HW 5

Name here

Please use D2L to turn in both the PDF output and your R Markdown file.

Q1.

After enjoying numerous Bridger Bowl powder days during your time at MSU, you have received a job offer in New Mexico. Your decision hinges on the quality of snow at your potential new home mountain, Taos.

According to https://www.onthesnow.com/new-mexico/taos-ski-valley/historical-snowfall.html annual snowfall totals can be obtained.

Below are the snowfall totals for Taos and Bridger Bowl for the last nine years.

```
taos <- c(78,192,169,179,191,204,197,116,195)
bridger <- c(271,209,228,166,316,254,344,319,247)
```

Specifically the BSF is interested in computing three probabilistic statements.

1. $Pr[\theta_{bridger} > 250]$
2. $Pr[\theta_{taos} > 180]$
3. $Pr[\theta_{bridger} > \theta_{taos}]$ where $\theta_{bridger}$ is the mean annual snow fall at Bridger Bowl and θ_{taos} is the mean annual snow fall at Taos.

a. (5 pts)

How would you go about addressing the questions in a classical framework? Would you be able to compute these probabilities?

b. (10 pts)

Using the prior structure where $p(\sigma^2_{bridger}, \theta_{bridger}) = p(\theta_{bridger}|\sigma^2_{bridger})p(\sigma^2_{bridger})$ and $p(\sigma^2_{taos}, \theta_{taos}) = p(\theta_{taos}|\sigma^2_{taos})p(\sigma^2_{taos})$ compute the marginal posterior distributions $p(\theta_{bridger}|y_{bridger,1}, \ldots, y_{bridger,15})$ and \$p(\theta_{taos}|y_{taos,1}, \ldots, y_{taos,9})$, where σ^2_1 and σ^2_{taos} are the variances for snowfall (in inches) a, and $y_{i,j}$ is the observed snowfall at location i for reading j. Then using posterior samples from each distribution compute the three values specified above.

Q2.

a. (5 points)

Sketch out the steps for a Gibbs sampler algorithm.

b. (5 pts)

Simulating data is a key step in verifying your algorithms are working correctly. This will be more apparent as we start studying sophisticated hierarchical models.

Simulate 100 observations from a standard normal distribution and plot a histogram of your data.

c. (5 pts)

Select and state prior distributions for θ the mean of the normal distribution and σ^2 the variance (or alternatively you may parameterize your model using the precision term).

d. (5 pts)

Implement an MCMC algorithm (using a Gibbs sampler or JAGS/stan) to simulate from the joint posterior distribution $p(\theta, \sigma^2 | y_1, \ldots, y_{100})$. Plot trace plots and histograms of the marginal posterior distributions for θ and σ^2. Include the true values on these figures. Comment on the figures.