Question3-3

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Abstract

In this assignment, we use the probability density function to find the log-likelihood function and the method-of-moments estimator of θ. Then we will discuss the influence of different initial value when using Newton-Raphson method to estimate the Maximum Likelihood Estimation for θ.

1 Question

Modeling beetle data The counts of a floor beetle at various time points (in days) are given in a dataset.

```
beetles <- data.frame(
    days    = c(0,  8,  28,  41,  63,  69,   97, 117,  135,  154),
    beetles = c(2, 47, 192, 256, 768, 896, 1120, 896, 1184, 1024))
```

A simple model for population growth is the logistic model given by

$$\frac{dN}{dt} = rN(1 - \frac{N}{K}),$$

where N is the population size, t is time, r is an unknown growth rate parameter, and K is an unknown parameter that represents the population carrying capacity of the environment. The solution to the differential equation is given by

$$N_t = f(t) = \frac{KN_0}{N_0 + (K - N_0)\exp(-rt)},$$

where N_t denotes the population size at time t. - Fit the population growth model to the beetles data using the Gauss-Newton approach, to minimize the sum of squared errors between model predictions and observed counts. - Show the contour plot of the sum of squared errors. - In many population modeling application, an assumption of lognormality is adopted. That is , we assume that $\log N_t$ are independent and normally distributed with mean $\log f(t)$ and variance σ^2. Find the maximum likelihood estimators of $\theta = (r, K, \sigma^2)$ using any suitable method of your choice. Estimate the variance your parameter estimates.