

# Normal mixture revisited HW7

Yichu Li.

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## Abstract

In the project, we want design an MCMC using the Gibbs sampling approach to estimate five parameters and then plot the histogram of the result.

### 6.3.1 Normal mixture revisited

The likelihood function

$$(x; \delta, \mu_1, \sigma_1^2, \mu_2, \sigma_2^2) = \prod_{i=1}^n [\delta N(\mu_1, \sigma_1^2) + (1 - \delta) N(\mu_2, \sigma_2^2)]$$

```
delta <- 0.7
n <- 100
set.seed(123)
u <- rbinom(n, prob = delta, size = 1)
sample <- rnorm(n, ifelse(u == 1, 7, 10), 0.5)

library("invgamma")
library("HI")

log.pos <- function(u1, u2, s1, s2, d, x = sample){
  p1 <- d * dnorm(x, u1, sqrt(s1))
  p2 <- (1 - d) * dnorm(x, u2, sqrt(s2))
  logL <- sum(log(p1 + p2))
  prior.u1 <- dnorm(u1, 0, 10)
  prior.u2 <- dnorm(u2, 0, 10)
  prior.s1 <- dinvgamma(s1, 0.5, 10)
  prior.s2 <- dinvgamma(s2, 0.5, 10)
  sum(logL + log(prior.u1) + log(prior.u2) + log(prior.s1) + log(prior.s2))
}

gib_function <- function(d_, u1_, u2_, s1_, s2_, x = sample, n){
  gib <- matrix(nrow = n, ncol = 5)
  ini <- c(d_, u1_, u2_, s1_, s2_)
  for(i in 1:n){
    gib[i, 1] <- arms(d_, log.pos, function(x, ...) (x > 0) * (x < 1), 1, u1 = ini[2], u2 = ini[3], s1 = ini[4], s2 = ini[5])
    ini[1] <- gib[i, 1]
    gib[i, 2] <- arms(u1_, log.pos, function(x, ...) (x > -50) * (x < 50), 1, d = ini[1], u2 = ini[3], s1 = ini[4], s2 = ini[5])
    ini[2] <- gib[i, 2]
    gib[i, 3] <- arms(u2_, log.pos, function(x, ...) (x > -50) * (x < 50), 1, d = ini[1], u1 = ini[2], s1 = ini[4], s2 = ini[5])
    ini[3] <- gib[i, 3]
    gib[i, 4] <- arms(s1_, log.pos, function(x, ...) (x > 0.001) * (x < 10), 1, d = ini[1], u1 = ini[2], u2 = ini[3], s1 = ini[4], s2 = ini[5])
    ini[4] <- gib[i, 4]
    gib[i, 5] <- arms(s2_, log.pos, function(x, ...) (x > 0.001) * (x < 10), 1, d = ini[1], u1 = ini[2], u2 = ini[3], s1 = ini[4], s2 = ini[5])
    ini[5] <- gib[i, 5]
  }
  gib
}
```

```

ini[3] <- gib[i,3]
gib[i,4] <- arms(s1_,log.pos,function(x,...)(x>0)*(x<50),1,d=ini[1],u1=ini[2],u2=ini[3],s
ini[4] <- gib[i,4]
gib[i,5] <- arms(s2_,log.pos,function(x,...)(x>0)*(x<50),1,d=ini[1],u1=ini[2],u2=ini[3],s
ini[5] <- gib[i,5]
}
gib
}
gibbr<-gib_function(0.5,5,5,1,1,sample,10000)[- (1:1500),]
par(mfrow=c(2,3))
hist(gibbr[,1],main="Histogram",xlab="delta")
hist(gibbr[,2],main="Histogram",xlab="mu[1]")
hist(gibbr[,3],main="Histogram",xlab="mu[2]")
hist(gibbr[,4],main="Histogram",xlab="sigma[1]")
hist(gibbr[,5],main="Histogram",xlab="sigma[2]")

```

