Approximation of Normal Distribution by Monte Carlo in R Markdown

Yiyi Xu

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Conside approximation of Normal Distribution by the Monte Carlo Methods.

1 Introduction

The main goal of this report is to use bookdown to produce a report for considring approximation of Normal Distribution By Monte Carlo Methods. The report will contain a table about the experiment with the approximation at $n \in 10^2, 10^3, 10^4$ at $t \in$ 0.0,0.67,0.84,1.28,1.65,2.32,2.58,3.09,3.72. Also, the report will contain boxplot to compare how n value effect the result bias of the experiment.

2 Math Equation

Distribution Function of N(0,1)

$$\Phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$$

Monte Carlo methods

$$\widehat{\Phi}(t) = \frac{1}{n} \sum_{i=1}^{n} I(X_i \le t)$$

3 Table

Define a function which satisfy Monte Carlo Method. Create N iid r.v., use Monte Carlo method to find the probability of Xi>t

```
F <- function(n,t){
  set.seed(1)
  (length(which(rnorm(n,0,1)<t)))/n
}
```

As we create the previouse function, we plug in different n and t values and get results. By create a table of the results, we can easily compare the result value with the true value.

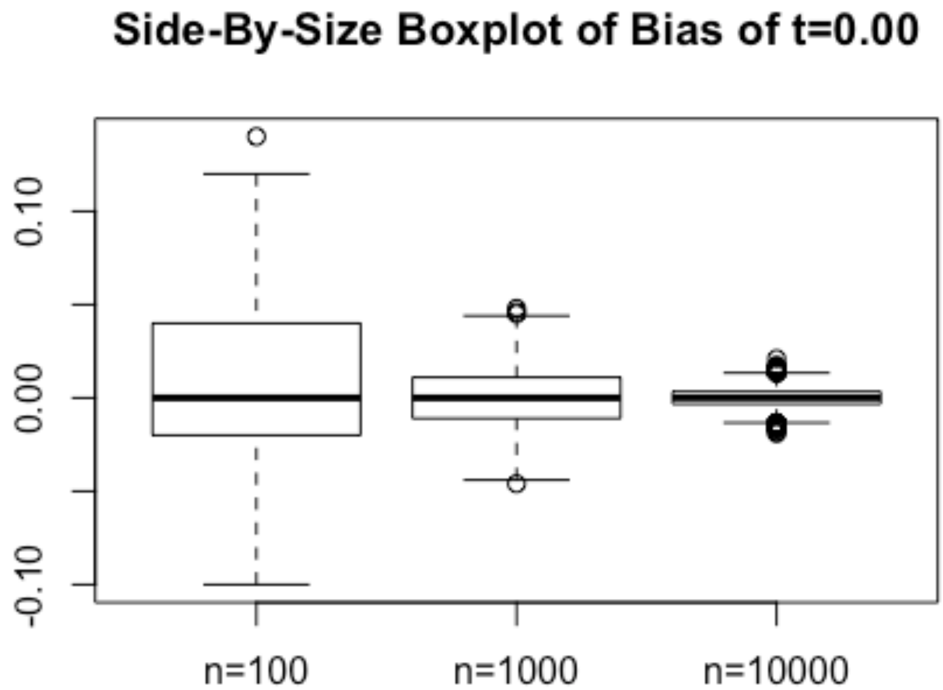
$\widehat{\Phi}(t)$	t=0.00	t=0.67	t=0.84	t=1.28	t=1.65	t=2.32	t=2.580	t=3.09	t=3.72

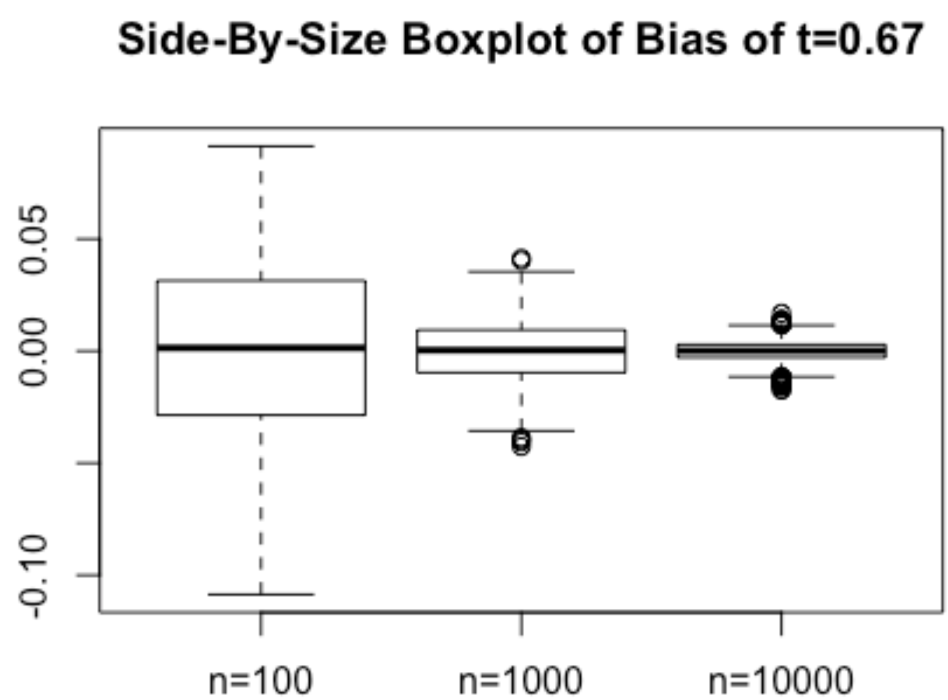
n=10	0.4600	0.7399	0.8199	0.9100	0.9699	0.9899	1	1	1
n=100	0.5180	0.7419	0.8040	0.8920	0.9409	0.9899	0.9959	0.9989	0.9989
n=1000	0.5068	0.7481	0.8015	0.8941	0.9494	0.9906	0.9959	0.9992	0.9999
$\Phi(t)$	0.5000	0.7486	0.7995	0.8997	0.9505	0.9898	0.9951	0.9990	0.9999

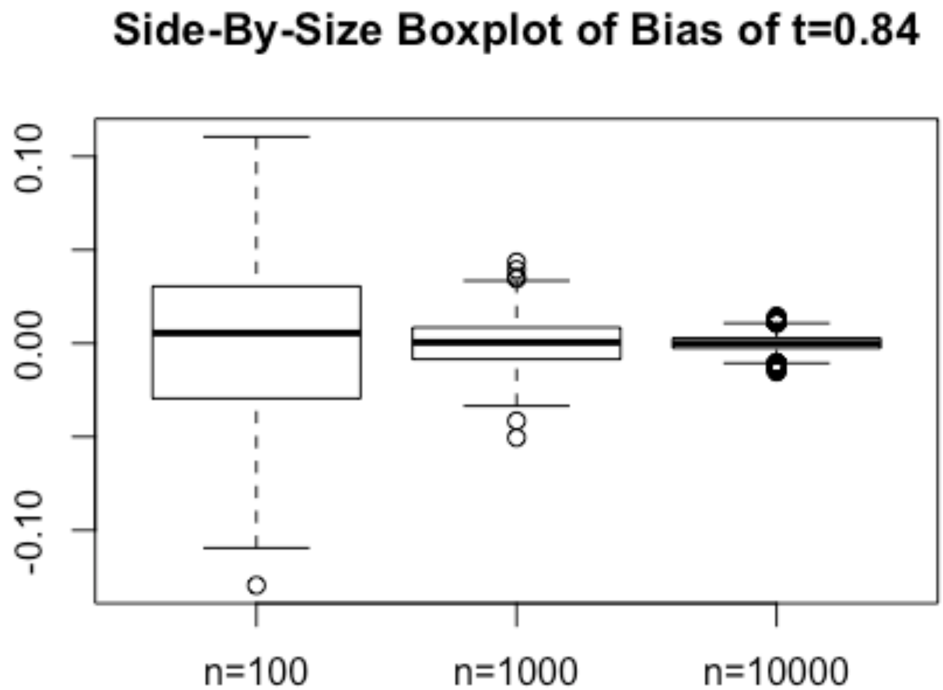
4 Figure- BoxPlot

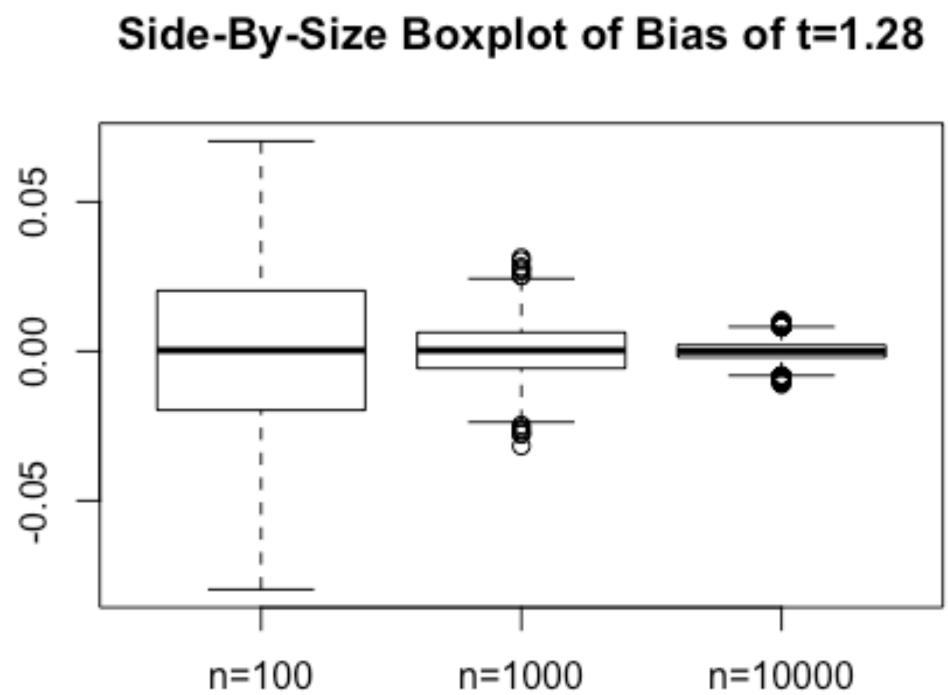
At first, create a G function to repeat each experiment at 100 times to create a bias vector. Then plug in different t and n values in order to see how the bias changes as n value changes.

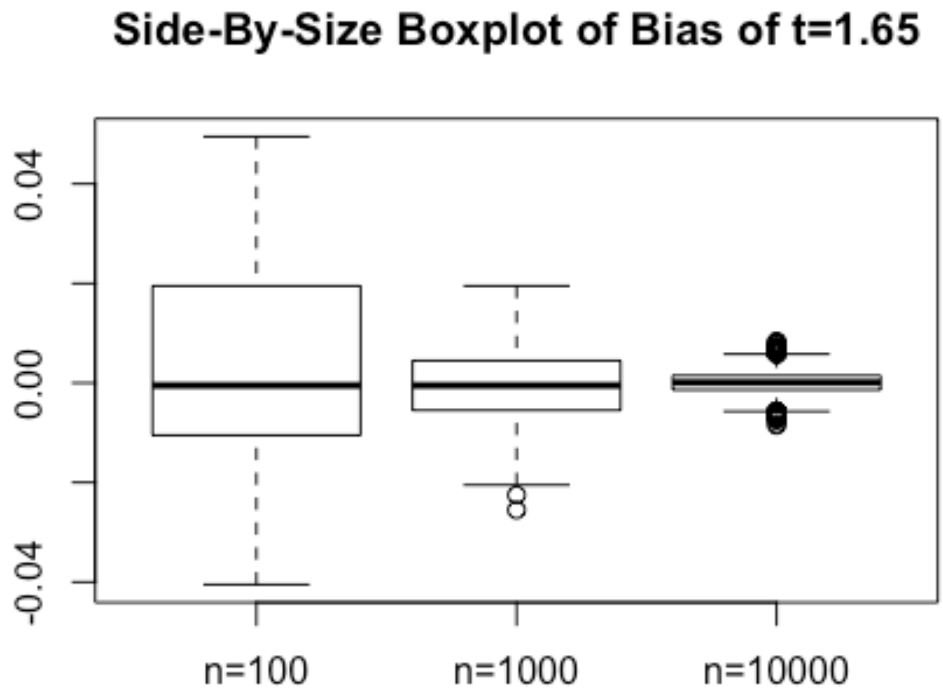
```
G <- function(n,t,p){
  a <- c(0)
  for(i in 1:n){
    a[i] <- ((length(which(rnorm(n,0,1)<t)))/n)-p
  }
  a
}
```


Side-By-Size Boxplot of Bias of t=0.00
0.10
0.00
-0.10
n=100
n=1000
n=10000

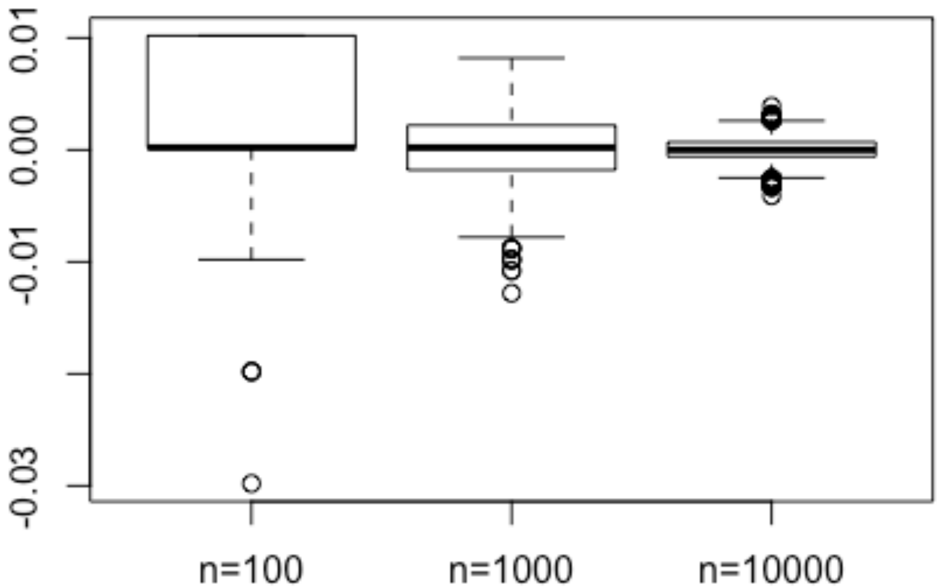

Side-By-Size Boxplot of Bias of t=0.67
0.05
0.00
-0.10
n=100
n=1000
n=10000

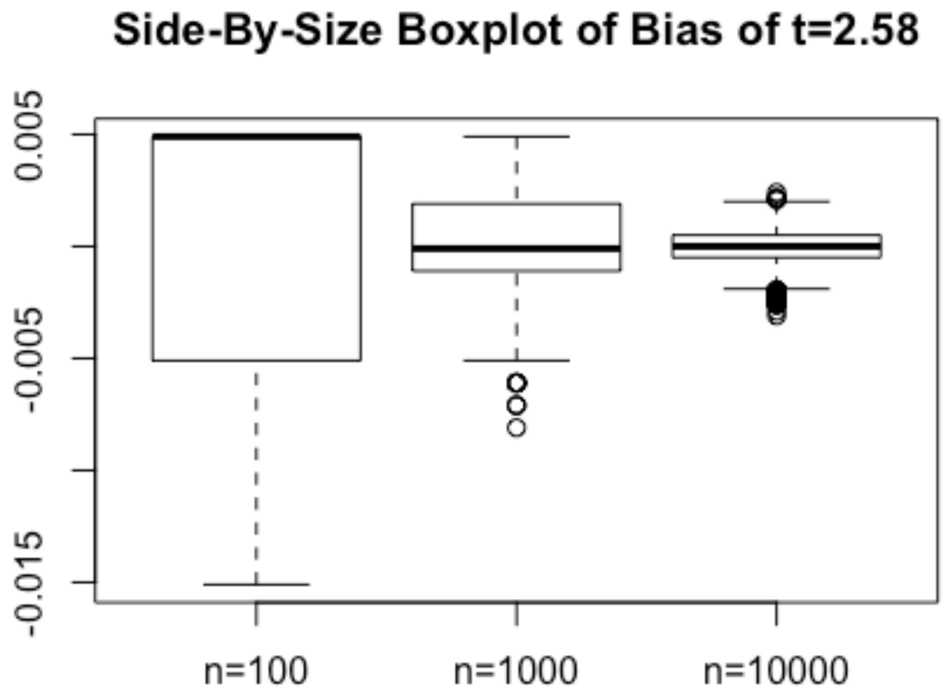

Side-By-Size Boxplot of Bias of t=0.84
0.10
0.00
-0.10
n=100
n=1000
n=10000


Side-By-Size Boxplot of Bias of t=1.28
0.05
0.00
-0.05
n=100
n=1000
n=10000


Side-By-Size Boxplot of Bias of t=1.65
0.04
0.00
-0.04
n=100
n=1000
n=10000

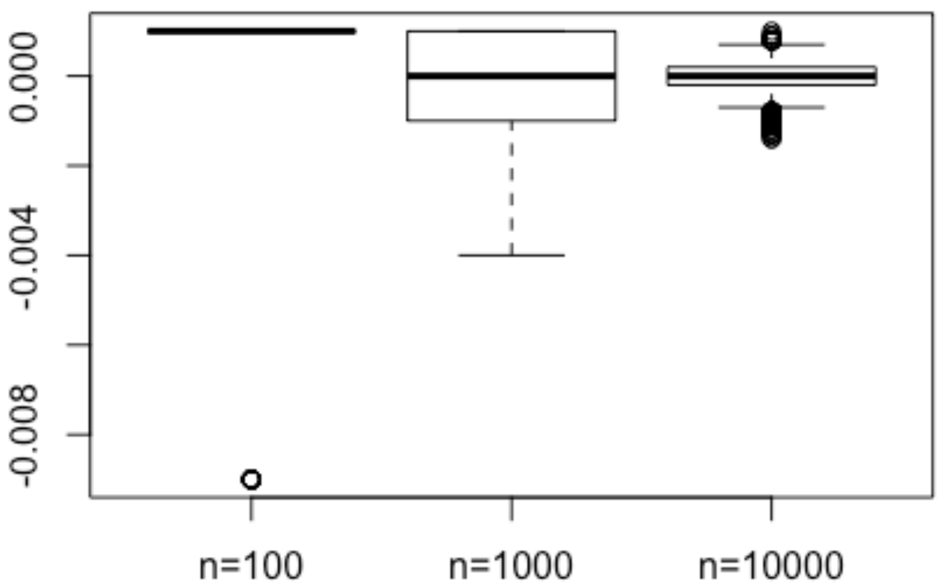
Side-By-Size Boxplot of Bias of t=2.32

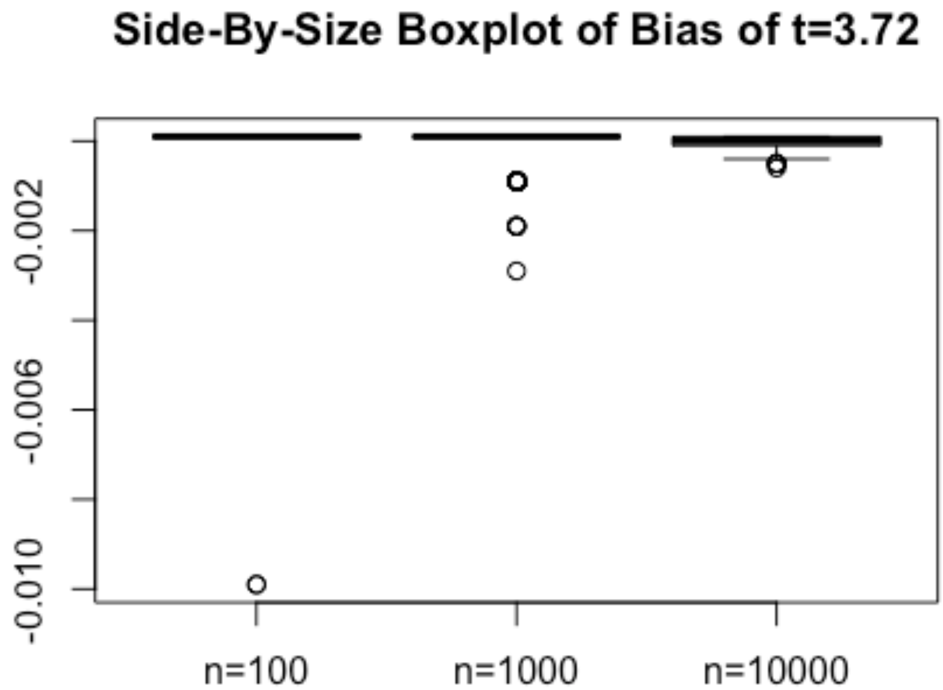




Side-By-Size Boxplot of Bias of t=2.58
0.005
-0.005
-0.015
n=100
n=1000
n=10000

Side-By-Size Boxplot of Bias of t=3.09





5 Summary

By Monte Carlo Methods, we obtain numerial result by repeating random sampling. By analysis the boxplot, as the N (number of Monte Carlo Methods repeat) increase, the bias of the result is smaller.