# Question3-3

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**Abstract**

In this assignment, we will use the Modeling beetle data to estimate the K and r using two different methods, and compare them.

## 1 Question

Modeling beetle data The counts of a floor beetle at various time points (in days) are given in a dataset.

```r
beetles <- data.frame(
    days    = c(0,  8,  28,  41,  63,  69,   97, 117,  135,  154),
    beetles = c(2, 47, 192, 256, 768, 896, 1120, 896, 1184, 1024))
```

A simple model for population growth is the logistic model given by

$$\frac{dN}{dt} = rN(1 - \frac{N}{K}),$$

where $N$ is the population size, $t$ is time, $r$ is an unknown growth rate parameter, and $K$ is an unknown parameter that represents the population carrying capacity of the environment. The solution to the differential equation is given by

$$N_t = f(t) = \frac{KN_0}{N_0 + (K - N_0)\exp(-rt)},$$

where $N_t$ denotes the population size at time $t$.

- Fit the population growth model to the beetles data using the Gauss-Newton approach, to minimize the sum of squared errors between model predictions and observed counts.
- Show the contour plot of the sum of squared errors.
- In many population modeling application, an assumption of lognormality is adopted. That is , we assume that $\log N_t$ are independent and normally distributed with mean $\log f(t)$ and variance $\sigma^2$. Find the maximum likelihood estimators of $\theta = (r, K, \sigma^2)$ using any suitable method of your choice. Estimate the variance your parameter estimates.

## 2 Fit the growth model to the beetles

We've already known the $N(t)$ equation

$$N_t = f(t) = \frac{KN_0}{N_0 + (K - N_0)\exp(-rt)},$$

So r is given by

$$r = \frac{1}{t}log(\frac{N_t(k - N_0)}{N_0(k - N_t)}$$

When $t = 0$, $N_t = N_0 = 2$

$$r = \frac{1}{t}log(\frac{N_t(k - 2)}{2(k - N_t)}$$

To soleve this question, choose $K = 1500$

```
K <- 1300
r <- log((beetles$beetles*(K-2))/(K - beetles$beetles)*2)/beetles$days
min(r[2:10])
max(r[2:10])
mean(r[2:10])
```

So we get $min(r) = 0.05956362$ , $max(r) = 0.5723223$ , $mean(r) = 0.1681074$

Now lets calculate the squared errors

$$SE = \sum[N_t - \frac{2K}{2 + (K-2)e^{-rt}}]^2$$

```
K <- seq(500,1500,length=1000)
R <- seq(0.05,0.6,length=1000)
SE <- function(k,r){
  se = sum((beetles$beetles-2*k/(2+(k-2)*exp(-r*beetles$days)))^2)
  se
}
```

Now let's try to minimize the sum of squared errors by cahnging K and R, we will use function nls() to do this work. Using the extimated $K$ and $mean(r)$

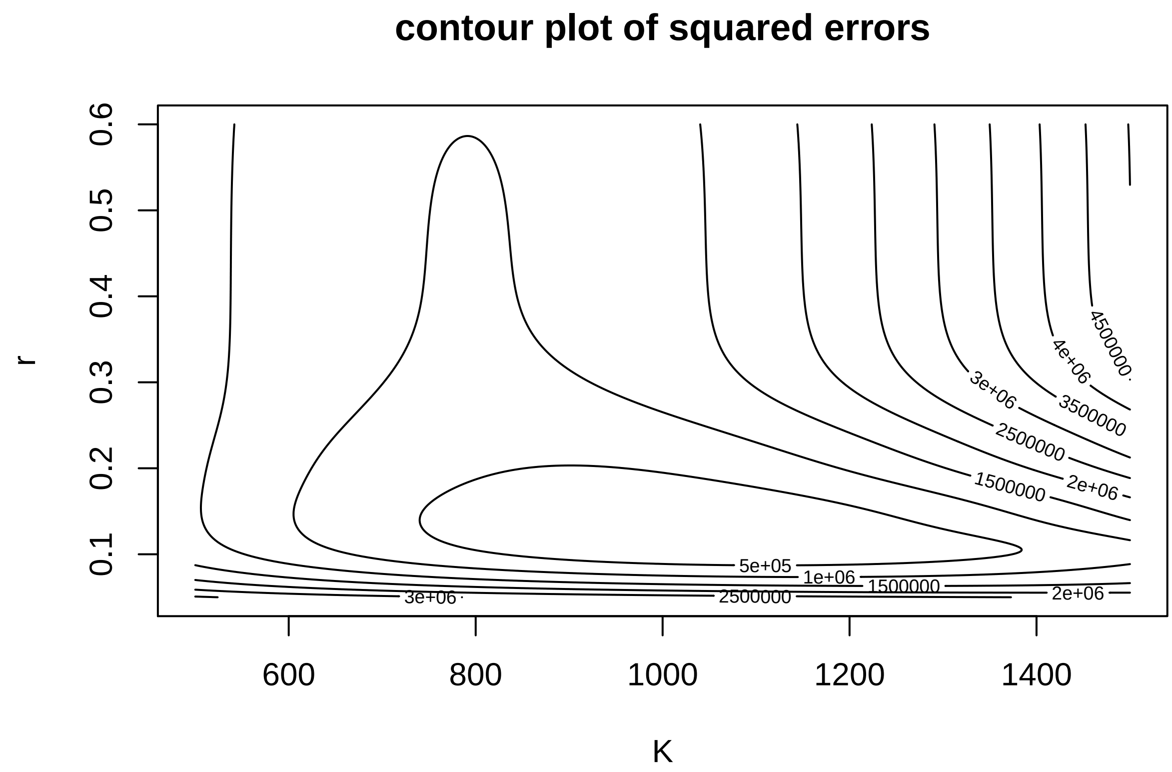
```
nls(beetles ~ N*2/(2+(N-2)*exp((-r)*days)),start = list(N = 1300, r = 0.1681074),data = beetles)
```

```
## Nonlinear regression model
##   model: beetles ~ N * 2/(2 + (N - 2) * exp((-r) * days))
##    data: beetles
##         N         r
## 1049.4069    0.1183
##  residual sum-of-squares: 73420
##
## Number of iterations to convergence: 8
## Achieved convergence tolerance: 4.426e-06
```

In this way we get $K = 1049.4$ and $r = 0.1183$

# 3 Contour plot of the sum of squared errors

```
b <- matrix(0,1000,1000)
for(j in 1:length(K)){
  for(i in 1:length(R)){
    b[j,i] = SE(K[j],R[i])
  }
}
contour(K,R,b,xlab="K",ylab="r",main="contour plot of squared errors")
```




## 4 MLE using an assumption of lognormality

First of all, we could get the log-likelihood using normal distribution

$$\ell(r,k,\sigma^2) = \sum_{1}^{N} log(N[logf(t),\sigma^2])$$

```
loglike <- function(x){
  k <- x[1]
  r <- x[2]
  sigma_2 <- x[3]
  sum=0
  for(i in 1:10){
  sum=sum+(-(log(2*pi*sigma_2))/2-(log(beetles$beetles[i])-log((2*k)/(2+(k-2)*exp(-r*beetles$days[i])))
  }
  -sum
}
options(warn=-1)
opt=optim(c(1300,0.168,4),loglike,method="BFGS",hessian=TRUE)
opt
```

```
## $par
## [1] 1293.9196460    0.1601082    0.5118147
```

```
##
## $value
## [1] 10.83829
##
## $counts
## function gradient
##      190      100
##
## $convergence
## [1] 1
##
## $message
## NULL
##
## $hessian
##               [,1]         [,2]         [,3]
## [1,]  4.479972e-06 3.896185e-02 -0.006332357
## [2,]  3.896185e-02 1.167523e+03  0.034527544
## [3,] -6.332357e-03 3.452754e-02 19.071481248
```

In this way, we can get $K = 1293.9$, $r = 0.160$, $\sigma^2 = 0.511$, these answers are not quite consistent with what we get before(we get $K = 1049.4$ and $r = 0.1183$). That may show the assumption is not so good.