MCMC Project

Yiyi Xu

10/25/2018

Given μ_1, μ_2, δ, σ_1 ,σ_2 , the distribution of the normal mixture is following:

$$f(x) = \delta N(\mu_1, \sigma_1^2) + (1-\delta)N(\mu_2, \sigma_2^2)$$

use $\mu_1 = 7$, $\mu_2 = 10$, $\delta = 0.7$, $\sigma_1 = 0.5$,$\sigma_2 = 0.5$ to get sample.

```
delta <- 0.7 # true value to be estimated based on the data
n <- 100
set.seed(123)
u <- rbinom(n, prob = delta, size = 1)
sample <- rnorm(n, ifelse(u == 1, 7, 10), 0.5)
```

Define Loglikehood function:

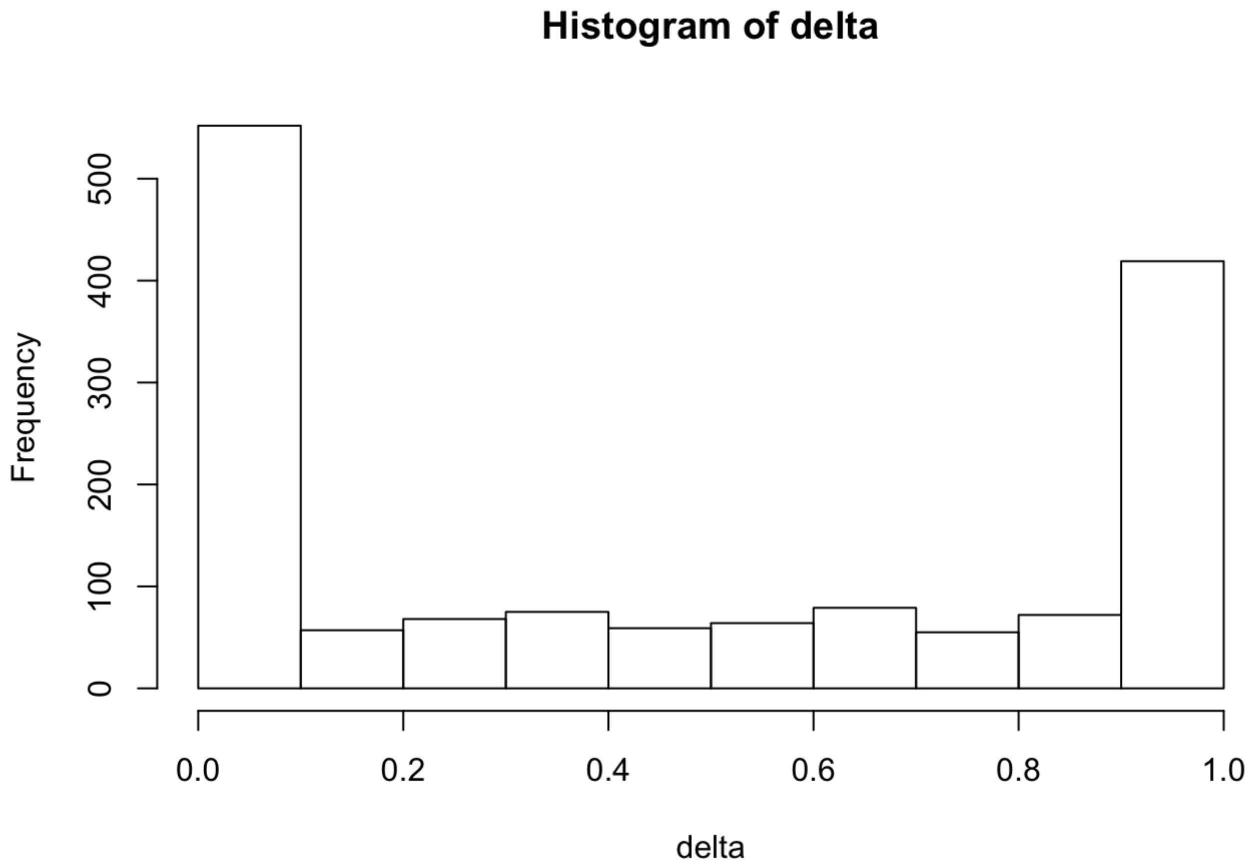
$$L(\mu_1, \mu_2, \delta, \sigma_1, \sigma_2; x) = \prod_{i=1}^{n}[\delta\frac{1}{\sqrt{2\pi}\sigma_1}e^{-\frac{(x_i-\mu_1)^2}{2\sigma_1^2}} + (1-\delta)\frac{1}{\sqrt{2\pi}\sigma_2}e^{-\frac{(x_i-\mu_2)^2}{2\sigma_2^2}}]$$

$$l(\mu_1, \mu_2, \delta, \sigma_1, \sigma_2; x) = logL(\mu_1, \mu_2, \delta, \sigma_1, \sigma_2; x) = \sum_{i=1}^{n}\log[\delta\frac{1}{\sqrt{2\pi}\sigma_1}e^{-\frac{(x_i-\mu_1)^2}{2\sigma_1^2}} + (1-\delta)\frac{1}{\sqrt{2\pi}\sigma_2}e^{-\frac{(x_i-\mu_2)^2}{2\sigma_2^2}}]$$

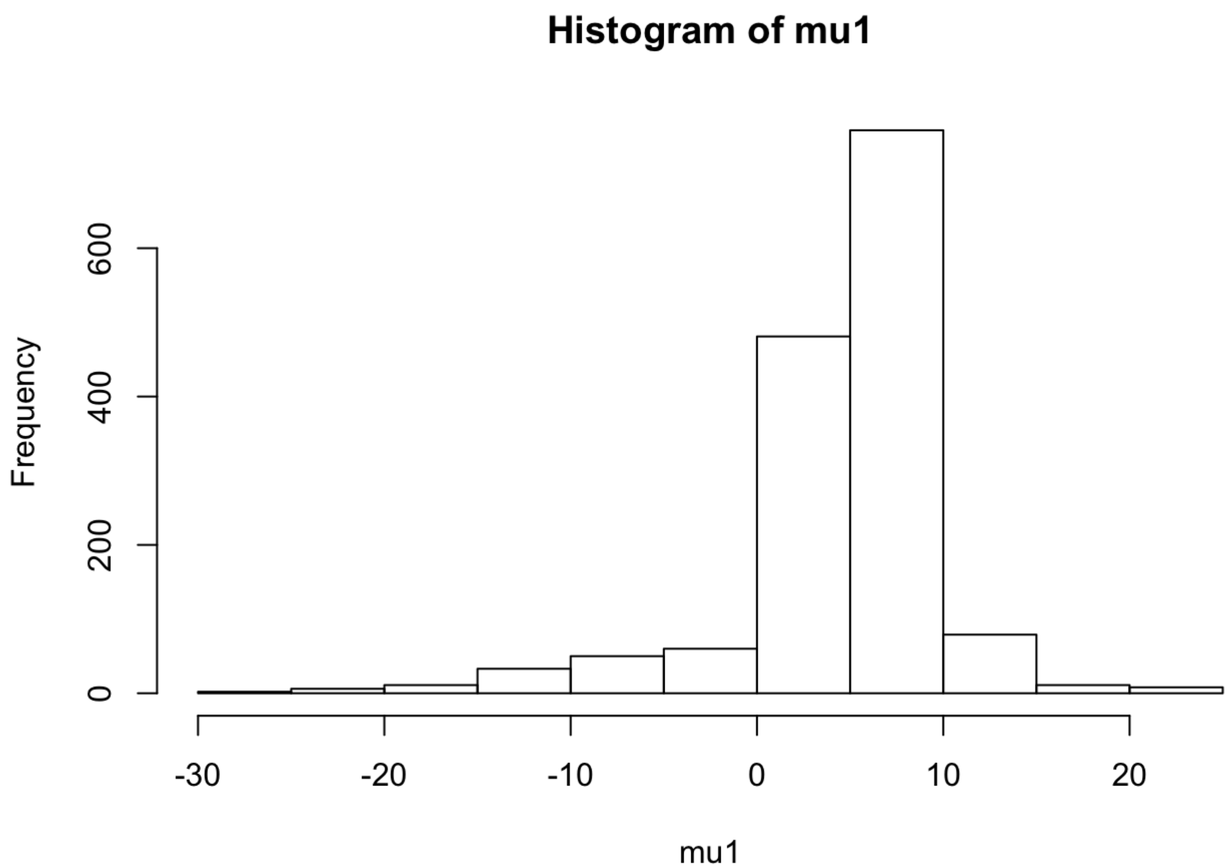
``` Define prior function: as prior for $\mu_1$ and $\mu_2$ are $N(0, 10^2)$, prior for $\sigma_1$ ,$\sigma_2$ are $IG(0.5, 10)$, the prior function is following:

$$\pi(\mu_1) = \frac{1}{\sqrt{2\pi}10}e^{-\frac{(\mu_1)^2}{2*10^2}}$$

$$\pi(\mu_2) = \frac{1}{\sqrt{2\pi}10}e^{-\frac{(\mu_2)^2}{2*10^2}}$$

$$\pi(\sigma_1^2) = \frac{10^{0.5}}{\Gamma(0.5)}(\sigma_1^2)^{-0.5-1}e^{-\frac{10}{\sigma_1^2}}$$

$$\pi(\sigma_2^2) = \frac{10^{0.5}}{\Gamma(0.5)}(\sigma_2^2)^{-0.5-1}e^{-\frac{10}{\sigma_2^2}}$$

Define log posterior function

$$\pi(\mu_1, \mu_2, \delta, \sigma_1^2, \sigma_2^2; x) \propto L(\mu_1, \mu_2, \delta, \sigma_1^2, \sigma_2^2; x)\pi(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2)$$

$$\log\pi(\mu_1, \mu_2, \delta, \sigma_1^2, \sigma_2^2; x) = \log L + \log\pi(\mu_1) + \log\pi(\mu_2) + \log\pi(\sigma_1^2) + \log\pi(\sigma_2^2)$$
```

```
library("invgamma")
log.pos <-function(u1,u2,s1,s2,d,x=sample){
p1<-d*dnorm(x,u1,sqrt(s1))
p2<-(1-d)*dnorm(x,u2,sqrt(s2))
 logL <- sum(log(p1+p2))
 prior.u1 <- dnorm(u1,0,10)
 prior.u2 <- dnorm(u2,0,10)
 prior.s1 <- dinvgamma(s1,0.5,10)
 prior.s2 <- dinvgamma(s2,0.5,10)
 sum(logL+log(prior.u1)+log(prior.u2)+log(prior.s1)+log(prior.s2))
}
```

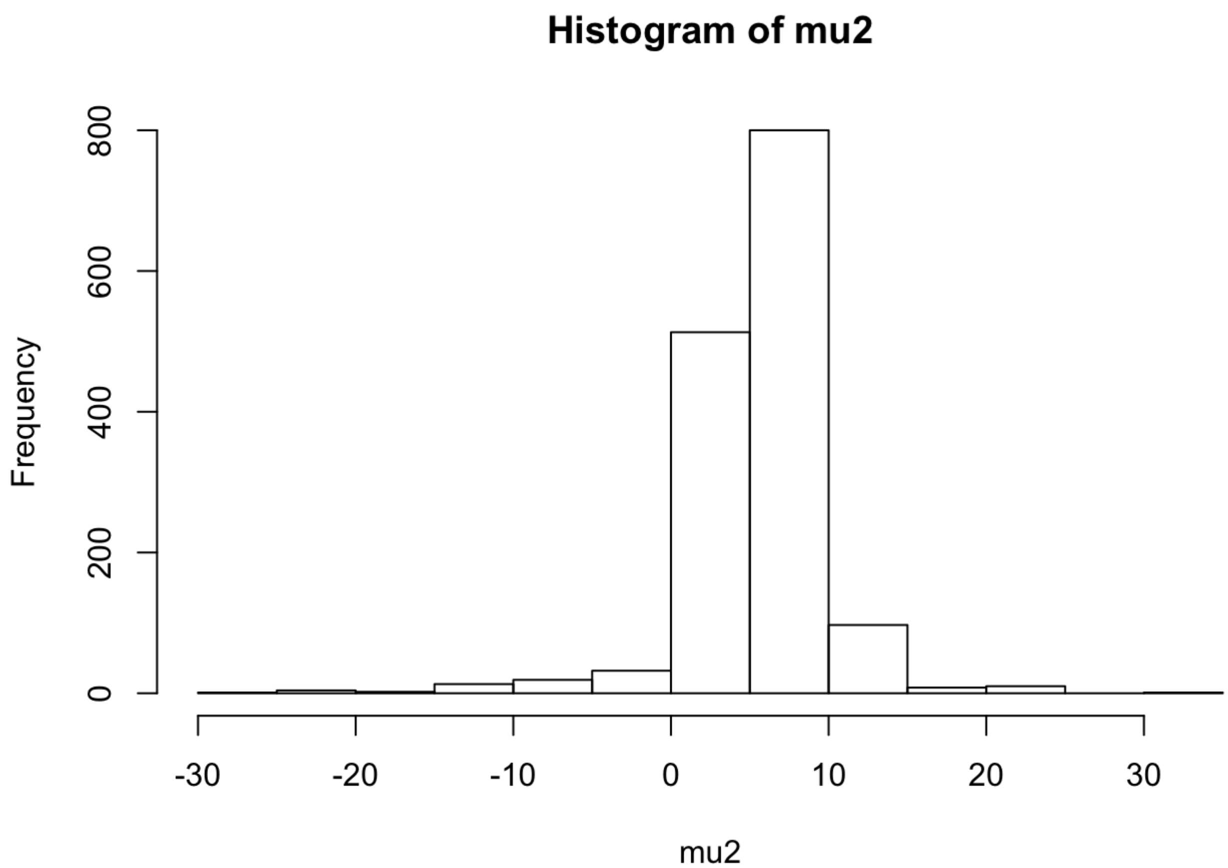
```
library("HI")
gibb_fun <- function(d_,u1_,u2_,s1_,s2_,x=sample,n){
  gibb<- matrix(nrow=n, ncol=5)
  ini <- c(d_,u1_,u2_,s1_,s2_)
  for(i in 1:n ){
     gibb[i,1] <- arms(d_,log.pos,function(x,...)(x>0)*(x<1),1,u1=ini[2],u2=ini[3],s1=in
i[4],s2=ini[5])
     ini[1] <-gibb[i,1]

     gibb[i,2] <- arms(u1_,log.pos,function(x,...)(x>-50)*(x<50),1,d=ini[1],u2=ini[3],s1
=ini[4],s2=ini[5])
     ini[2] <-gibb[i,2]

     gibb[i,3] <- arms(u2_,log.pos,function(x,...)(x>-50)*(x<50),1,d=ini[1],u1=ini[2],s1
=ini[4],s2=ini[5])
     ini[3] <-gibb[i,3]

     gibb[i,4] <- arms(s1_,log.pos,function(x,...)(x>0)*(x<50),1,d=ini[1],u1=ini[2],u2=i
ni[3],s2=ini[5])
     ini[4] <-gibb[i,4]

     gibb[i,5] <- arms(s2_,log.pos,function(x,...)(x>0)*(x<50),1,d=ini[1],u1=ini[2],u2=i
ni[3],s1=ini[4])
     ini[5] <-gibb[i,5]
  }
     gibb
}

gibb.result<-gibb_fun(0.5,5,5,1,1,sample,3000)[-(1:1500),]
hist(gibb.result[,1],main="Histogram of delta",xlab="delta")
```



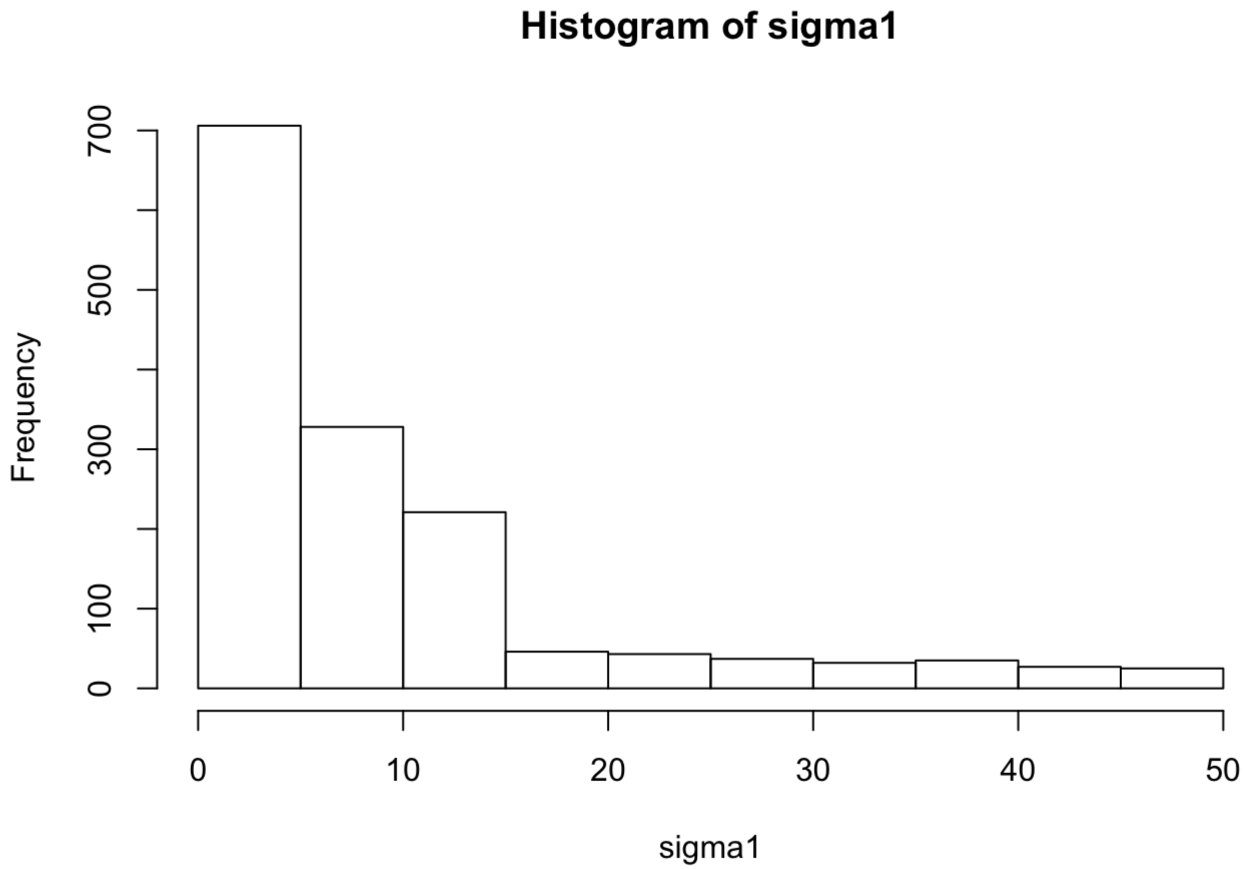

```
hist(gibb.result[,2],main="Histogram of mu1",xlab="mu1")
```



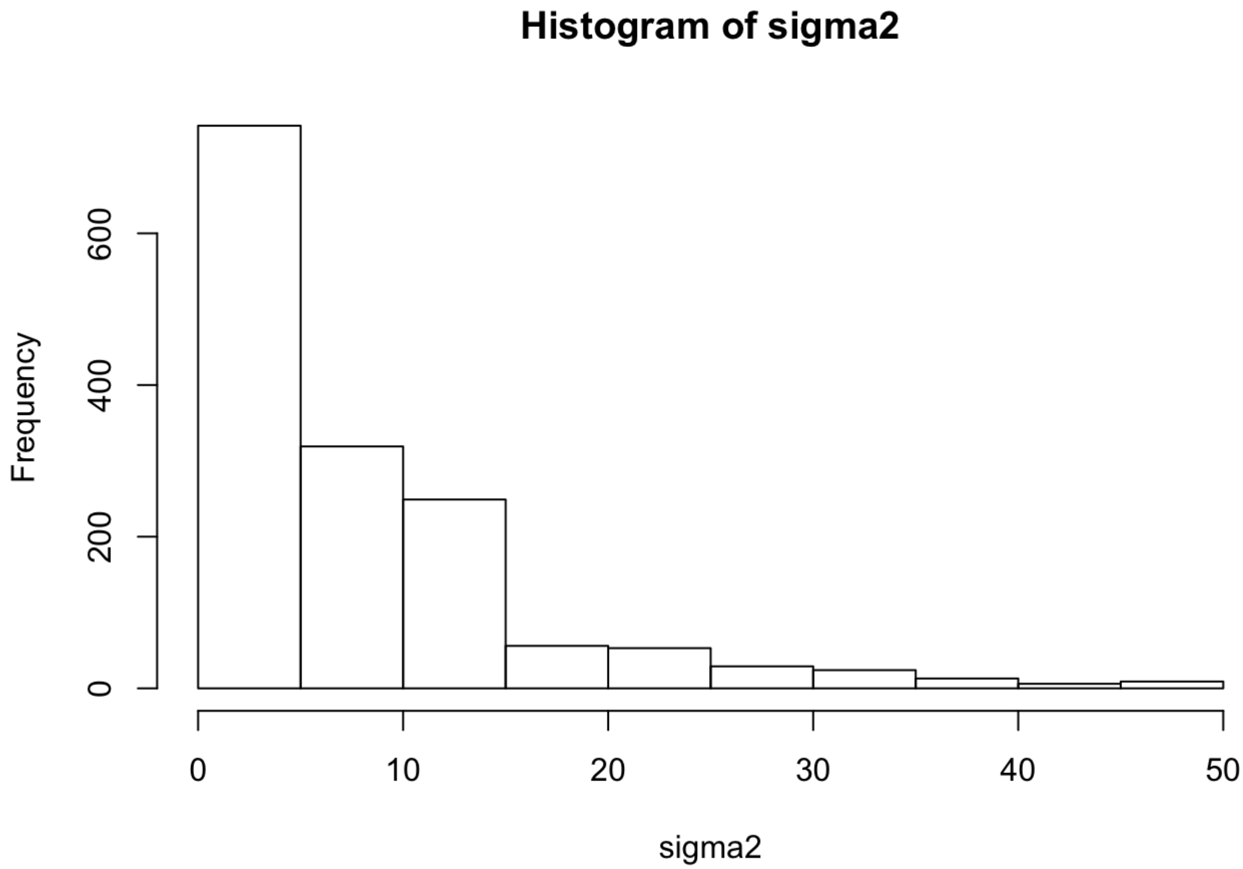

```
hist(gibb.result[,3],main="Histogram of mu2",xlab="mu2")
```




```
hist(gibb.result[,4],main="Histogram of sigma1",xlab="sigma1")
```




```
hist(gibb.result[,5],main="Histogram of sigma2",xlab="sigma2")
```


Histogram of sigma2
Frequency
0
200
400
600
0
10
20
30
40
50
sigma2