

HW4

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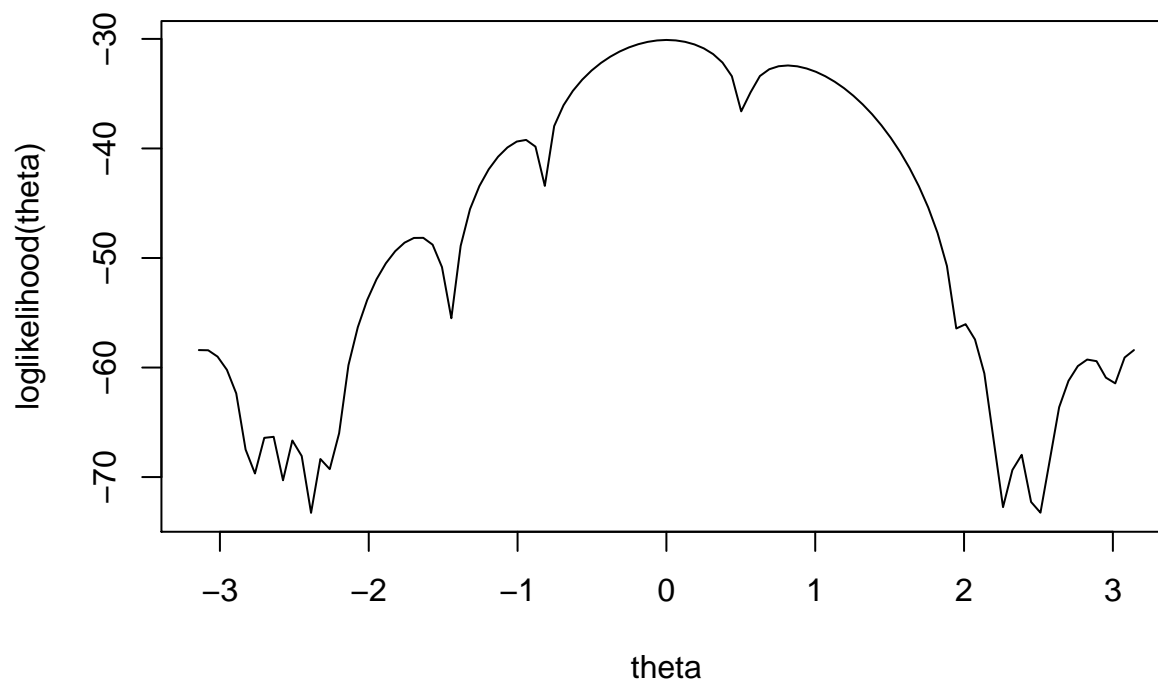
Abstract

Consider estimating the location parameter of a Cauchy distribution with a known scale parameter.

Many local maxima

loglikelihood

```
loglkh <- function(theta){  
  sum(log((1 - cos(x - theta))/(2 * pi), base = exp(1)))  
}  
loglikelihood <- Vectorize(loglkh)  
curve(loglikelihood, -pi, pi, xname = 'theta')
```



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MOM

$$\begin{aligned} E(X|\theta) &= \int_{x=0}^{2\pi} \frac{1 - \cos(x - \theta)}{2\pi} x dx = \int_{x=0}^{2\pi} \frac{x}{2\pi} dx - \frac{1}{2\pi} \int_0^{2\pi} x \cos(x - \theta) dx = \pi - \frac{1}{2\pi} (x \sin(x - \theta) + \cos(x - \theta)) \Big|_0^{2\pi} \\ &= \pi - \frac{1}{2\pi} (-2\pi \sin(\theta)) = \pi + \sin(\theta) = \bar{X} = 3.236842 \end{aligned}$$

```
asin(mean(x)-pi)
```

```
## [1] 0.09539407
```

So $\theta = 0.09539407$.

MLE using Newton-Raphson

```
loglikh.1 <- function(theta){  
  sum(sin(theta-x)/(1-cos(theta-x)))  
}  
newtonRaphson(loglikh.1, asin(mean(x)-pi), dfun = NULL)$root
```

```
## [1] 0.003118157
```

start at -2.7 and 2.7

```
newtonRaphson(loglikh.1, -2.7, dfun = NULL)$root
```

```
## [1] -2.668857
```

```
newtonRaphson(loglikh.1, 2.7, dfun = NULL)$root
```

```
## [1] 2.848415
```

repeat 200 using start values between -pi to pi

```
start <- seq(-pi, pi, length.out = 200)  
nr <- double(200)  
for (i in start){  
  nr[which(start == i)] <- newtonRaphson(loglikh.1, i, dfun = NULL)$root  
}  
nrtble <- data.table(start = start, MLE = nr)  
data.table(cn = names(nrtble), transpose(nrtble))
```

##	cn	V1	V2	V3	V4	V5	V6
## 1:	start	-3.141593	-3.110019	-3.078445	-3.046871	-3.015297	-2.983724
## 2:	MLE	-3.112471	-3.112471	-3.112471	-3.112471	-3.112471	-3.112471
##	V7	V8	V9	V10	V11	V12	V13
## 1:	-2.952150	-2.920576	-2.889002	-2.857428	-2.825855	-2.794281	-2.762707
## 2:	-3.112471	-3.112471	-3.112471	-3.112471	-3.112471	-2.786557	-2.786557
##	V14	V15	V16	V17	V18	V19	V20
## 1:	-2.731133	-2.699560	-2.667986	-2.636412	-2.604838	-2.573264	-2.541691
## 2:	-2.668857	-2.668857	-2.668857	-2.668857	-2.668857	-2.509356	-2.509356
##	V21	V22	V23	V24	V25	V26	V27
## 1:	-2.510117	-2.478543	-2.446969	-2.415395	-2.383822	-2.352248	-2.320674
## 2:	-2.509356	-2.509356	-2.509356	-2.509356	-2.388267	-2.297926	-2.297926
##	V28	V29	V30	V31	V32	V33	V34
## 1:	-2.289100	-2.257526	-2.225953	-2.194379	-2.162805	-2.131231	-2.099657
## 2:	-2.297926	-2.297926	-2.232192	-1.662712	-1.662712	-1.662712	-1.662712
##	V35	V36	V37	V38	V39	V40	V41
## 1:	-2.068084	-2.036510	-2.004936	-1.973362	-1.941788	-1.910215	-1.878641
## 2:	-1.662712	-1.662712	-1.662712	-1.662712	-1.662712	-1.662712	-1.662712
##	V42	V43	V44	V45	V46	V47	V48
## 1:	-1.847067	-1.815493	-1.783919	-1.752346	-1.720772	-1.689198	-1.657624
## 2:	-1.662712	-1.662712	-1.662712	-1.662712	-1.662712	-1.662712	-1.662712
##	V49	V50	V51	V52	V53	V54	V55
## 1:	-1.626050	-1.594477	-1.562903	-1.531329	-1.499755	-1.468181	-1.436608
## 2:	-1.662712	-1.662712	-1.662712	-1.662712	-1.662712	-1.662712	-1.447503
##	V56	V57	V58	V59	V60	V61	
## 1:	-1.4050339	-1.3734601	-1.3418863	-1.3103125	-1.2787387	-1.2471649	
## 2:	-0.9544058	-0.9544058	-0.9544058	-0.9544058	-0.9544058	-0.9544058	
##	V62	V63	V64	V65	V66	V67	
## 1:	-1.2155911	-1.1840173	-1.1524435	-1.1208697	-1.0892959	-1.0577221	
## 2:	-0.9544058	-0.9544058	-0.9544058	-0.9544058	-0.9544058	-0.9544058	
##	V68	V69	V70	V71	V72	V73	
## 1:	-1.0261484	-0.9945746	-0.9630008	-0.9314270	-0.8998532	-0.8682794	
## 2:	-0.9544058	-0.9544058	-0.9544058	-0.9544058	-0.9544058	-0.9544058	
##	V74	V75	V76	V77	V78		
## 1:	-0.8367056	-0.805131786	-0.773557990	-0.741984195	-0.710410399		
## 2:	-0.9544058	0.003118157	0.003118157	0.003118157	0.003118157		
##	V79	V80	V81	V82	V83		
## 1:	-0.678836604	-0.647262808	-0.615689013	-0.584115217	-0.552541421		
## 2:	0.003118157	0.003118157	0.003118157	0.003118157	0.003118157		
##	V84	V85	V86	V87	V88		
## 1:	-0.520967626	-0.489393830	-0.457820035	-0.426246239	-0.394672444		
## 2:	0.003118157	0.003118157	0.003118157	0.003118157	0.003118157		
##	V89	V90	V91	V92	V93		
## 1:	-0.363098648	-0.331524853	-0.299951057	-0.268377262	-0.236803466		
## 2:	0.003118157	0.003118157	0.003118157	0.003118157	0.003118157		
##	V94	V95	V96	V97	V98		
## 1:	-0.205229671	-0.173655875	-0.142082080	-0.110508284	-0.078934489		
## 2:	0.003118157	0.003118157	0.003118157	0.003118157	0.003118157		

##	V99	V100	V101	V102	V103		
## 1:	-0.047360693	-0.015786898	0.015786898	0.047360693	0.078934489		
## 2:	0.003118157	0.003118157	0.003118157	0.003118157	0.003118157		
##	V104	V105	V106	V107	V108	V109	
## 1:	0.110508284	0.142082080	0.173655875	0.205229671	0.236803466	0.268377262	
## 2:	0.003118157	0.003118157	0.003118157	0.003118157	0.003118157	0.003118157	
##	V110	V111	V112	V113	V114	V115	
## 1:	0.299951057	0.331524853	0.363098648	0.394672444	0.426246239	0.457820035	
## 2:	0.003118157	0.003118157	0.003118157	0.003118157	0.003118157	0.003118157	
##	V116	V117	V118	V119	V120	V121	V122
## 1:	0.489393830	0.5209676	0.5525414	0.5841152	0.6156890	0.6472628	0.6788366
## 2:	0.003118157	0.8126374	0.8126374	0.8126374	0.8126374	0.8126374	0.8126374
##	V123	V124	V125	V126	V127	V128	V129
## 1:	0.7104104	0.7419842	0.7735580	0.8051318	0.8367056	0.8682794	0.8998532
## 2:	0.8126374	0.8126374	0.8126374	0.8126374	0.8126374	0.8126374	0.8126374
##	V130	V131	V132	V133	V134	V135	V136
## 1:	0.9314270	0.9630008	0.9945746	1.0261484	1.0577221	1.0892959	1.1208697
## 2:	0.8126374	0.8126374	0.8126374	0.8126374	0.8126374	0.8126374	0.8126374
##	V137	V138	V139	V140	V141	V142	V143
## 1:	1.1524435	1.1840173	1.2155911	1.2471649	1.2787387	1.3103125	1.3418863
## 2:	0.8126374	0.8126374	0.8126374	0.8126374	0.8126374	0.8126374	0.8126374
##	V144	V145	V146	V147	V148	V149	V150
## 1:	1.3734601	1.4050339	1.4366077	1.4681815	1.4997553	1.5313291	1.5629029
## 2:	0.8126374	0.8126374	0.8126374	0.8126374	0.8126374	0.8126374	0.8126374
##	V151	V152	V153	V154	V155	V156	V157
## 1:	1.5944767	1.6260505	1.6576243	1.6891981	1.7207719	1.7523457	1.7839194
## 2:	0.8126374	0.8126374	0.8126374	0.8126374	0.8126374	0.8126374	0.8126374
##	V158	V159	V160	V161	V162	V163	V164
## 1:	1.8154932	1.8470670	1.8786408	1.9102146	1.9417884	1.973362	2.004936
## 2:	0.8126374	0.8126374	0.8126374	0.8126374	0.8126374	2.007223	2.007223
##	V165	V166	V167	V168	V169	V170	V171
## 1:	2.036510	2.068084	2.099657	2.131231	2.162805	2.194379	2.225953
## 2:	2.007223	2.007223	2.007223	2.007223	2.007223	2.007223	2.237013
##	V173	V174	V175	V176	V177	V178	V179
## 1:	2.289100	2.320674	2.352248	2.383822	2.415395	2.446969	2.478543
## 2:	2.374712	2.374712	2.374712	2.374712	2.374712	2.374712	2.488450
##	V181	V182	V183	V184	V185	V186	V187
## 1:	2.541691	2.573264	2.604838	2.636412	2.667986	2.699560	2.731133
## 2:	2.848415	2.848415	2.848415	2.848415	2.848415	2.848415	2.848415
##	V189	V190	V191	V192	V193	V194	V195
## 1:	2.794281	2.825855	2.857428	2.889002	2.920576	2.952150	2.983724
## 2:	2.848415	2.848415	2.848415	2.848415	2.848415	2.848415	3.170715
##	V197	V198	V199	V200			
## 1:	3.046871	3.078445	3.110019	3.141593			
## 2:	3.170715	3.170715	3.170715	3.170715			

Reference

[jun-yan/stat-5361]<https://github.com/jun-yan/stat-5361>