

Homework 5

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1 Finite mixture regression

1.1 Follow the lecture notes to verify the validity of the provided E-step and M-step.

Based on the lecture notes, we have

$$Q(\Psi|\Psi^{k+1}) = E(l_n^c(\Psi)|x, y, \Psi^{k+1})$$

Since the complete log-likelihood can be written as

$$l_n^c(\Psi) = \sum_{i=1}^n \sum_{j=1}^m z_{ij} \log\{\pi_j \phi(y_i - x_i' \beta_j; 0, \sigma^2)\},$$

and notice that, only z_{ij} is the unknown part in the complete data, the expectation with respect to the complete data will affect z_{ij} only. So we have

$$\begin{aligned} Q(\Psi|\Psi^{k+1}) &= \sum_{i=1}^n \sum_{j=1}^m E(z_{ij}|x_i, y_i, \Psi^k) \log\{\pi_j \phi(y_i - x_i' \beta_j; 0, \sigma^2)\} \\ &= \sum_{i=1}^n \sum_{j=1}^m p_{ij}^{(k+1)} \log\{\pi_j \phi(y_i - x_i' \beta_j; 0, \sigma^2)\}. \end{aligned}$$

Then we consider $p_{ij}^{(k+1)}$, since z_{ij} only takes two values, 1 and 0, its conditional expectation is just the conditional probability of taking value 1. Then based on Bayes rule, we have

$$\begin{aligned} p_{ij}^{(k+1)} &= E(z_{ij}|x_i, y_i, \Psi^k) \\ &= p(x_{ij}|x_i, y_i, \Psi^k) \\ &= \frac{\pi_j^{(k)} \phi(y_i - x_i' \beta_j; 0, \sigma^{2(k)})}{\sum_{j=1}^m \pi_j^{(k)} \phi(y_i - x_i' \beta_j; 0, \sigma^{2(k)})}. \end{aligned}$$

Then by taking first order derivative of function $Q(\Psi|\Psi^{k+1})$ with respect to $\{\pi_j\}$, $\beta_j, j = 1, \dots, m$ and σ^2 separately, and set them to be 0, we can get the MLE of these parameters.

1.2 Implement this algorithm in R

```
regmix_em <- function(y,xmat,pi.init,beta.init,sigma.init,
                      control=list(max.ite,con.tol)){
  n <- length(y)
  p <- ncol(xmat)
  k <- ncol(beta.init)
  err <- 100
  ite <- 0
  conver <- 0
  max.ite <- control[[1]]
  con.tol <- control[[2]]
  xmat <- as.matrix(xmat)
  while ((err > con.tol)&(ite < max.ite)) {
    p.mat <- matrix(nrow = n, ncol = k)
    for (i in 1:n){
      for (j in 1:k){
        p.mat[i,j] <- pi.init[j]*dnorm(y[i]-t(xmat[i,])%*%beta.init[,j],0,sigma.init)/sum(pi.i
      }
    }
    pi.ite <- apply(p.mat,2,mean)
    beta.ite <- matrix(nrow = p, ncol = k)
    for (j in 1:k){
      upp <- 0
      low <- 0
      for (i in 1:n){
        upp <- upp+xmat[i,]%*%t(xmat[i,])*p.mat[i,j]
        low <- low+xmat[i,]*y[i]*p.mat[i,j]
      }
      beta.ite[,j] <- solve(upp)%*%low
    }
    sigma2.ite <- 0
    for (j in 1:k){
      sigma2.ite <- sigma2.ite+sum((p.mat[,j]*(y-xmat%*%beta.ite[,j]))^2)
    }
    sigma.ite <- sqrt(sigma2.ite/n)
    #err <- sqrt(sum((pi.ite-pi.init)^2)+sum((beta.ite-beta.init)^2)+
    #          sum((sigma.ite-sigma.init)^2))
    err <- sum(abs(pi.ite-pi.init))+sum(abs(beta.ite-beta.init))+
            sum(abs(sigma.ite-sigma.init))
    pi.init <- pi.ite
    beta.init <- beta.ite
    sigma.init <- sigma.ite
    ite <- ite+1
  }
  if (ite >= max.ite) conver <- 1
  return(list(pi.est=pi.init, beta.est=beta.init,
```

```

        sigma.est=sigma.init, converge=conver))
}

```

1.3 Generate data to test the algorithm

```

regmix_sim <- function(n, pi, beta, sigma) {
  K <- ncol(beta)
  p <- nrow(beta)
  xmat <- matrix(rnorm(n * p), n, p) # normal covaraites
  error <- matrix(rnorm(n * K, sd = sigma), n, K)
  ymat <- xmat %*% beta + error # n by K matrix
  ind <- t(rmultinom(n, size = 1, prob = pi))
  y <- rowSums(ymat * ind)
  data.frame(y, xmat)
}

n <- 400
pi <- c(.3, .4, .3)
bet <- matrix(c( 1, 1, 1,
                -1, -1, -1), 2, 3)

sig <- 1
set.seed(1205)
dat <- regmix_sim(n, pi, bet, sig)
regmix_em(y = dat[,1], xmat = dat[,-1],
  pi.init = pi / pi / length(pi),
  #beta.init = matrix(c( 1, 2, 3,
  #                    -1, -1, -1), 2, 3),
  beta.init = bet * 0,
  sigma.init = sig / sig,
  control = list(max.ite = 500, con.tol = 1e-5))

## $pi.est
## [1] 0.3333333 0.3333333 0.3333333
##
## $beta.est
##          [,1]      [,2]      [,3]
## [1,] 0.3335660 0.3335660 0.3335660
## [2,] -0.4754645 -0.4754645 -0.4754645
##
## $sigma.est
## [1] 1.732492
##
## $converge
## [1] 0

```