

HW7

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$$f(x|\delta, \mu_1, \mu_2, \sigma_1^2, \sigma_2^2) = \delta \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} + (1-\delta) \frac{1}{\sqrt{2\pi}\sigma_2} e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}}$$

$$\mu_1 \sim N(0, 10^2), \mu_2 \sim N(0, 10^2), 1/\sigma_1^2 \sim \text{Gamma}(.5, 10), 1/\sigma_2^2 \sim \text{Gamma}(.5, 10).$$

$$f(\delta, \mu_1, \mu_2, \sigma_1^2, \sigma_2^2, x) = \left[\delta \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} + (1-\delta) \frac{1}{\sqrt{2\pi}\sigma_2} e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}} \right]$$

$$\frac{1}{2\pi} e^{-\frac{\mu_1^2 + \mu_2^2}{200}} \frac{1}{\Gamma(.5)^2 10} (1/\sigma_1^2)^{1.5} (1/\sigma_2^2)^{1.5} e^{-\frac{1}{10\sigma_1^2}} e^{-\frac{1}{10\sigma_2^2}}$$

$$f(\delta, \mu_1, \mu_2, \sigma_1^2, \sigma_2^2 | x) \propto \exp\left[\log\left(\frac{\delta}{\sigma_1} e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} + \frac{1-\delta}{\sigma_2} e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}}\right) - \mu_1^2/200 - \mu_2^2/200 - 3\log\sigma_1 - 3\log\sigma_2 - \frac{1}{10\sigma_1^2} - \frac{1}{10\sigma_2^2}\right]$$

estimate parameters step by step

```
# true value
delta <- 0.7
mu1 <- 2
mu2 <- 4
sigma1 <- .5
sigma2 <- .8
# sample
n <- 1000
u <- rbinom(n, prob = delta, size = 1)
dat <- rnorm(n, ifelse(u == 1, mu1, mu2), ifelse(u == 1, sigma1, sigma2))
mylike <- function(delta, mu1, mu2, sigma1, sigma2, x){
  sum(log(delta / sigma1 * exp(-(x - mu1) ^ 2 / (2 * sigma1 ^ 2)) + (1 - delta) / sigma2 * exp(-(x - mu2) ^ 2 / (2 * sigma2 ^ 2))))
}
mymcmc <- function(niter, init, data){
  v <- matrix(NA, niter, 5)
  for (i in 1:niter) {
    ## delta
    delta <- arms(init[1], function(x) mylike(x, mu1 = init[2], mu2 = init[3], sigma1 = init[4], sigma2 = init[5], x))
    v[i, 1] <- init[1] <- delta

    ## mu1
    mu1 <- arms(init[2], function(x) mylike(delta = init[1], x, mu2 = init[3], sigma1 = init[4], sigma2 = init[5], x))
    v[i, 2] <- init[2] <- mu1

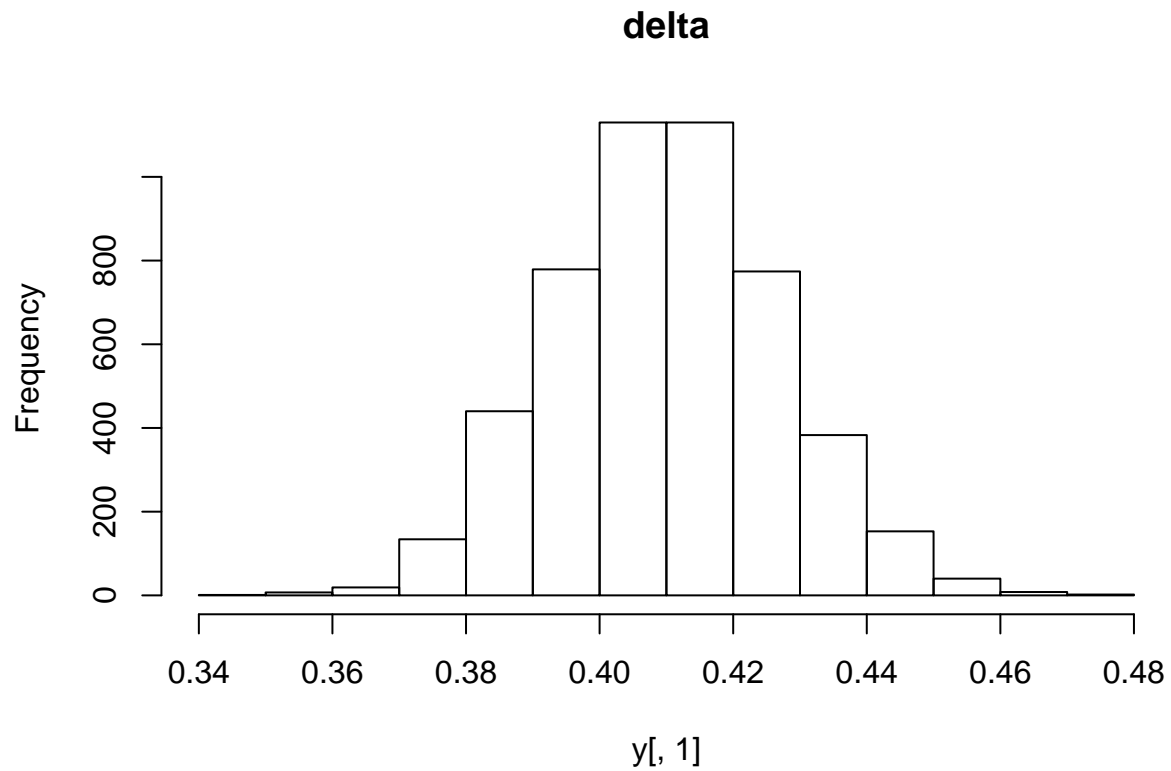
    ## mu2
    mu2 <- arms(init[3], function(x) mylike(delta = init[1], mu1 = init[2], x, sigma1 = init[4], sigma2 = init[5], x))
    v[i, 3] <- init[3] <- mu2

    ## sigma1
    sigma1 <- arms(init[4], function(x) mylike(delta = init[1], mu1 = init[2], mu2 = init[3], x, sigma2 = init[5], x))
    v[i, 4] <- init[4] <- sigma1
  }
}
```

```

## sigma2
sigma2 <- arms(init[5], function(x) mylike(delta = init[1], mu1 = init[2], mu2 = init[3], sigma1 = :
v[i, 5] <- init[5] <- sigma2
}
v
}
niter <- 10000
init <- c(.6, 3, 5, 3, 4)
y <- mymcmc(niter, init, dat)[- (1: 5000), ]
hist(y[, 1], main = "delta")

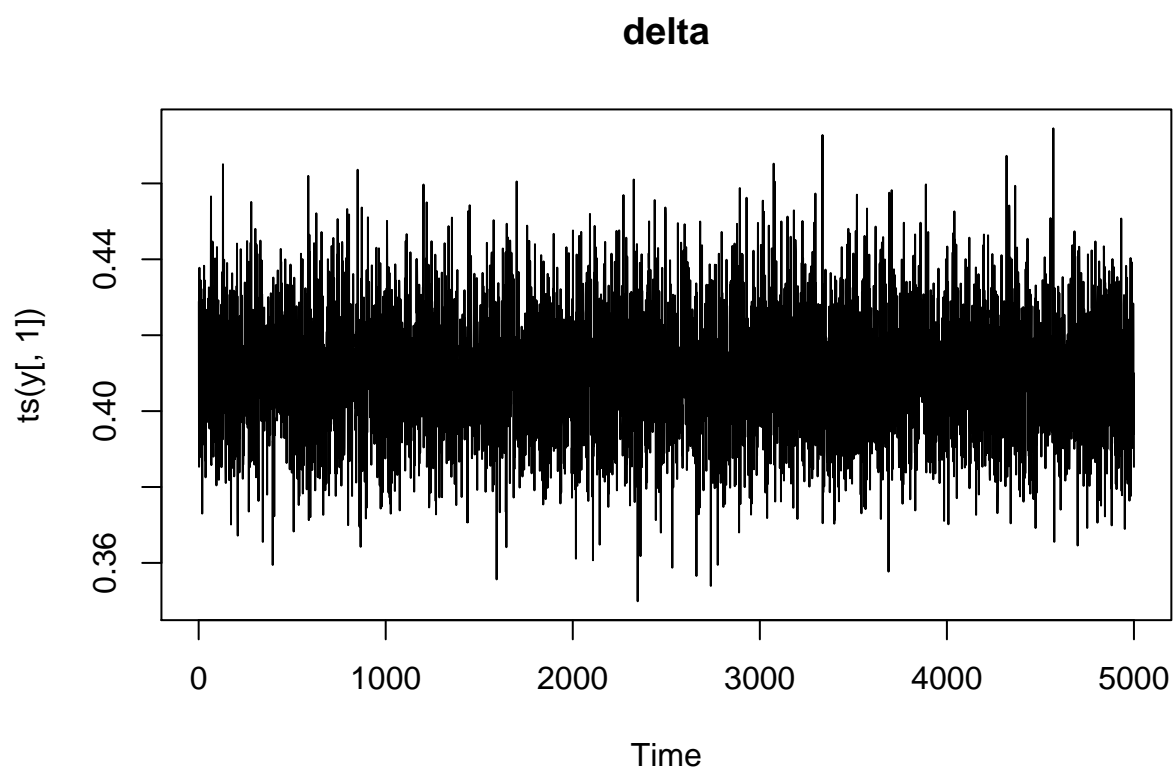
```



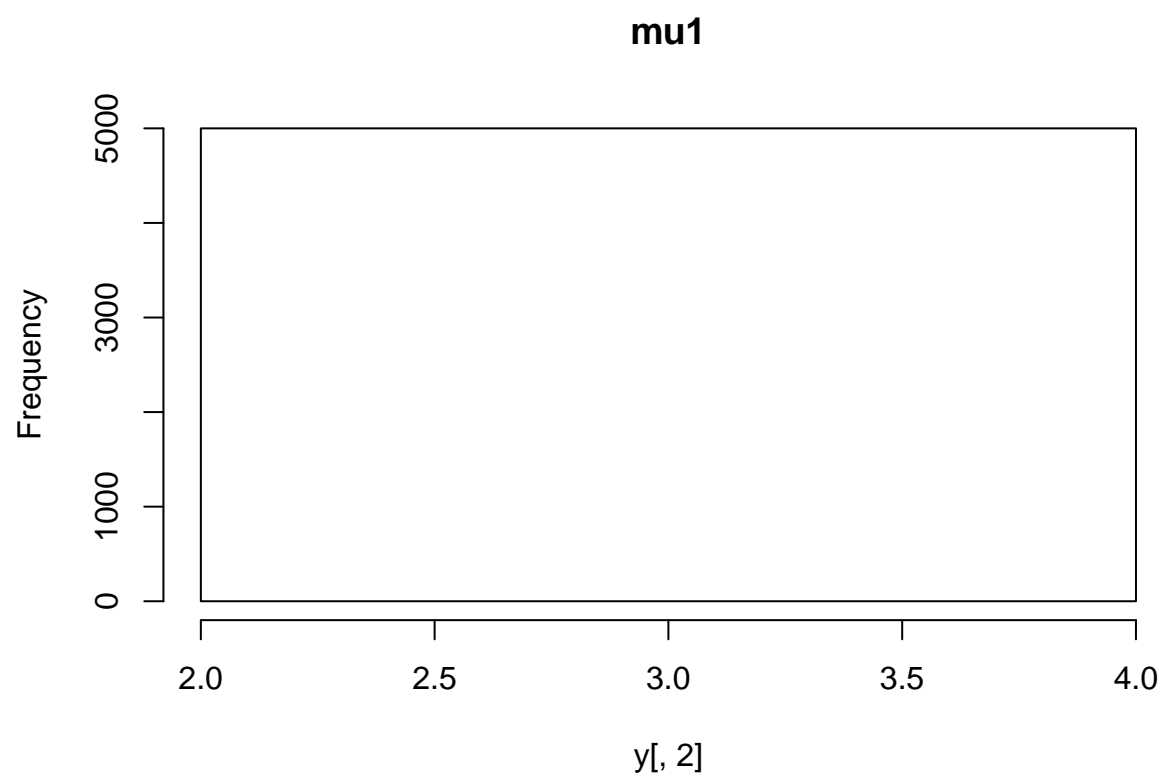
```

plot(ts(y[, 1]), main = "delta")

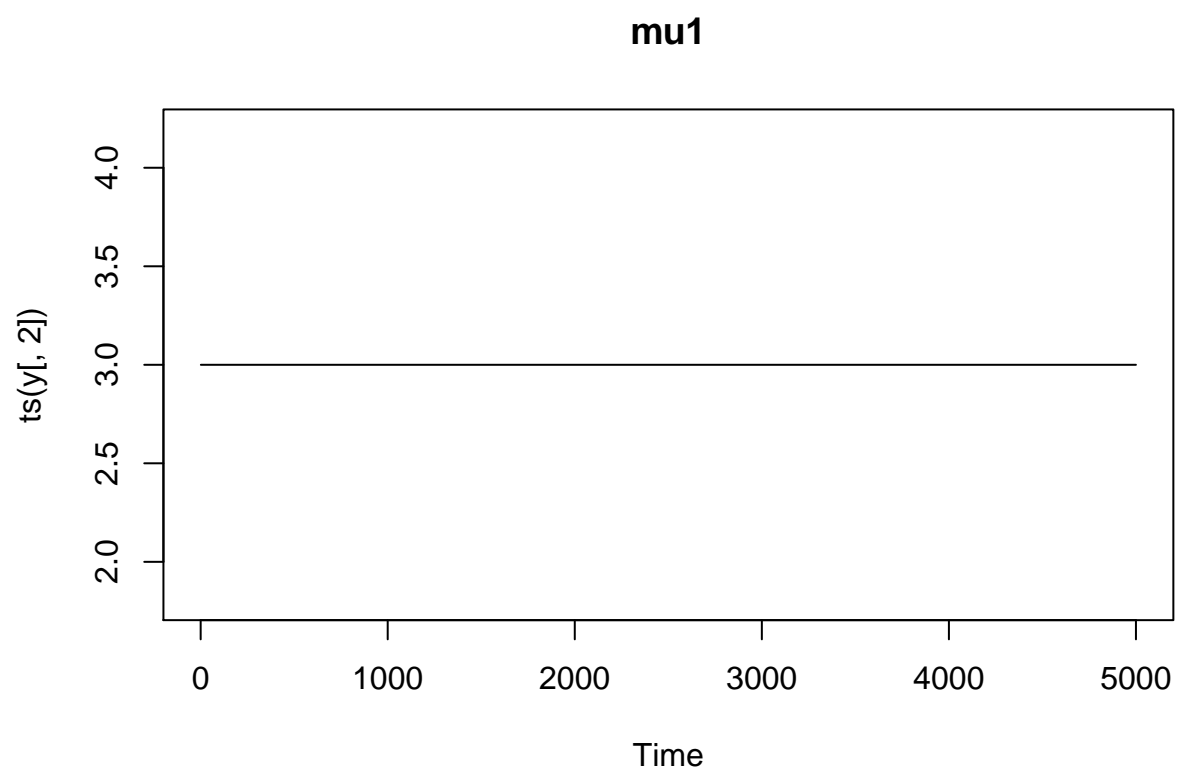
```



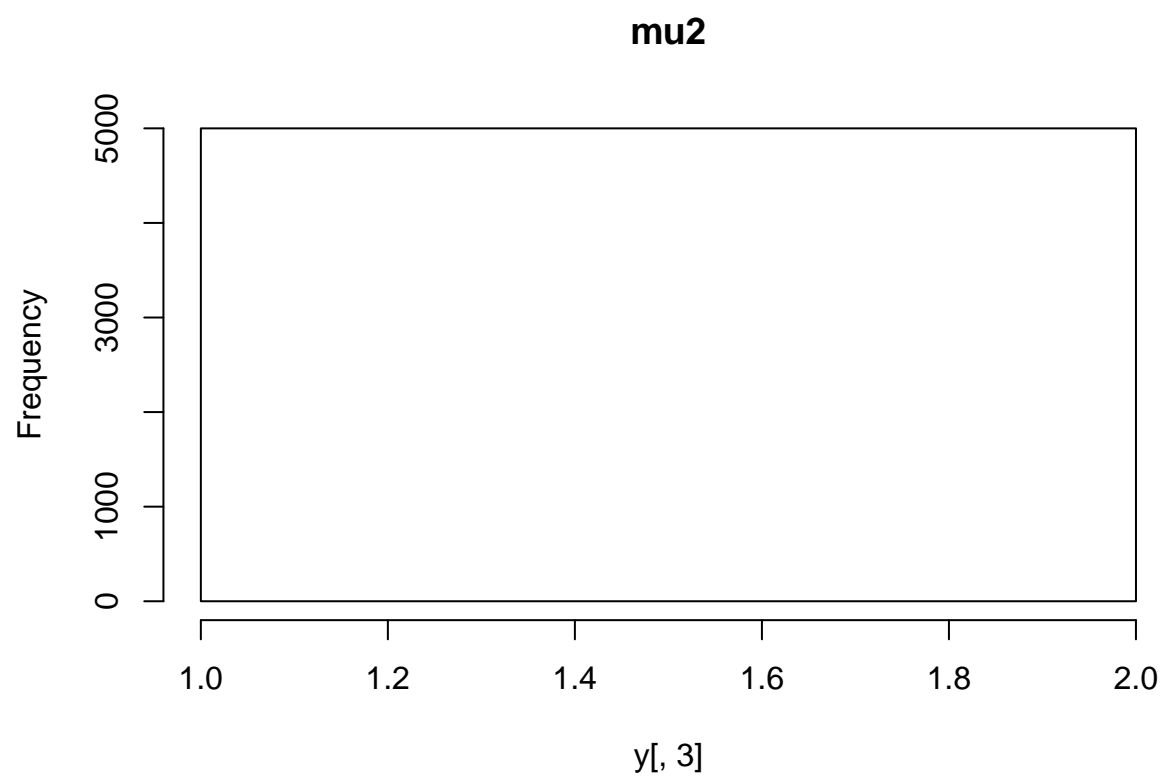
```
hist(y[, 2], main = "mu1")
```



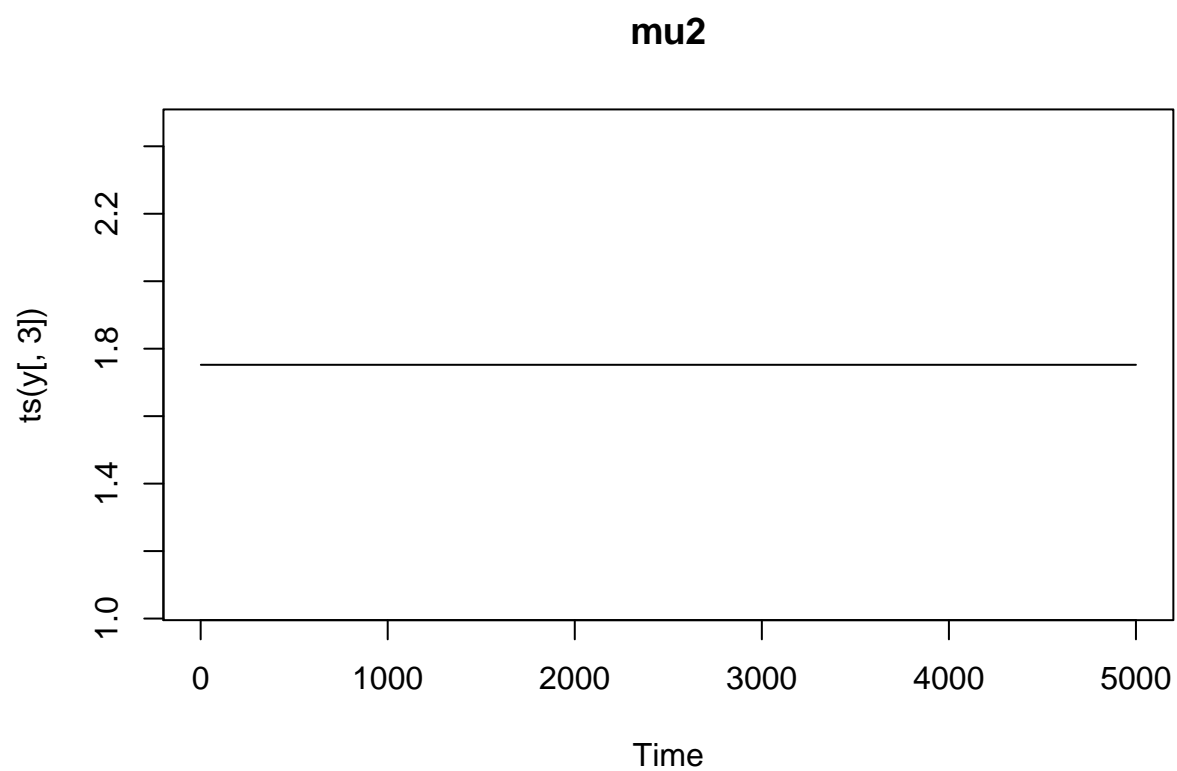
```
plot(ts(y[, 2]), main = "mu1")
```



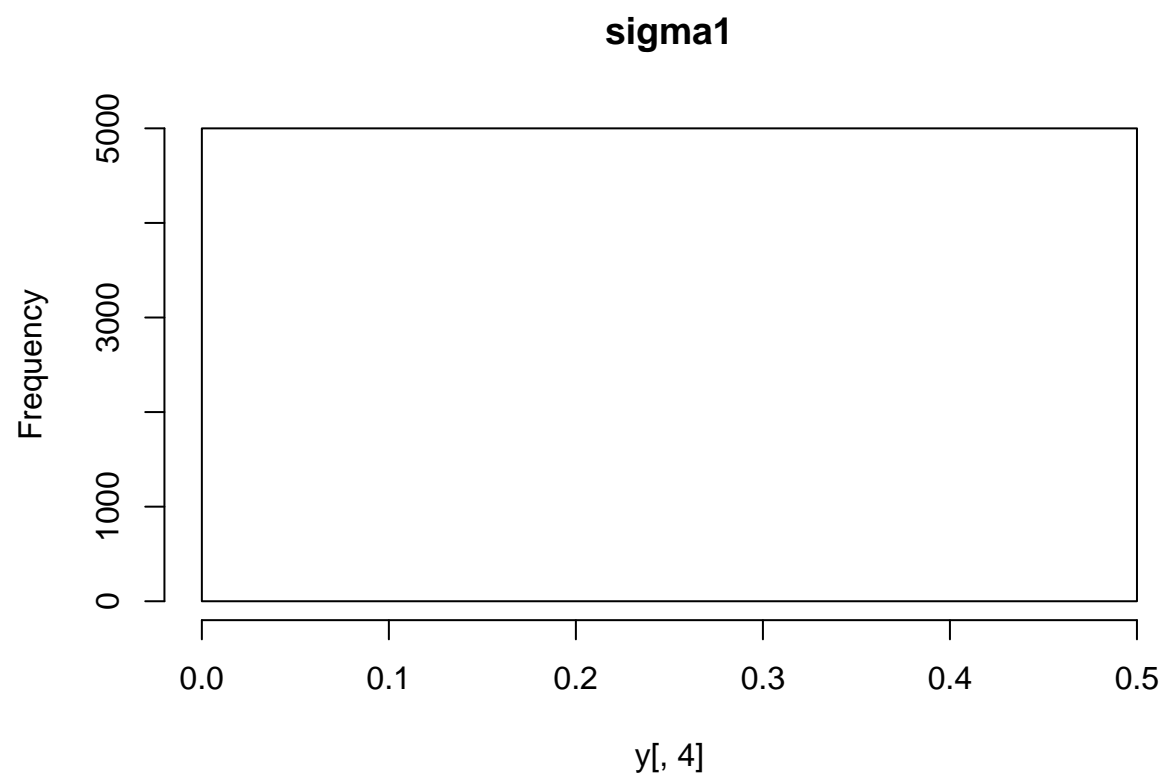
```
hist(y[, 3], main = "mu2")
```



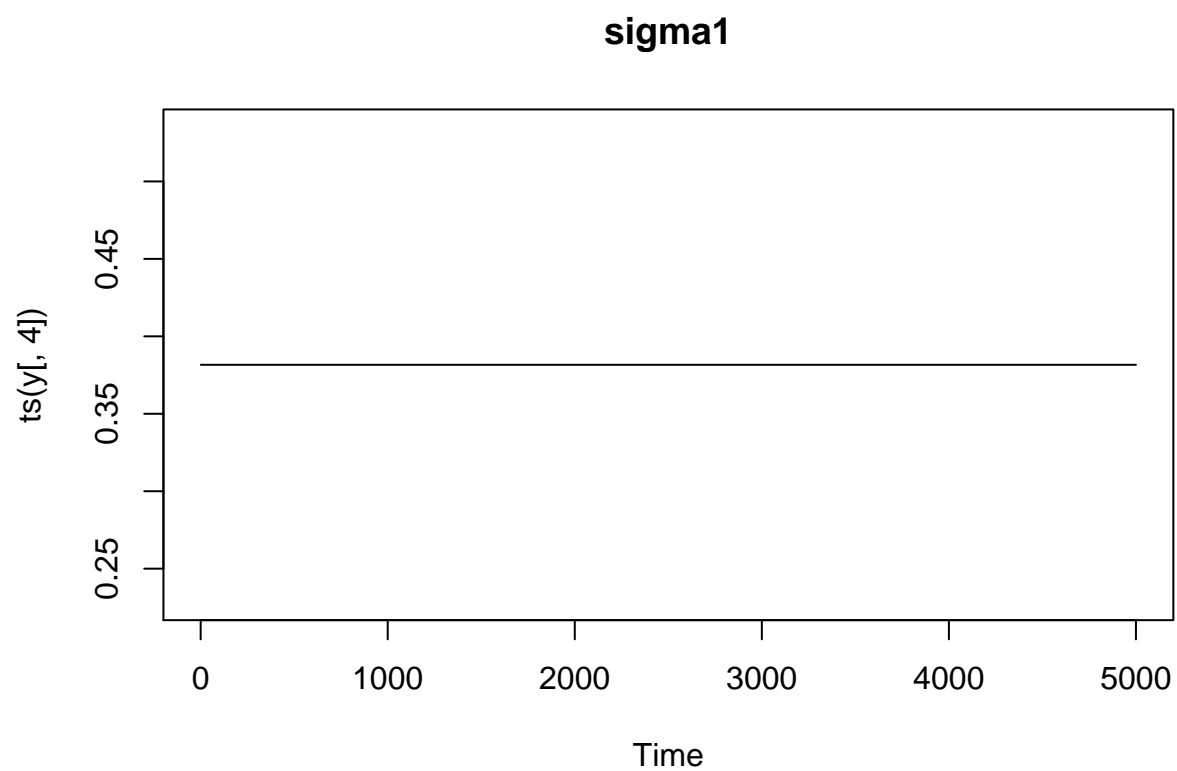
```
plot(ts(y[, 3]), main = "mu2")
```



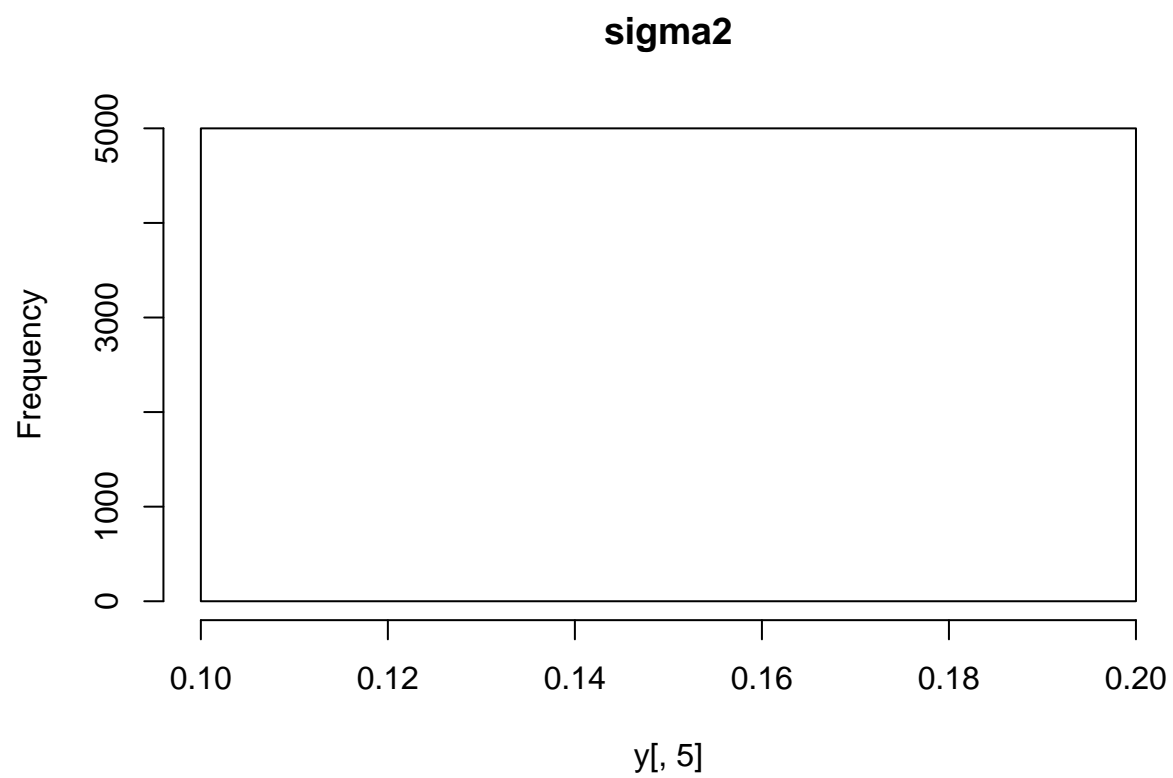
```
hist(y[, 4], main = "sigma1")
```



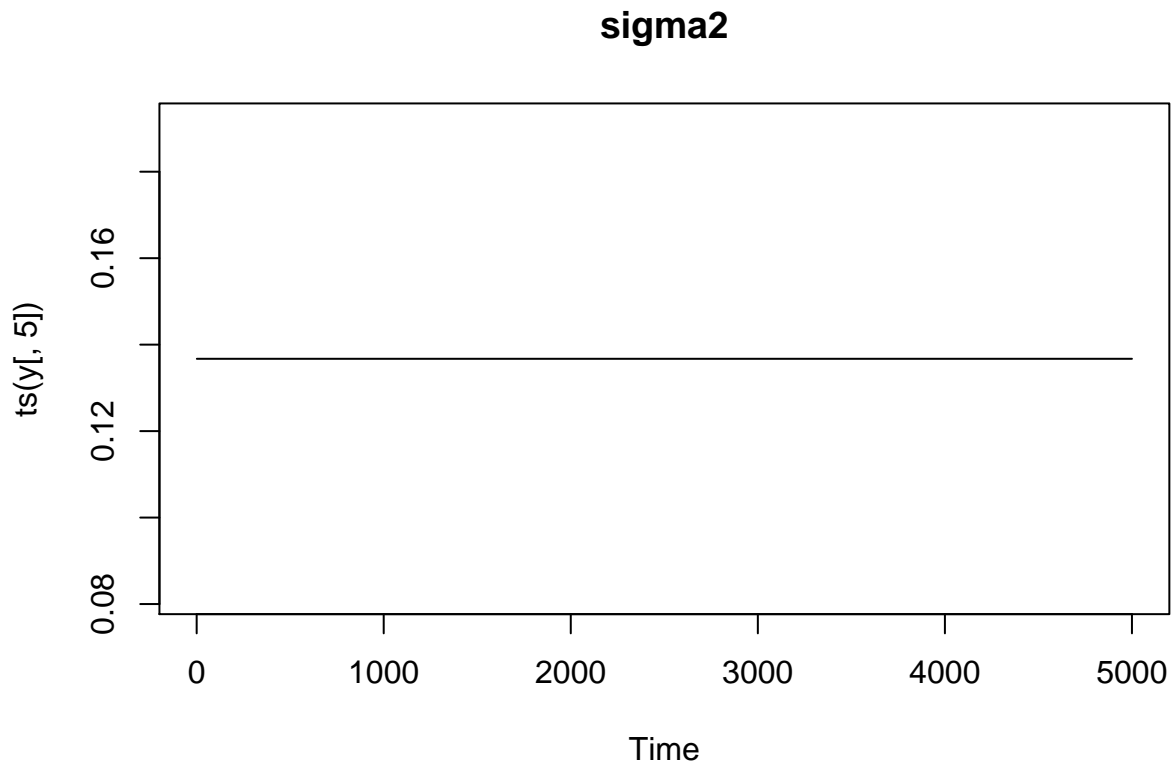
```
plot(ts(y[, 4]), main = "sigma1")
```

```
hist(y[, 5], main = "sigma2")
```

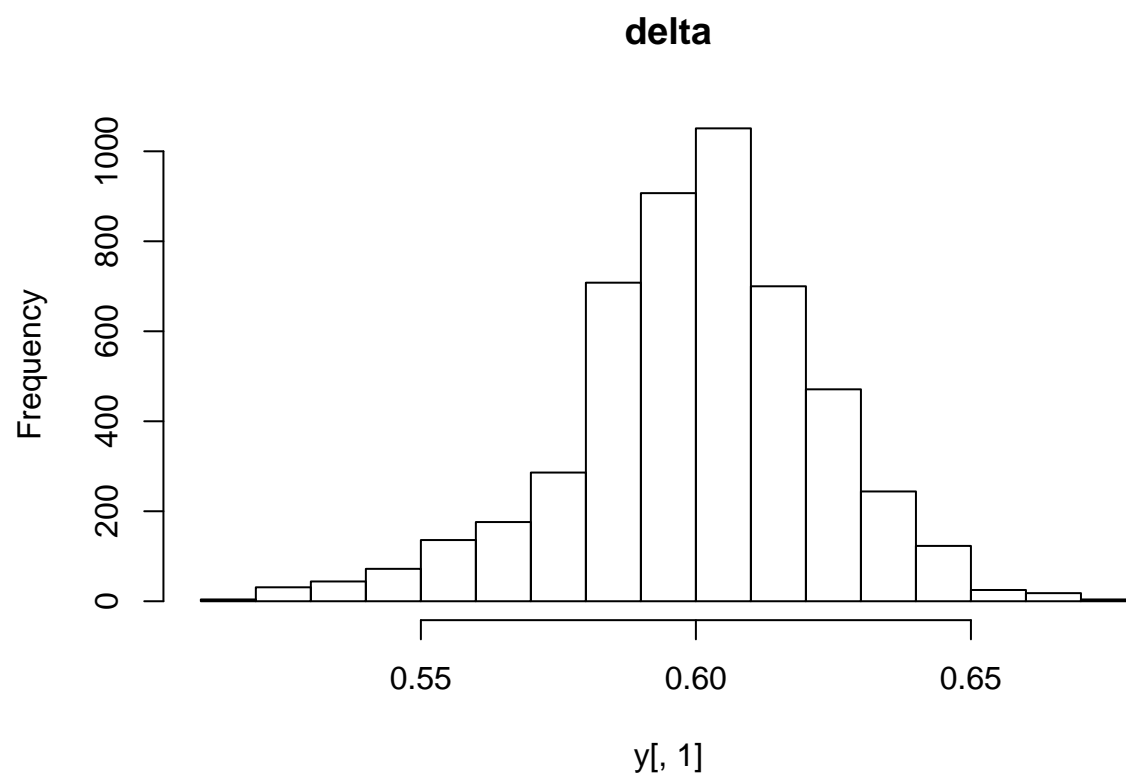


```
plot(ts(y[, 5]), main = "sigma2")
```

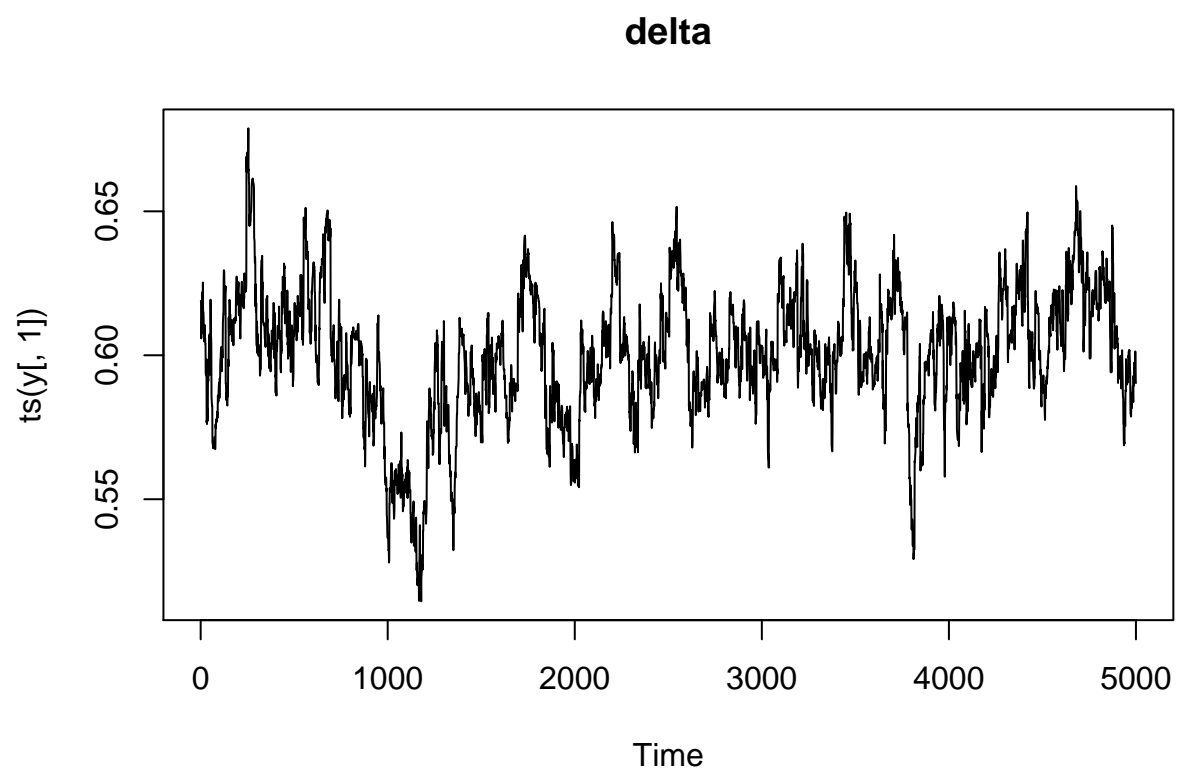


estimate parameters simultaneously

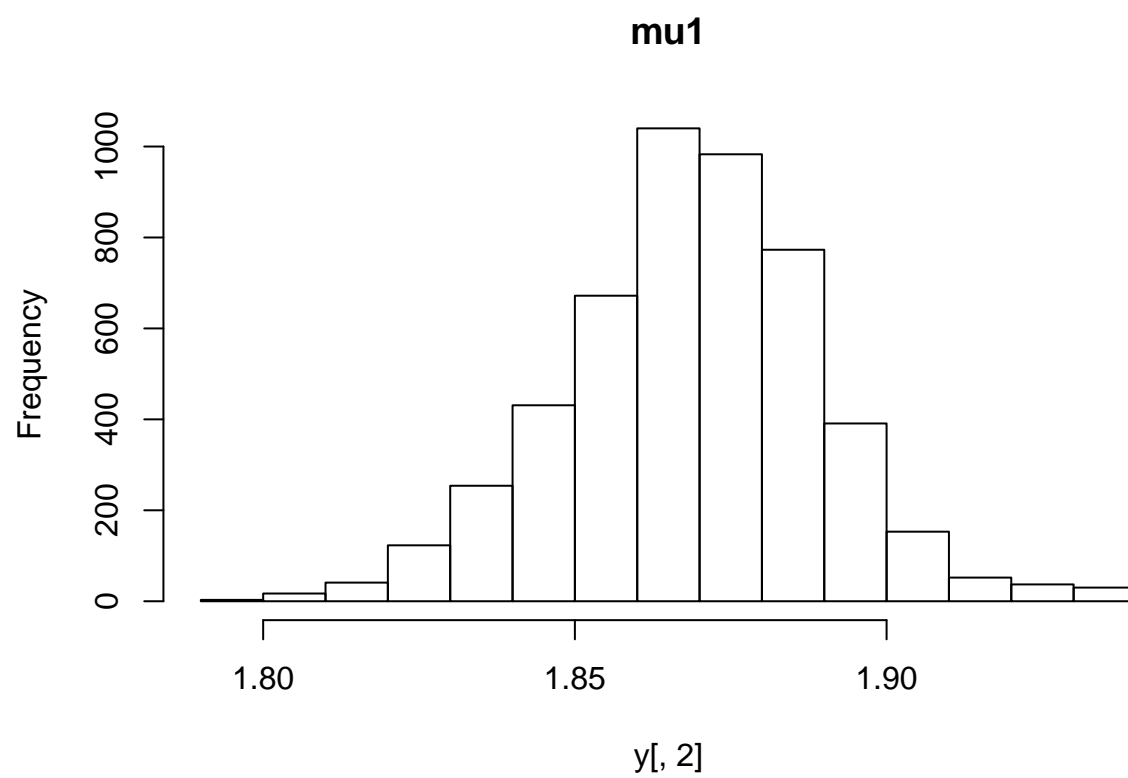
```
# true value
delta <- 0.7
mu1 <- 2
mu2 <- 4
sigma1 <- .5
sigma2 <- .8
# sample
n <- 1000
u <- rbinom(n, prob = delta, size = 1)
dat <- rnorm(n, ifelse(u == 1, mu1, mu2), ifelse(u == 1, sigma1, sigma2))
mylike <- function(delta, mu1, mu2, sigma1, sigma2, x){
  sum(log(delta / sigma1 * exp(-(x - mu1) ^ 2 / (2 * sigma1 ^ 2)) + (1 - delta) / sigma2 * exp(-(x - mu2) ^ 2 / (2 * sigma2 ^ 2))))
}
init <- c(.6, 3, 5, 3, 4)
y <- arms(init, function(x) mylike(x[1], x[2], x[3], x[4], x[5], x = dat), function(x) (x[1] > 1e-5) *
hist(y[, 1], main = "delta")
```



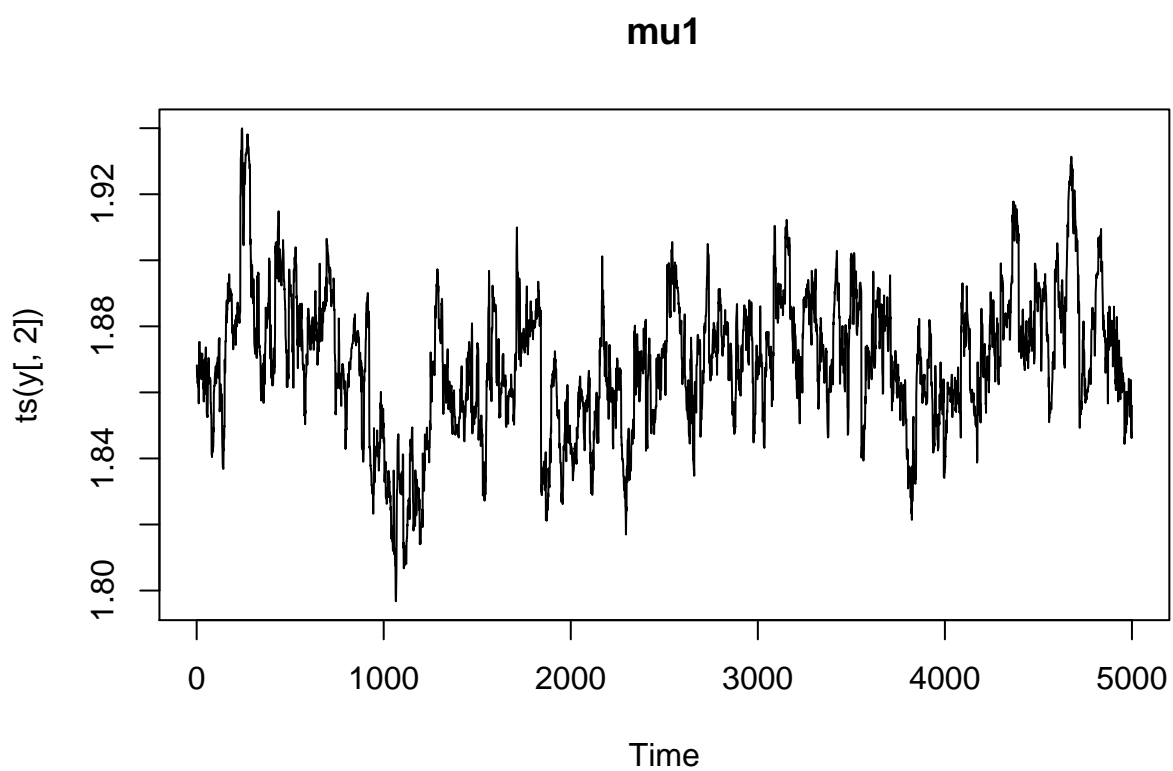
```
plot(ts(y[, 1]), main = "delta")
```



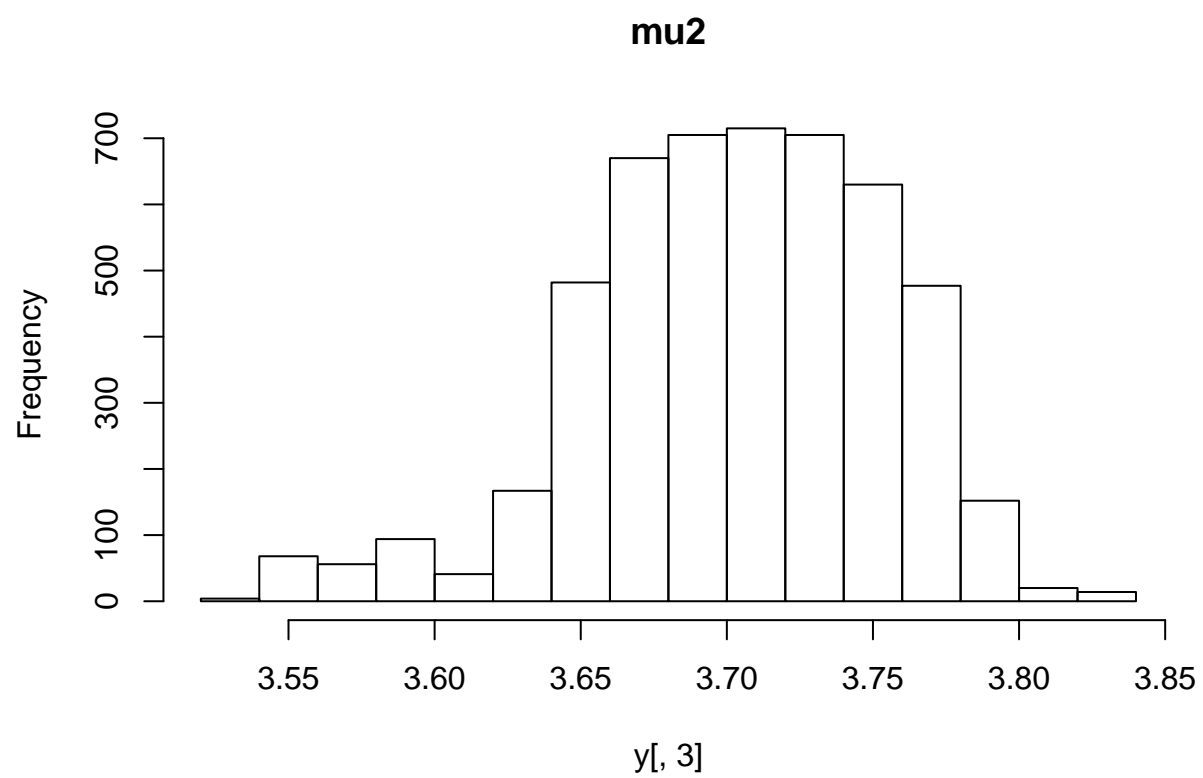
```
hist(y[, 2], main = "mu1")
```



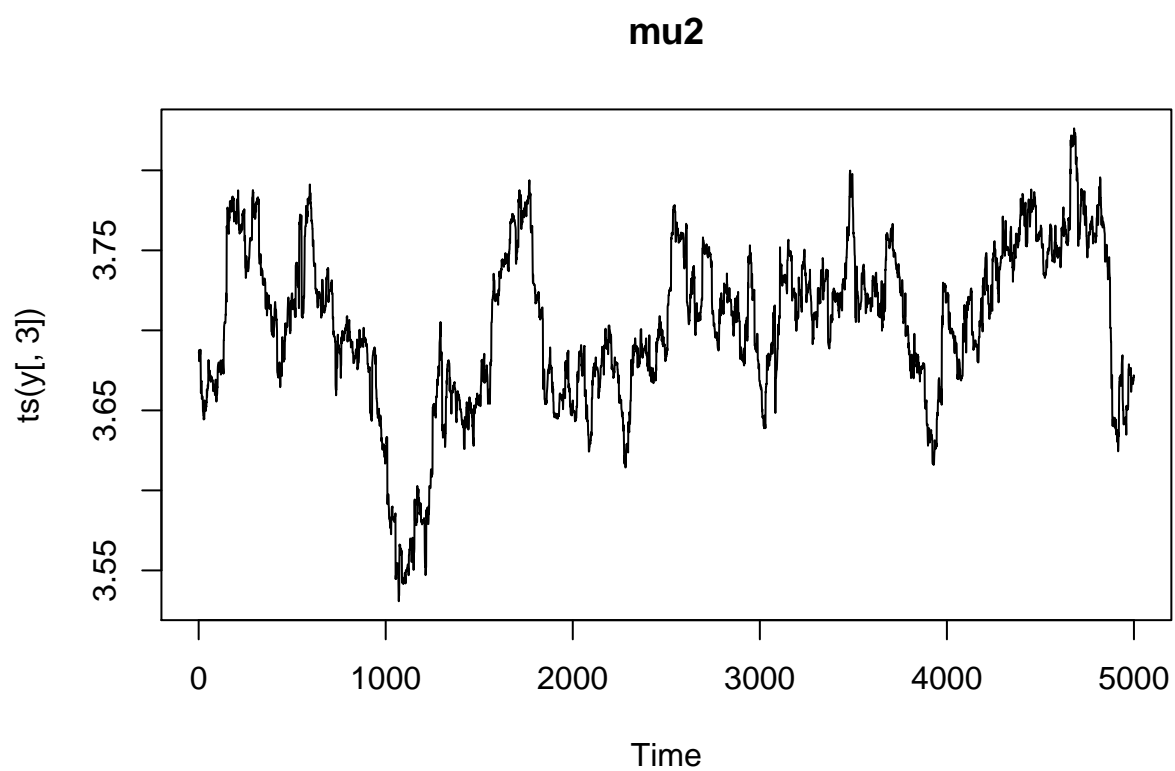
```
plot(ts(y[, 2]), main = "mu1")
```



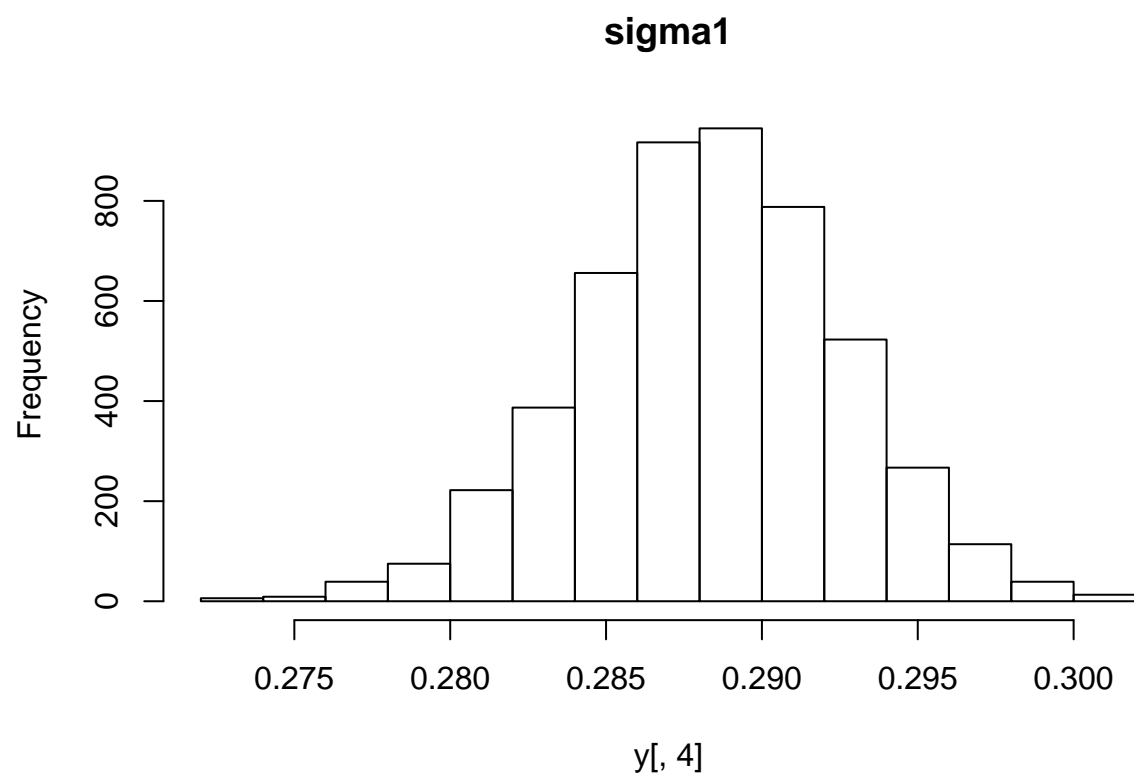
```
hist(y[, 3], main = "mu2")
```



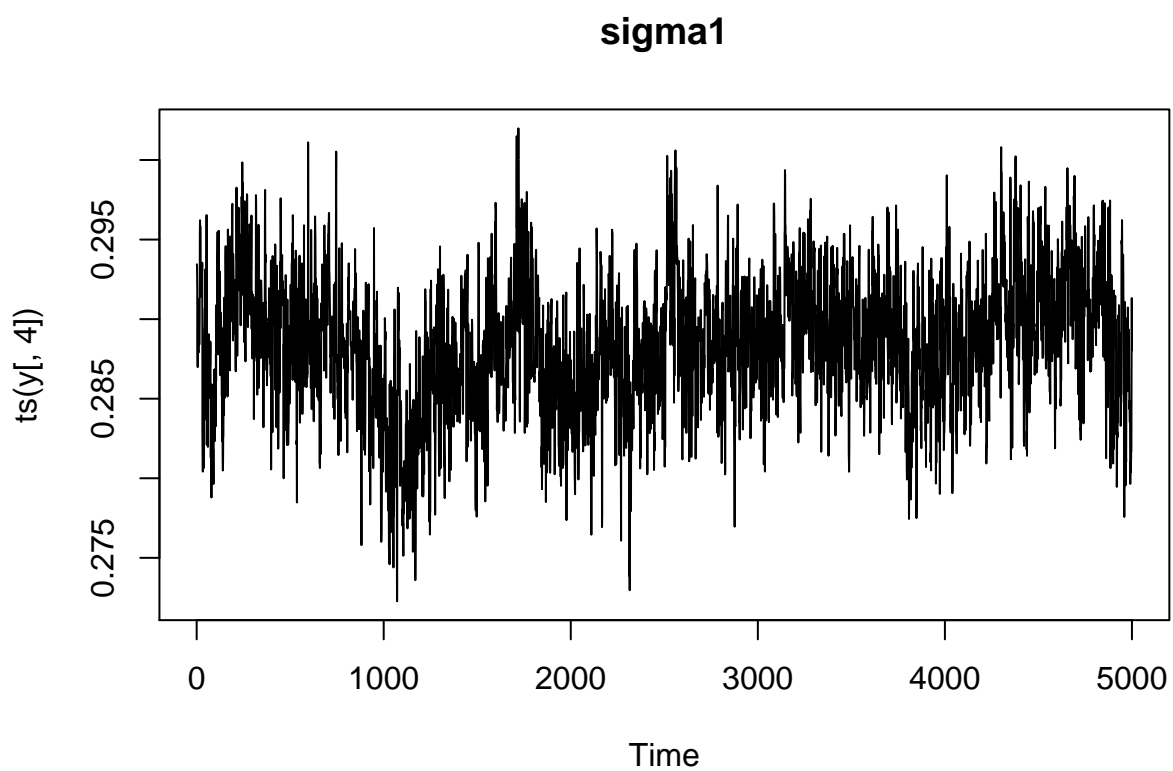
```
plot(ts(y[, 3]), main = "mu2")
```

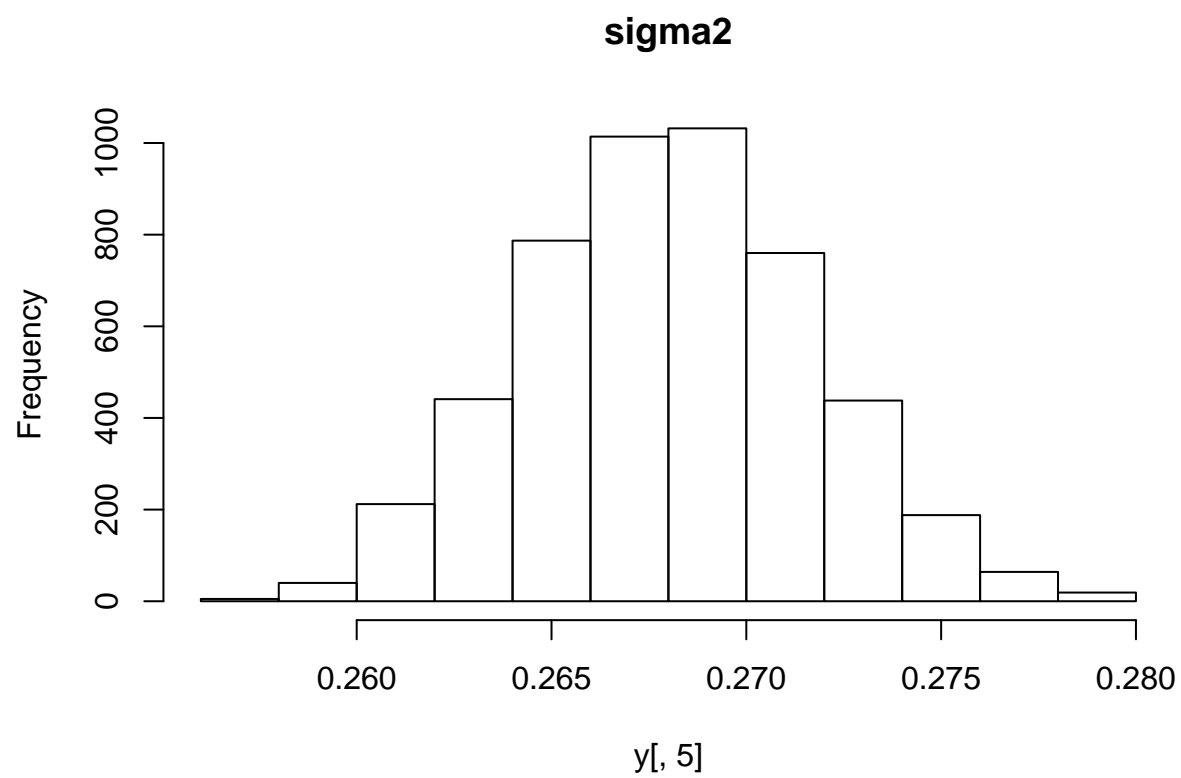
```
hist(y[, 4], main = "sigma1")
```



```
plot(ts(y[, 4]), main = "sigma1")
```



```
hist(y[, 5], main = "sigma2")
```



```
plot(ts(y[, 5]), main = "sigma2")
```

