HW7

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$$f(x|\delta,\mu_1,\mu_2,\sigma_1^2,\sigma_2^2) = \delta\frac{1}{\sqrt{2\pi}\sigma_1}e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} + (1-\delta)\frac{1}{\sqrt{2\pi}\sigma_2}e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}}$$

$\mu_1 \sim N(0,10^2)$, $\mu_2 \sim N(0,10^2)$, $1/\sigma_1^2 \sim Gamma(.5,10)$, $1/\sigma_2^2 \sim Gamma(.5,10)$.

$$f(\delta,\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,x) = [\delta\frac{1}{\sqrt{2\pi}\sigma_1}e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} + (1-\delta)\frac{1}{\sqrt{2\pi}\sigma_2}e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}}]$$

$$\frac{1}{2\pi}e^{-\frac{\mu_1^2+\mu_2^2}{200}}\frac{1}{\Gamma(.5)^2 10}(1/\sigma_1^2)^{1.5}(1/\sigma_2^2)^{1.5}e^{-\frac{1}{10\sigma_1^2}}e^{-\frac{1}{10\sigma_2^2}}$$

$$f(\delta,\mu_1,\mu_2,\sigma_1^2,\sigma_2^2|x) \propto \exp[\log(\frac{\delta}{\sigma_1}e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} + \frac{1-\delta}{\sigma_2}e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}}) - \mu_1^2/200 - \mu_2^2/200 - 3\log\sigma_1 - 3\log\sigma_2 - \frac{1}{10\sigma_1^2} - \frac{1}{10\sigma_2^2}]$$

using arms() univariate version in each step

```
# true value
delta <- 0.7
mu1 <- 2
mu2 <- 4
sigma1 <- 2
sigma2 <- 3
# sample
n <- 1000
u <- rbinom(n, prob = delta, size = 1)
dat <- rnorm(n, ifelse(u == 1, mu1, mu2), ifelse(u == 1, sigma1, sigma2))
mylike <- function(delta, mu1, mu2, sigma1, sigma2, x){
  sum(log(delta / sigma1 * exp(-(x - mu1) ^ 2 / (2 * sigma1 ^ 2)) + (1 - delta) / sigma2 * exp(-(x - mu
}
mymcmc <- function(niter, init, data){
    v <- matrix(NA, niter, 5)
    for (i in 1:niter) {
      ## delta
      delta <- arms(init[1], function(x) mylike(x,mu1 = init[2], mu2 = init[3], sigma1 = init[4], sigma2
      v[i, 1] <- init[1] <- delta

      ## mu1
      mu1 <- arms(init[2], function(x) mylike(delta = init[1], x, mu2 = init[3], sigma1 = init[4], sigma
      v[i, 2] <- init[2] <- mu1

      ## mu2
      mu2 <- arms(init[3], function(x) mylike(delta = init[1], mu1 = init[2], x, sigma1 = init[4], sigma
      v[i, 3] <- init[3] <- mu2

      ## sigma1
```

```
    sigma1 <- arms(init[4], function(x) mylike(delta = init[1], mu1 = init[2], mu2 = init[3], x, sigma
    v[i, 4] <- init[4] <- sigma1

    ## sigma2
    sigma2 <- arms(init[5], function(x) mylike(delta = init[1], mu1 = init[2], mu2 = init[3], sigma1 =
    v[i, 5] <- init[5] <- sigma2
  }
  v
}
niter <- 10000
init <- c(.6, 3, 5, 3, 4)
y <- mymcmc(niter, init, dat)[-(1: 5000), ]
hist(y[, 1], main = "delta")
```
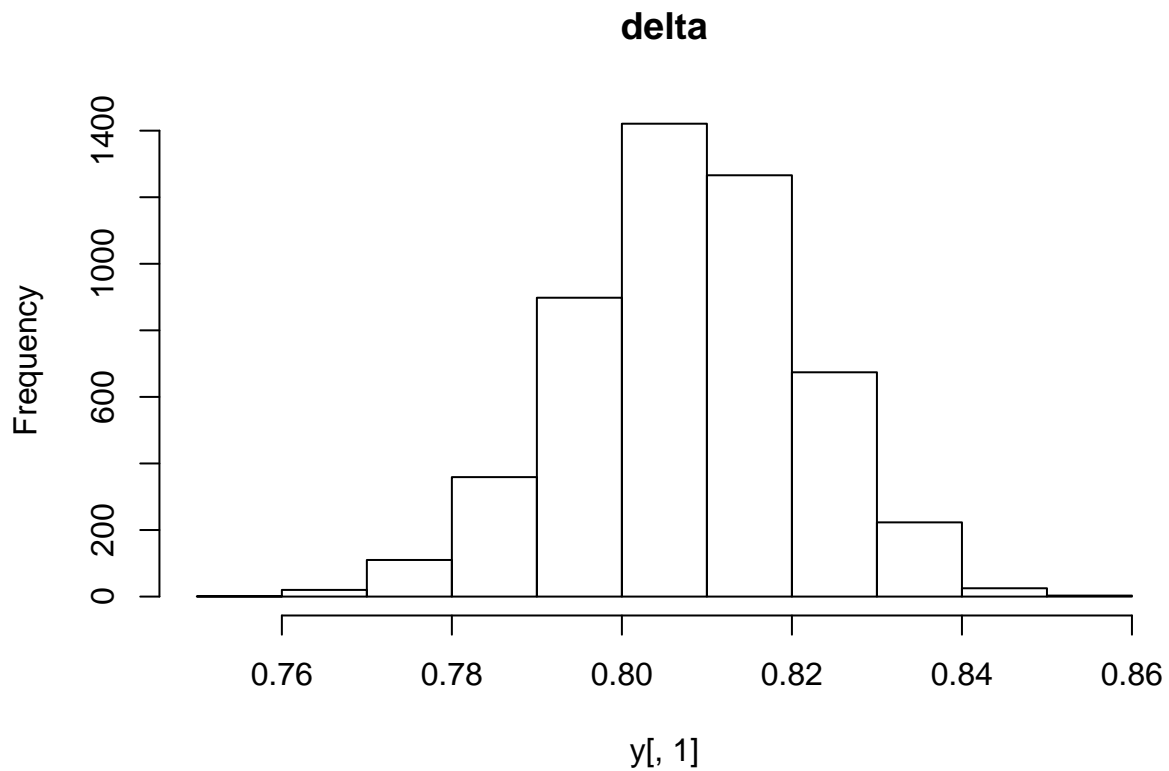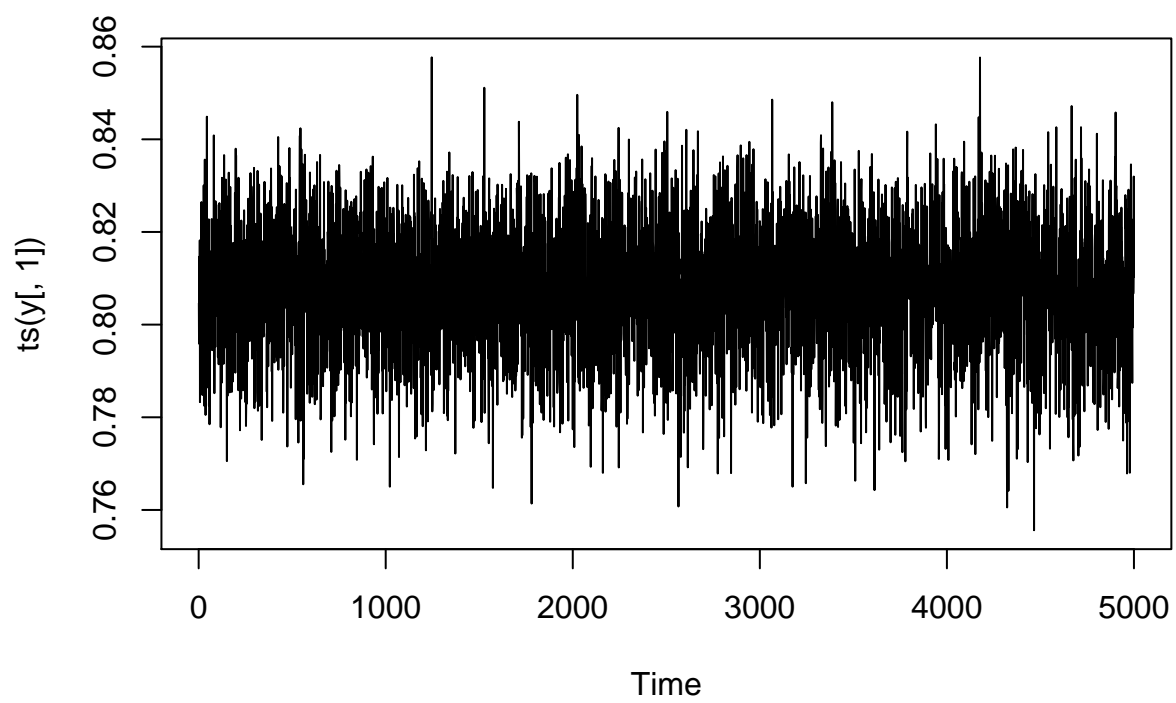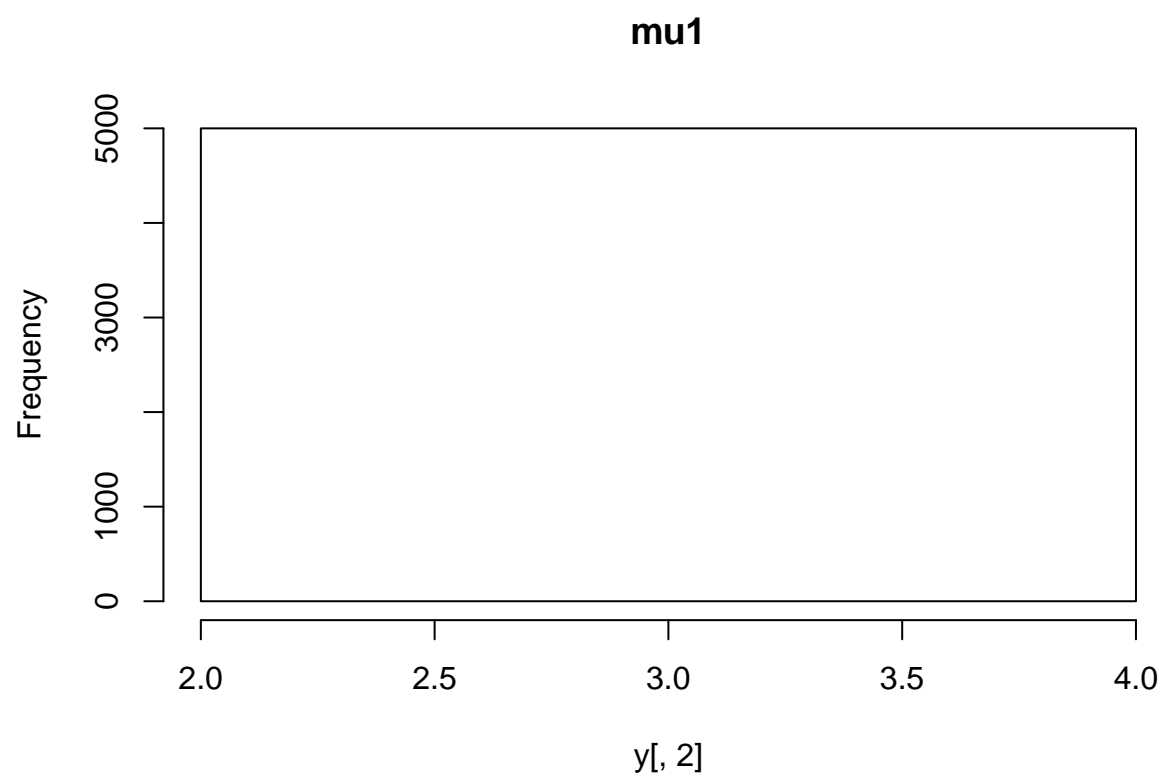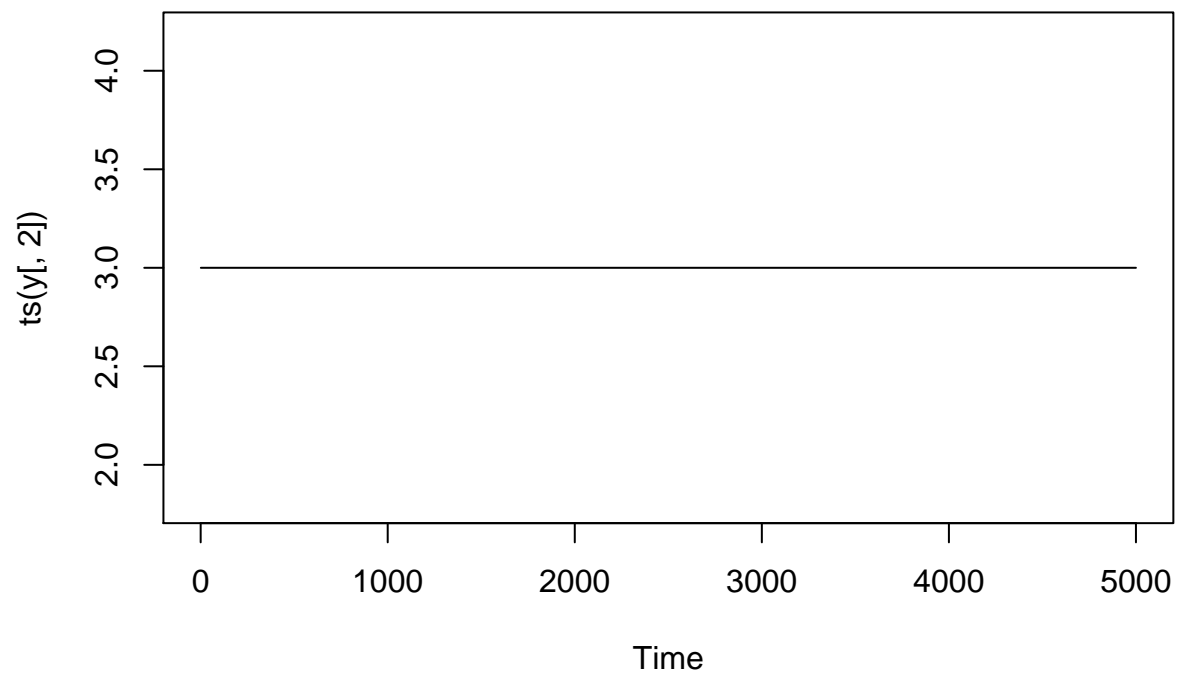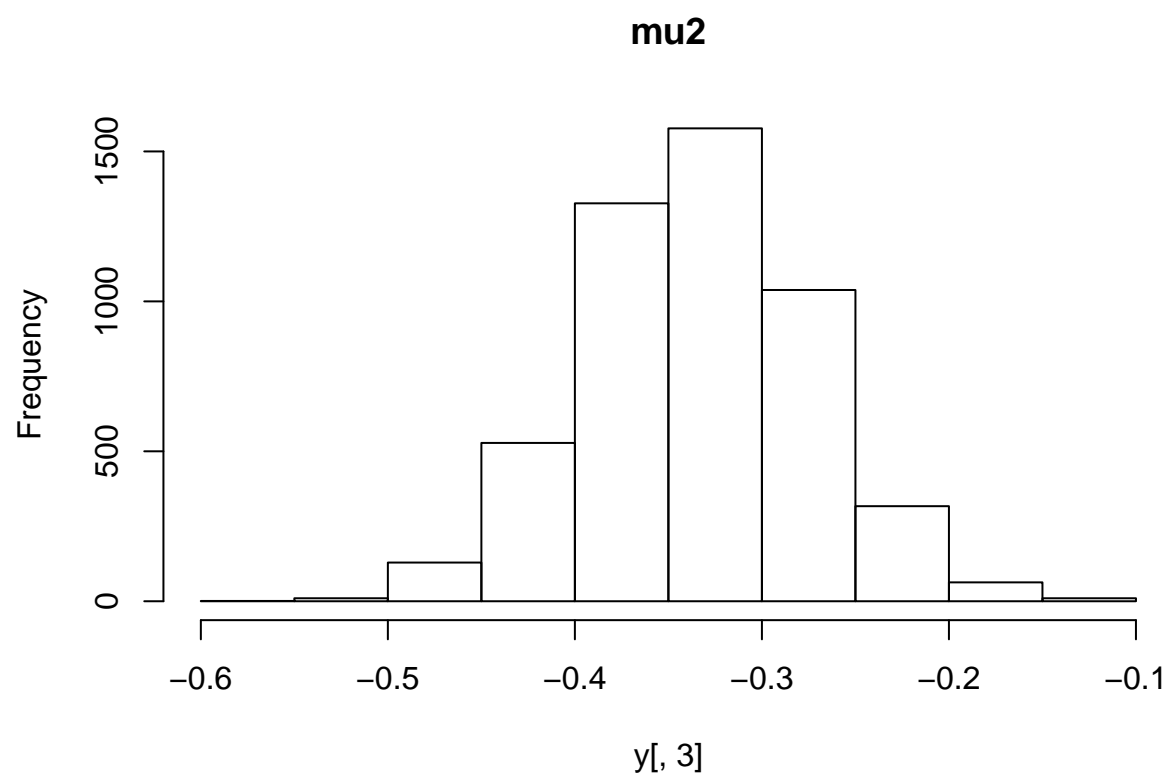

delta

```
plot(ts(y[, 1]))
```

```r
hist(y[, 2], main = "mu1")
```
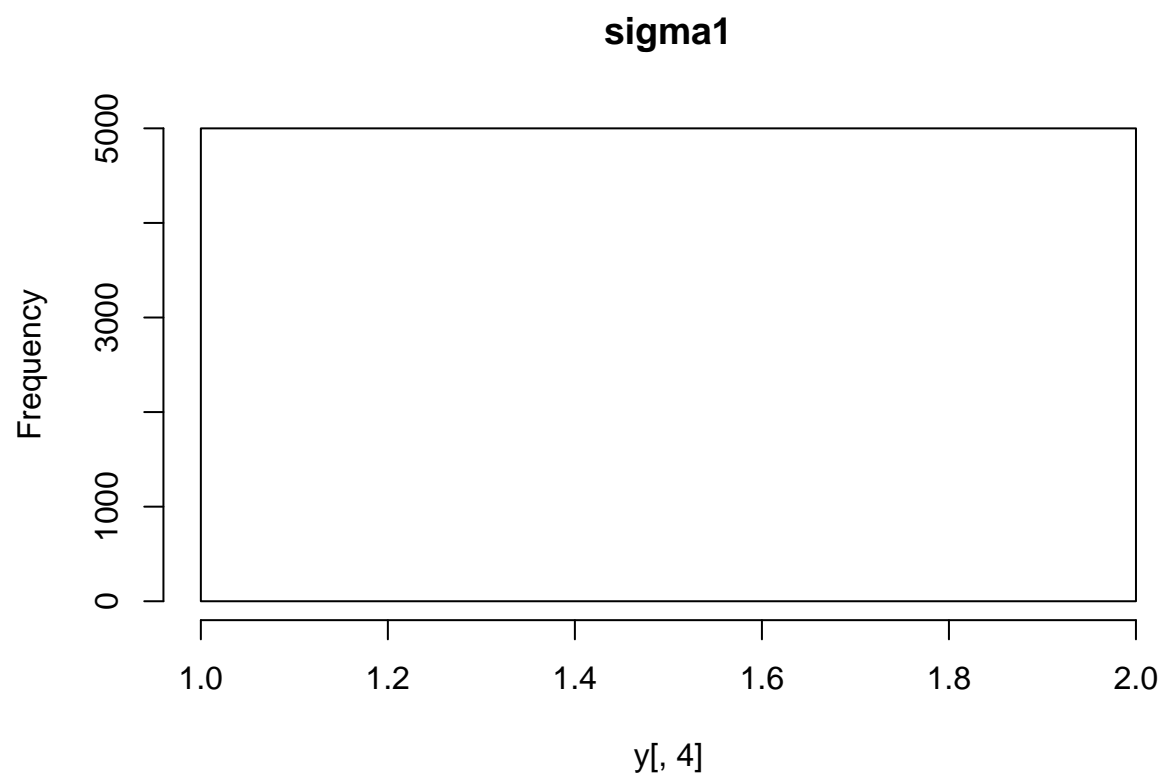
mu1


```
plot(ts(y[, 2]))
```

```r
hist(y[, 3], main = "mu2")
```
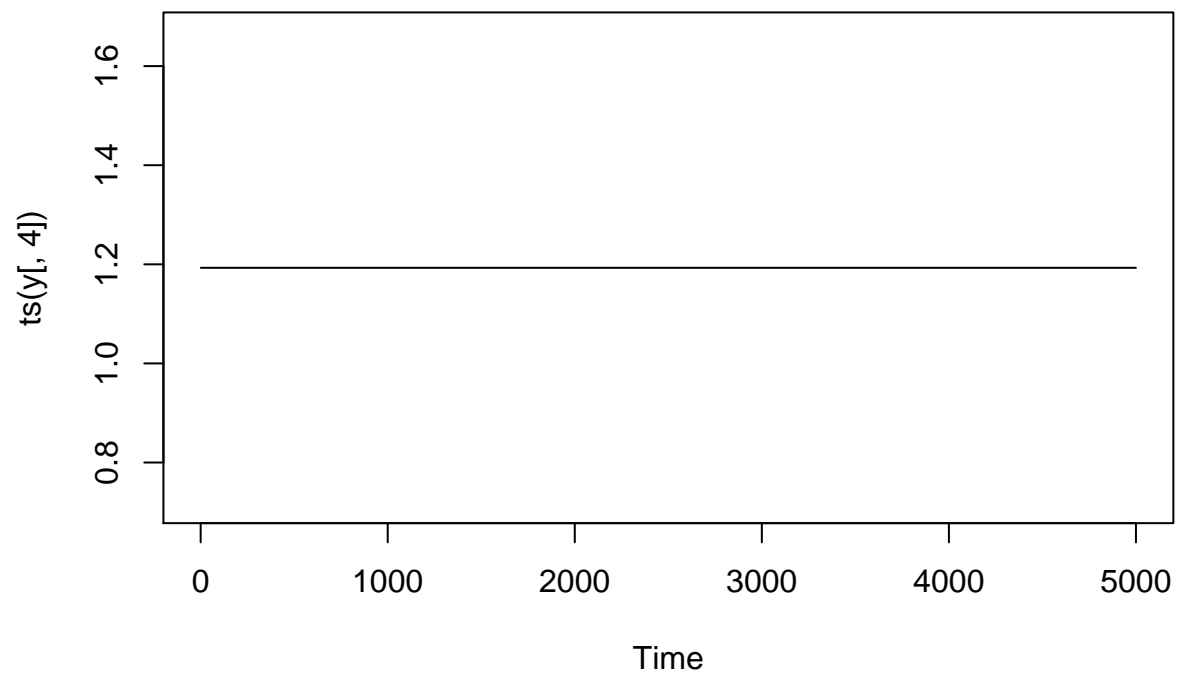
mu2


```r
plot(ts(y[, 3]))
```

```r
hist(y[, 4], main = "sigma1")
```

sigma1
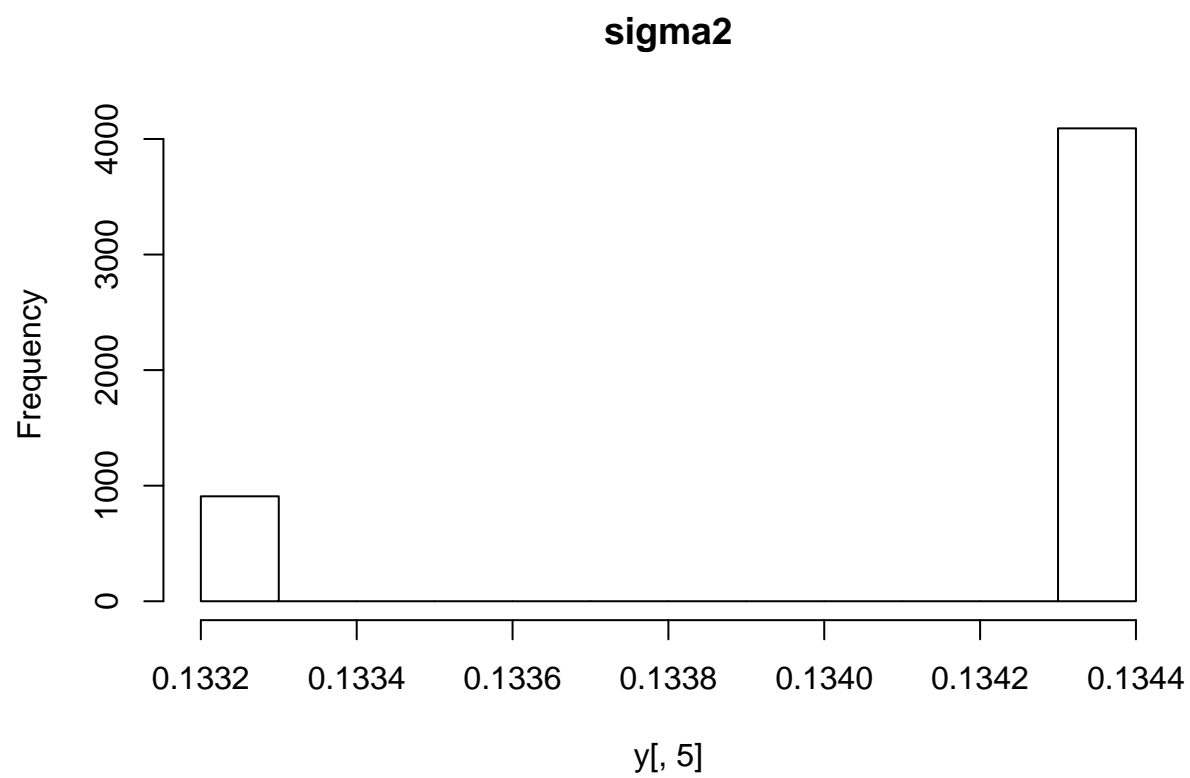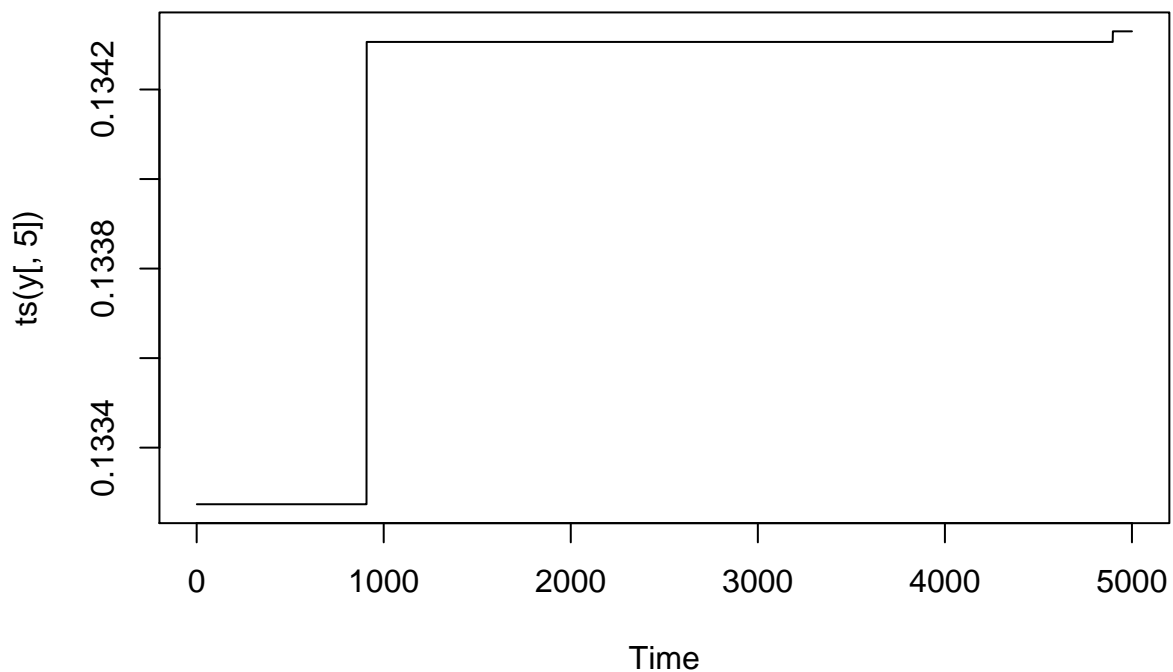


```
plot(ts(y[, 4]))
```

```r
hist(y[, 5], main = "sigma2")
```
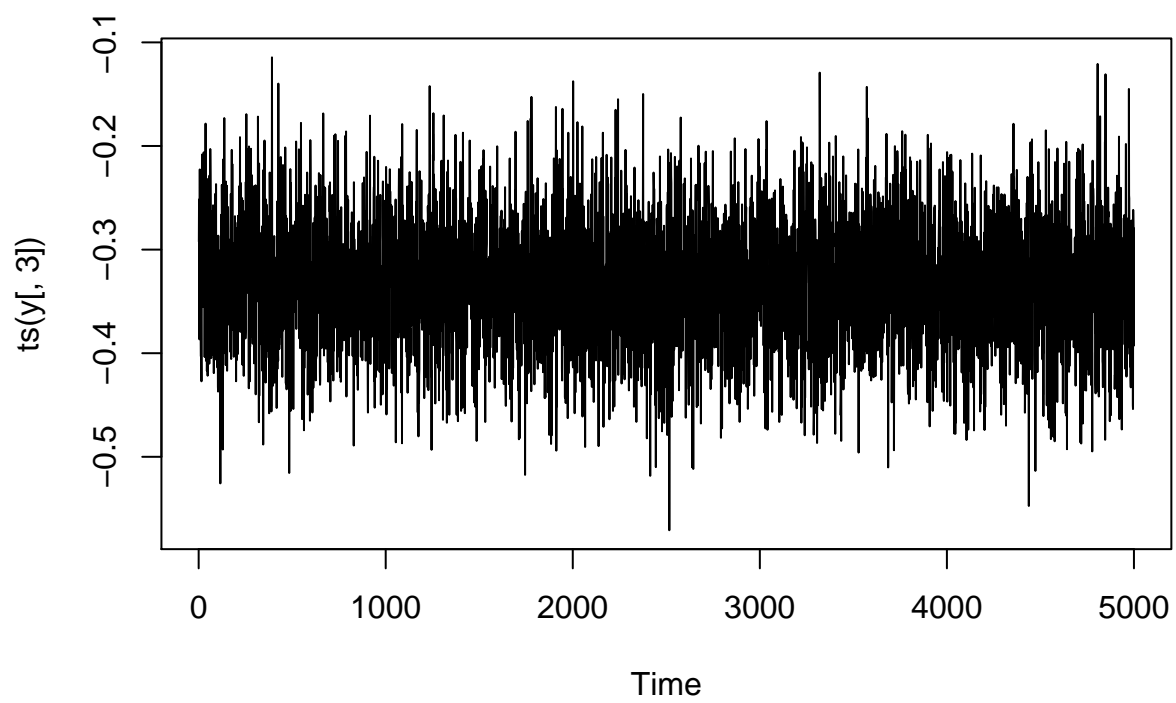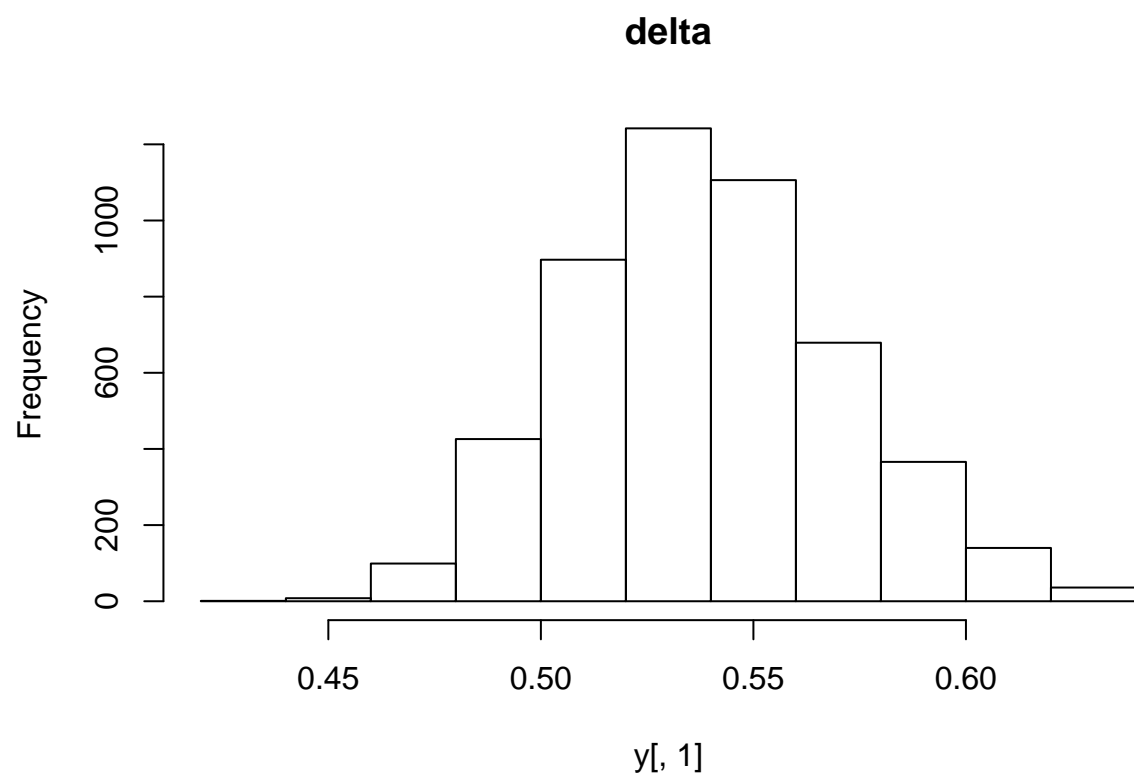
sigma2

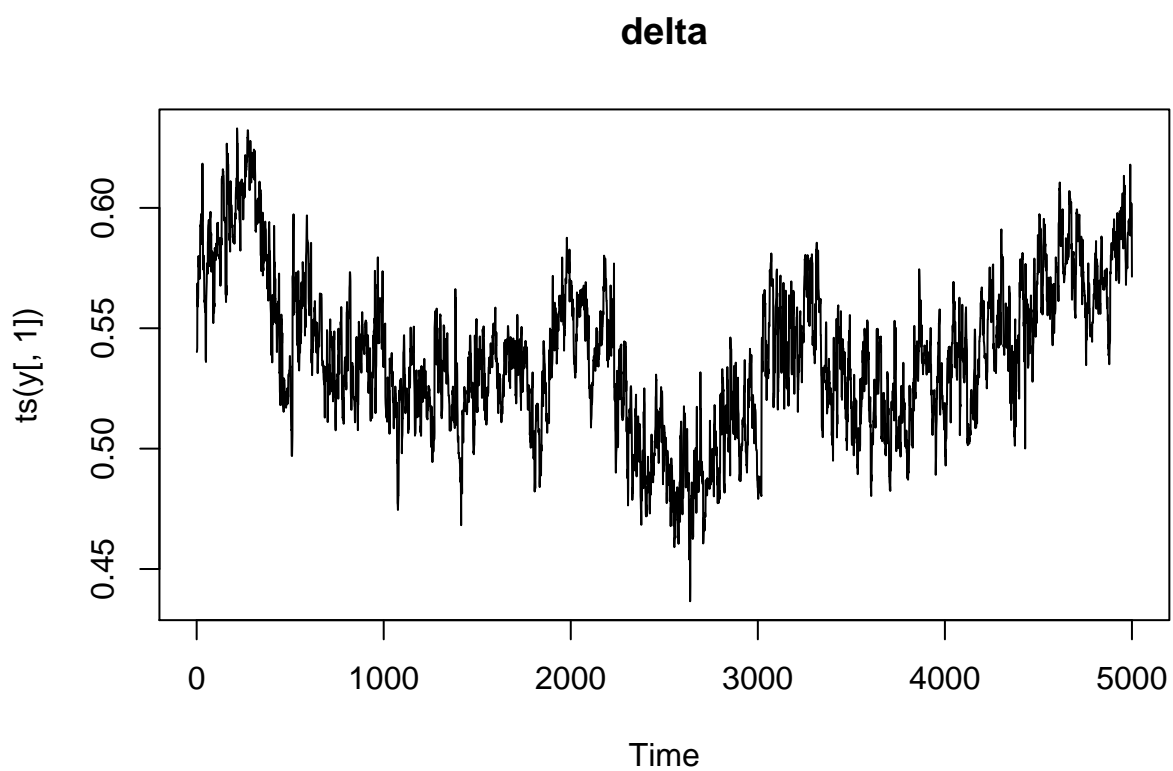```
plot(ts(y[, 5]))
```

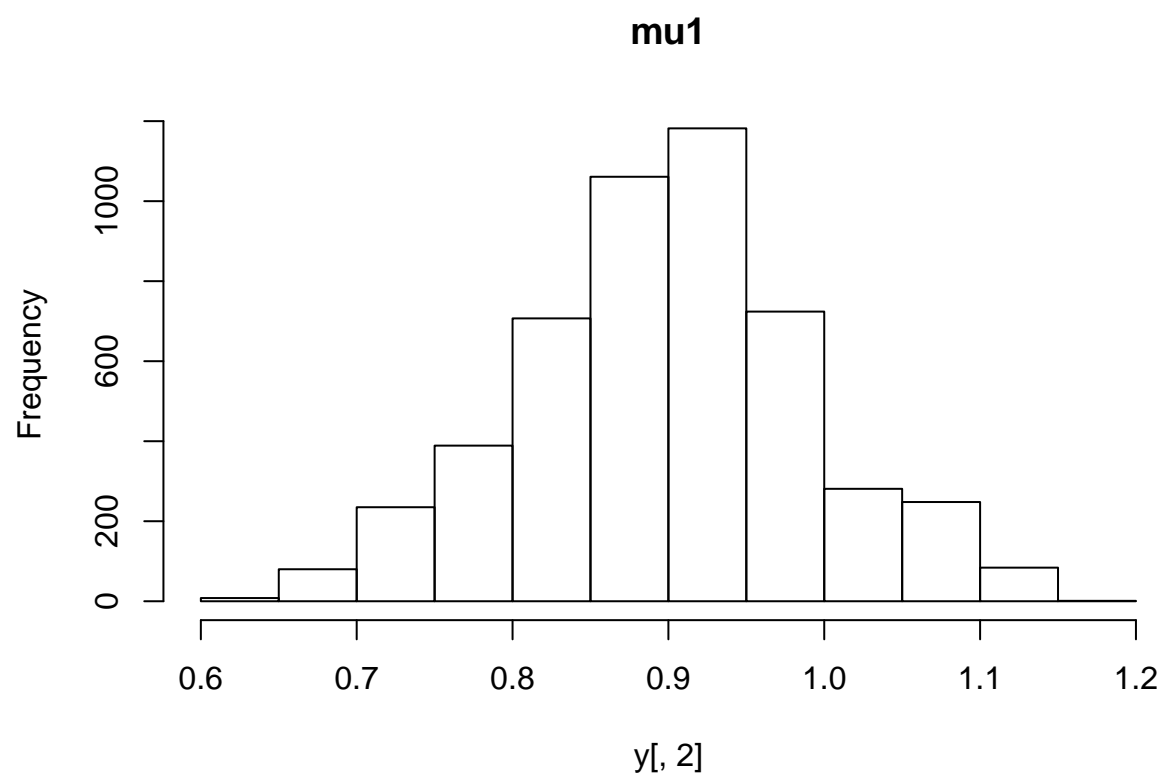**using arms() multivariate version**

```r
# true value
delta <- 0.7
mu1 <- 2
mu2 <- 4
sigma1 <- 2
sigma2 <- 3
# sample
n <- 1000
u <- rbinom(n, prob = delta, size = 1)
dat <- rnorm(n, ifelse(u == 1, mu1, mu2), ifelse(u == 1, sigma1, sigma2))
mylike <- function(delta, mu1, mu2, sigma1, sigma2, x){
  sum(log(delta / sigma1 * exp(-(x - mu1) ^ 2 / (2 * sigma1 ^ 2)) + (1 - delta) / sigma2 * exp(-(x - mu2
}
init <- c(.6, 3, 5, 3, 4)
y <- arms(init, function(x) mylike(x[1], x[2], x[3], x[4], x[5], x = dat), function(x) (x[1] > 1e-5) *
hist(y[, 1], main = "delta")
```
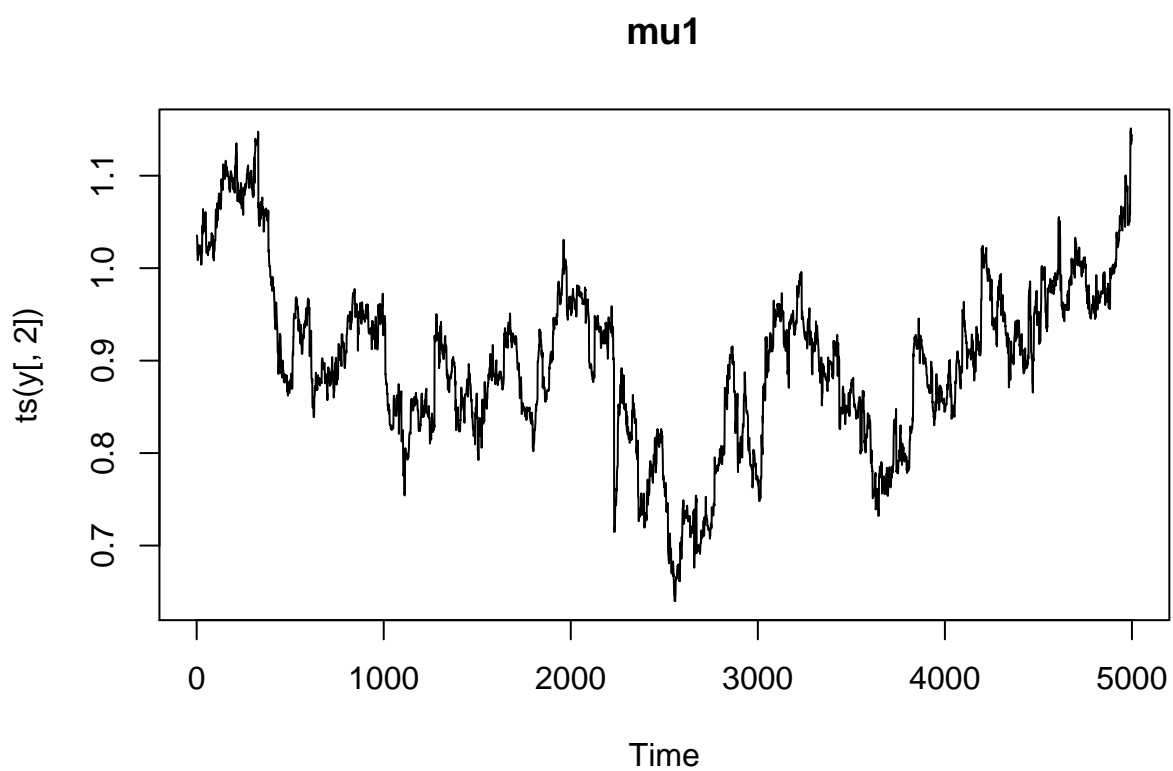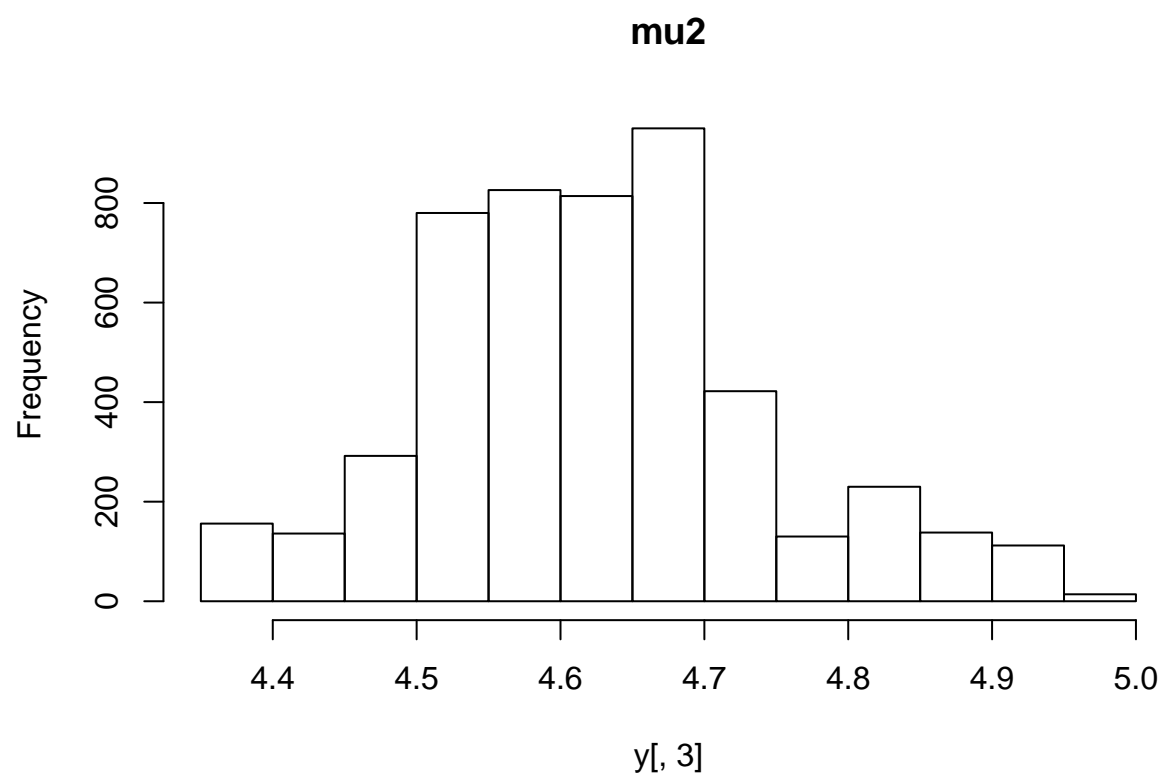
delta

```
plot(ts(y[, 1]), main = "delta")
```

delta


```r
hist(y[, 2], main = "mu1")
```

mu1

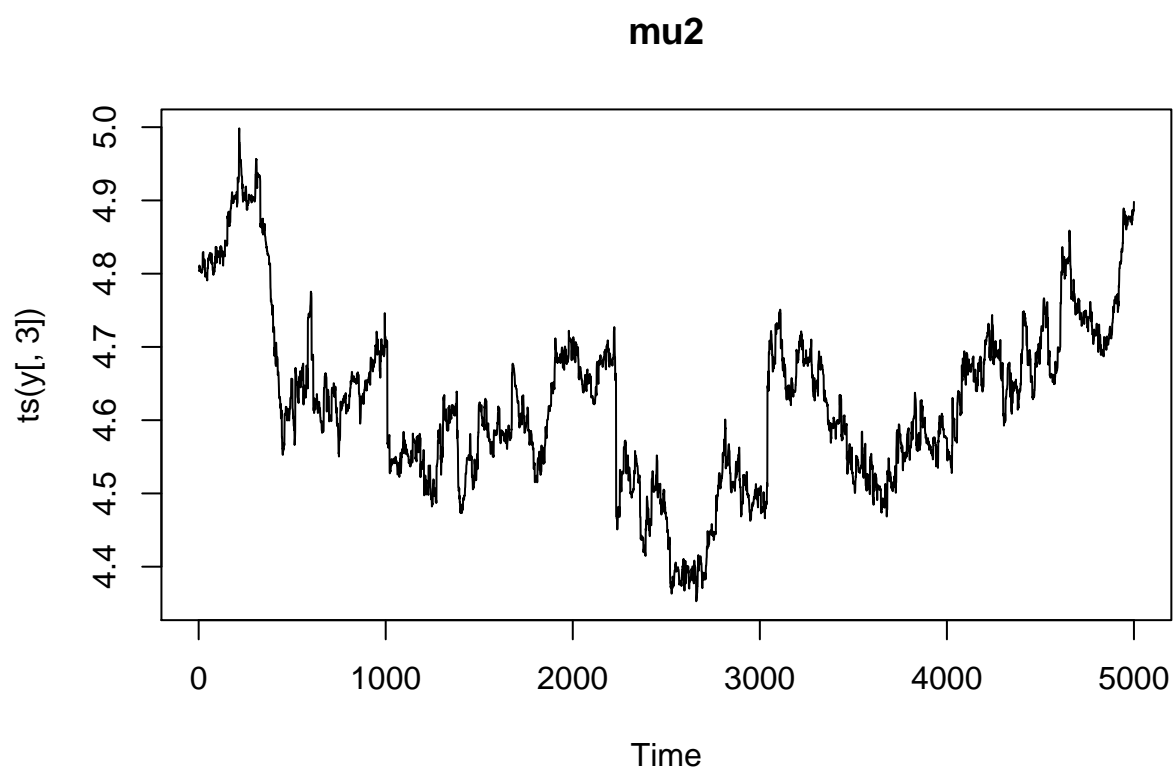

```r
plot(ts(y[, 2]), main = "mu1")
```

mu1


```r
hist(y[, 3], main = "mu2")
```
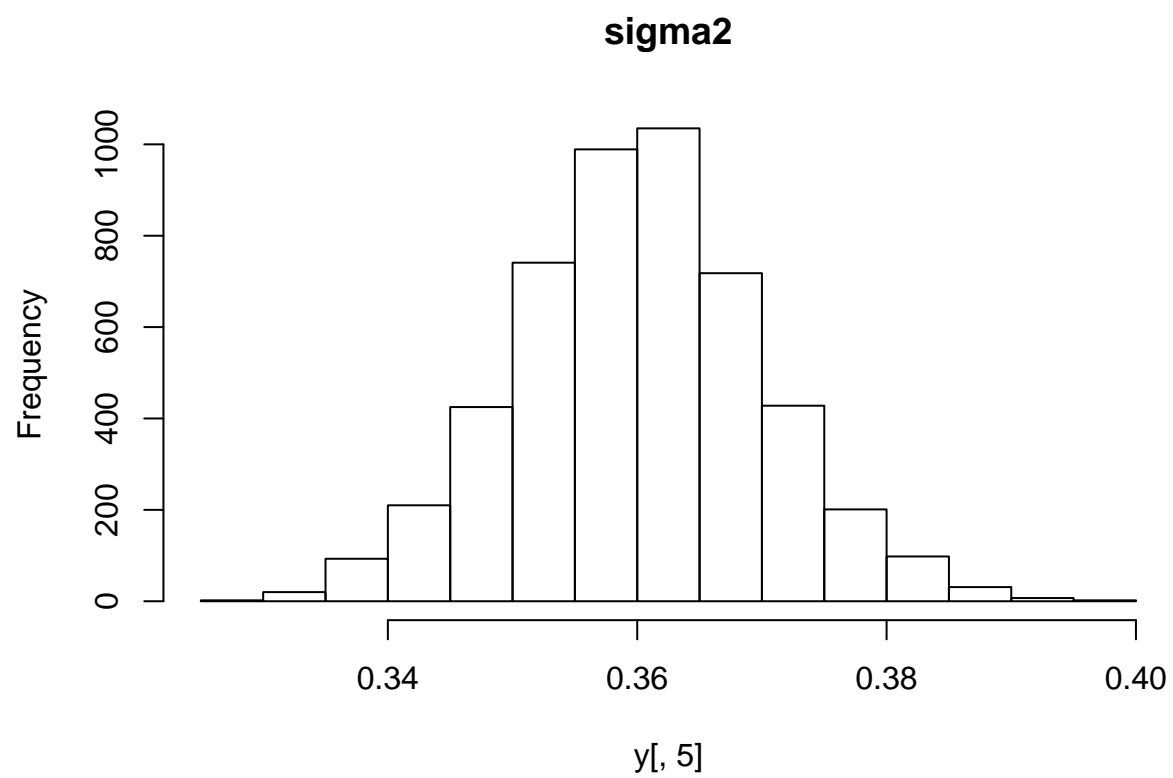
mu2


```
plot(ts(y[, 3]), main = "mu2")
```

mu2


```r
hist(y[, 4], main = "sigma1")
```
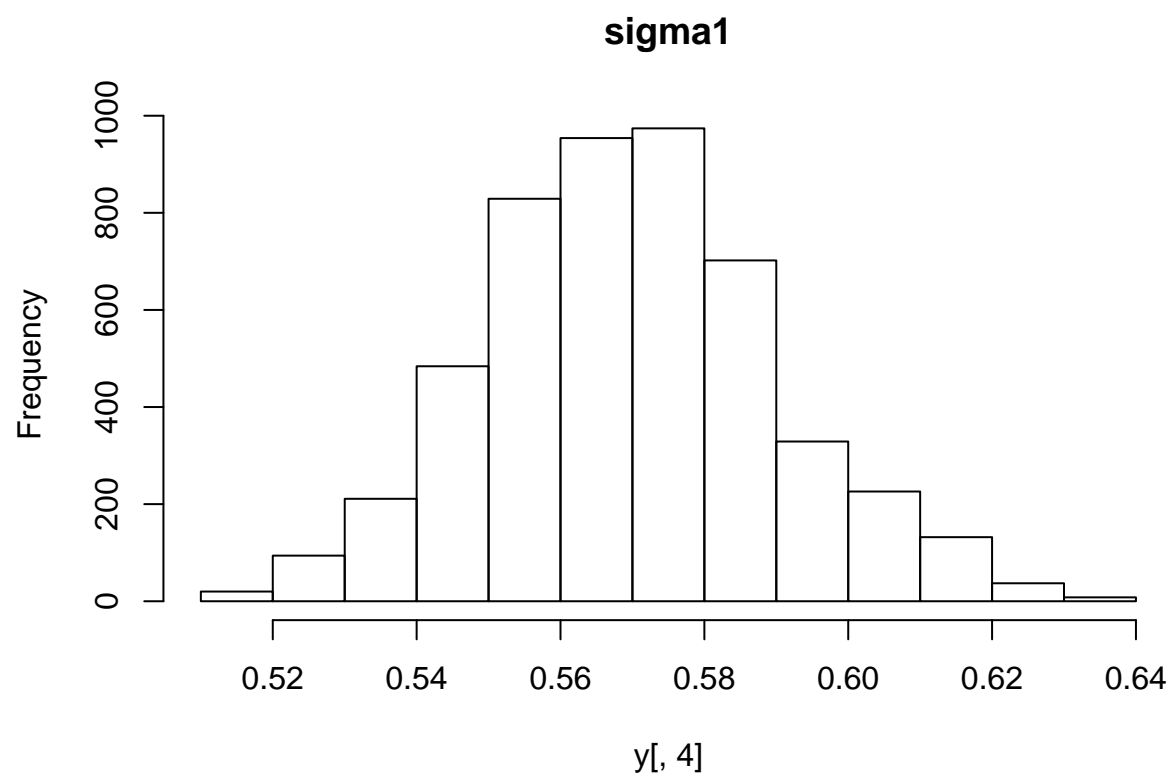
sigma1


```r
plot(ts(y[, 4]), main = "sigma1")
```

sigma1


```r
hist(y[, 5], main = "sigma2")
```

sigma2


```r
plot(ts(y[, 5]), main = "sigma2")
```

sigma2