Ex7

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1. MCMC With Gibbs Sampling

We have parameters below.

$$\mu_1 \sim N(0, 10^2)$$
$$\mu_2 \sim N(0, 10^2)$$
$$\frac{1}{\sigma_1^2} \sim \Gamma(a, b)$$
$$\frac{1}{\sigma_2^2} \sim \Gamma(a, b)$$
$$a = 0.5,\ \ b = 10, \lambda = \frac{1}{b} = \frac{1}{10}$$
$$\sigma_1^2 \sim IG(0.5, 10)$$
$$\sigma_2^2 \sim IG(0.5, 10)$$
$$\delta \sim U(0, 1)$$

The density function of X is a mixture normal distribution.

$$f(x) = \delta \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} + (1-\delta)\frac{1}{\sqrt{2\pi}\sigma_2} e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}}$$

The Likelihood function is as follow.

$$L(x; \delta, \mu_1, \sigma_1^2, \mu_2, \sigma_2^2) = \prod_{i=1}^{n} (\delta \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} + (1-\delta)\frac{1}{\sqrt{2\pi}\sigma_2} e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}})$$

The joint posterior probability is as follow.

$$\begin{aligned} P(\delta, \mu_1, \sigma_1^2, \mu_2, \sigma_2^2|x) &\propto L(x; \delta, \mu_1, \sigma_1^2, \mu_2, \sigma_2^2) * f_1(\mu_1) * f_2(\mu_2) * f_3(\sigma_1^2) * f_4(\sigma_2^2) \\ &= \prod_{i=1}^{n} (\delta \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} + (1-\delta)\frac{1}{\sqrt{2\pi}\sigma_2} e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}}) \\ &\times 1 \times \frac{1}{10\sqrt{2\pi}} e^{-\frac{\mu_1^2}{2\times 10^2}} \times \frac{1}{\sqrt{10}\sqrt{2\pi}} \frac{1}{(\sigma_1^2)^{\frac{3}{2}}} e^{-\frac{1}{10\sigma_1^2}} \times \frac{1}{10\sqrt{2\pi}} e^{-\frac{\mu_2^2}{2\times 10^2}} \times \frac{1}{\sqrt{10}\sqrt{2\pi}} \frac{1}{(\sigma_2^2)^{\frac{3}{2}}} e^{-\frac{1}{10\sigma_2^2}} \end{aligned}$$

We need to get

$$P(\delta|x, \mu_1, \sigma_1^2, \mu_2, \sigma_2^2), P(\mu_1|x, \delta, \sigma_1^2, \mu_2, \sigma_2^2), P(\sigma_1^2|x, \delta, \mu_1, \mu_2, \sigma_2^2), P(\mu_2|x, \delta, \mu_1, \sigma_1^2, \sigma_2^2), P(\sigma_2^2|x, \delta, \mu_1, \sigma_1^2, \mu_2)$$

2. Code And Graph

```
library(actuar)
```

```
##
## Attaching package: 'actuar'

## The following object is masked from 'package:grDevices':
##
##     cm
```

```
delta <- 0.7 # true value to be estimated based on the data
n <- 100
set.seed(123)
u <- rbinom(n, prob = delta, size = 1)
x <- rnorm(n, ifelse(u == 1, 7, 10), 0.5)

loglikelihood=function(delta,mu1,mu2,var1,var2){
  sum(log(delta*dnorm(x,mu1,sqrt(var1))+(1-delta)*dnorm(x,mu2,sqrt(var2))))
}
prior=function(delta,mu1,mu2,var1,var2){
  delta.prior=1
  mu1.prior=dnorm(mu1,0,10)
  mu2.prior=dnorm(mu2,0,10)
  var1.prior=dinvgamma(var1,0.5,10)
  var2.prior=dinvgamma(var2,0.5,10)
  log(mu1.prior*mu2.prior*var1.prior*var2.prior)
}
all=function(delta,mu1,mu2,var1,var2){
  loglikelihood(delta,mu1,mu2,var1,var2)+prior(delta,mu1,mu2,var1,var2)
}
library(HI)
mcmcgibbs=function(delta.start,mu1.start,mu2.start,var1.start,var2.start,count){
  delta.new=rep(0,count)
  mu1.new=rep(0,count)
  mu2.new=rep(0,count)
  var1.new=rep(0,count)
  var2.new=rep(0,count)
  for(i in 1:count){
    delta.new[i]=arms(delta.start,all,function(x,mu1,mu2,var1,var2)(x>0)*(x<1),1,
                      mu1=mu1.start,mu2=mu2.start,var1=var1.start,var2=var2.start)
    mu1.new[i]=arms(mu1.start,all,function(delta,x,mu2,var1,var2)(x>-50)*(x<50),1,
                    delta=delta.start,mu2=mu2.start,var1=var1.start,var2=var2.start)
    mu2.new[i]=arms(mu2.start,all,function(delta,mu1,x,var1,var2)(x>-50)*(x<50),1,
                    mu1=mu1.start,delta=delta.start,var1=var1.start,var2=var2.start)
    var1.new[i]=arms(var1.start,all,function(delta,mu1,mu2,x,var2)(x>0.00001)*(x<100),1,
                     mu1=mu1.start,mu2=mu2.start,delta=delta.start,var2=var2.start)
    var2.new[i]=arms(var2.start,all,function(delta,mu1,mu2,var1,x)(x>0.00001)*(x<100),1,
                     mu1=mu1.start,mu2=mu2.start,var1=var1.start,delta=delta.start)
    delta.start=delta.new[i]
    mu1.start=mu1.new[i]
    mu2.start=mu2.new[i]
    var1.start=var1.new[i]
    var2.start=var2.new[i]
```

```
  }
  rbind(delta.new,mu1.new,mu2.new,var1.new,var2.new)
}
```

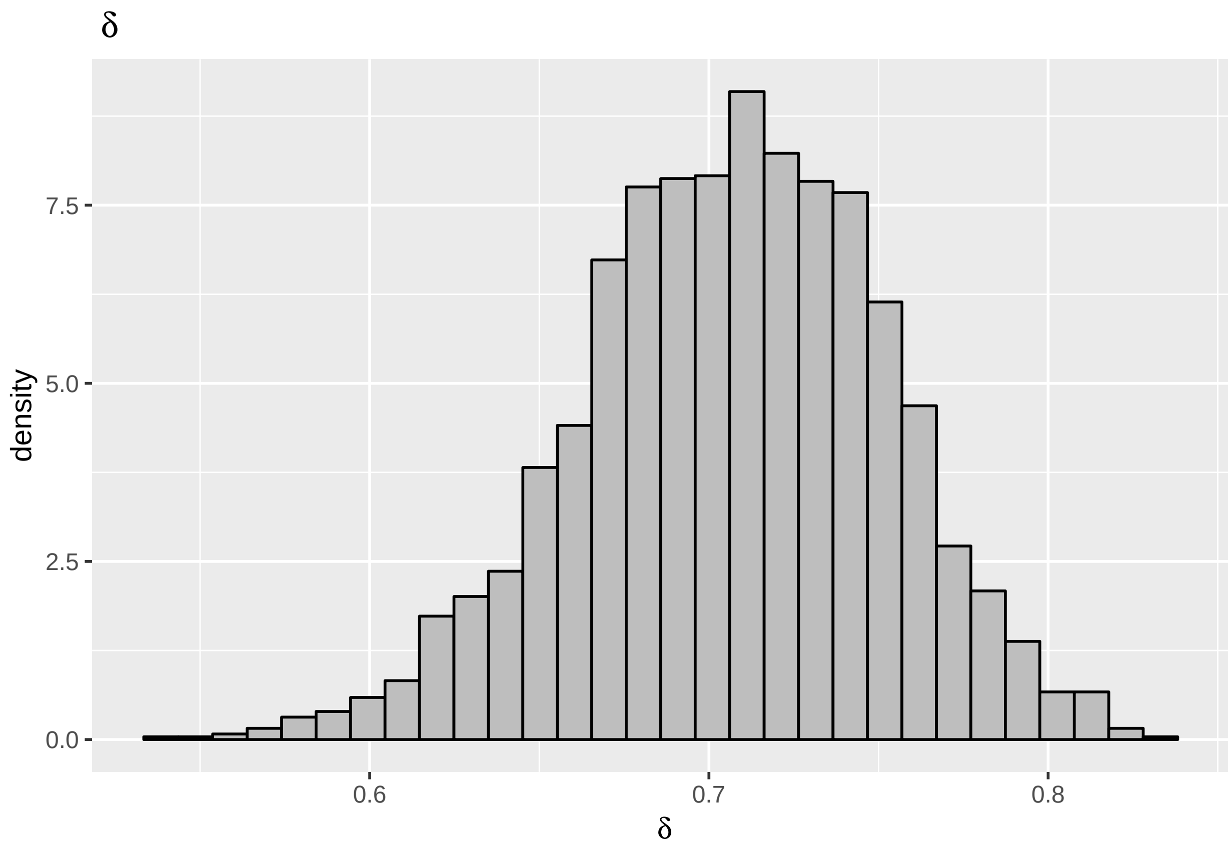
With initial value$\delta^{(0)} = 0.5, \mu_1^{(0)} = 0, \sigma_1^{2(0)} = 0, \mu_2^{(0)} = 10, \sigma_2^{2(0)} = 10$, we run the 5000 times and throw away the first 2500.

```
data=mcmcgibbs(0.5,0,0,10,10,5000)[,-(1:2500)]
```

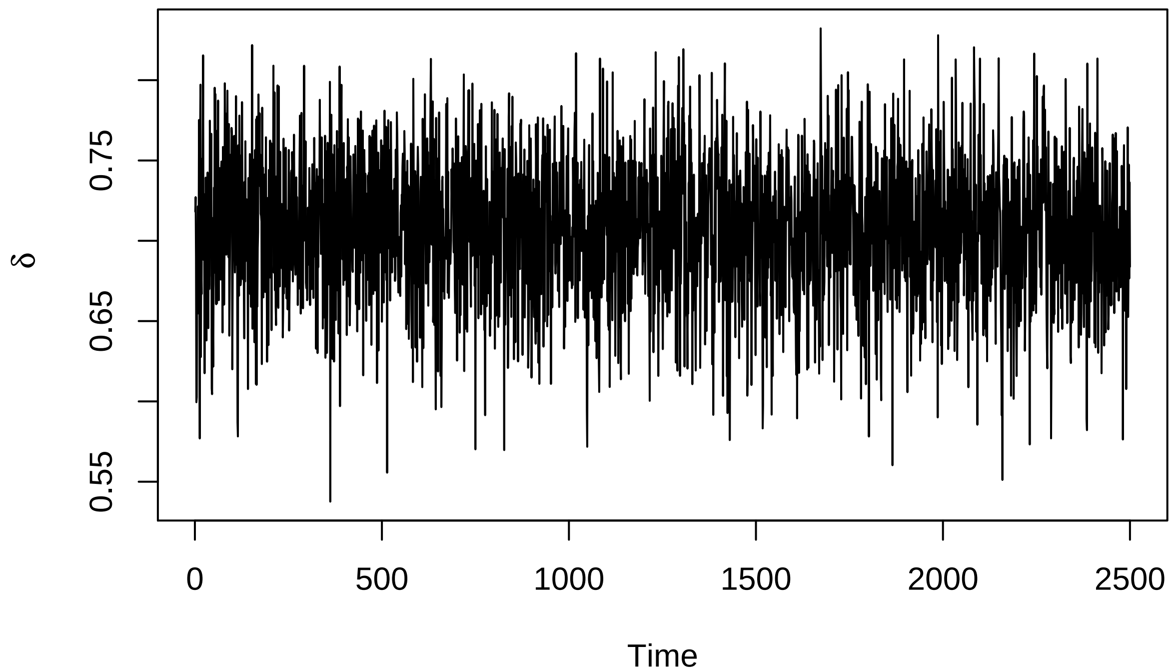
```
library(ggplot2)

ggplot(data.frame(x=data[1,]),aes(x=x))+
xlab(~delta)+
ylab("density")+
ggtitle(~delta)+
geom_histogram(aes(y=..density..),fill="gray", colour="black")
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

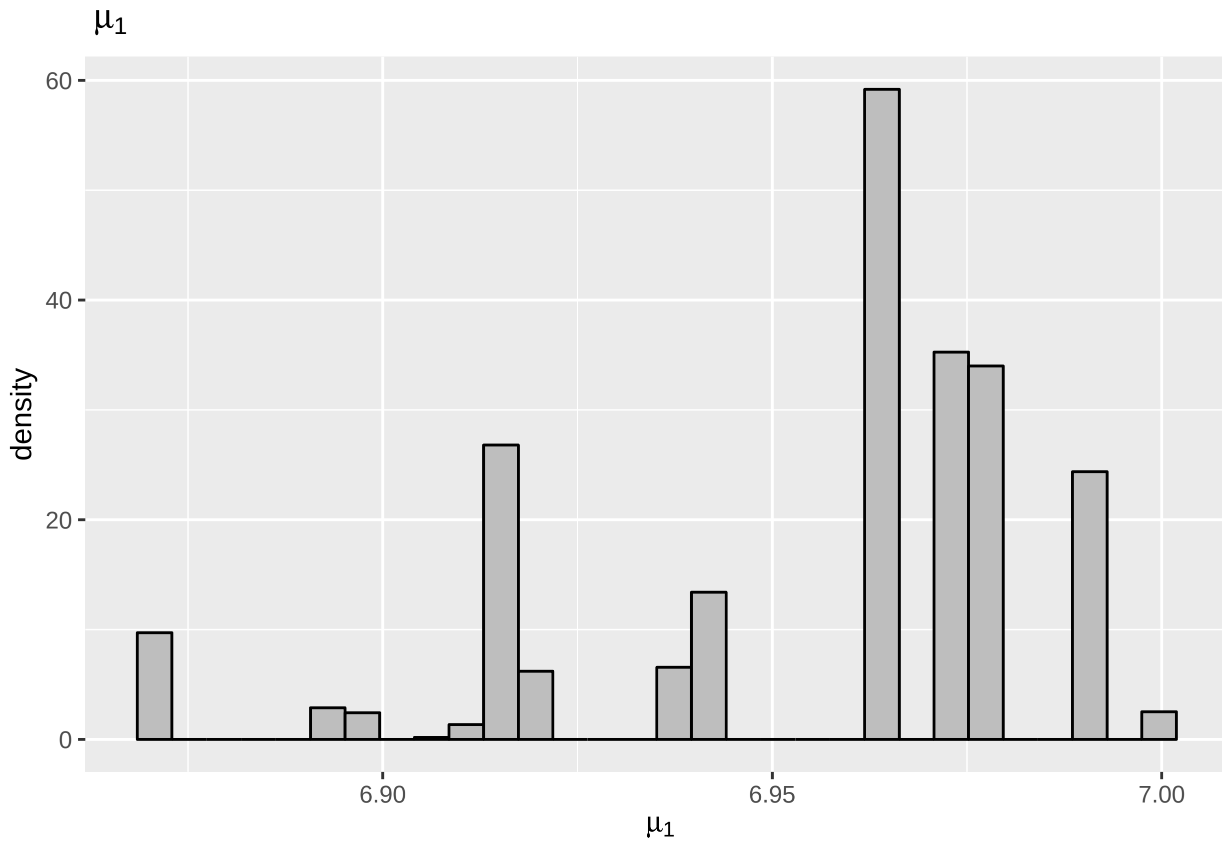



```
plot(ts(data[1,]),xlab="Time",ylab=~delta)
```

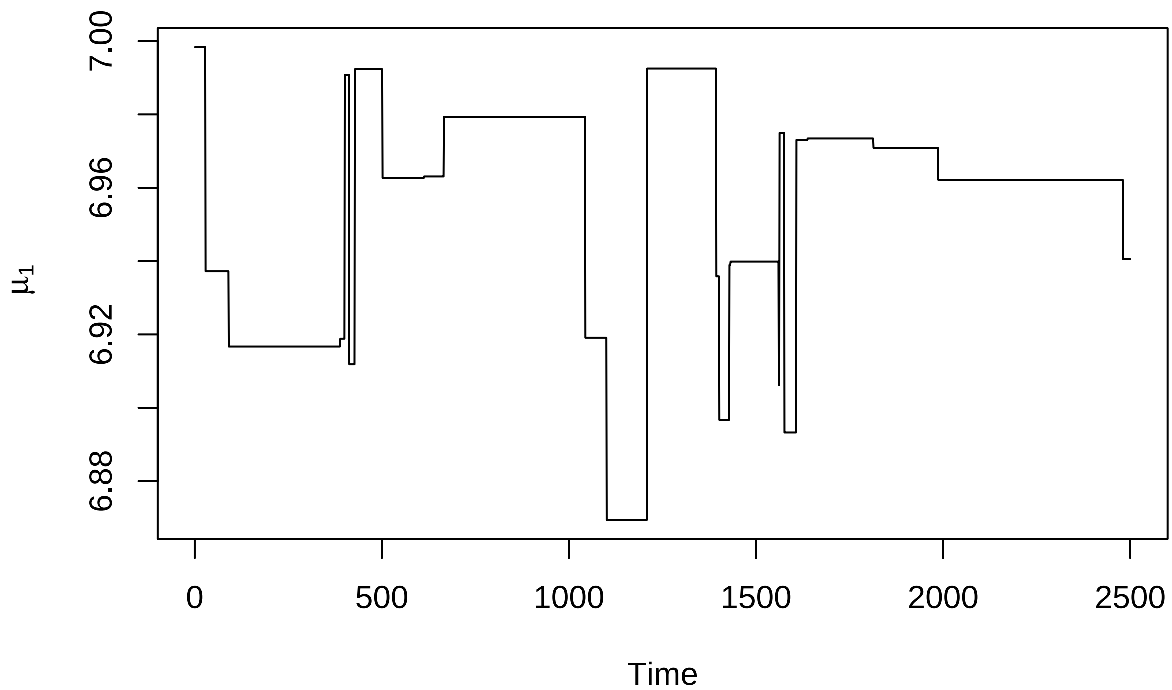



```r
ggplot(data.frame(x=data[2,]),aes(x=x))+
xlab(~mu[1])+
ylab("density")+
ggtitle(~mu[1])+
geom_histogram(aes(y=..density..),fill="gray", colour="black")
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

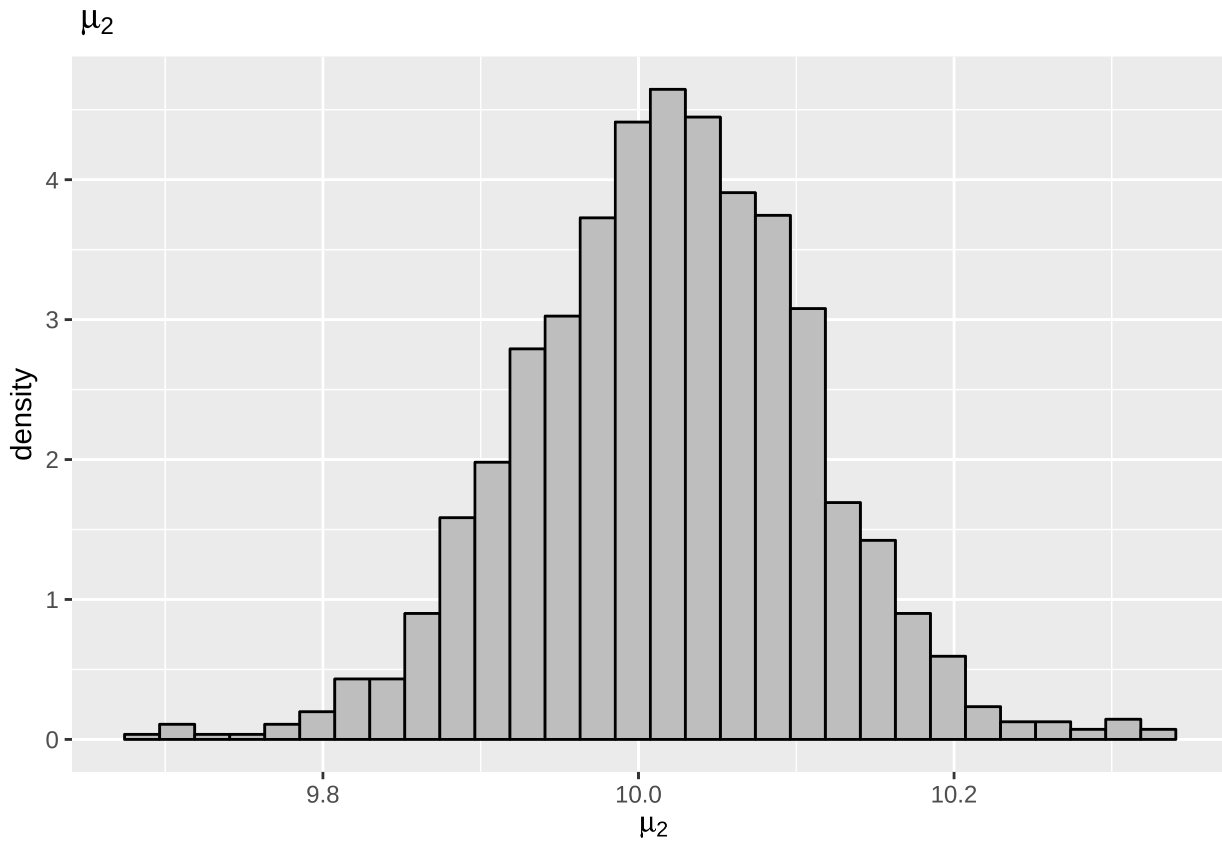



```
plot(ts(data[2,]),xlab="Time",ylab=~mu[1])
```

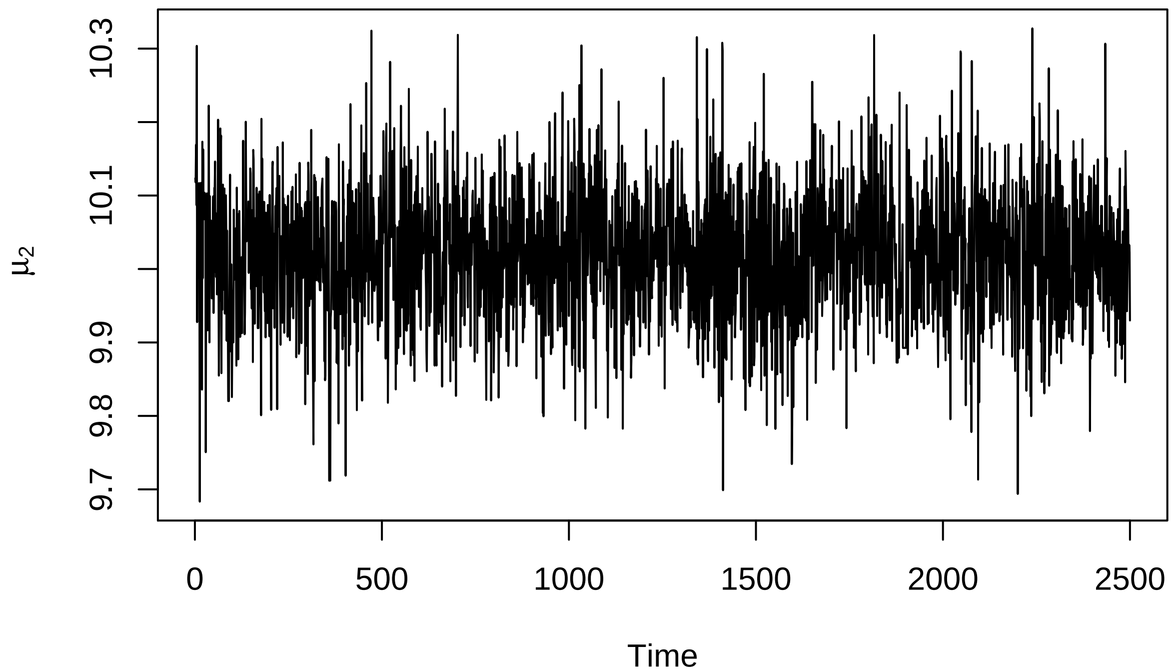



```
ggplot(data.frame(x=data[3,]),aes(x=x))+
xlab(~mu[2])+
ylab("density")+
ggtitle(~mu[2])+
geom_histogram(aes(y=..density..),fill="gray", colour="black")
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

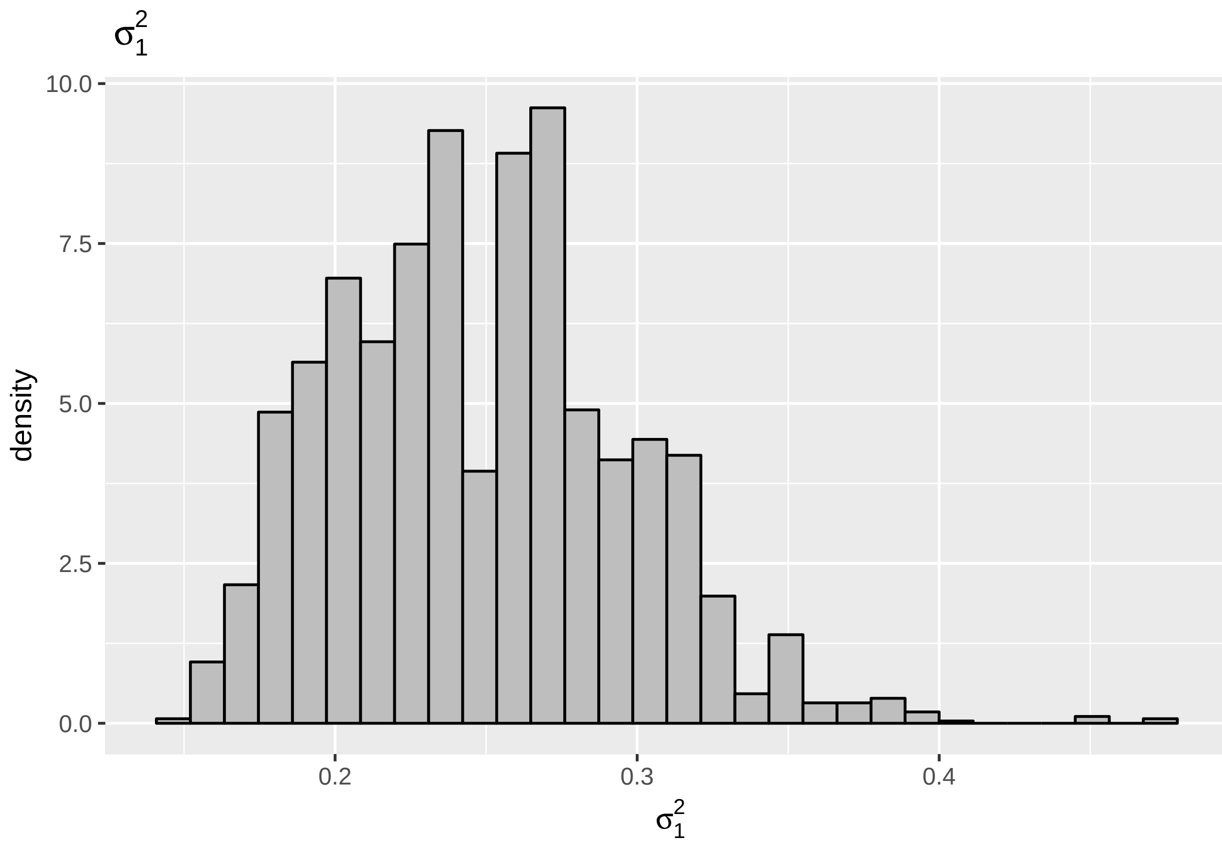



```
plot(ts(data[3,]),xlab="Time",ylab=~mu[2])
```

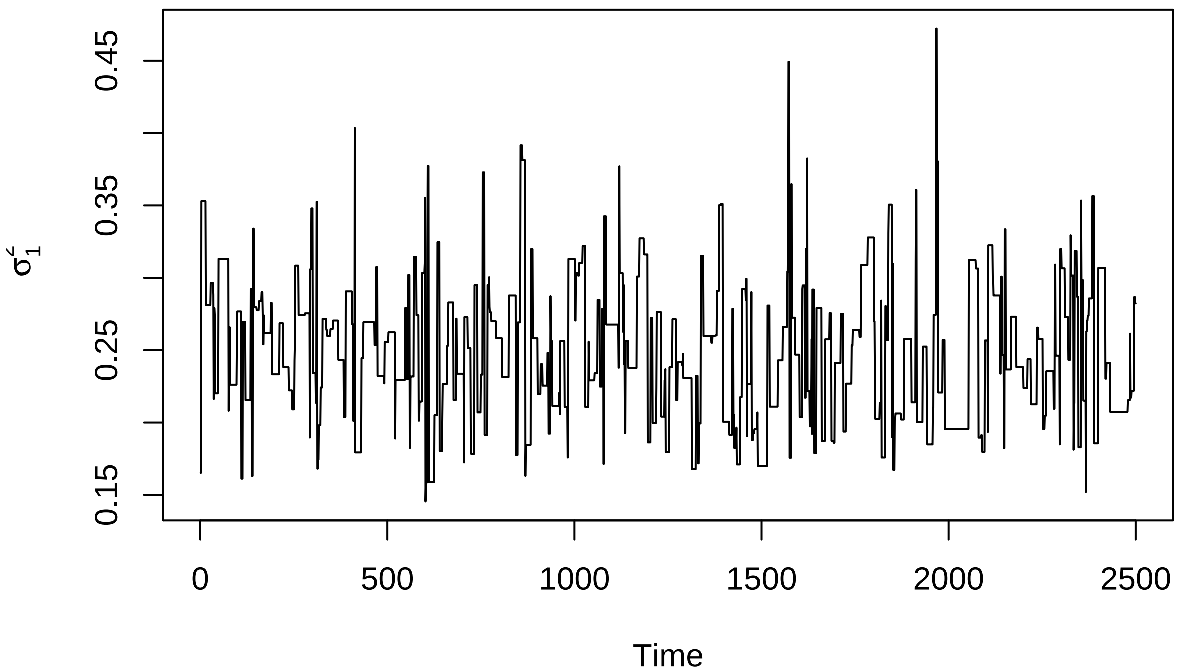



```
ggplot(data.frame(x=data[4,]),aes(x=x))+
xlab(~sigma[1]^2)+
ylab("density")+
ggtitle(~sigma[1]^2)+
geom_histogram(aes(y=..density..),fill="gray", colour="black")
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

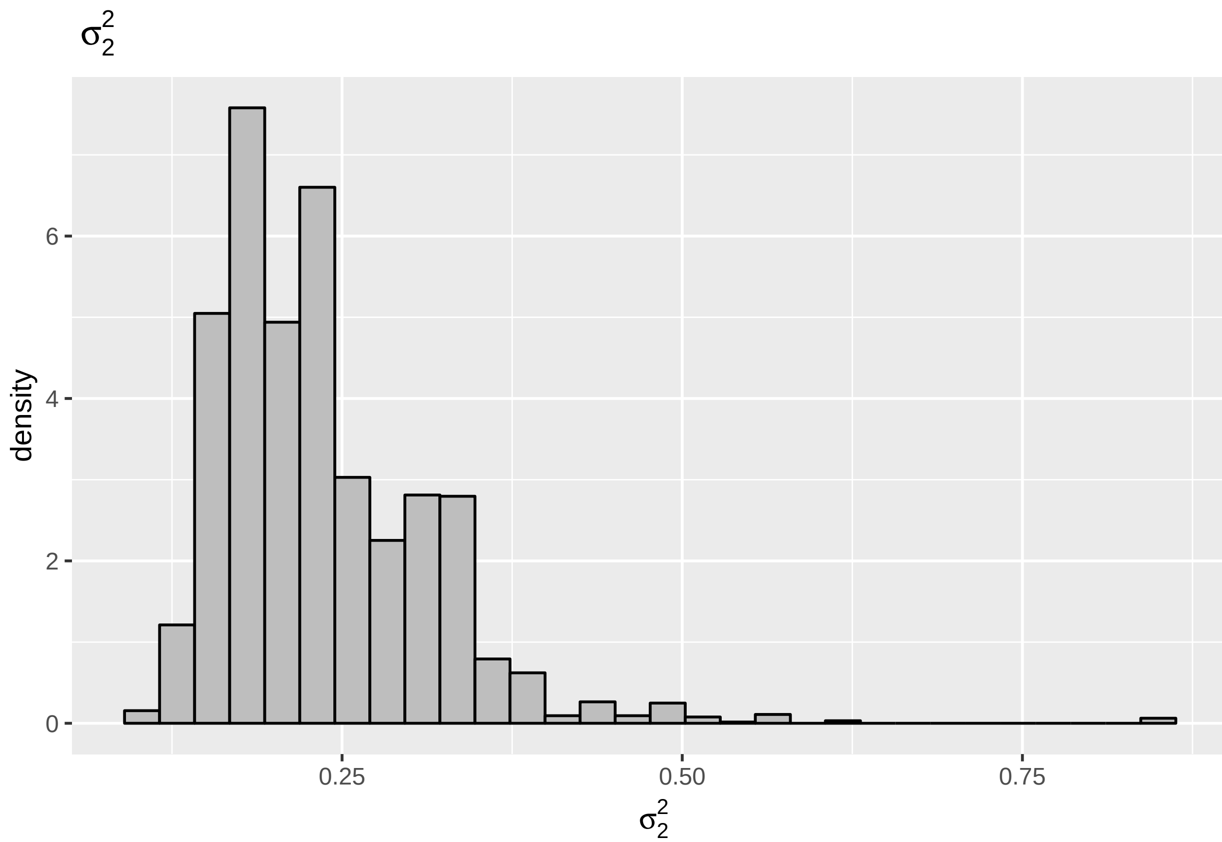



```
plot(ts(data[4,]),xlab="Time",ylab=~sigma[1]^2)
```

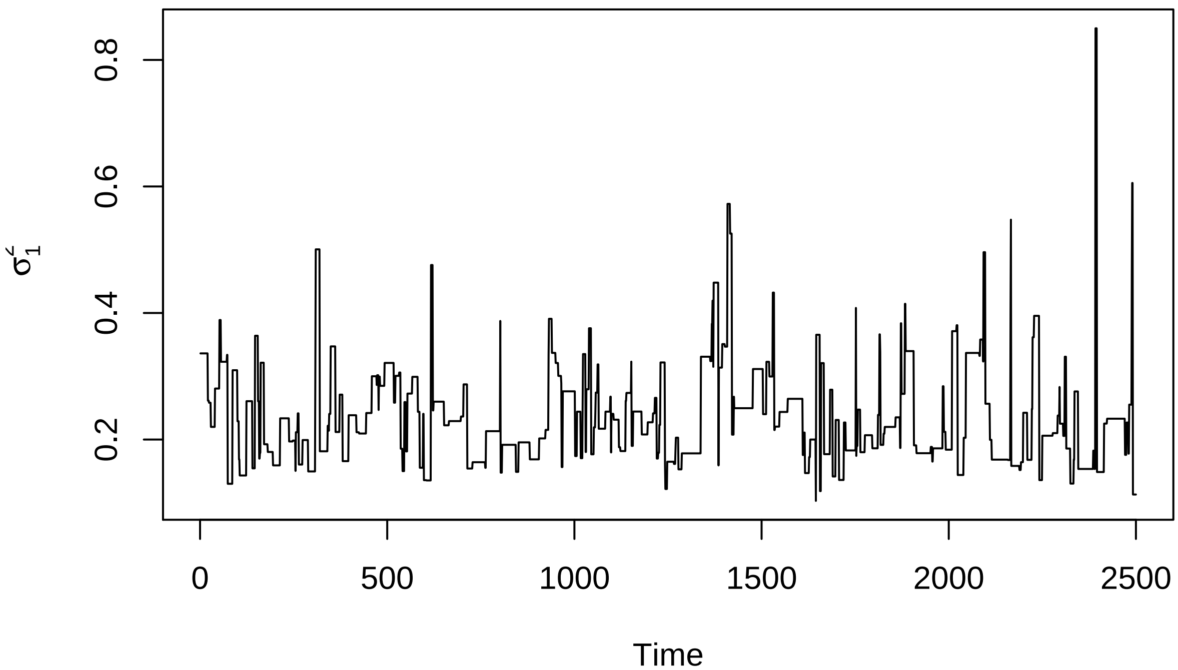



```
ggplot(data.frame(x=data[5,]),aes(x=x))+
xlab(~sigma[2]^2)+
ylab("density")+
ggtitle(~sigma[2]^2)+
geom_histogram(aes(y=..density..),fill="gray", colour="black")
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```




```
plot(ts(data[5,]),xlab="Time",ylab=~sigma[1]^2)
```


0.2
0.4
0.6
0.8
$\hat{\sigma}_1$
0
500
1000
1500
2000
2500
Time