Normal Mixture Revisited

5361 Homework 7

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1 Normal Mixture Revisited

Consider a normal mixture, prior for $\mu_1, \mu_2 \sim N(0, 10^2)$, and the prior for $1/\sigma_1^2, 1/\sigma_2^2 \sim \Gamma(0.5, 10)$. Further, all the priors are independent.

First, the mixture normal distribution is

$$f(x) = \delta N(\mu_1, \sigma_1^2) + (1 - \delta) N(\mu_2, \sigma_2^2)$$

Since $\frac{1}{\sigma_1^2}, \frac{1}{\sigma_2^2} \sim \Gamma(0.5, 10)$, $\sigma_1^2, \sigma_2^2 \sim$ Inv-Gamma(0.5, 10). So the posterior density is

$$\pi(\theta|\mathbf{x}) \propto \prod_{i=1}^{n} f(x_i) \times \frac{1}{\Gamma(0.5)10^{0.5}} (\frac{1}{\sigma_1^2})^{1.5} e^{-\frac{1}{10\sigma_1^2}} \times \frac{1}{\Gamma(0.5)10^{0.5}} (\frac{1}{\sigma_2^2})^{1.5} e^{-\frac{1}{10\sigma_2^2}} \times \frac{1}{10\sqrt{2\pi}} e^{-\frac{\mu_1^2}{2 \times 10^2}} \times \frac{1}{10\sqrt{2\pi}} e^{-\frac{\mu_2^2}{2 \times 10^2}}$$

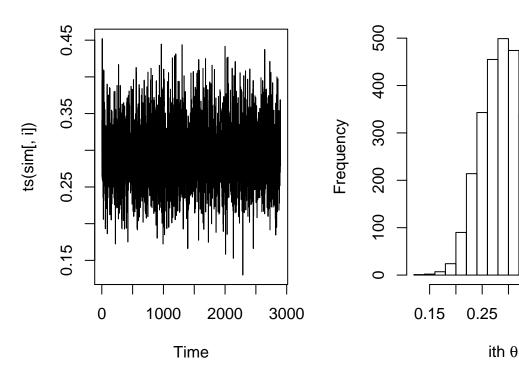where $\theta = (\delta, \mu_1, \mu_2, \sigma_1^2, \sigma_2^2)$.

```
library('invgamma')
library('HI')
delta <- 0.7
n <- 100
set.seed(123)
u <- rbinom(n, prob = delta, size = 1)
x <- rnorm(n, ifelse(u == 1, 7, 10), 0.5)

logll <- function(theta, x){
  delta <- theta[1]
  mu1 <- theta[2]
  mu2 <- theta[3]
  sg1 <- theta[4]
  sg2 <- theta[5]

  sum(log(delta*dnorm(x, mu1, sg1^0.5)+(1-delta)*dnorm(x, mu2, sg2^0.5)))
}

loglr <- function(theta){
  delta <- theta[1]
  mu1 <- theta[2]
```
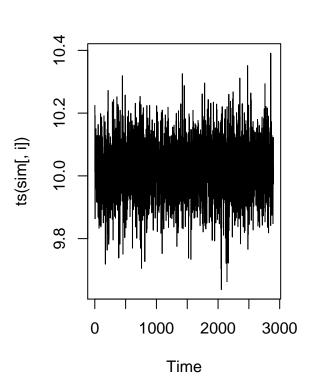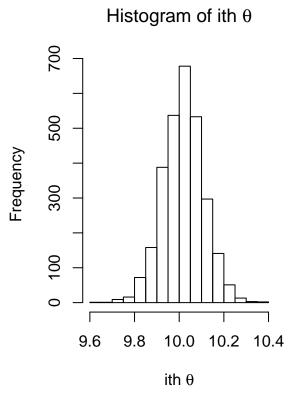
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```r
    mu2 <- theta[3]
    sg1 <- theta[4]
    sg2 <- theta[5]

    return(dnorm(mu1, 0, 10, log = T)+dnorm(mu2, 0, 10, log = T)+
          dinvgamma(sg1, shape = 0.5, scale = 10, log = T)+dinvgamma(sg2, shape = 0.5, scale = 10
    )
    }

logpost <- function(theta, x) {
    delta <- theta[1]
    mu1 <- theta[2]
    mu2 <- theta[3]
    sg1 <- theta[4]
    sg2 <- theta[5]

    return(logll(theta, x)+loglr(theta))
}

mymcmc <- function(niter, thetaInit, x, nburn= 100) {
    p <- length(thetaInit)
    thetaCurrent <- thetaInit
    logFC <- function(th, idx) {
      theta <- thetaCurrent
      theta[idx] <- th
      logpost(theta, x)
    }
    out <- matrix(thetaInit, niter, p, byrow = TRUE)
    ## Gibbs sampling
    for (i in 2:niter) {
      for (j in 1:p) {
      if (j == 1 | j == 4 | j == 5){
        out[i, j] <- thetaCurrent[j] <-
          HI::arms(thetaCurrent[j], logFC,
                    function(x, idx) ((x > 0) * (x < 1)),
                    1, idx = j)
      } else if (j == 2 | j == 3) {
        out[i, j] <- thetaCurrent[j] <-
          HI::arms(thetaCurrent[j], logFC,
                    function(x, idx) ((x > -50) * (x < 50)),
                    1, idx = j)
      }
      }
    }
    out[-(1:nburn), ]
}
```

```
niter <- 3000
nburn <- 1000
thetaInit <- c(0.3, 10, 7, 0.25, 0.24)
sim <- mymcmc(niter, thetaInit, x)

par(mfrow = c(1,2))
for (i in 1:length(thetaInit)){
  plot(ts(sim[,i]))
  hist(sim[,i], main = expression(paste("Histogram of ith ", theta)),
       xlab = expression(paste("ith ", theta)))
}
```

Histogram of ith θ

Histogram of ith θ

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Histogram of ith θ

ts(sim[, i])

7.10
7.00
6.90
6.80

0   1000   2000   3000

Time

Frequency

800
600
400
200
0

6.75   6.85   6.95   7.05

ith θ

Histogram of ith θ

Histogram of ith θ