MLE of Location Parameter of a Cauchy Distribution by Using Different Algorithms

HW 3 of STAT 5361 Statistical Computing

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Placeholder

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Newton-Raphson Method

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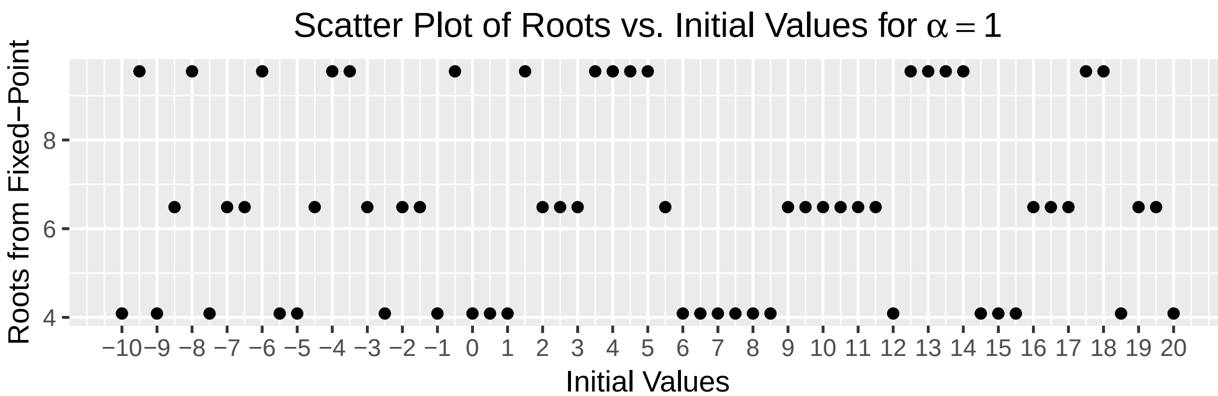
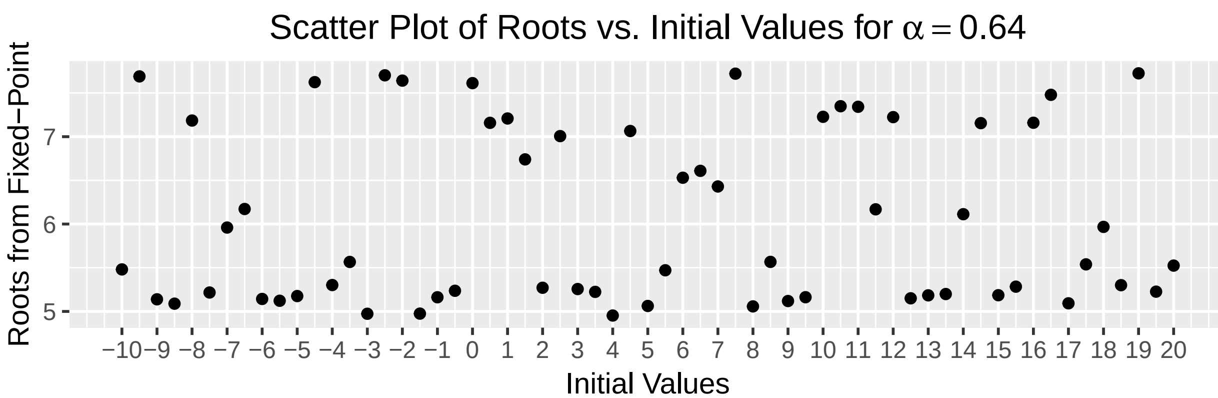
Chapter 3

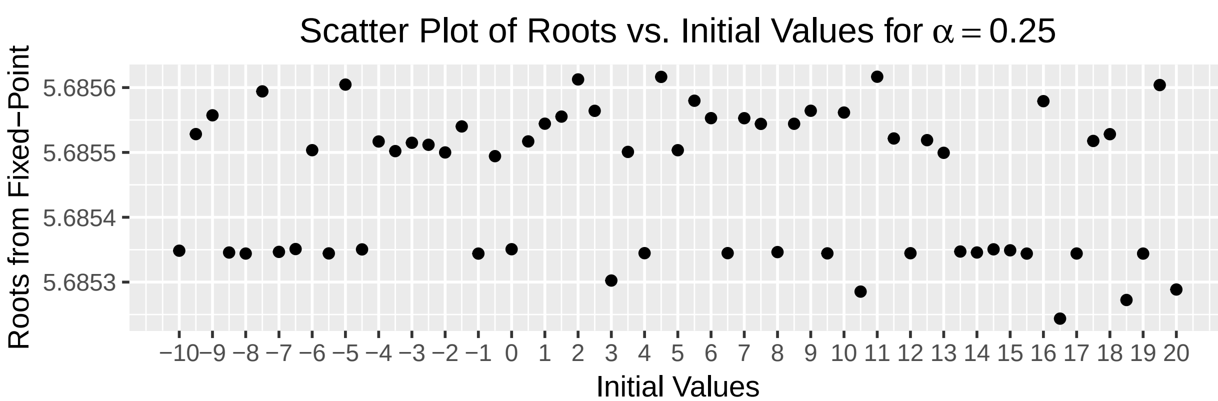
Fixed-Point Iterations

```r
set.seed(20180909)
sample <- rcauchy(10, 5)

dev1_log_sum <- function(x){
  dev1_log_sum <- 0
  for (i in 1:length(sample)) {
    dev1_log_sum <- dev1_log_sum - 2 * (x - sample[i])/(1 + (x - sample[i])^2)
  }
  dev1_log_sum
}

fixed.point <- function(init, fun, alpha, maxiter = 100, tol = .Machine$double.eps^0.2){
  x <- init
  for (i in 1:maxiter) {
    x1 <- alpha * fun(x) + x
    if(abs(x1 - x) < tol) break
    x <- x1
  }
  if(i == maxiter)
    message("Reached the maximum iteration!")

  return(data.frame(root = x1, iter = i))
}

init <- seq(-10, 20, by = 0.5)

res1 <- data.frame(init = init, root = rep(NA, length(init)))
res2 <- data.frame(init = init, root = rep(NA, length(init)))
res3 <- data.frame(init = init, root = rep(NA, length(init)))

for (i in 1:length(init)) {
  res1$root[i] <- fixed.point(init[i], dev1_log_sum, 1)$root
}
for (i in 1:length(init)) {
  res2$root[i] <- fixed.point(init[i], dev1_log_sum, 0.64)$root
}
for (i in 1:length(init)) {
```

```r
  res3$root[i] <- fixed.point(init[i], dev1_log_sum, 0.25)$root
}

library("ggplot2")
library("gridExtra")

p1 <- ggplot(res1, aes(x = init, y = root)) + geom_point() +
scale_x_continuous(breaks = round(seq(min(res1$init), max(res1$init), by = 1),1)) +
labs(x = "Initial Values", y = "Roots from Fixed-Point") +
theme(plot.title = element_text(hjust = 0.5)) +
ggtitle(expression("Scatter Plot of Roots vs. Initial Values for"~alpha == 1))
p2 <- ggplot(res2, aes(x = init, y = root)) + geom_point() +
scale_x_continuous(breaks = round(seq(min(res2$init), max(res2$init), by = 1),1)) +
labs(x = "Initial Values", y = "Roots from Fixed-Point") +
theme(plot.title = element_text(hjust = 0.5)) +
ggtitle(expression("Scatter Plot of Roots vs. Initial Values for"~alpha == 0.64))
p3 <- ggplot(res3, aes(x = init, y = root)) + geom_point() +
scale_x_continuous(breaks = round(seq(min(res3$init), max(res3$init), by = 1),1)) +
labs(x = "Initial Values", y = "Roots from Fixed-Point") +
theme(plot.title = element_text(hjust = 0.5)) +
ggtitle(expression("Scatter Plot of Roots vs. Initial Values for"~alpha == 0.25))

grid.arrange(p1, p2, p3, nrow = 3)
```







From the plots above, we can see when the α decreases, the outcomes become better.

Chapter 4

Fisher Scoring Method and Newton-Raphson Method

Chapter 5

Comments