

< STAT-5361 > HW#7

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1 Exercises 6.3.1

Given the conditions: The prior for μ_1 and μ_2 are $N(0, 10^2)$, that the prior for σ_1^2 and σ_2^2 are $\Gamma(a, b)$ with shape $a = .5$ and scale $b = 10, \pi(\delta), \pi(\mu_1), \pi(\mu_2), \pi(\sigma_1^2)$, and $\pi(\sigma_2^2)$.

Suppose that we consider finite mixture normal distribution, \mathbf{X}

$$f(x) = \delta N(\mu_1, \sigma_1^2) + (1 - \delta) N(\mu_2, \sigma_2^2)$$

$$f(x, \delta, \mu_1, \mu_2, \sigma_1^2, \sigma_2^2) \propto (\delta \frac{1}{\sigma_1} \exp\{-\frac{(x-\mu_1)^2}{2\sigma_1^2}\} + (1-\delta) \frac{1}{\sigma_2} \exp\{-\frac{(x-\mu_2)^2}{2\sigma_2^2}\}) \cdot f(\delta) \cdot f(\mu_1) \cdot f(\mu_2) \cdot f(\sigma_1^2) \cdot f(\sigma_2^2)$$

- 1) δ : $\pi(\delta|\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, x) \propto f(x|\delta)$
- 2) μ_1 : $\pi(\mu_1|\delta, \mu_2, \sigma_1^2, \sigma_2^2, x) \propto f(x|\delta) f_1(\mu_1)$
- 3) μ_2 : $\pi(\mu_2|\delta, \mu_1, \sigma_1^2, \sigma_2^2, x) \propto f(x|\delta) f_2(\mu_2)$
- 4) σ_1^2 : $\pi(\mu_2|\delta, \mu_1, \mu_2, \sigma_2^2, x) \propto f(x|\delta) f_3(\sigma_1^2)$
- 5) σ_2^2 : $\pi(\mu_2|\delta, \mu_1, \mu_2, \sigma_1^2, x) \propto f(x|\delta) f_4(\sigma_2^2)$

```
library(MCMCpack)
```

```
## Loading required package: coda
```

```
## Loading required package: MASS
```

```
## ##
```

```
## ## Markov Chain Monte Carlo Package (MCMCpack)
```

```
## ## Copyright (C) 2003-2018 Andrew D. Martin, Kevin M. Quinn, and Jong Hee Park
```

```
## ##
```

```
## ## Support provided by the U.S. National Science Foundation
```

```
## ## (Grants SES-0350646 and SES-0350613)
```

```
## ##
```

```
library(HI)
```

```
delta <- 0.7
```

```
n <- 100
```

```
set.seed(254)
```

```
u <- rbinom(n, prob = delta, size = 1)
```

```
x <- rnorm(n, ifelse(u == 1, 7, 10), 0.5)
```

```
loglik = function(x,mu1,mu2,sigma1,sigma2,delta)
{
```

```

Fisrt = log( ( delta * dnorm(x, mu1,sigma1) ) + ( (1-delta) * dnorm(x,mu2,sigma2) ) )
return(sum(Fisrt))
}

Function = function(x,ini.D,ini.mu1,ini.mu2, ini.sigma1,ini.sigma2, iter)
{
  inix=x
  Cur.D <- ini.D; Cur.mu1 <- ini.mu1; Cur.mu2 <- ini.mu2; Cur.sigma1 <- ini.sigma1; Cur.sigma2
  for(i in 1:iter)
  {
    D_v =function(delta) loglik(x =inix, mu1 = Cur.mu1, mu2 = Cur.mu2, sigma1 = Cur.sigma1, sigma2 = Cur.sigma2, delta = delta)
    N.D[i] = arms(Cur.D, D_v, function(delta) {(delta>0)*(delta<1)}, 1)

    mu1.v = function(mu1) loglik(x =inix, mu1 = mu1, mu2 = Cur.mu2, sigma1 = Cur.sigma1, sigma2 = Cur.sigma2, delta = N.D[i]) + log( dnorm(mu1,0,10^2))
    N.mu1[i] = arms(Cur.mu1, mu1.v, function(mu1) (mu1 > -30) * (mu1 < 30), 1)

    mu2.v = function(mu2) loglik(x =inix, mu1 = N.mu1[i], mu2 = mu2, sigma1 = Cur.sigma1, sigma2 = Cur.sigma2, delta = N.D[i]) + log( dnorm(mu2,0,10^2))
    N.mu2[i] = arms(Cur.mu2, mu2.v, function(mu2) (mu2 > -30) * (mu2 < 30), 1)

    sigma1.v = function(sigma1) loglik(x =inix, mu1 = N.mu1[i], mu2 = N.mu2[i], sigma1 = sigma1, sigma2 = Cur.sigma2, delta = N.D[i]) + log(dinvgamma(sigma1, 0.5, 100))
    N.sigma1[i] = arms(Cur.sigma1, sigma1.v, function(sigma1) (sigma1> 0) * (sigma1 < 100), 1)

    sigma2.v = function(sigma2) loglik(x =inix, mu1 = N.mu1[i], mu2 = N.mu2[i], sigma1 = N.sigma1[i], sigma2 = sigma2, delta = N.D[i]) + log(dinvgamma(sigma2, 0.5, 100))
    N.sigma2[i] = arms(Cur.sigma2, sigma2.v, function(sigma2) (sigma2 > 0) * (sigma2 < 100), 1)

    Cur.D = N.D[i]; Cur.mu1 = N.mu1[i]; Cur.mu2 = N.mu2[i];

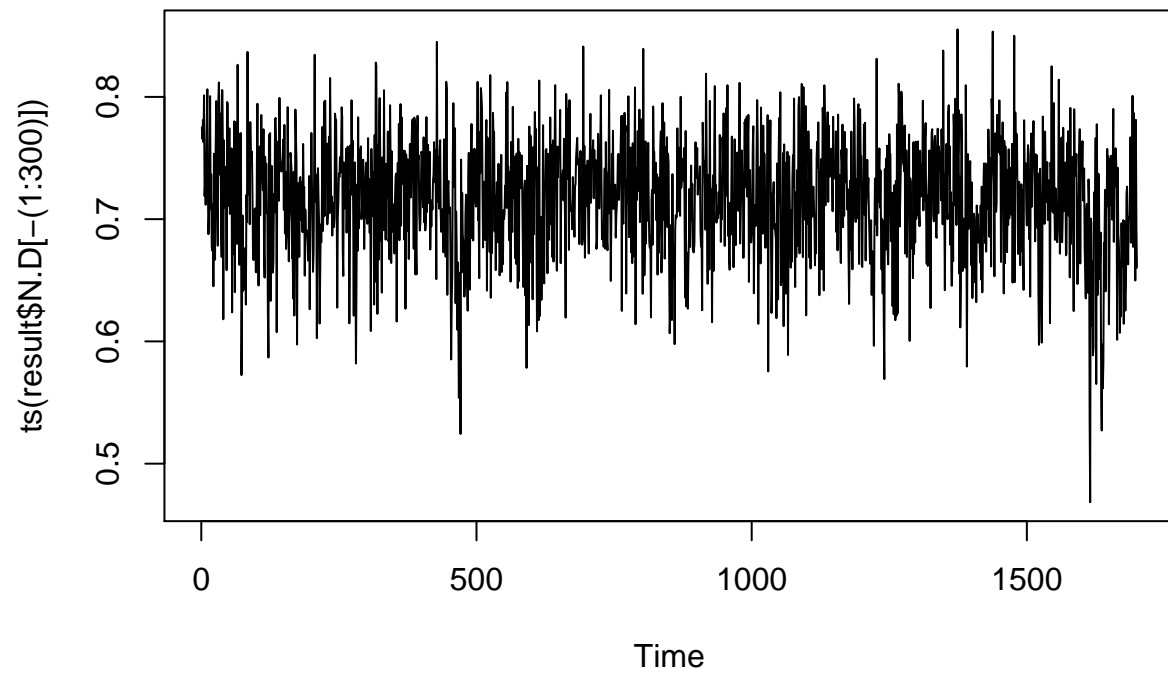
    Cur.sigma1 = N.sigma1[i]; Cur.sigma2 = N.sigma2[i]
  }

  list(N.D = N.D, N.mu1= N.mu1, N.mu2 = N.mu2, N.sigma1= N.sigma1, N.sigma2 = N.sigma2)
}

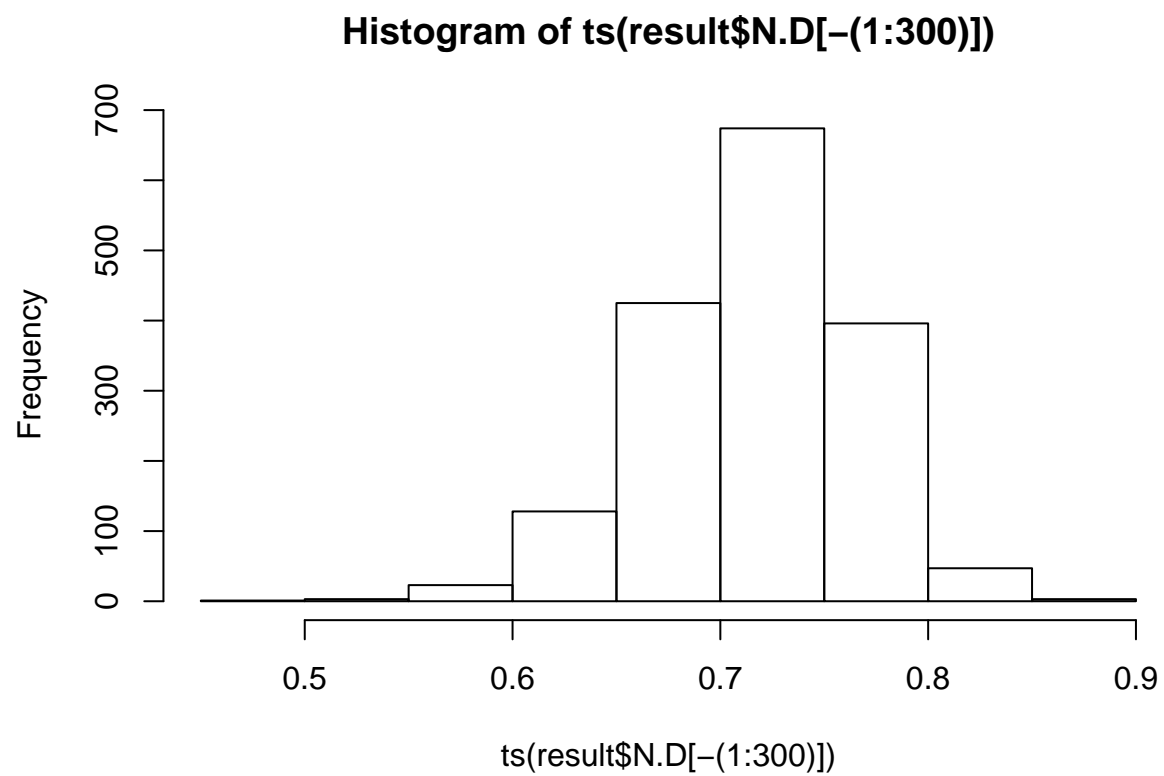
result = Function(x,0.5,1,1,1,1, iter = 2000)

plot(ts(result$N.D[-(1:300)]))

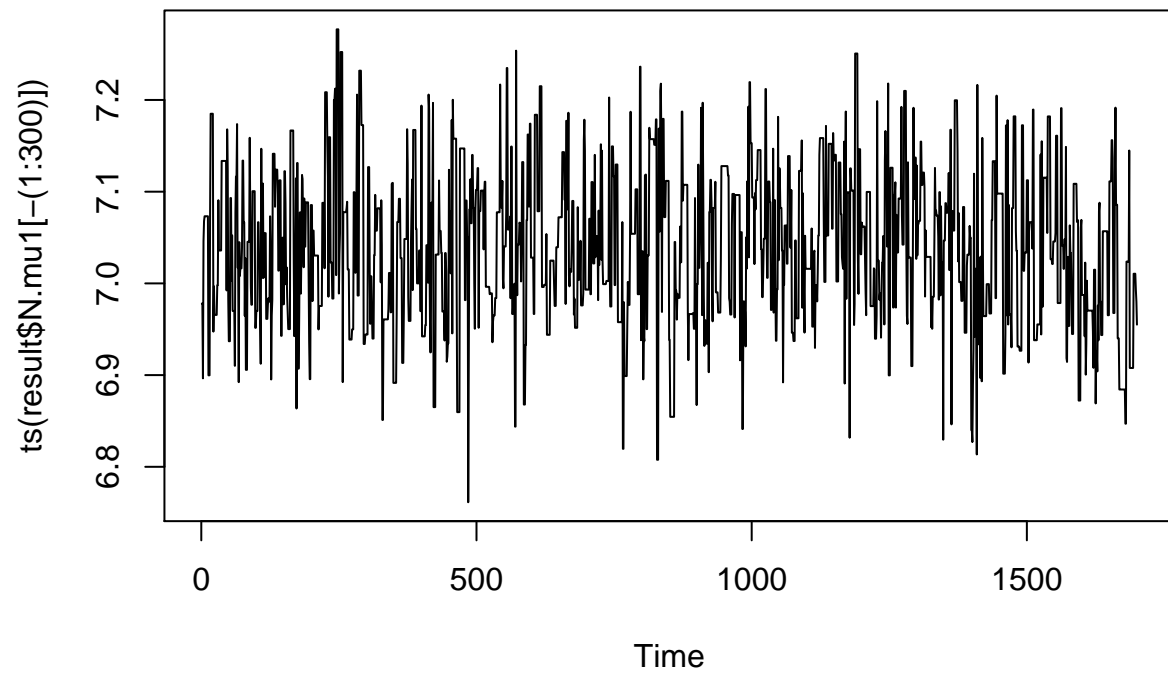
```



```
hist(ts(result$N.D[-(1:300)]))
```

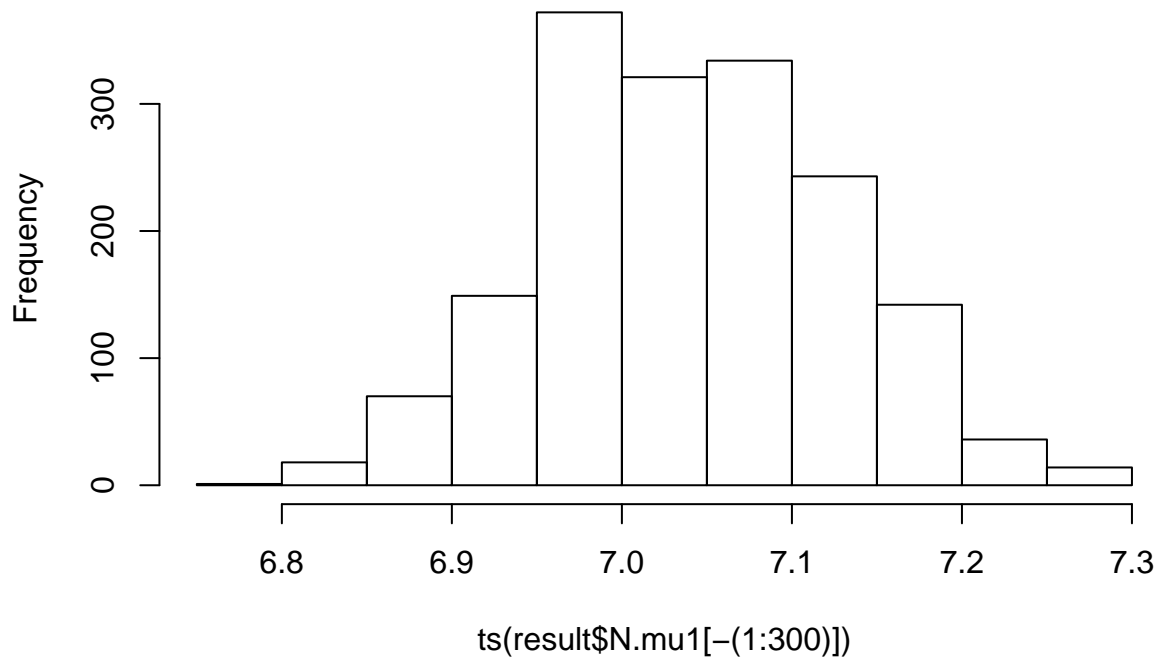


```
plot(ts(result$N.mu1[-(1:300)]))
```

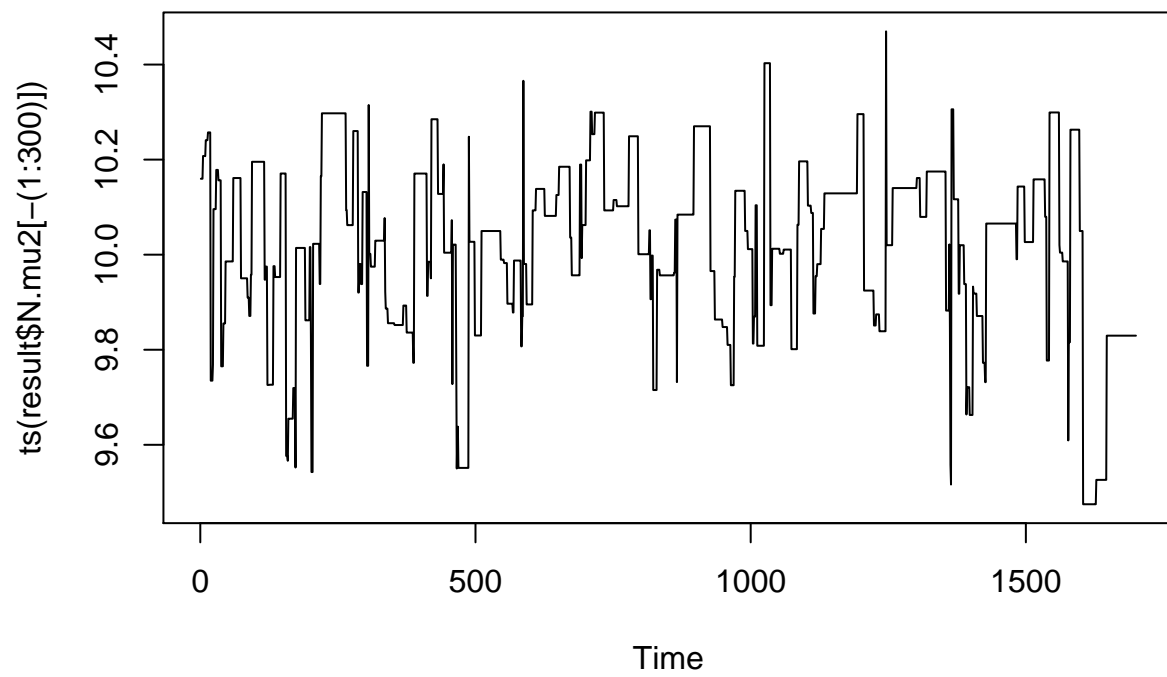


```
hist(ts(result$N.mu1[-(1:300)]))
```

Histogram of `ts(result$N.mu1[-(1:300)])`

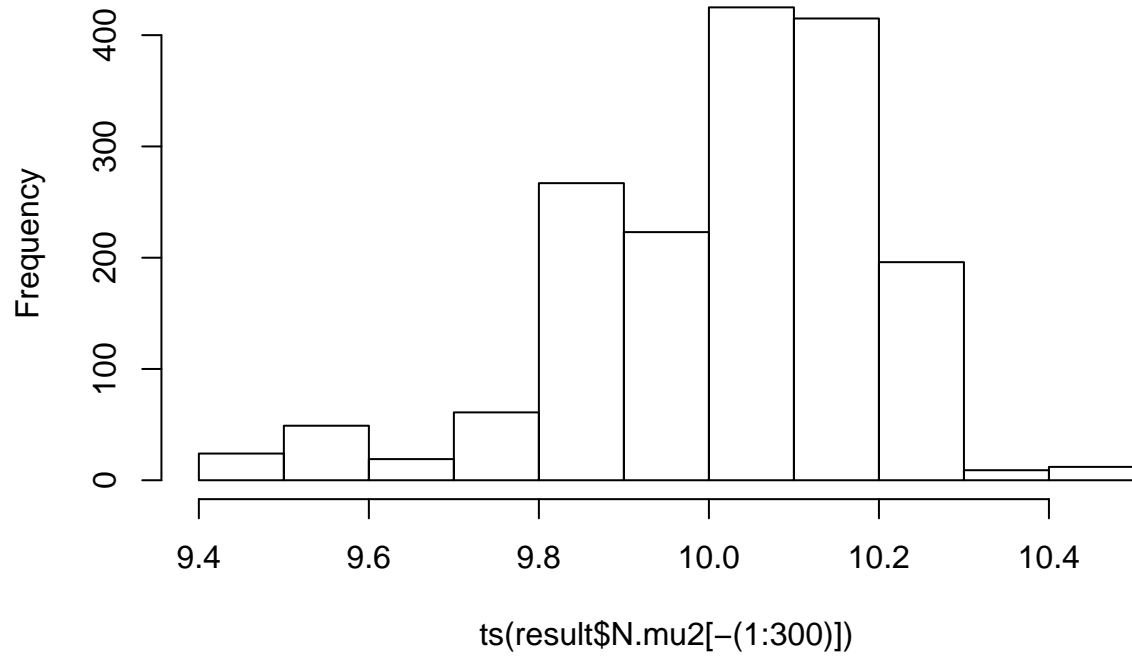


```
plot(ts(result$N.mu2[-(1:300)]))
```

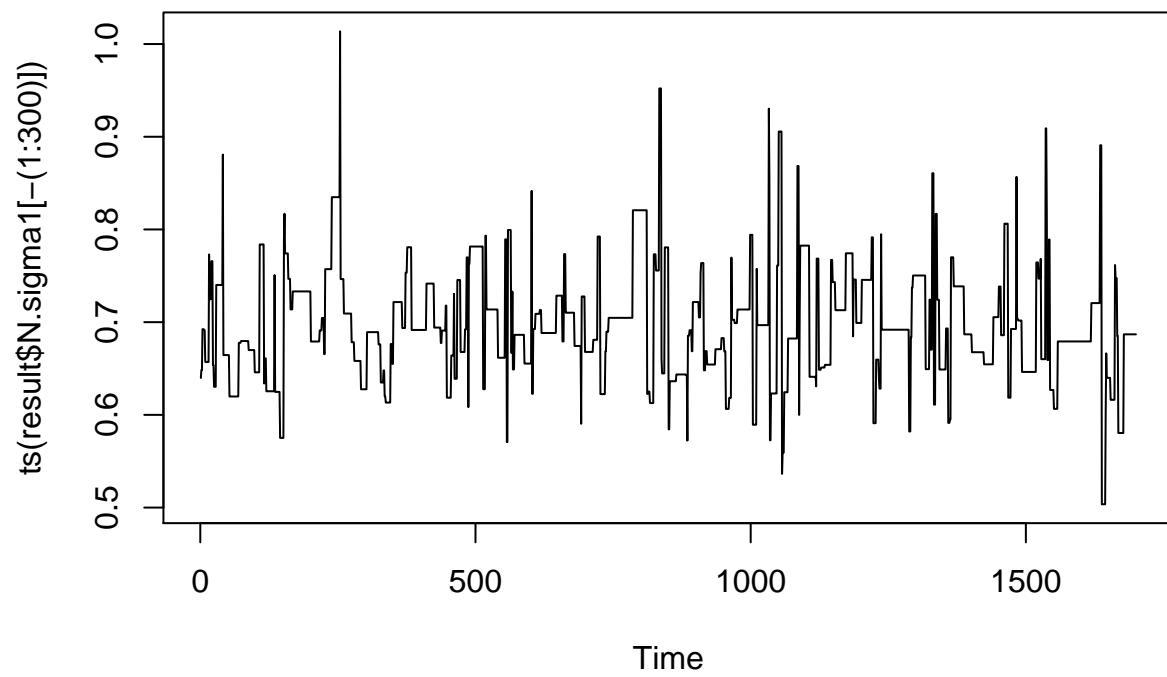


```
hist(ts(result$N.mu2[-(1:300)]))
```


Histogram of `ts(result$N.mu2[-(1:300)])`

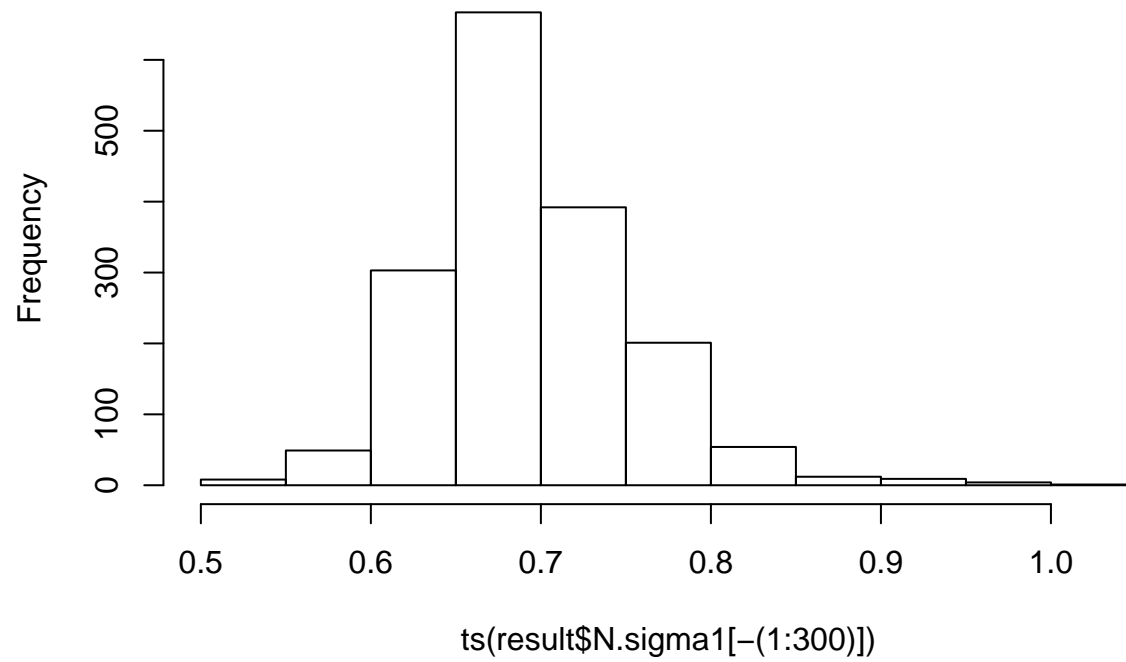


```
plot(ts(result$N.sigma1[-(1:300)]))
```

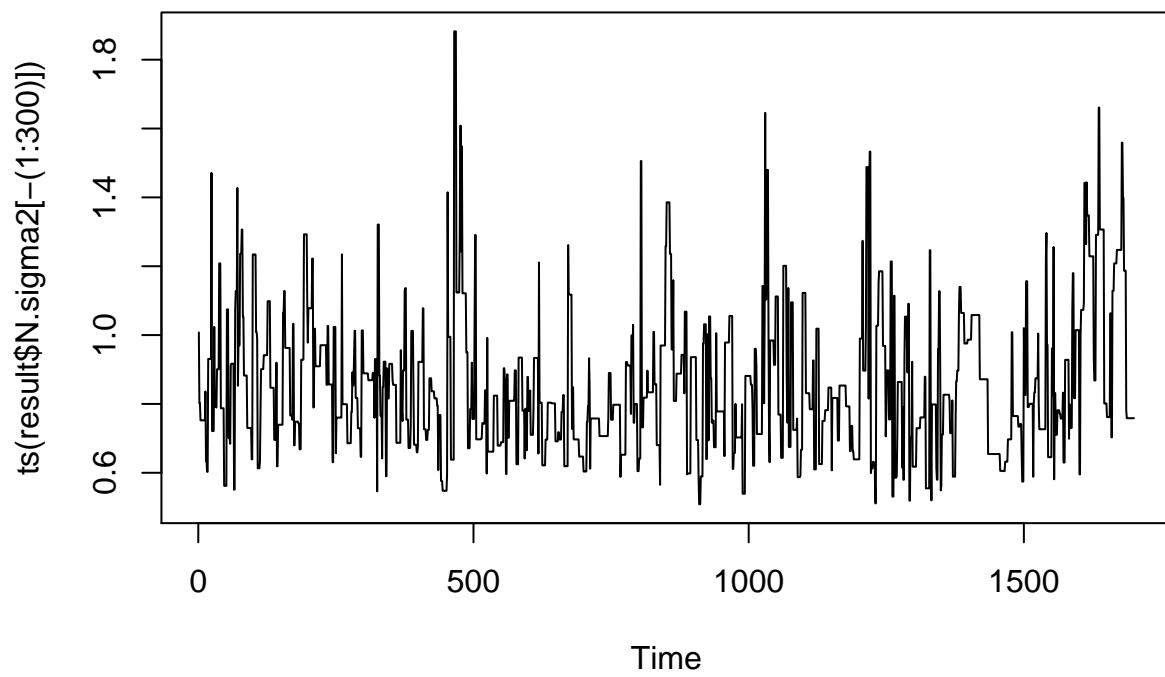


```
hist(ts(result$N.sigma1[-(1:300)]))
```

Histogram of `ts(result$N.sigma1[-(1:300)])`



```
plot(ts(result$N.sigma2[-(1:300)]))
```



```
hist(ts(result$N.sigma2[-(1:300)]))
```

