HW7

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$$f(x|\delta,\mu_1,\mu_2,\sigma_1^2,\sigma_2^2)=\delta\frac{1}{\sqrt{2\pi}\sigma_1}e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}}+(1-\delta)\frac{1}{\sqrt{2\pi}\sigma_2}e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}}$$

$\mu_1\sim N(0,10^2)$, $\mu_2\sim N(0,10^2)$, $1/\sigma_1^2\sim Gamma(.5,10)$, $1/\sigma_2^2\sim Gamma(.5,10)$.

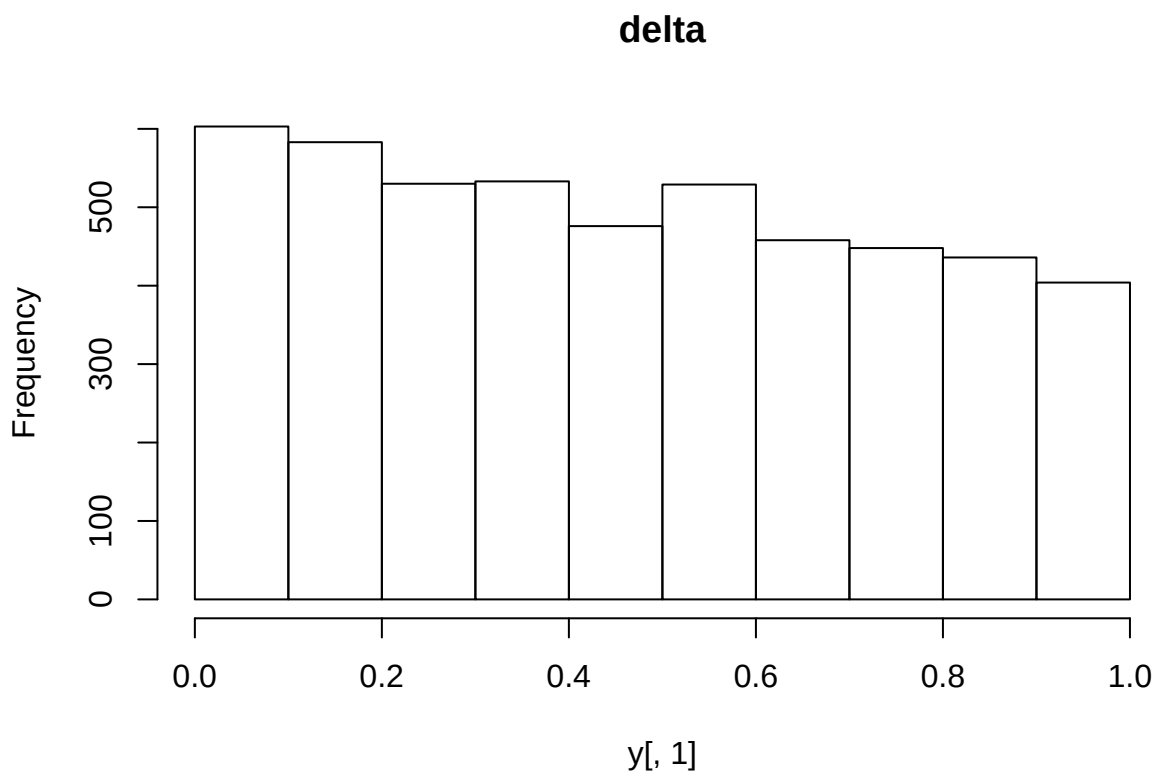
$$f(\delta,\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,x)=[\delta\frac{1}{\sqrt{2\pi}\sigma_1}e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}}+(1-\delta)\frac{1}{\sqrt{2\pi}\sigma_2}e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}}]$$
$$\frac{1}{2\pi}e^{-\frac{\mu_1^2+\mu_2^2}{200}}\frac{1}{\Gamma(.5)^2 10}(1/\sigma_1^2)^{1.5}(1/\sigma_2^2)^{1.5}e^{-\frac{1}{10\sigma_1^2}}e^{-\frac{1}{10\sigma_2^2}}$$

$$f(\delta,\mu_1,\mu_2,\sigma_1^2,\sigma_2^2|x)\propto\exp[\log(\frac{\delta}{\sigma_1}e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}}+\frac{1-\delta}{\sigma_2}e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}})-\mu_1^2/200-\mu_2^2/200-3\log\sigma_1-3\log\sigma_2-\frac{1}{10\sigma_1^2}-\frac{1}{10\sigma_2^2}]$$

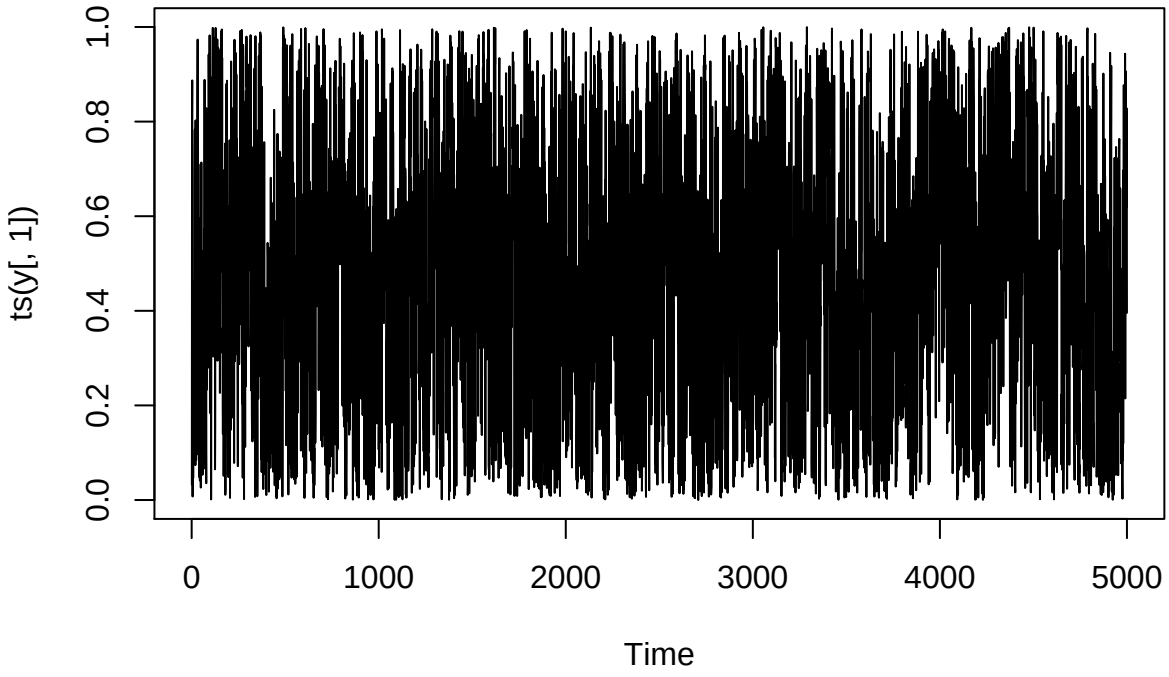
using arms() univariate version in each step

```
# true value

delta <- 0.7
mu1 <- 2
mu2 <- 4
sigma1 <- 2
sigma2 <- 3



# sample
n <- 1000
u <- rbinom(n, prob = delta, size = 1)
dat <- rnorm(n, ifelse(u == 1, mu1, mu2), ifelse(u == 1, sigma1, sigma2))


mylike <- function(delta, mu1, mu2, sigma1, sigma2, x){
  log(delta / sigma1 * exp(-(x - mu1) ^ 2 / (2 * sigma1 ^ 2)) + (1 - delta) / sigma2 * exp(-(x - mu2) ^
}


mymcmc <- function(niter, init, data){
    v <- matrix(NA, niter, 5)
    for (i in 1:niter) {
      ## delta
      delta <- arms(init[1], function(x) mylike(x,mu1 = init[2], mu2 = init[3], sigma1 = init[4], sigma
      v[i, 1] <- init[1] <- delta

      ## mu1
```

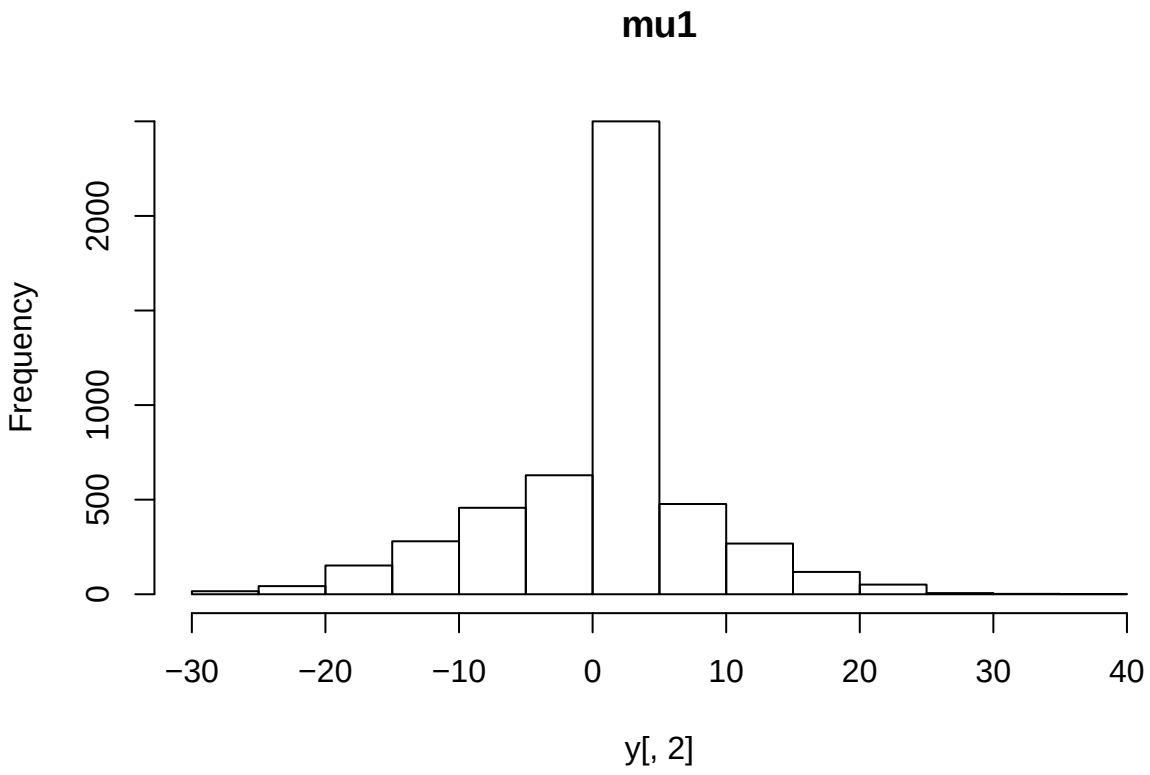
```
      mu1 <- arms(init[2], function(x) mylike(delta = init[1], x, mu2 = init[3], sigma1 = init[4], sigma
      v[i, 2] <- init[2] <- mu1

      ## mu2
      mu2 <- arms(init[3], function(x) mylike(delta = init[1], mu1 = init[2], x, sigma1 = init[4], sigma
      v[i, 3] <- init[3] <- mu2

      ## sigma1
      sigma1 <- arms(init[4], function(x) mylike(delta = init[1], mu1 = init[2], mu2 = init[3], x, sigma
      v[i, 4] <- init[4] <- sigma1

      ## sigma2
      sigma2 <- arms(init[5], function(x) mylike(delta = init[1], mu1 = init[2], mu2 = init[3], sigma1 =
      v[i, 5] <- init[5] <- sigma2
    }
    v
}


niter <- 10000
init <- c(.6, 3, 5, 3, 4)
y <- mymcmc(niter, init, dat)[-(1: 5000), ]
hist(y[, 1], main = "delta")
```



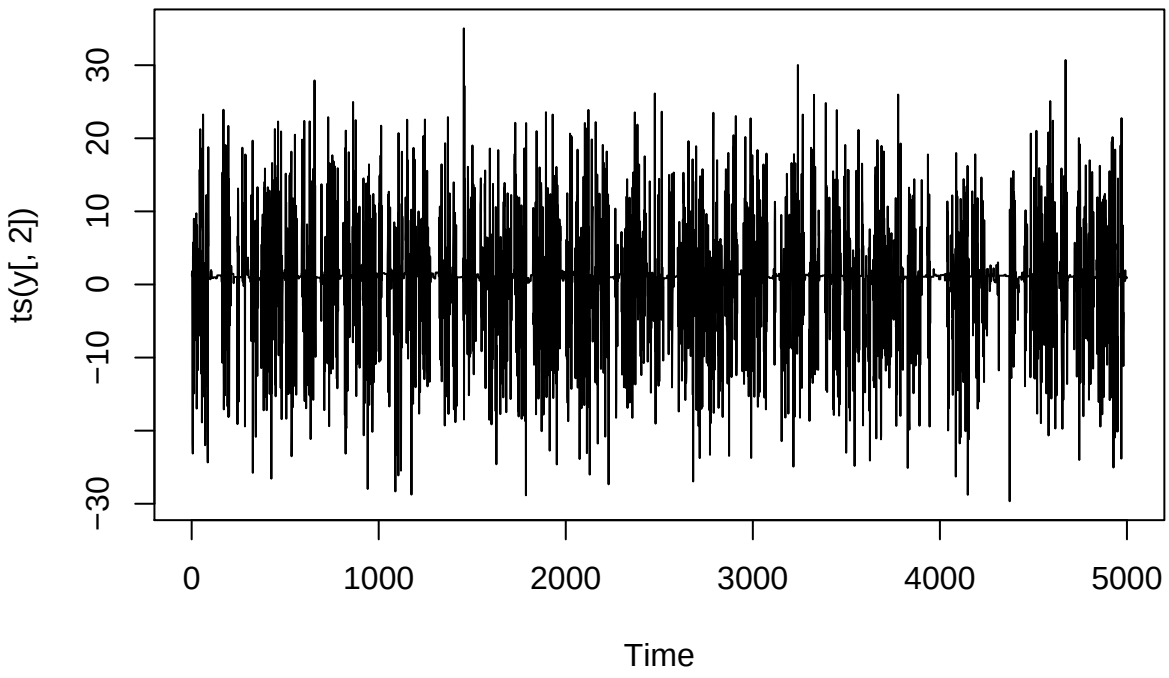
```r
plot(ts(y[, 1]))
```



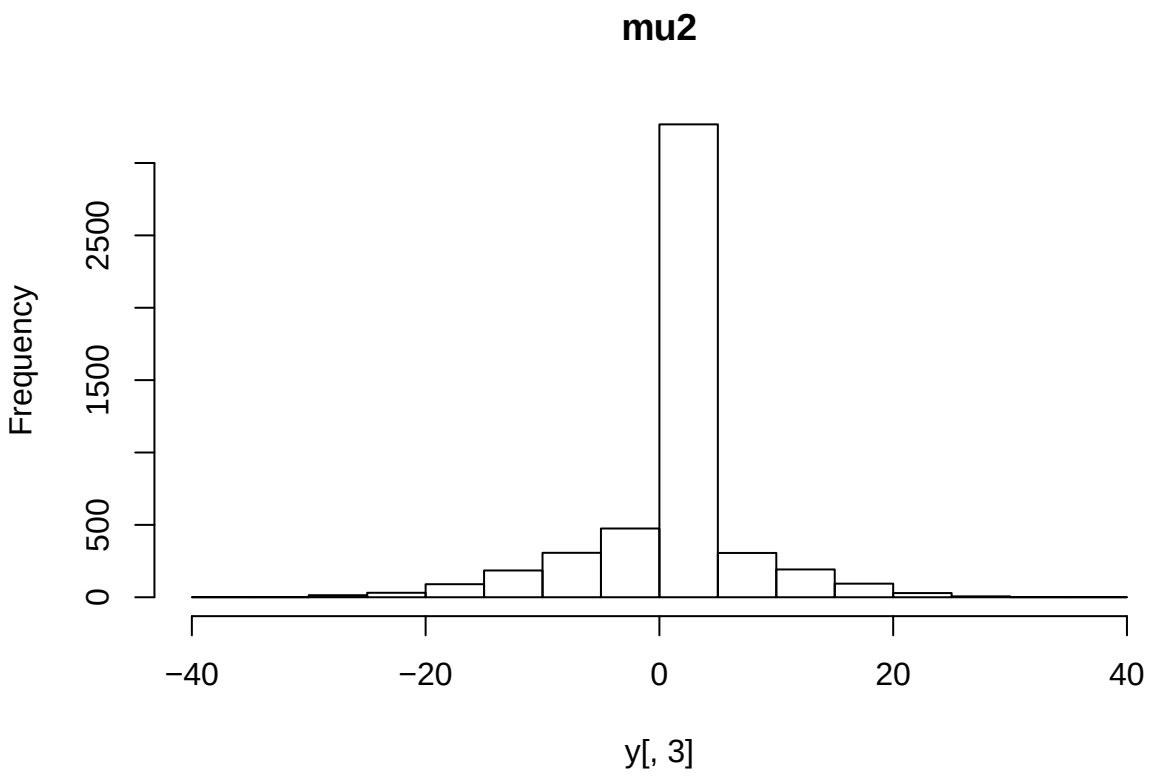

```r
hist(y[, 2], main = "mu1")
```



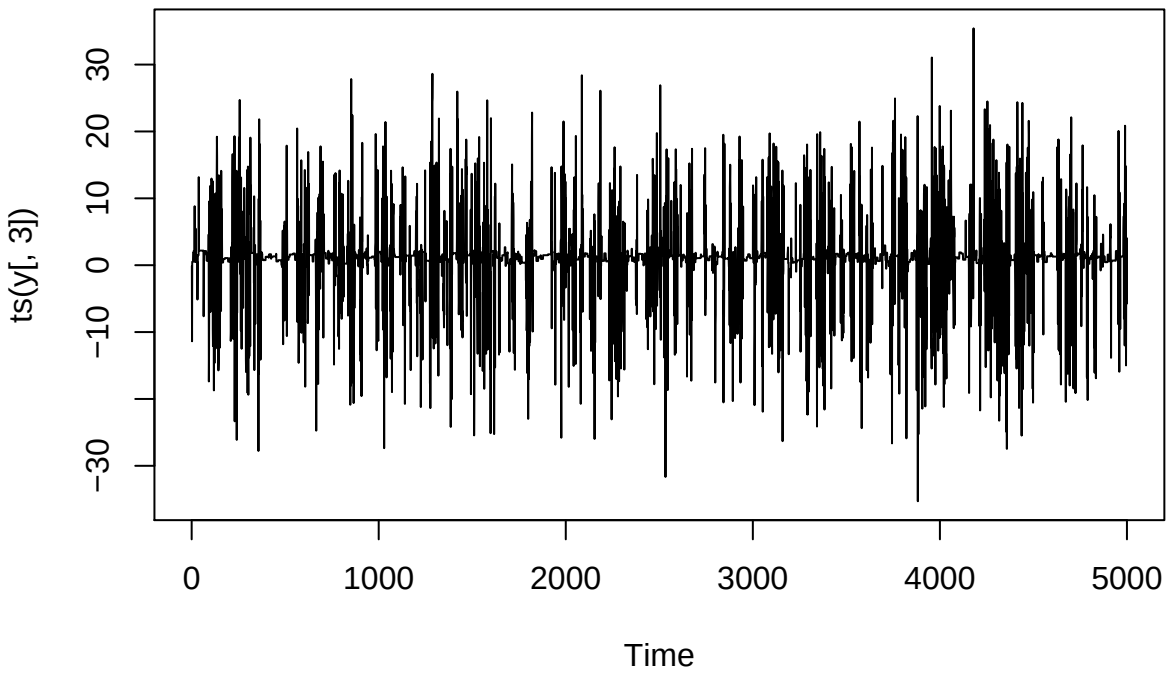

```
plot(ts(y[, 2]))
```



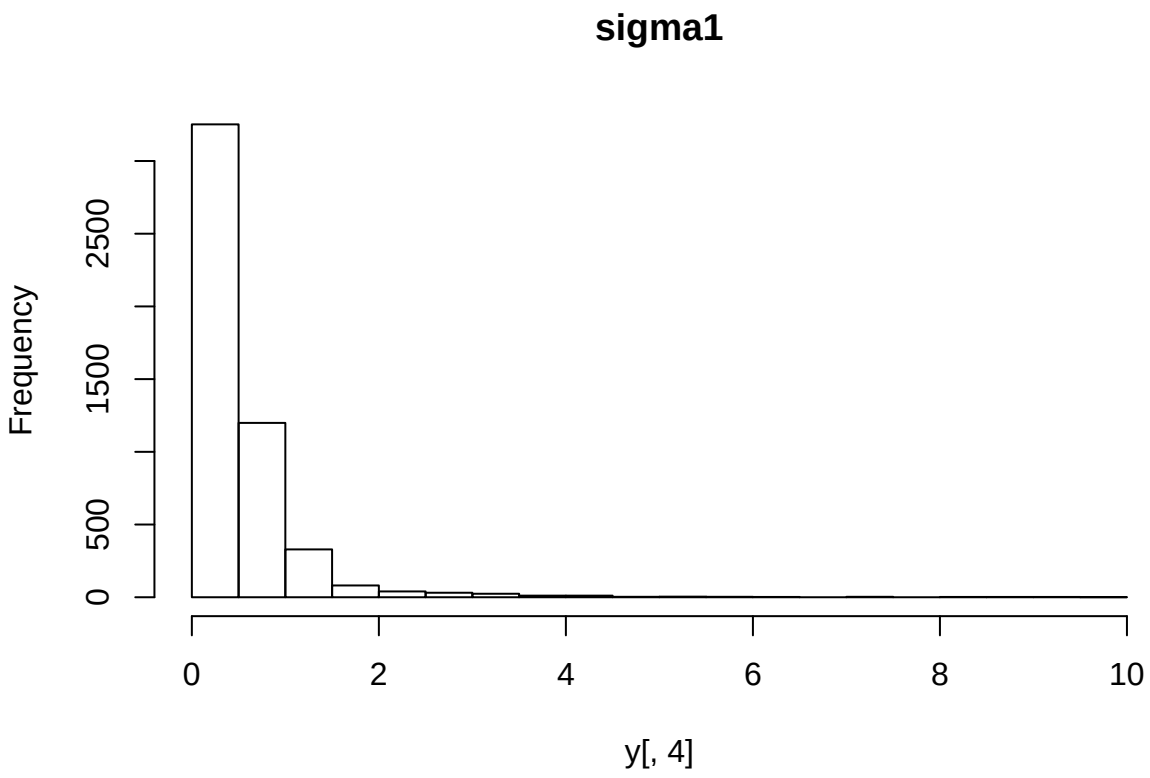

```
hist(y[, 3], main = "mu2")
```



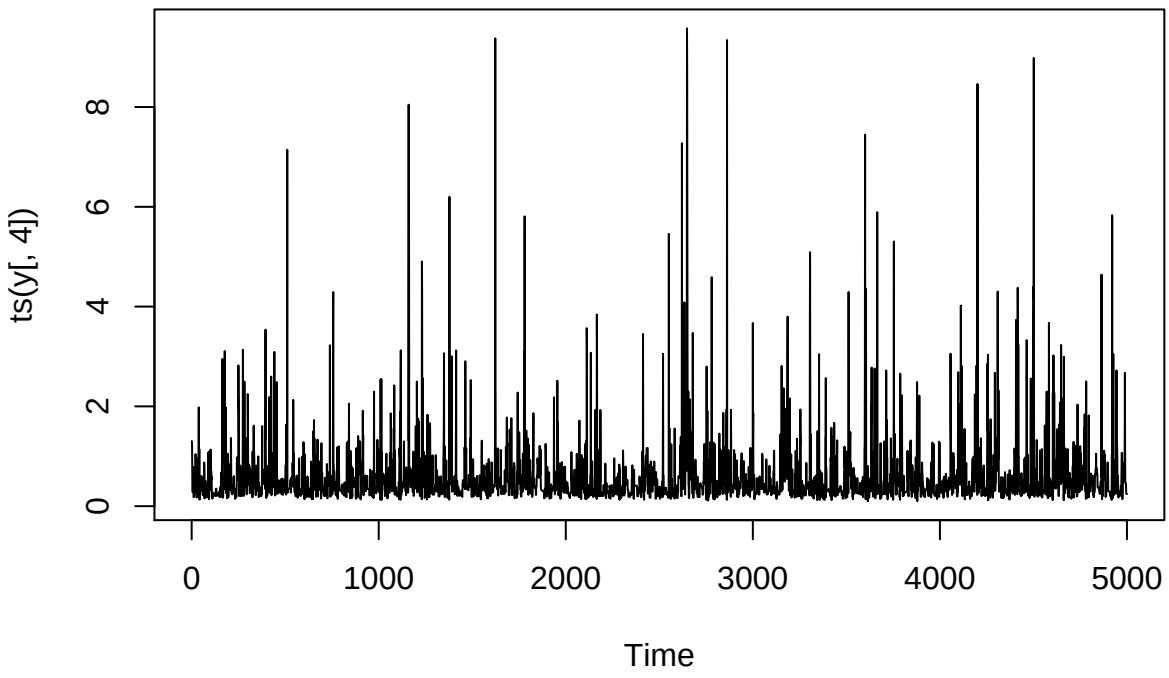

```
plot(ts(y[, 3]))
```



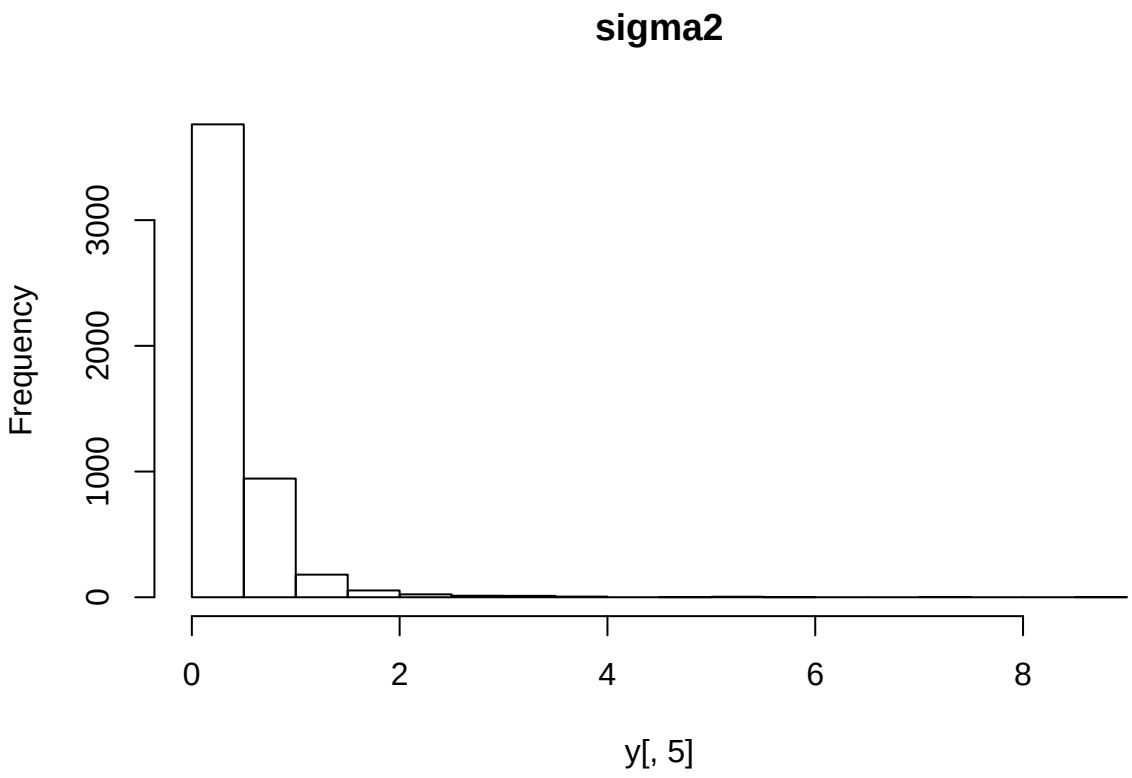

```r
hist(y[, 4], main = "sigma1")
```



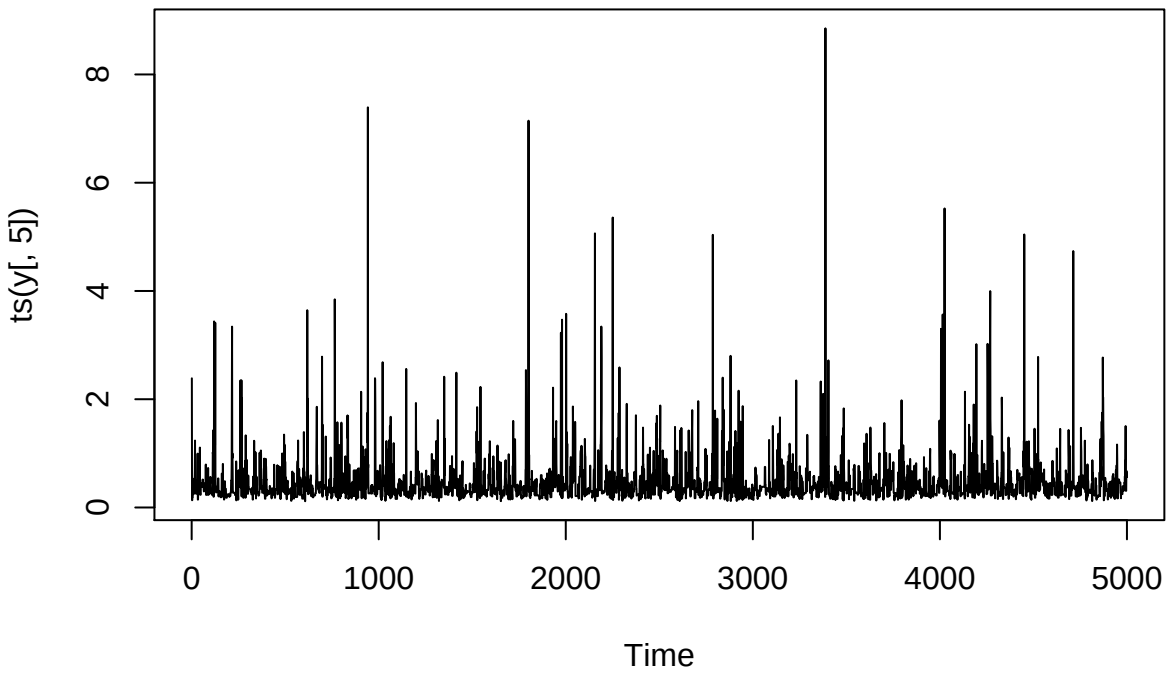

```
plot(ts(y[, 4]))
```




```
hist(y[, 5], main = "sigma2")
```



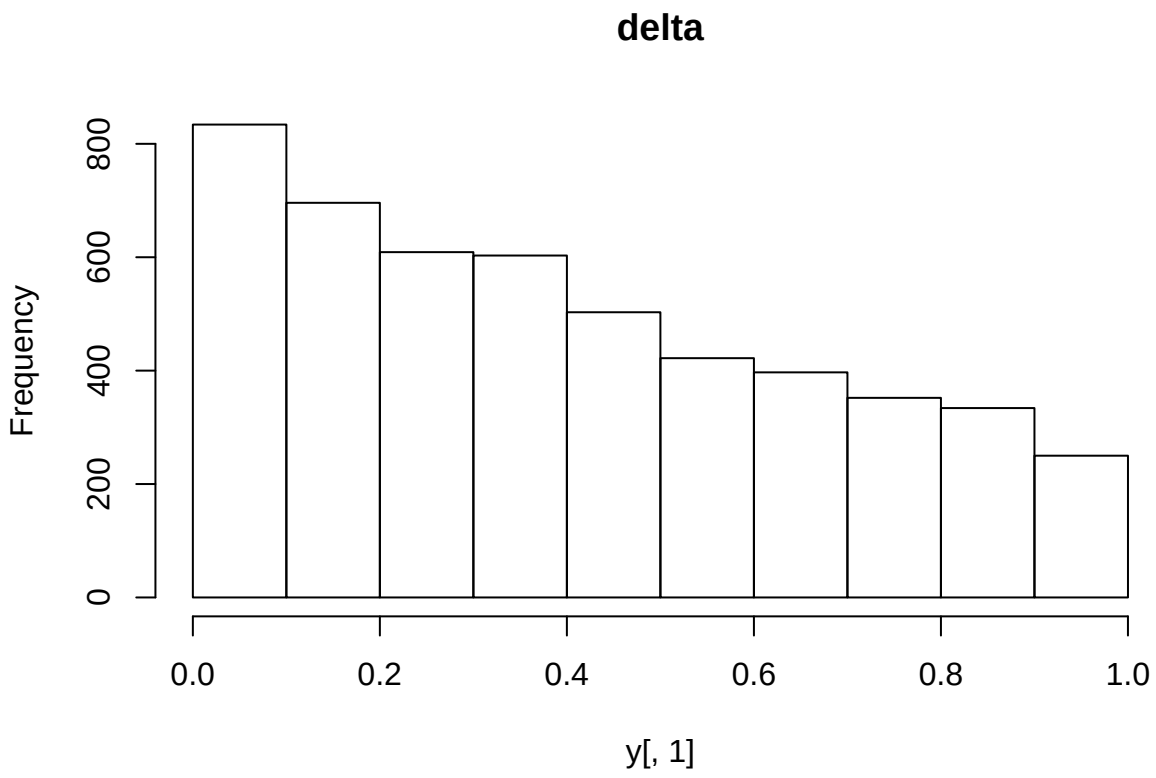

```
plot(ts(y[, 5]))
```



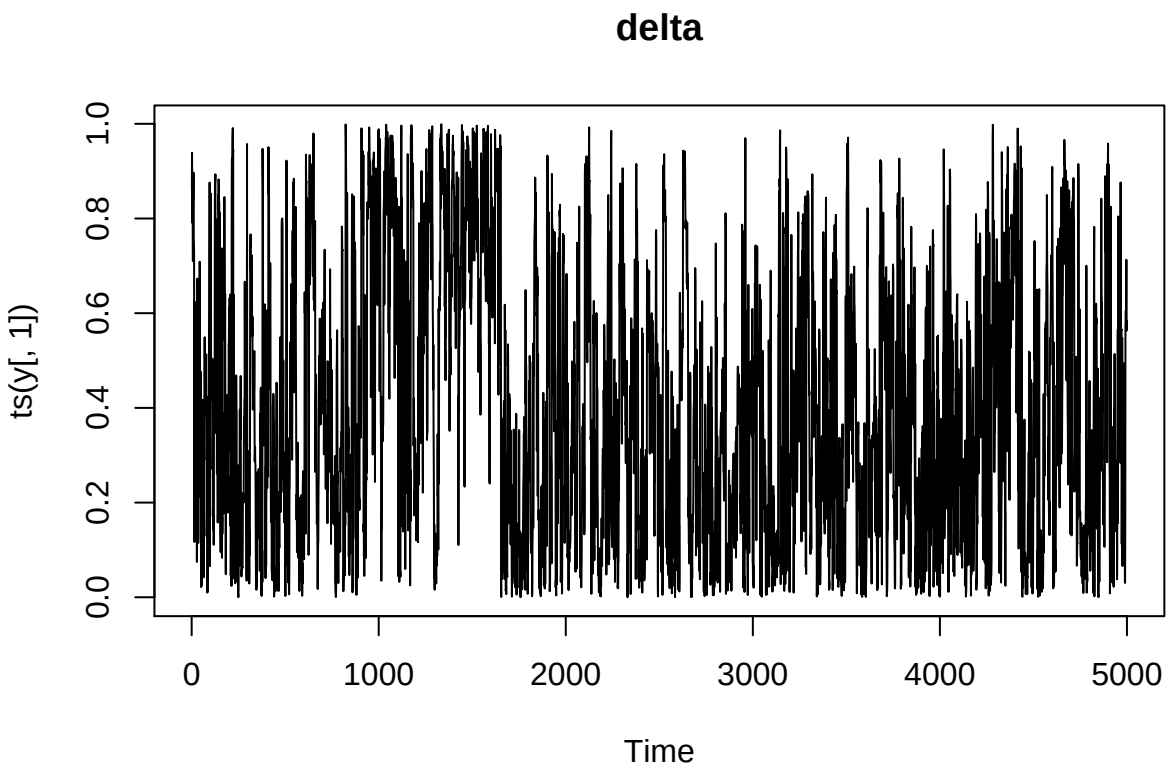
using arms() multivariate version

```
# true value
delta <- 0.7
mu1 <- 2
mu2 <- 4
sigma1 <- 2
sigma2 <- 3



# sample
n <- 1000
u <- rbinom(n, prob = delta, size = 1)
dat <- rnorm(n, ifelse(u == 1, mu1, mu2), ifelse(u == 1, sigma1, sigma2))


mylike <- function(delta, mu1, mu2, sigma1, sigma2, x){
  log(delta / sigma1 * exp(-(x - mu1) ^ 2 / (2 * sigma1 ^ 2)) + (1 - delta) / sigma2 * exp(-(x - mu2) ^
}

init <- c(.6, 3, 5, 3, 4)

y <- arms(init, function(x) mylike(x[1], x[2], x[3], x[4], x[5], x = dat), function(x) (x[1] > 1e-5) *
```

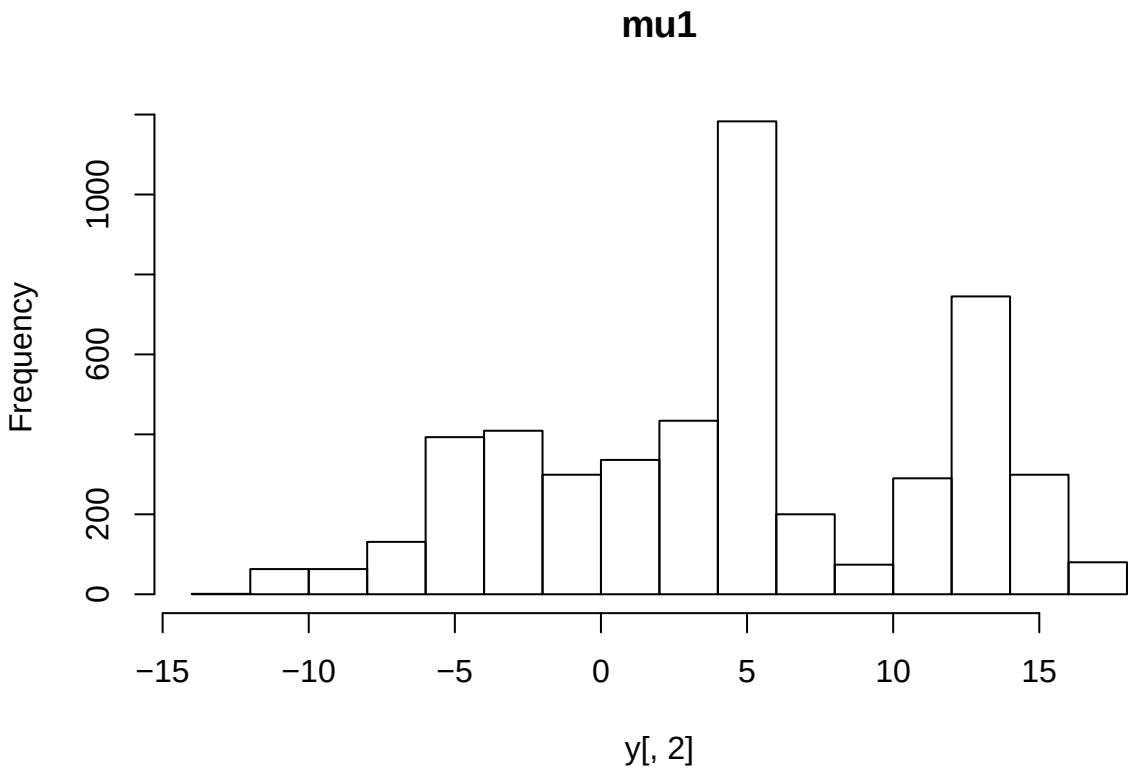
```r
hist(y[, 1], main = "delta")
```



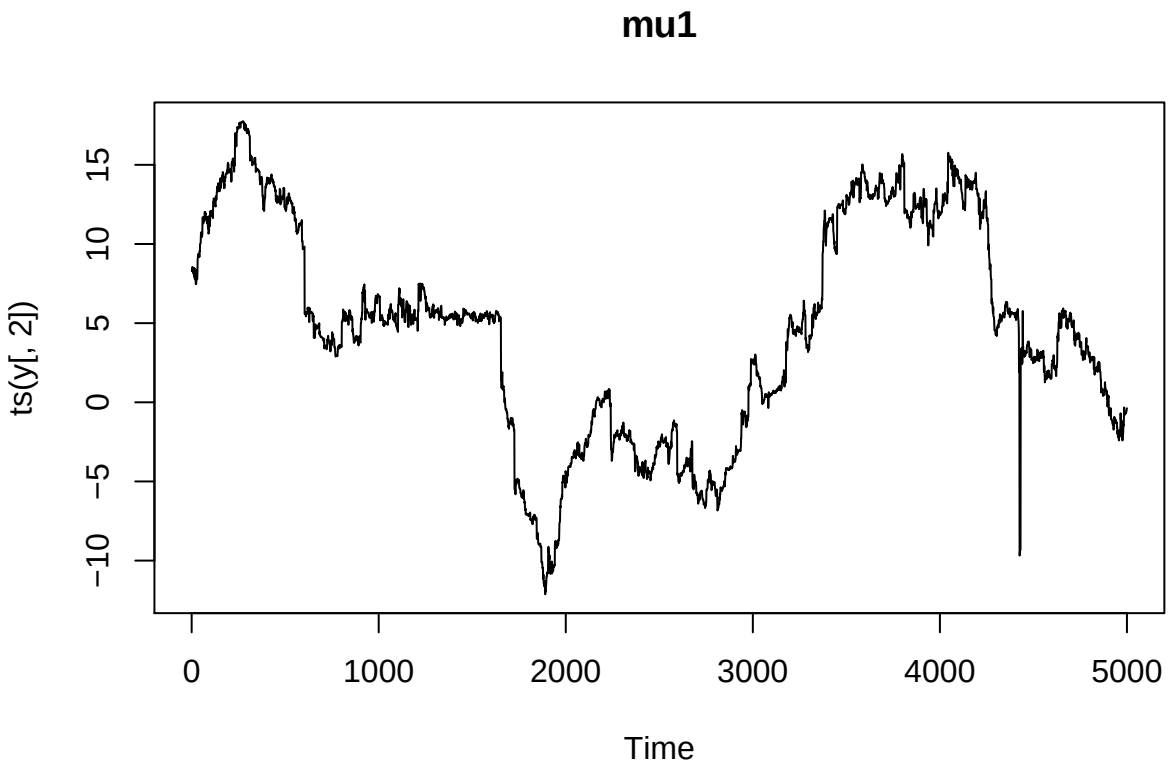

```r
plot(ts(y[, 1]), main = "delta")
```



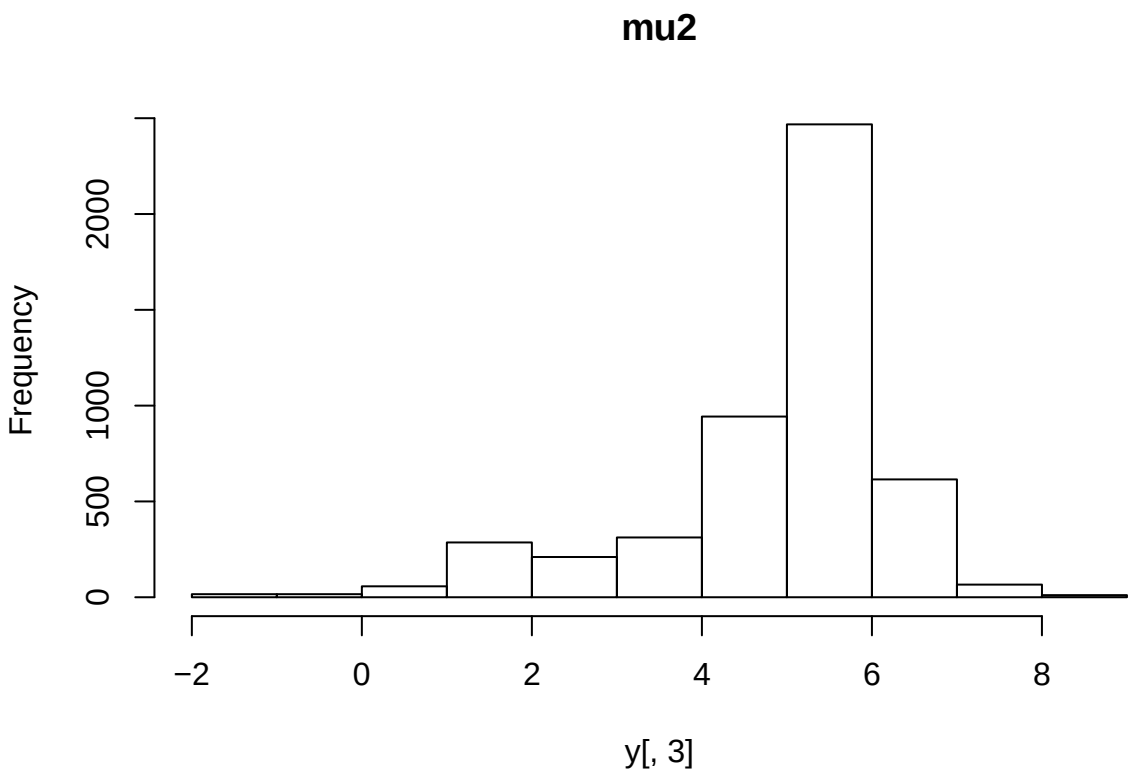

```r
hist(y[, 2], main = "mu1")
```



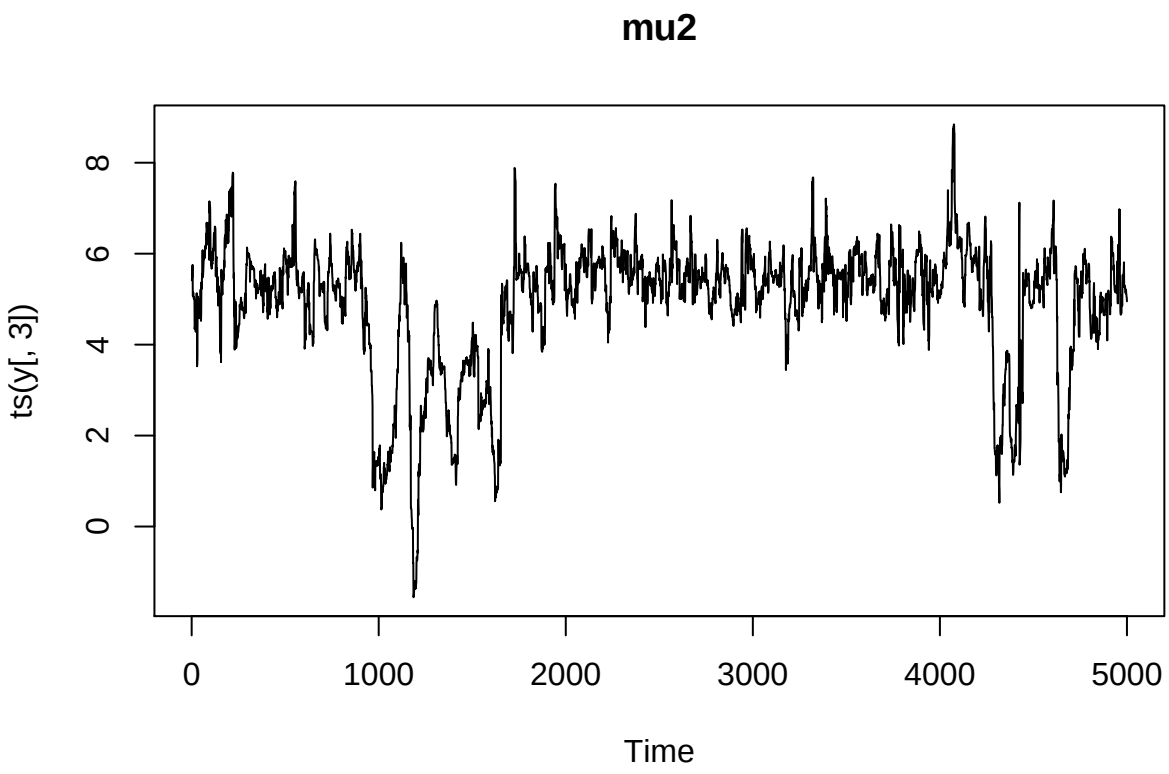

```
plot(ts(y[, 2]), main = "mu1")
```



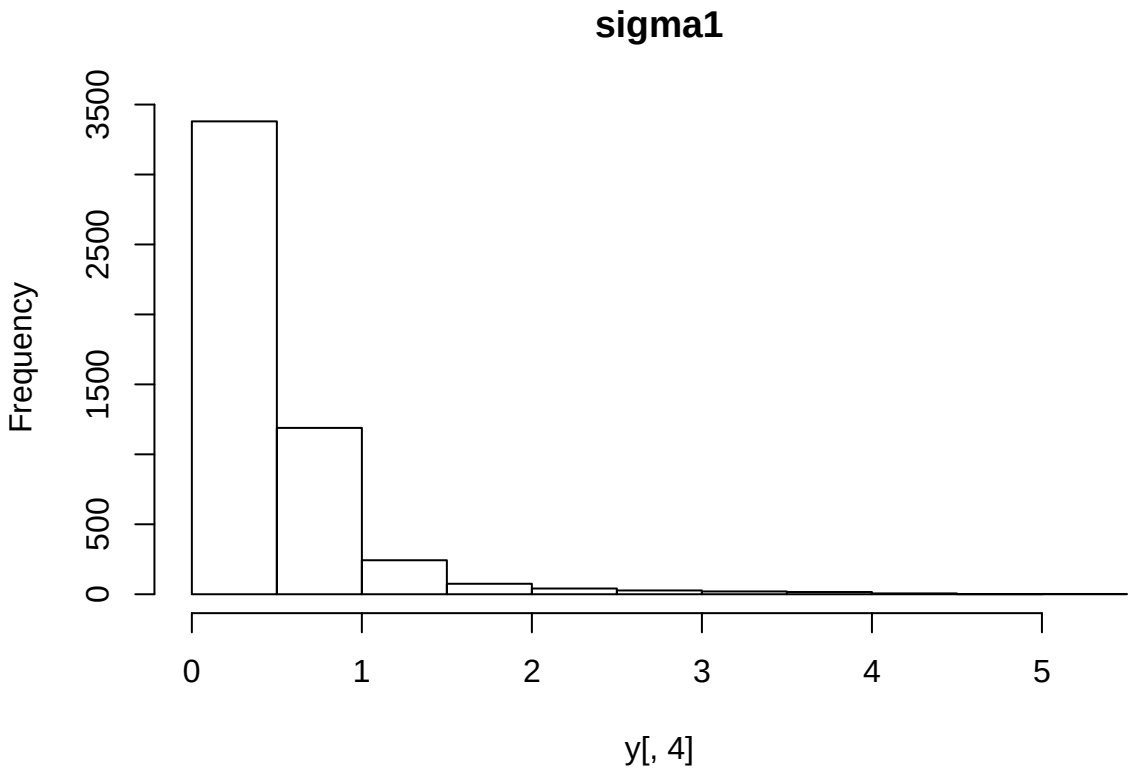

```r
hist(y[, 3], main = "mu2")
```



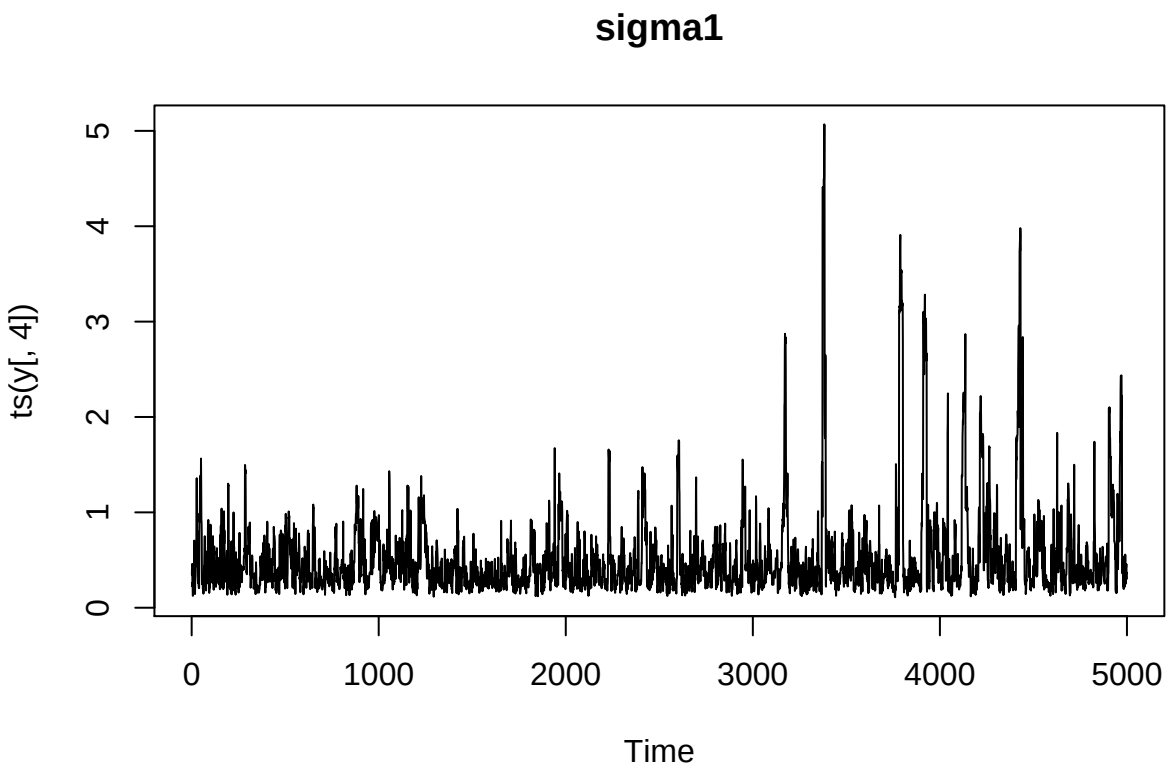

```
plot(ts(y[, 3]), main = "mu2")
```



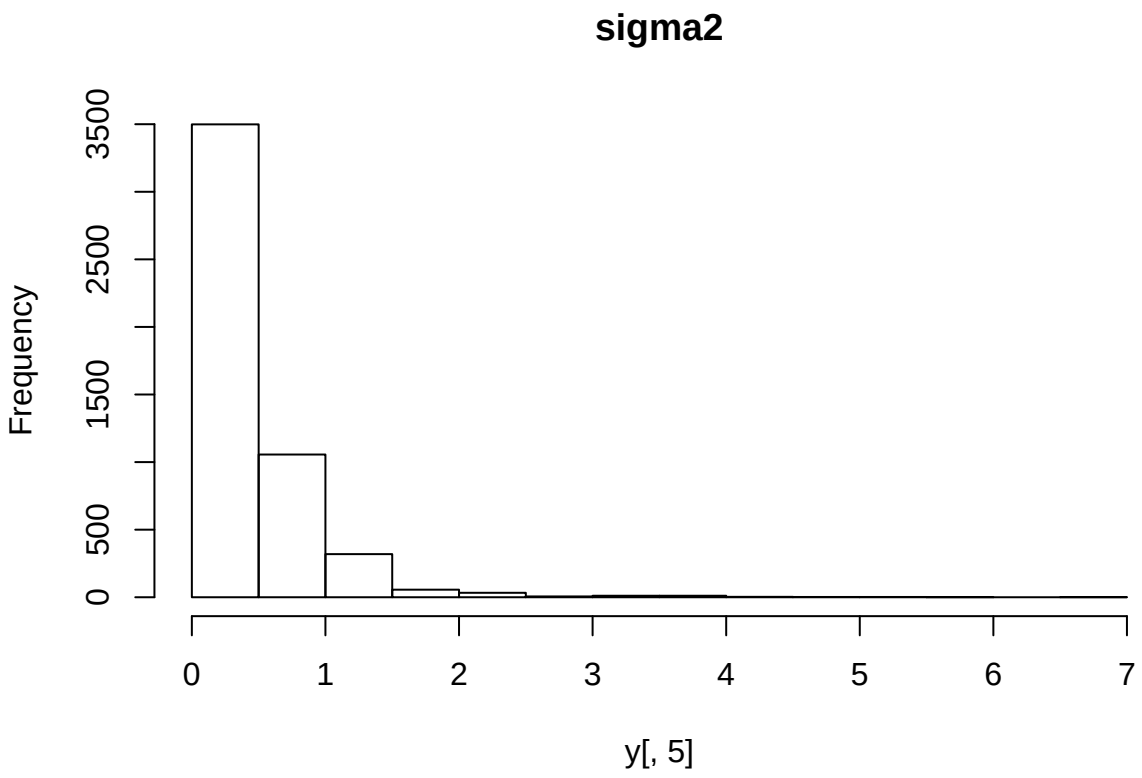

```
hist(y[, 4], main = "sigma1")
```



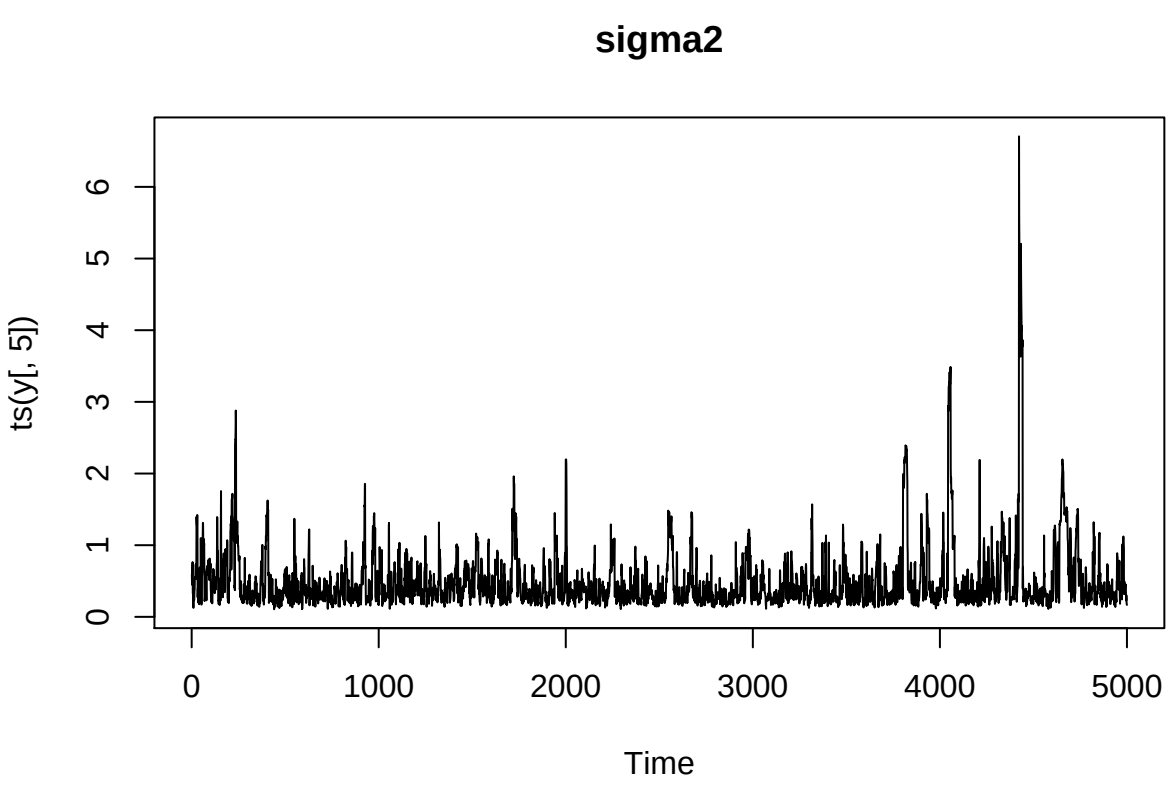

```
plot(ts(y[, 4]), main = "sigma1")
```




```
hist(y[, 5], main = "sigma2")
```




```
plot(ts(y[, 5]), main = "sigma2")
```


sigma2
ts(y[, 5])
0
1
2
3
4
5
6
0
1000
2000
3000
4000
5000
Time