HW7

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6.3.1) Given a normal distribution $f(x) = \delta N(\mu_1, \sigma_1^2) + (1 - \delta)N(\mu_2, \sigma_2^2)$ The log-likelihood function becomes $(\theta = (\mu_1, \mu_2, \sigma_1, \sigma_2))$:

$$\ell(\theta) = \Pi[_{i=1}^n \delta \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} + (1 - \delta)\frac{1}{\sqrt{2\pi}\sigma_2} e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}}]$$

The priors for μ_1, μ_2 are $N(0, 100)$ and the priors for σ are $\sigma_1^{-2} = \sigma_2^{-2} = \Gamma(.5, 10)$ which is an inverse gamma And assumed parameters are $\delta = 0.7, \sigma_1 = \sigma_2 = 0.5, \mu_1 = 7, \mu_2 = 10$

Therefore, the posterior distribution becomes (using $\ell(\theta)$ as above):

$$q(x|\theta) = \ell(\theta) * \Gamma_1^{-1}(0.5, 10) * \Gamma_2^{-1}(0.5, 10) * N_1(0, 100) * N_2(0, 100)$$

```r
delta <- 0.7 # true value to be estimated based on the data
n <- 100
set.seed(123)
u <- rbinom(n, prob = delta, size = 1)
y <- rnorm(n, ifelse(u == 1, 7, 10), 0.5)


library("invgamma")
mc.post <- function(delta, mu1, mu2, sigma1, sigma2, x=y){
  llk <- sum(log((delta * dnorm(x, mu1, sqrt(sigma1))))) + sum(log((delta * dnorm(x, mu2, sqrt(sigma2)))
  mc.mu1 <- dnorm(mu1, 0, 10, log = TRUE)
  mc.mu2 <- dnorm(mu2, 0, 10, log = TRUE)
  mc.sigma1 <- dinvgamma(sigma1, 0.5, 10, log = TRUE)
  mc.sigma2 <- dinvgamma(sigma2, 0.5, 10, log = TRUE)

  return(llk + mc.mu1 + mc.mu2 + mc.sigma1 + mc.sigma2)
}

library("HI")
gibbs <- function(delta.g, mu1.g, mu2.g, sigma1.g, sigma2.g, x=y, n){
  gib.mat <- matrix(nrow = n, ncol = 5)
  init.g <- c(delta.g, mu1.g, mu2.g, sigma1.g, sigma2.g)
  for (i in 1:n) {
    gib.mat[i,1] <- arms(delta.g, mc.post, function(x,...)(x>0)*(x<1), 1, mu1 = init.g[2], mu2 = init.g
    init.g[1] <- gib.mat[i,1]

    gib.mat[i,2] <- arms(mu1.g, mc.post, function(x,...)(x>-100)*(x<100), 1, d = init.g[1], mu2 = init.g
    init.g[2] <- gib.mat[i,2]

    gib.mat[i,3] <- arms(mu2.g, mc.post, function(x,...)(x>-100)*(x<100), 1, d = init.g[1], mu1 = init.g
    init.g[3] <- gib.mat[i,3]

    gib.mat[i,4] <- arms(sigma1.g, mc.post, function(x,...)(x>0)*(x<100), 1, d = init.g[1], mu2 = init.g
    init.g[4] <- gib.mat[i,4]

    gib.mat[i,5] <- arms(sigma2.g, mc.post, function(x,...)(x>0)*(x<100), 1, d = init.g[1], mu2 = init.g
    init.g[5] <- gib.mat[i,5]
```
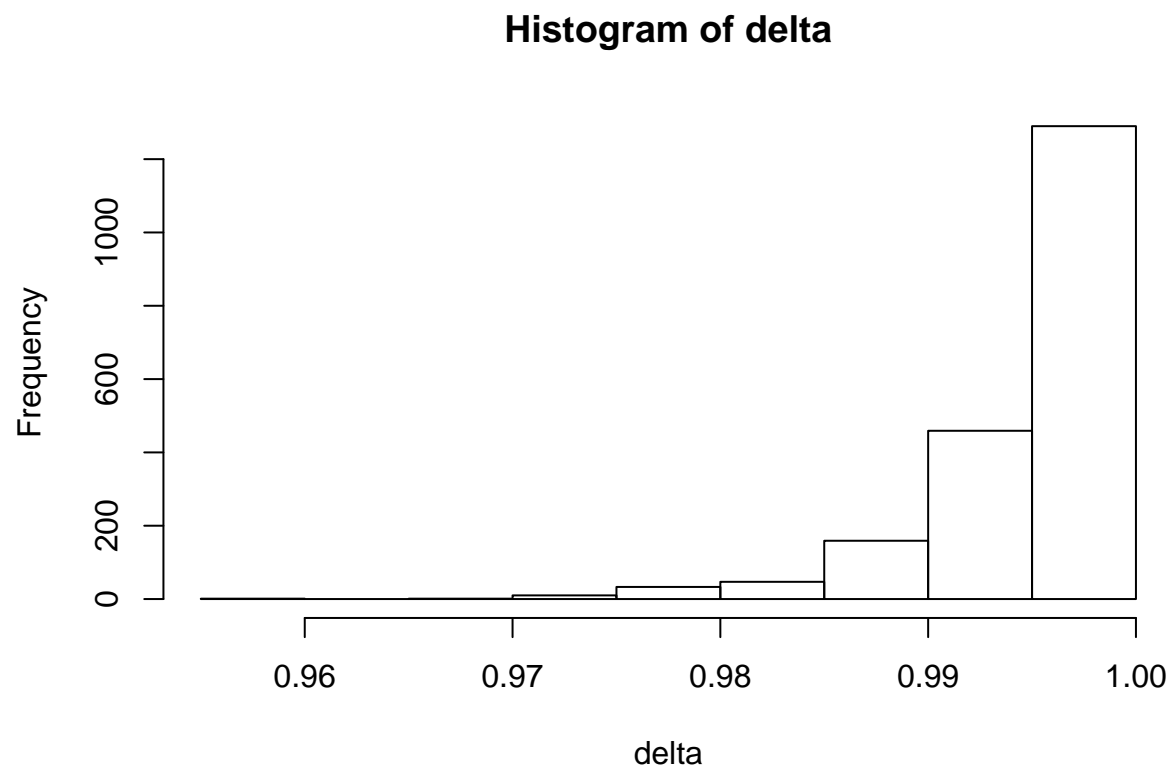
```
  }
  gib.mat
}


mygibbs <- gibbs(0.5, 5, 5, 1, 1, x=x, 2500)[-(1:500),]    #burn-in = 500
hist(mygibbs[,1], xlab = "delta", main = "Histogram of delta")
```
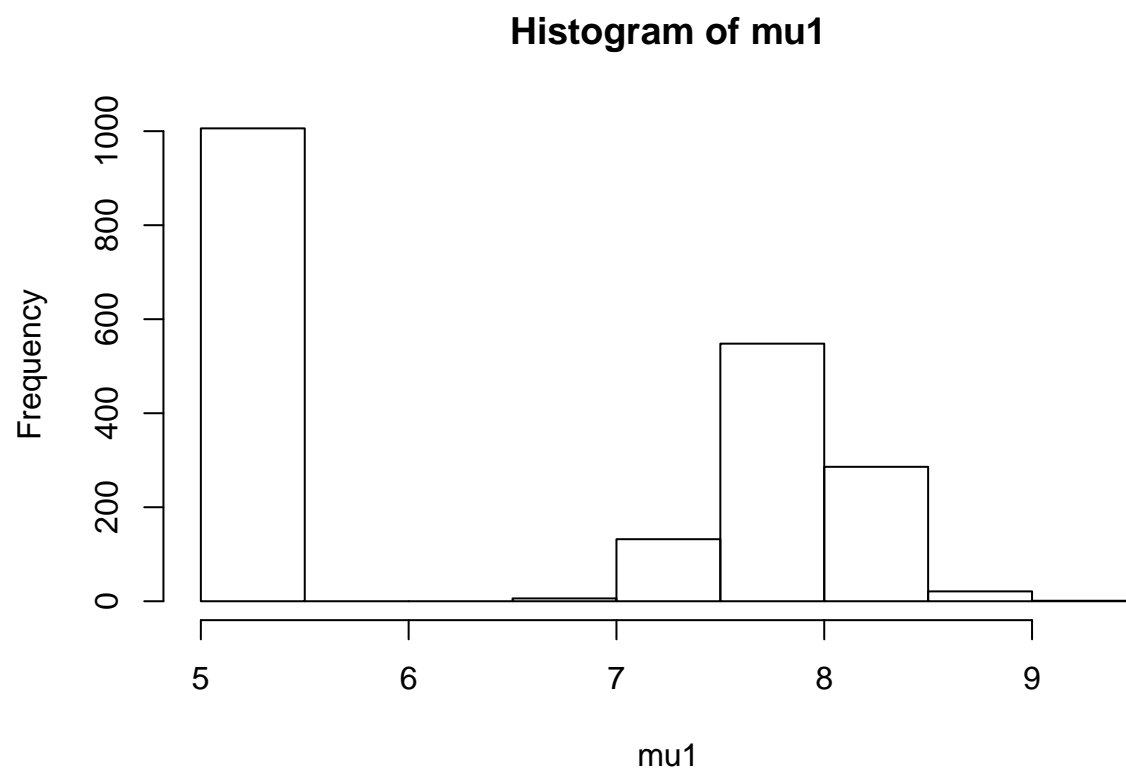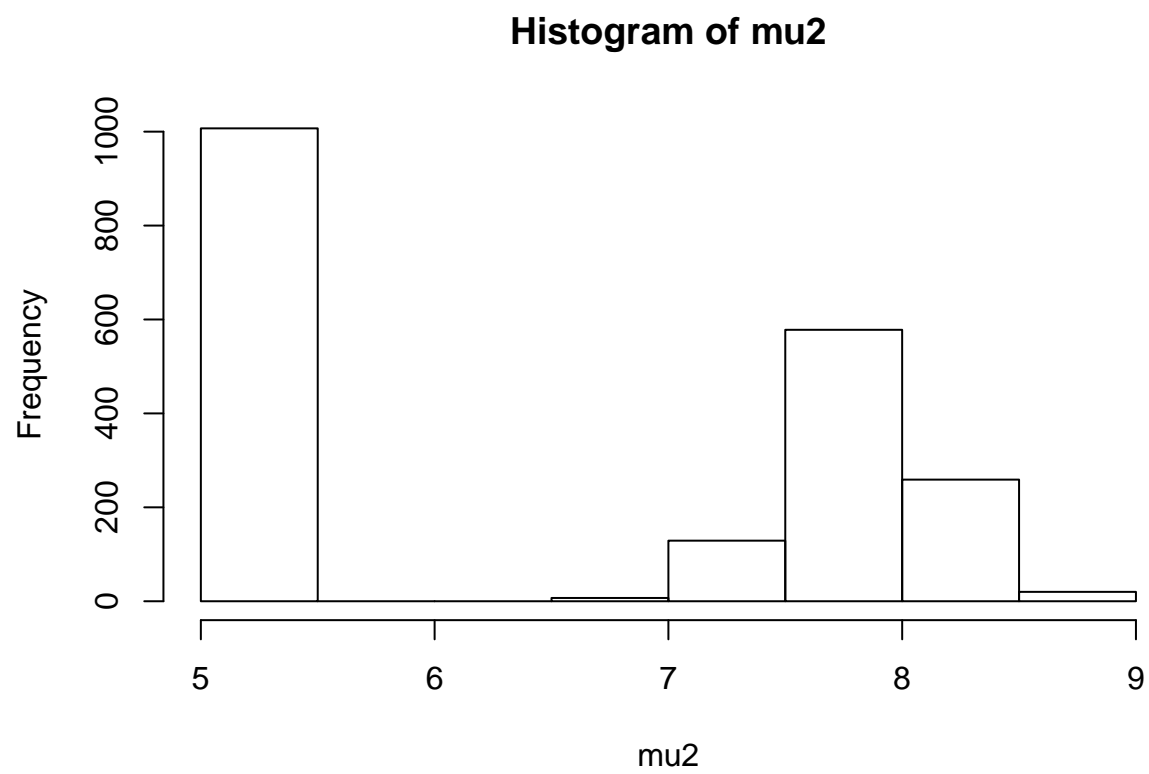
Histogram of delta


```
hist(mygibbs[,2], xlab = "mu1", main = "Histogram of mu1")
```
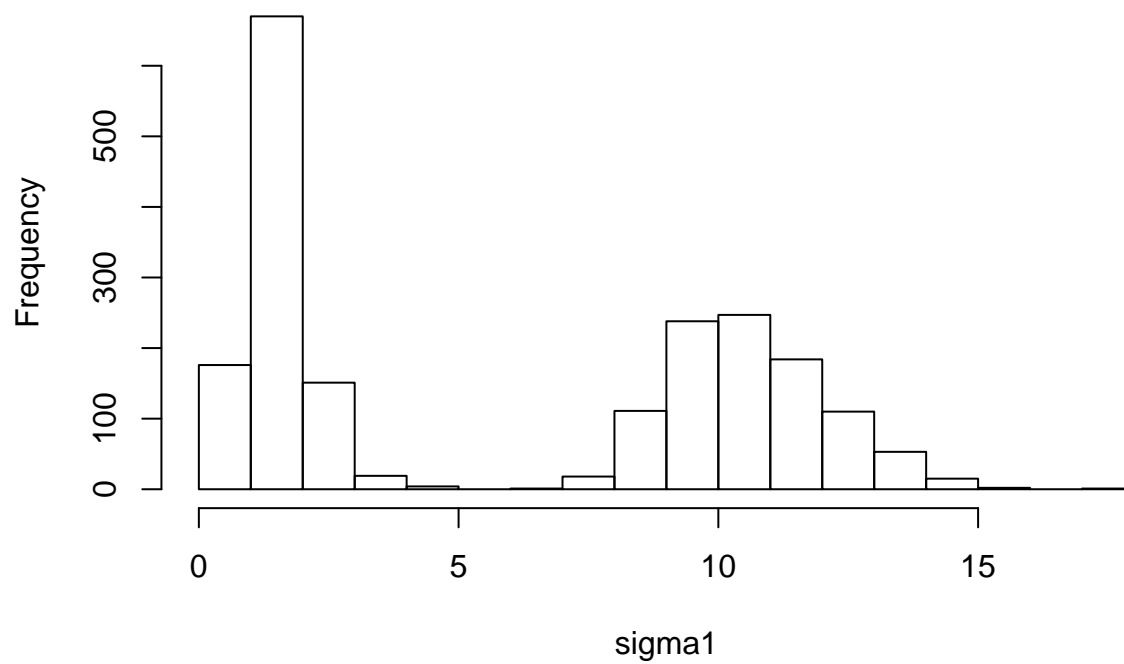
Histogram of mu1


```
hist(mygibbs[,3], xlab = "mu2", main = "Histogram of mu2")
```

Histogram of mu2


```r
hist(mygibbs[,4], xlab = "sigma1", main = "Histogram of sigma1")
```

Histogram of sigma1


```r
hist(mygibbs[,5], xlab = "sigma2", main = "Histogram of sigma2")
```

Histogram of sigma2