Approximation of Monte Carlo Method

5361 Homework 2

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Abstract

Consider approximation of standard normal distribution by the Monte Carlo methods.

1 Exercise 1.2

1.1 Functions

I'm going to compare standard Normal distribution:

$$\Phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$$

with the Monte Carlo methods, where X_i's are iid $N(0,1)$ variables:

$$\hat{\Phi}(t) = \frac{1}{n} \sum_{i=1}^{n} I(X_i \leq t)$$

, when n is large and goes larger.

Experiment with the approximation at $n \in \{10^2, 10^3, 10^4\}$ at $t \in \{0.0, 0.67, 0.84, 1.28, 1.65, 2.32, 2.58, 3.09, 3.72\}$

```
library(knitr)
library(kableExtra)

t <- c(0.0, 0.67, 0.84, 1.28, 1.65, 2.32, 2.58, 3.09, 3.72)
a <- array(data=NA, dim=length(t))

for (i in 1:length(t)){
a[i] <- pnorm(t[i], mean=0, sd=1)
}
t <- c("t=", t)
a <- c("F(x)=", a)

table1 <- array(c(t, a), dim = c(10, 2))

set.seed(20180913)
norm100 <- rnorm(100)
norm1000 <- rnorm(1000)
norm10000 <- rnorm(10000)

t <- c(0.0, 0.67, 0.84, 1.28, 1.65, 2.32, 2.58, 3.09, 3.72)
b <- array(data=NA, dim=length(t))
c <- array(data=NA, dim=length(t))
```

Table 1: The CDF of Standard Normal Distribution

t=	F(x)=
0	0.5
0.67	0.74857110490469
0.84	0.79954580673955
1.28	0.899727432045558
1.65	0.950528531966352
2.32	0.98982956133128
2.58	0.995059984242229
3.09	0.998999217523386
3.72	0.999900388611024

Table 2: The Summation of Monte Carlo Methods

t=	Sum= (n=100)	Sum= (n=1000)	Sum= (n=10000)
0	0.61	0.512	0.5016
0.67	0.8	0.759	0.7498
0.84	0.84	0.805	0.7993
1.28	0.93	0.899	0.8974
1.65	0.97	0.95	0.9492
2.32	0.98	0.99	0.9889
2.58	0.99	0.995	0.9933
3.09	1	0.999	0.9988
3.72	1	1	0.9997

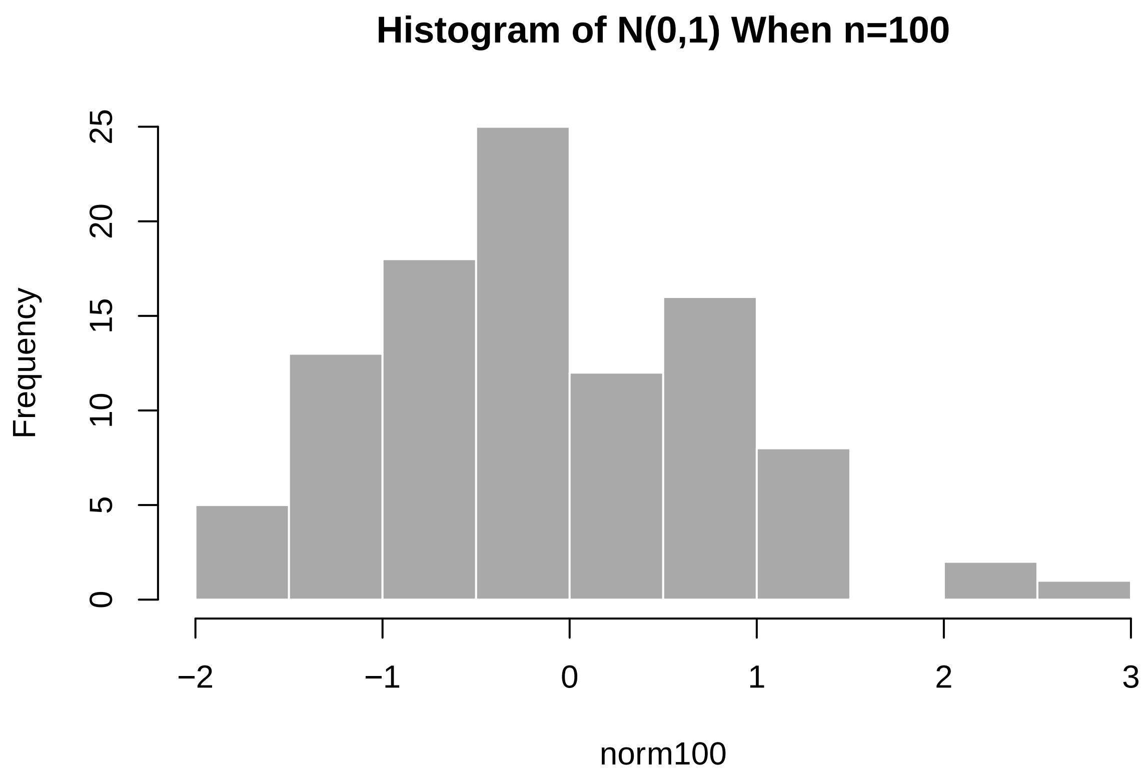
```r
d <- array(data=NA, dim=length(t))

for (i in 1:length(t)){
b[i] <- sum(norm100 <= t[i])/100
c[i] <- sum(norm1000 <= t[i])/1000
d[i] <- sum(norm10000 <= t[i])/10000
}
t <- c("t=", t)
b <- c("Sum= (n=100)", b)
c <- c("Sum= (n=1000)", c)
d <- c("Sum= (n=10000)", d)

table2 <- array(c(t, b, c, d), dim = c(10, 4))

kable(table1, caption = 'The CDF of Standard Normal Distribution', booktabs=T) %>%
  row_spec(1, bold = T, hline_after = T)
```

```r
kable(table2, caption = 'The Summation of Monte Carlo Methods', booktabs=T) %>%
  row_spec(1, bold = T, hline_after = T)
```

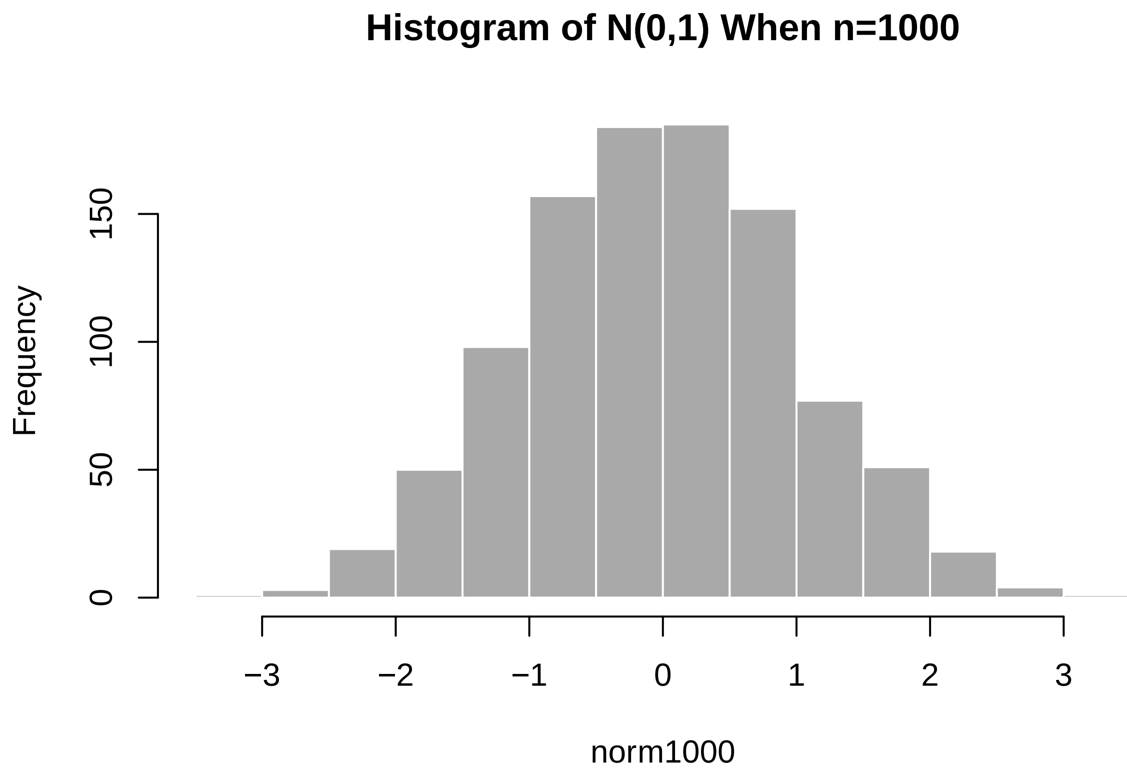
1.2 Figures

It is visual from graphs of two equations:

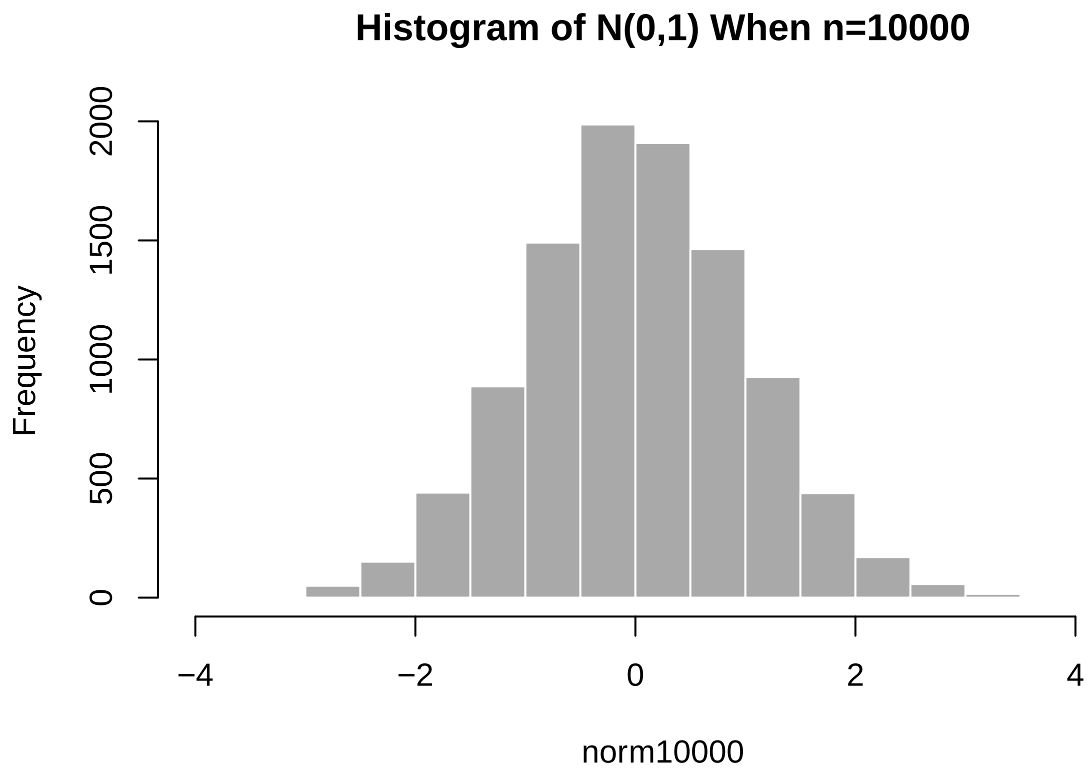
```
hist(norm100, col = 'darkgray', border = 'white', main = "Histogram of N(0,1) When n=100")
```




```
hist(norm1000, col = 'darkgray', border = 'white', main = "Histogram of N(0,1) When n=1000")
```

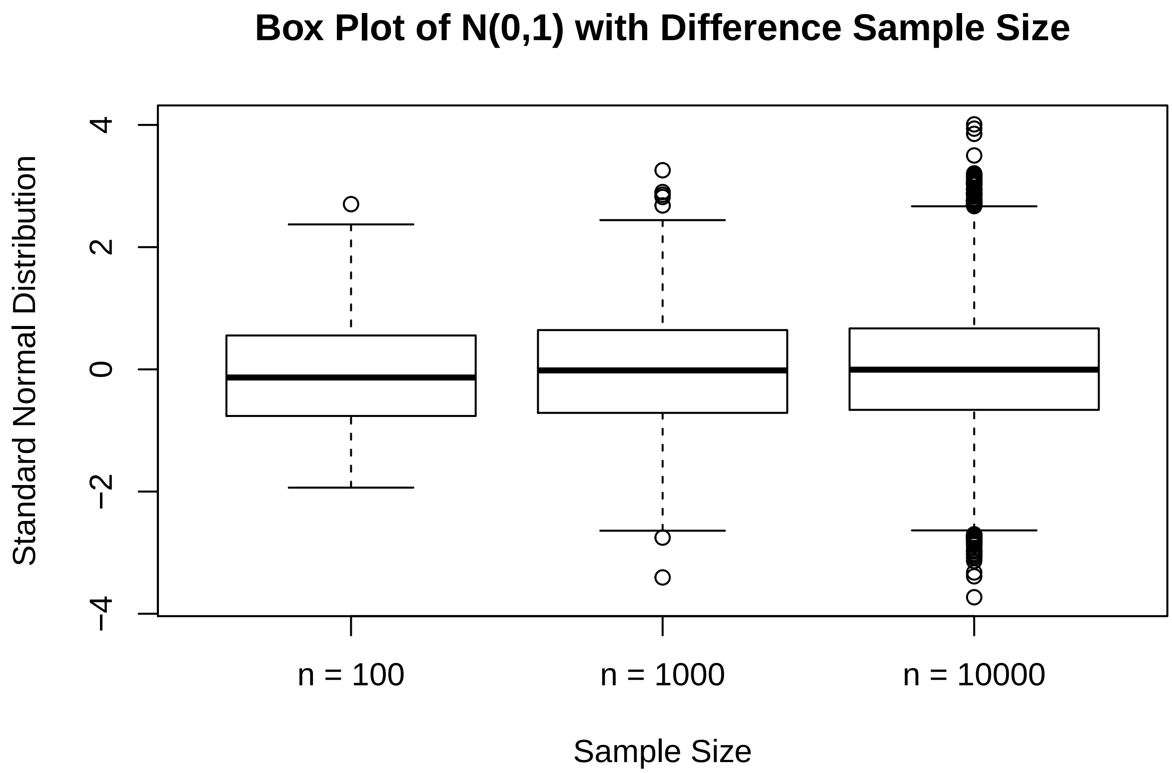



```r
hist(norm10000, col = 'darkgray', border = 'white', main = "Histogram of N(0,1) When n=10000")
```



Which is obvious that when n increasing, the distribution of samples random chose from standard normal distribution is more close to standard normal distribution, which is a continues distribution.

```
group <- array(NA, dim=11100)
for (x in 1:100){
  group[x] <- "n = 100"
}
for (x in 101:1100){
  group[x] <- "n = 1000"
}
for (x in 1101:11100){
  group[x] <- "n = 10000"
}

norm <- c(norm100, norm1000, norm10000)
boxplot(norm~group, data = NULL,
        main = "Box Plot of N(0,1) with Difference Sample Size",
        xlab = "Sample Size",
        ylab = "Standard Normal Distribution")
```



The box plots show that when n goes larger and larger, the distribution approaches a standard normal distribution. So the mean is more and more close to 0, and distribution is symmetric on 0.

Since the highly similarity of two distributions, cdf of $N(0,1)$ can be simulated by Monte Carlo method in calculation.

2 Exercise 1.3

The double-precision floating-point format is is a computer number format, usually occupying 64 bits in computer memory.

$$(-1)^{sign}(1+\sum_{i=1}^{52} b_{52-i}2^{-i}) \times 2^{e-1023}$$

2.1 .Machine$double.xmax

Which is the largest normalized floating-point number. In decimal system, it can be denoted as

$$(-1)^0(1+(1-2^{-52}))2^{1023}$$

In binary system, it can be denoted as $0\ 11111111110\ 1111111111111111111111111111111111111111111111111111_2$

```
.Machine$double.xmax
```

```
## [1] 1.797693e+308
```

2.2 .Machine$double.xmin

Which is the smallest non-vanishing normalized floating-point power of the radix. In decimal system, it can be denoted as

$$(-1)^0(1+1)2^{-1023}$$

In binary system, it can be denoted as $0\ 00000000001\ 0000000000000000000000000000000000000000000000000000_2$

```
.Machine$double.xmin
```

```
## [1] 2.225074e-308
```

2.3 .Machine$double.eps

Which is the smallest positive floating-point number x such that $1 + x\ != 1$. In decimal system, it can be denoted as

$$2^{-52}$$

In binary system, it can be denoted as 00000000001_2

```
.Machine$double.eps
```

```
## [1] 2.220446e-16
```

2.4 .Machine$double.neg.eps

Which is a small positive floating-point number x such that $1 - x\ != 1$. In decimal system, it can be denoted as

$$2^{-53}$$

In binary system, it can be denoted as 00000000000_2

```
.Machine$double.neg.eps
```

```
## [1] 1.110223e-16
```