Question2

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Abstract

In this article, we talk about the solution of queation2. I will show you my way to compute the approximation, how to store the data and how to get the plots.

Question

The approximation of the distribution function of $N(0,1)$,

$$\Phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy, (\#eq:cdf) \tag{1}$$

The Monte Carlo methods:

$$\hat{\Phi}(t) = \frac{1}{n} \sum_{i=1}^{n} I(X_i \leq t), \tag{2}$$

where X_i's are iid $N(0,1)$ variables. Experiment with the approximation at $n \in \{10^2, 10^3, 10^4\}$ at $t \in \{0.0, 0.67, 0.84, 1.28, 1.65, 2.32, 2.58, 3.09, 3.72\}$ to form a table. The table should include the true value for comparison. Further, repeat the experiment 100 times. Draw box plots of the bias at all t.

Solution

In order to solve this question, I'd like to use function *rnorm*() to generate random numbers for normal distribution with mean equal to *mean* and standard deviation equal to *sd*. And then I calculated all the values at $t \in \{0.0, 0.67, 0.84, 1.28, 1.65, 2.32, 2.58, 3.09, 3.72\}$ with the approximation at $n \in \{10^2, 10^3, 10^4\}$.

Following is my source code to calculate the approximation:

```
t<- c(0.0, 0.67, 0.84, 1.28, 1.65, 2.32, 2.58, 3.09, 3.72)
n<- c(10^2,10^3,10^4)
result<- array(0, c(3,9,100))
for (i in 1:length(n)){
  for(j in 1:length(t)){
    a<- rnorm(n[i],0,1)
    result[i,j,1]<- sum(a<=t[j])/n[i]
  }
}
```

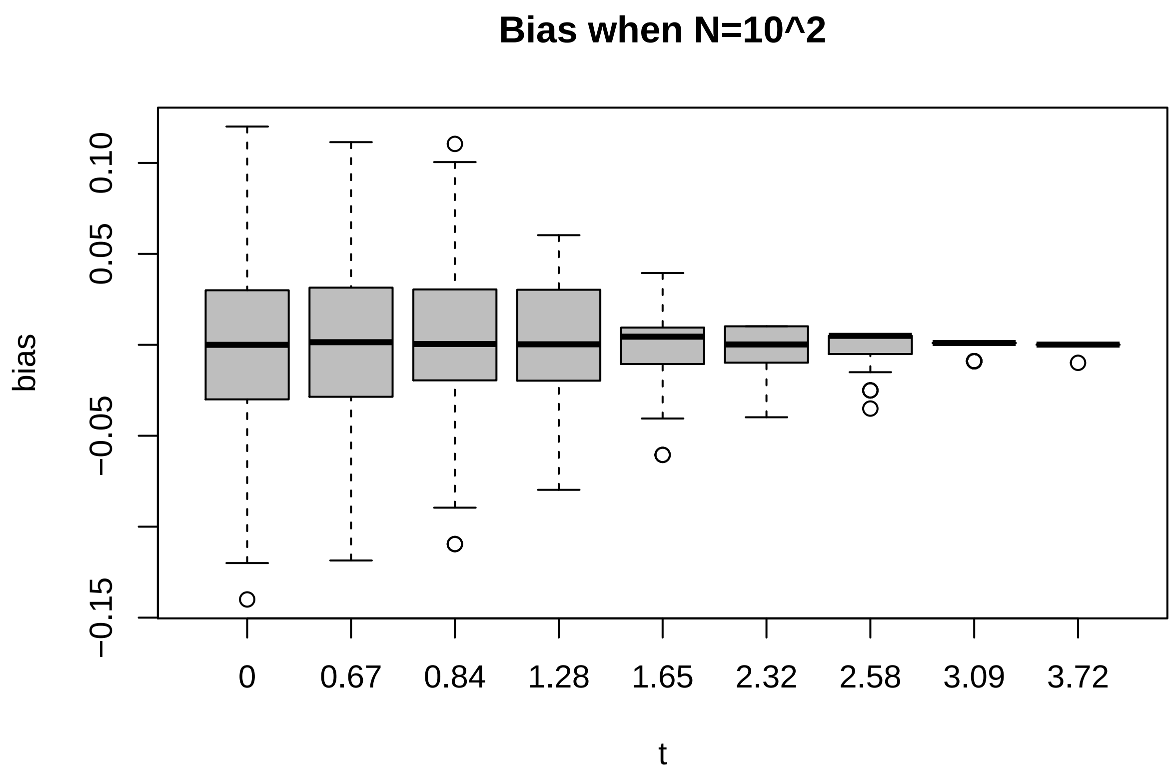
The approximation will be stored in a 3-dimensional array named *result*. And we were required to repeat the experiment 100 times to get the bias at all t.

```
t<- c(0.0, 0.67, 0.84, 1.28, 1.65, 2.32, 2.58, 3.09, 3.72)
n<- c(10^2,10^3,10^4)
result<- array(0, c(3,9,100))
for (k in 1:100){
  for (i in 1:length(n)){
```

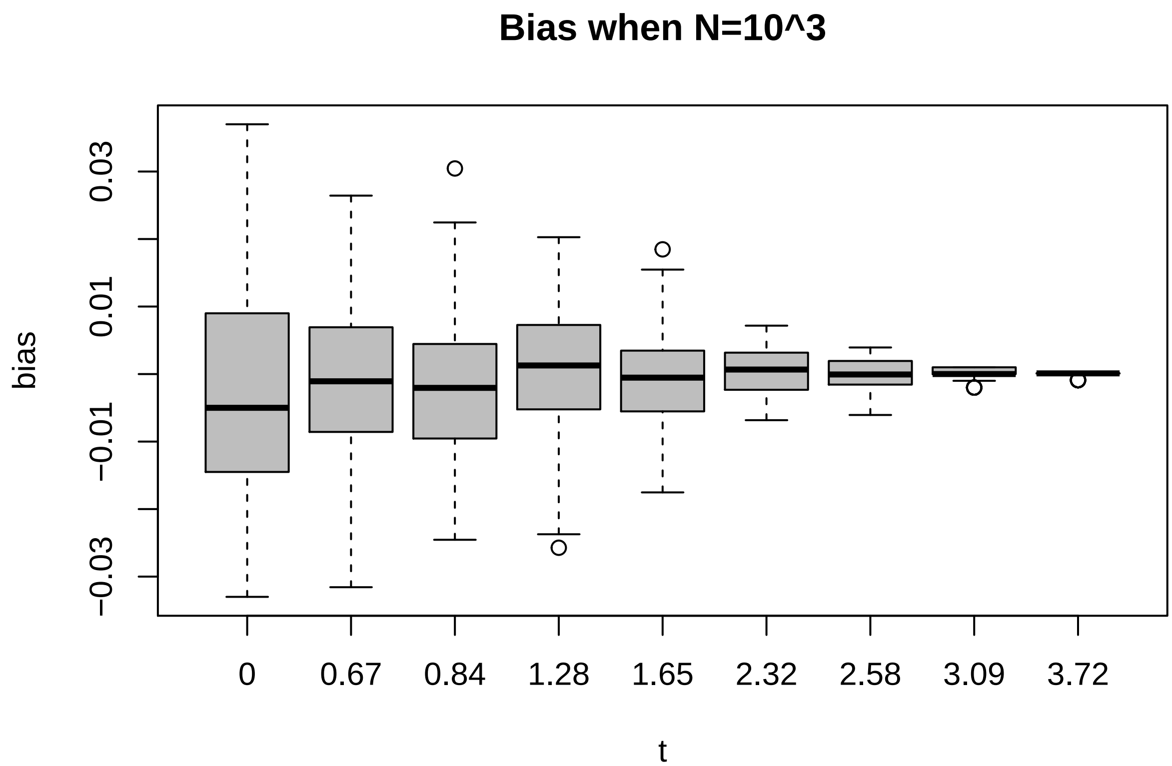
```
    for(j in 1:length(t)){
      a<- rnorm(n[i],0,1)
      result[i,j,k]<- sum(a<=t[j])/n[i]
    }
  }
}
```

After that, we draw box plots of the bias at all t in different n. In order to get the true value of the the distribution function, I would like to use function $pnorm()$ to generate it.

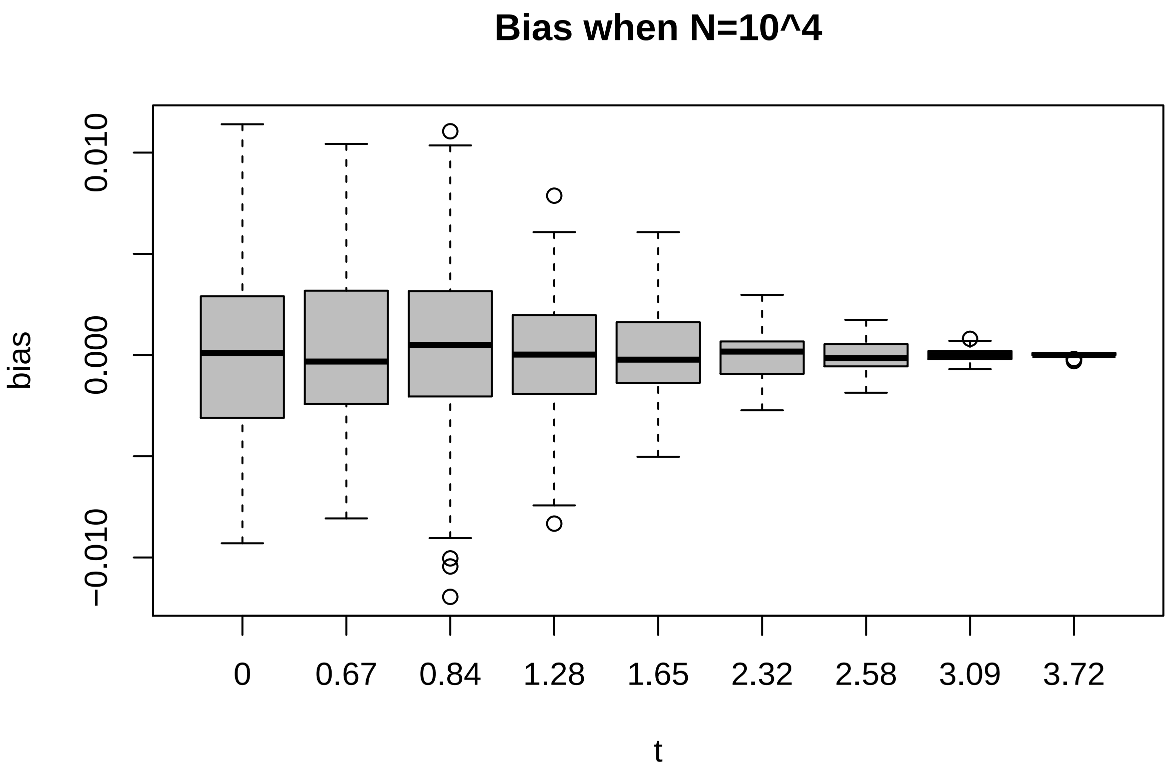
```
True_value<- pnorm(t)
boxplot(result[1,1,]-True_value[1],result[1,2,]-True_value[2],result[1,3,]-True_value[3],result[1,4,]-T
        names = t,
        main = 'Bias when N=10^2',
        xlab = 't',
        ylab = 'bias',
        col = 'grey')
```




```
boxplot(result[2,1,]-True_value[1],result[2,2,]-True_value[2],result[2,3,]-True_value[3],result[2,4,]-T
        names = t,
        main = 'Bias when N=10^3',
        xlab = 't',
        ylab = 'bias',
        col = 'grey')
```




```
boxplot(result[3,1,]-True_value[1],result[3,2,]-True_value[2],result[3,3,]-True_value[3],result[3,4,]-Tr
        names = t,
        main = 'Bias when N=10^4',
        xlab = 't',
        ylab = 'bias',
        col = 'grey')
```


Bias when N=10^4
bias
0.010
0.000
−0.010
0
0.67
0.84
1.28
1.65
2.32
2.58
3.09
3.72
t