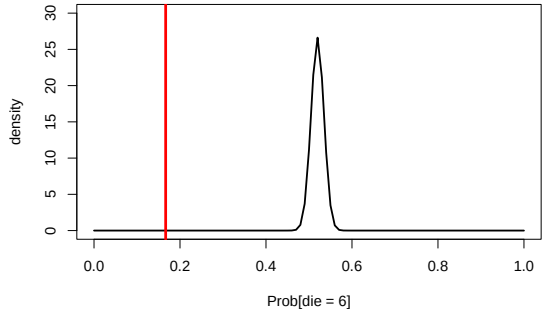
Now assume we are using a fair die and update the probabilities after the following ten rolls: 6, 2, 5, 6, 4, 6, 6, 4, 6, 6. Our distribution can be updated as



Now we observe 100 more rolls of the die, which results in 52 rolls of 6.



After 1000 more rolls of the die, which results in 519 rolls of 6.



Steps of Bayesian Data Analysis

For a Bayesian analysis we will follow these steps:

1. **Identify**

2. **Define**

3. **Specify**

4. **Use**

5. **Check**