Bias of Monte Carlo Method in Standard Normal Distribution

Guanting Wei

Sep.13.2018

Abstract

This experiment uses Monte Carlo Method to estimate the real value of Standard Normal Distribution and get the bias.

Introduction

It is hard to calculate distribution function of N(0,1) directly.

$$\Phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$$

Instead, we can use Monte Carlo methods to estimate its value:

$$\hat{\Phi}(t) = \frac{1}{n} \sum_{i=1}^{n} I(X_i \leq t)$$

In this experiment, we calculate when $t \in 0.0, 0.67, 0.84, 1.28, 1.65, 2.32, 2.58, 3.09, 3.72$ n times($n \in 10^2, 10^3, 10^4$)

Code

I use rnorm to get a random number "a" according to Normal Distribution. The proportion of a which is smaller than t is similar to the number we want. Repeat the experimant 100 times and store all data in one array T.

```
n <- c(100,1000,10000)
t <- c(0.0,0.67,0.84,1.28,1.65,2.32,2.58,3.09,3.72)
T=array(0,dim=c(length(t),length(n),100),
        dimnames=list(c("0.0","0.67","0.84","1.28","1.65","2.32","2.58","3.09","3.72"),
                      c("10^2","10^3","10^4")))
for(i in 1:100){
  for(j in 1:length(t)){
    for(k in 1:length(n)){
      for (l in 1:n[k]){
      a <- rnorm(1, mean=0, sd=1)
      if(a<=t[j])
      {b=1}
      else
      {b=0}
```

```
      T[j,k,i]=T[j,k,i]+b
    }
    T[j,k,i]=T[j,k,i]/n[k]
    }
  }
}
```

True Value

I use pnorm to calculate the true value and store the data in an array V.

```
V=array(0,dim=c(9,1),dimname=list(c("0.0","0.67","0.84","1.28","1.65","2.32","2.58","3.09","3.
                                  c("true")))
V[1]=pnorm(0)
V[2]=pnorm(0.67)
V[3]=pnorm(0.84)
V[4]=pnorm(1.28)
V[5]=pnorm(1.65)
V[6]=pnorm(2.32)
V[7]=pnorm(2.58)
V[8]=pnorm(3.09)
V[9]=pnorm(3.72)
```

Table

```
knitr::kable(T[ , ,1], booktabs = TRUE, caption = 'one of the 100 experiments')
```

Table 1: one of the 100 experiments

	10^2	10^3	10^4
0.0	0.58	0.488	0.4975
0.67	0.68	0.746	0.7448
0.84	0.75	0.797	0.8003
1.28	0.89	0.884	0.8964
1.65	0.95	0.941	0.9511
2.32	1.00	0.984	0.9891
2.58	0.99	0.994	0.9948
3.09	1.00	0.998	0.9989
3.72	1.00	1.000	0.9999

```
knitr::kable(V, booktabs = TRUE, caption = 'True Value')
```

Table 2: True Value

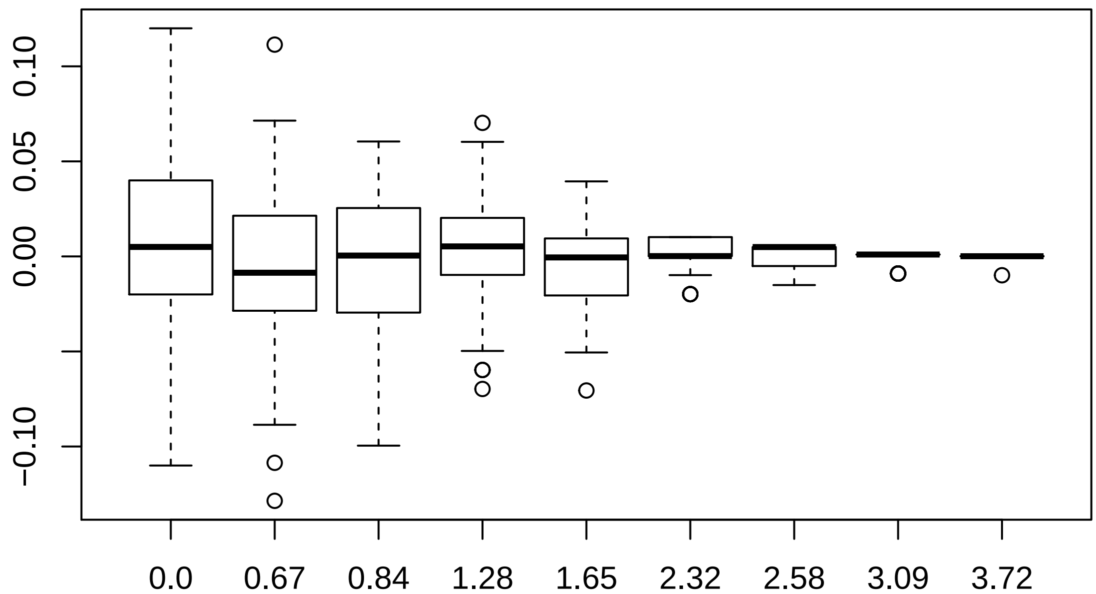
	true
0.0	0.5000000
0.67	0.7485711
0.84	0.7995458
1.28	0.8997274
1.65	0.9505285
2.32	0.9898296
2.58	0.9950600
3.09	0.9989992
3.72	0.9999004

Boxplot of Bias

First, we need to calculate all the bias and use those bias to make box plots.

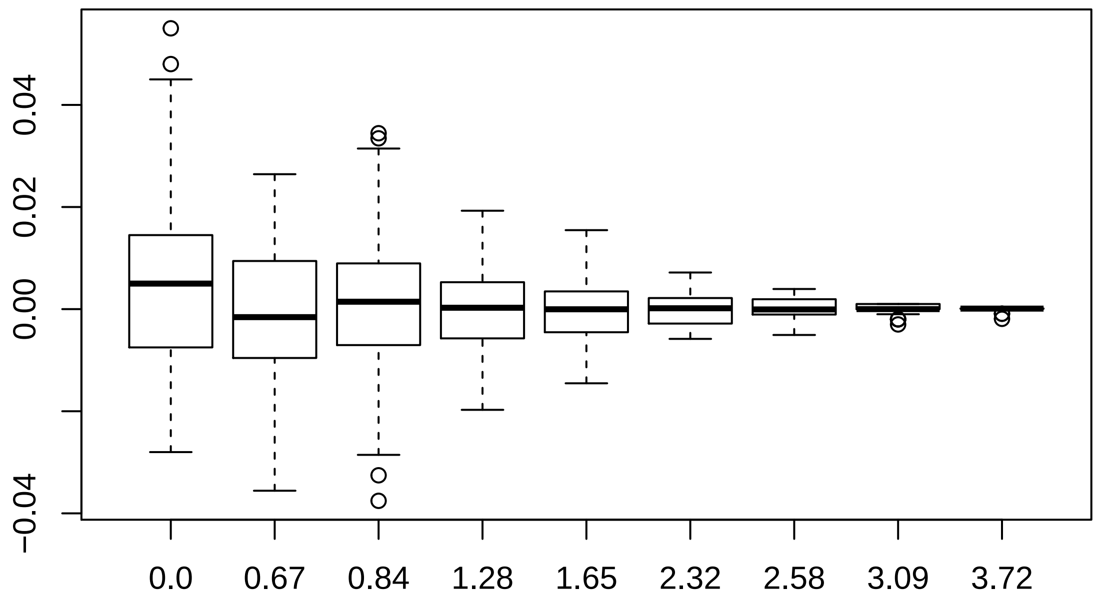
n=10^2

```
boxplot(T[1,1, ]-V[1],T[2,1, ]-V[2],T[3,1, ]-V[3],T[4,1, ]-V[4],T[5,1, ]-V[5],T[6,1, ]-V[6],
        T[7,1, ]-V[7],T[8,1, ]-V[8],T[9,1, ]-V[9],
        names = c("0.0","0.67","0.84","1.28","1.65","2.32","2.58","3.09","3.72"))
```



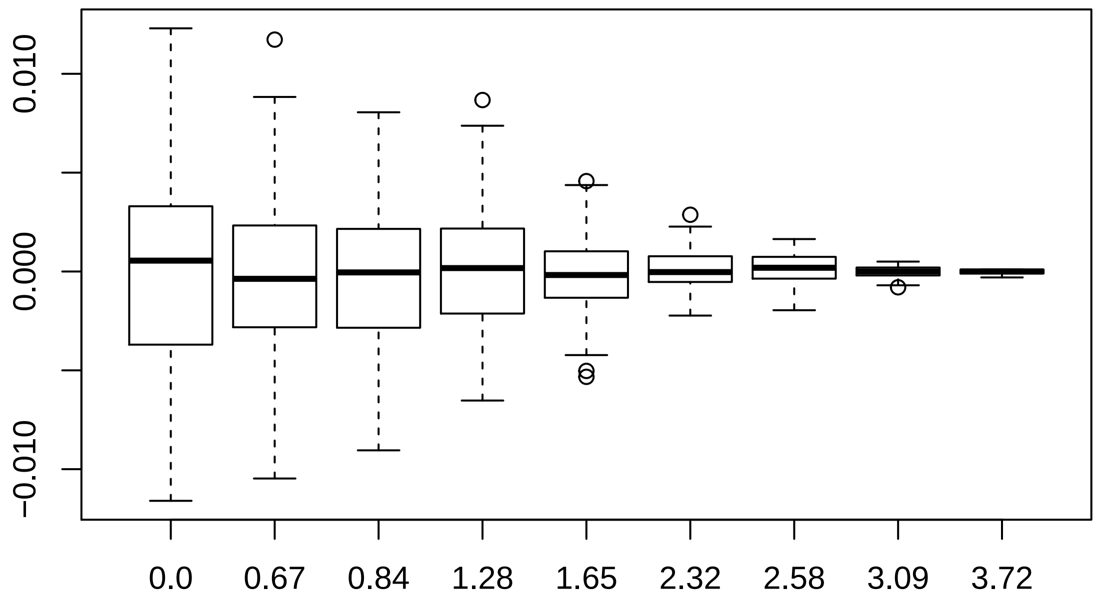
n=10^3

```
boxplot(T[1,2, ]-V[1],T[2,2, ]-V[2],T[3,2, ]-V[3],T[4,2, ]-V[4],T[5,2, ]-V[5],T[6,2, ]-V[6],
        T[7,2, ]-V[7],T[8,2, ]-V[8],T[9,2, ]-V[9],
        names = c("0.0","0.67","0.84","1.28","1.65","2.32","2.58","3.09","3.72"))
```



n=10^4

```
boxplot(T[1,3, ]-V[1],T[2,3, ]-V[2],T[3,3, ]-V[3],T[4,3, ]-V[4],T[5,3, ]-V[5],T[6,3, ]-V[6],
        T[7,3, ]-V[7],T[8,3, ]-V[8],T[9,3, ]-V[9],
        names = c("0.0","0.67","0.84","1.28","1.65","2.32","2.58","3.09","3.72"))
```



0.010
0.000
−0.010
0.0
0.67
0.84
1.28
1.65
2.32
2.58
3.09
3.72