Statistical Computing Homework 4, Chapter 3

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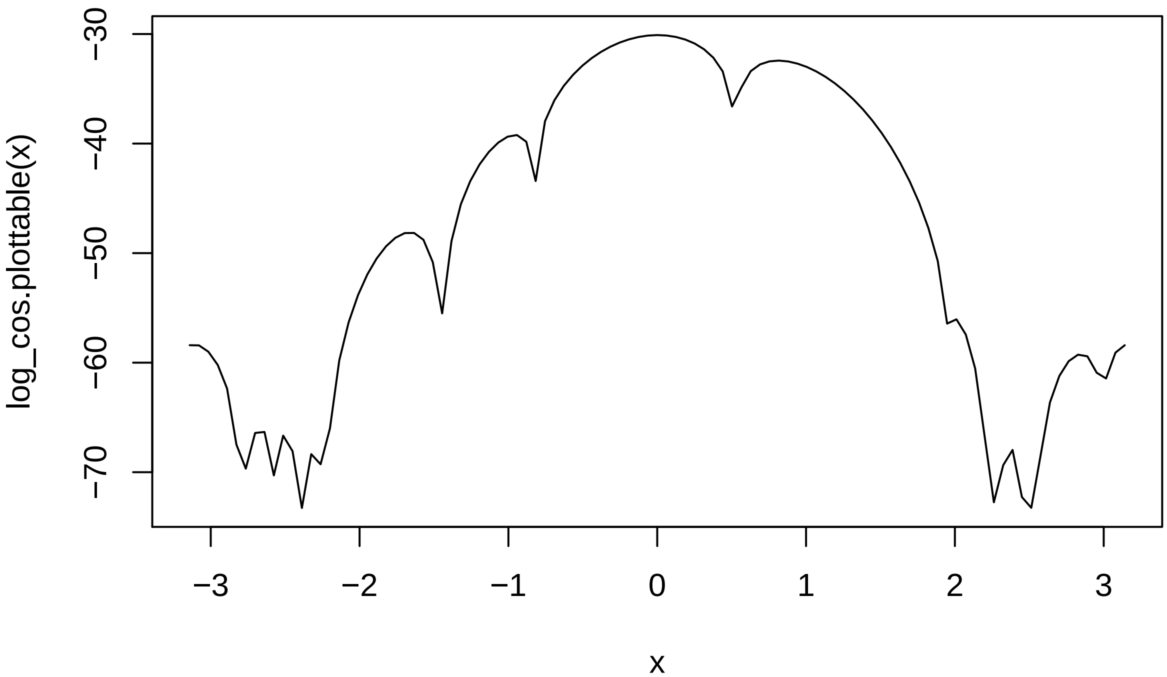
Contents

3.3.2 Many local maxima

(a) log-likelihood

```
x <- c(3.91, 4.85, 2.28, 4.06, 3.70, 4.04, 5.46, 3.53, 2.28, 1.96,
       2.53, 3.88, 2.22, 3.47, 4.82, 2.46, 2.99, 2.54, 0.52)
log_cos <- function(theta) {
  sum( log(1 - cos(x - theta)) ) - length(x)*log(2*pi)
}
log_cos.plottable <- function(x) {return(sapply(x, log_cos))}
curve(log_cos.plottable, from = -pi, to = pi)
```



(b) Methods of moment

Let

$$\frac{1}{2\pi}\int_0^{2\pi}\left[x - x\cos(x-\theta)\right]dx = \bar{X}$$

so evetually, we have $\pi + \sin(\theta) = \bar{X}$, so MOM $\tilde{\theta}_n = \arcsin(\bar{X} - \pi) = 0.0953941$

(c) Find MLE using Newton-Raphson

```
log_cos_D1 <- function(theta) {
  sum( sin(theta-x)/(1-cos(x-theta)) )
}

log_cos_D2 <- function(theta) {
  sum( 1+cos(theta-x)/(1-cos(theta-x))^2 )
}

para <- c(-2.7, 2.7);  temp <- rep(0, length(para))
for (i in 1:length(para)) {
  iter <- 0;  epsilon <- 0.001;
  temp[i] <- para[i] - 1
  while( (iter <= 1000)&(abs(para[i]-temp[i])/(abs(temp[i])) > epsilon) ) {
    temp[i] <- para[i]
    para[i] <- para[i] - log_cos_D1(para[i])/log_cos_D2(para[i])
    if(abs(para[i]) == Inf) {break}
    iter <- iter + 1
    #print(para[i])
  }
}
print(para)
```

```
## [1] -2.700049  2.697756
```

We can see that if we start from -2.7 and 2.7, the Newton_Raphson algorithm will give us the similar result with the initial values.

(d)

```
para <- seq(-pi, pi, length.out = 200);  temp <- rep(0, length(para))
for (i in 1:length(para)) {
  iter <- 0;  epsilon <- 0.001;
  temp[i] <- para[i] - 1
  while( (iter <= 1000)&(abs(para[i]-temp[i])/(abs(temp[i])) > epsilon) ) {
    temp[i] <- para[i]
    para[i] <- para[i] - log_cos_D1(para[i])/log_cos_D2(para[i])
    if(abs(para[i]) == Inf) {break}
    iter <- iter + 1
    #print(para[i])
  }
}
print(para)
```

```
##   [1] -3.1421470 -3.1099375 -3.0769175 -3.0438883 -2.9862293 -2.9808953
##   [7] -2.9503023 -2.9196235 -2.8886503 -2.8573610 -2.8258534 -2.7942822
##  [13] -2.7627067 -2.7311387 -2.6996089 -2.6679818 -2.6363612 -2.6048332
##  [19] -2.5732648 -2.5417143 -2.5101200 -2.4784189 -2.4469208 -2.4153942
##  [25] -2.3838213 -2.3522535 -2.3207197 -2.2890875 -2.2575248 -2.2259526
##  [31] -2.1944008 -2.1630228 -2.1320151 -2.1015594 -2.0984893 -2.0979572
##  [37] -2.0983081 -2.0991544 -2.0993606 -2.0982989 -2.0995181 -2.0987066
##  [43] -2.0994312 -2.0991218 -2.0985298 -2.0983966 -1.6903913 -1.6574795
##  [49] -1.6254471 -1.5938975 -1.5625352 -1.5311748 -1.4997261 -1.4681814
##  [55] -1.4366077 -1.4050512 -1.3736357 -1.3425156 -1.3186971 -1.3180609
```

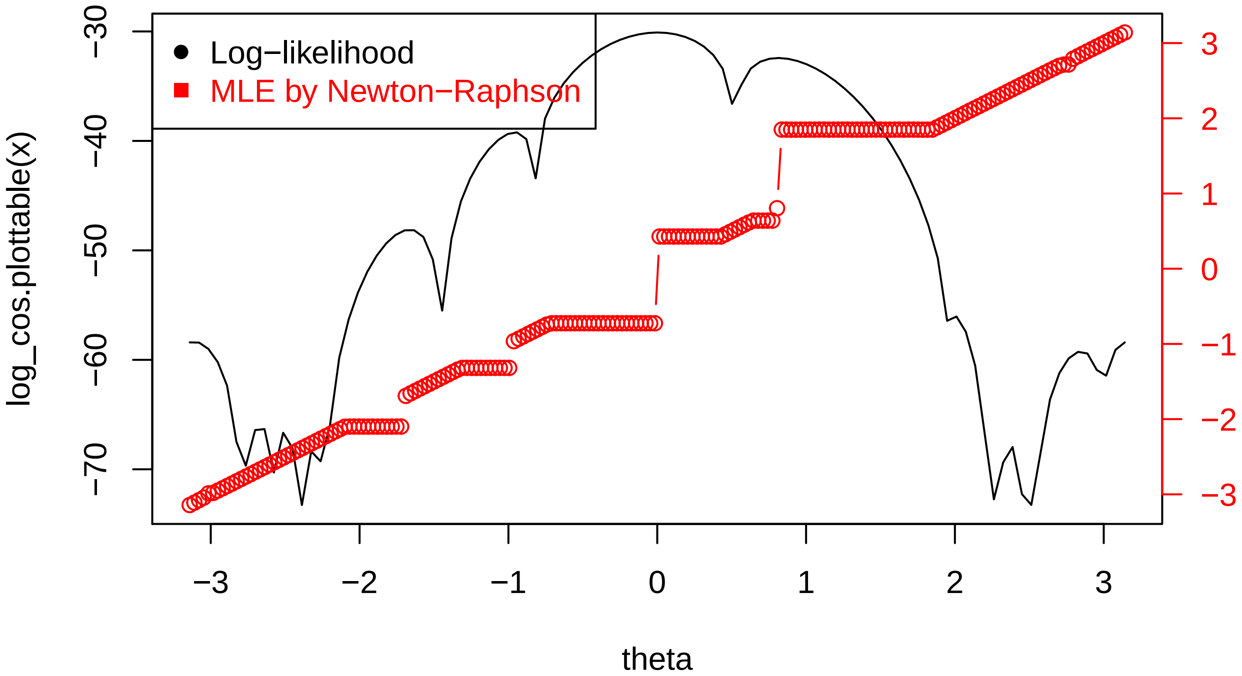
```
##  [61] -1.3182941 -1.3185565 -1.3181618 -1.3186324 -1.3180132 -1.3181516
##  [67] -1.3179187 -1.3183730 -1.3185563 -0.9631142 -0.9312931 -0.8997462
##  [73] -0.8682469 -0.8367044 -0.8051350 -0.7736355 -0.7423622 -0.7254113
##  [79] -0.7247663 -0.7247849 -0.7248769 -0.7249449 -0.7250450 -0.7248600
##  [85] -0.7249321 -0.7251640 -0.7247881 -0.7251865 -0.7249138 -0.7253687
##  [91] -0.7251252 -0.7248758 -0.7253206 -0.7250327 -0.7254081 -0.7250131
##  [97] -0.7252162 -0.7252278 -0.7248338 -0.7252505  0.4283557  0.4280697
## [103]  0.4283589  0.4279754  0.4283668  0.4283586  0.4280268  0.4283044
## [109]  0.4280597  0.4283605  0.4283655  0.4281215  0.4281008  0.4279924
## [115]  0.4579482  0.4894087  0.5209676  0.5525251  0.5839990  0.6153368
## [121]  0.6397172  0.6396775  0.6394579  0.6395375  0.6395949  0.8046583
## [127]  1.8501991  1.8497966  1.8490294  1.8489455  1.8493700  1.8490720
## [133]  1.8501415  1.8492247  1.8498832  1.8503813  1.8491017  1.8495115
## [139]  1.8498470  1.8501305  1.8503907  1.8489066  1.8491647  1.8494102
## [145]  1.8496399  1.8498559  1.8500669  1.8502871  1.8487940  1.8491130
## [151]  1.8495217  1.8500669  1.8490903  1.8502084  1.8501620  1.8493903
## [157]  1.8488009  1.8488807  1.8489031  1.8792375  1.9103261  1.9417924
## [163]  1.9733613  2.0049335  2.0366779  2.0689210  2.1009061  2.1319497
## [169]  2.1630079  2.1943941  2.2259525  2.2575309  2.2890998  2.3206470
## [175]  2.3521779  2.3838648  2.4154385  2.4469704  2.4785414  2.5101204
## [181]  2.5416905  2.5732348  2.6046310  2.6357777  2.6666448  2.6973277
## [187]  2.7140480  2.7145678  2.7922908  2.8252431  2.8575600  2.8892428
## [193]  2.9206980  2.9521743  2.9837238  3.0152893  3.0467834  3.0781594
## [199]  3.1094882  3.1410383
```

```r
#par(pty="s")
curve(log_cos.plottable, from = -pi, to = pi, xlab="theta")
## Allow a second plot on the same graph
par(new=TRUE)

## Plot the second plot and put axis scale on right
plot(seq(-pi, pi, length.out = 200), para, pch=1,  xlab="", ylab="",
    axes=FALSE, type="b", col="red")

## a little farther out (line=4) to make room for labels
mtext("MLE",side=3,col="red",line=4)
axis(4, ylim=c(-4,4), col="red",col.axis="red",las=1)


## Add Legend
legend("topleft",legend=c("Log-likelihood","MLE by Newton-Raphson"),
  text.col=c("black","red"),pch=c(16,15),col=c("black","red"))
```




```
#par(pty="m")
```

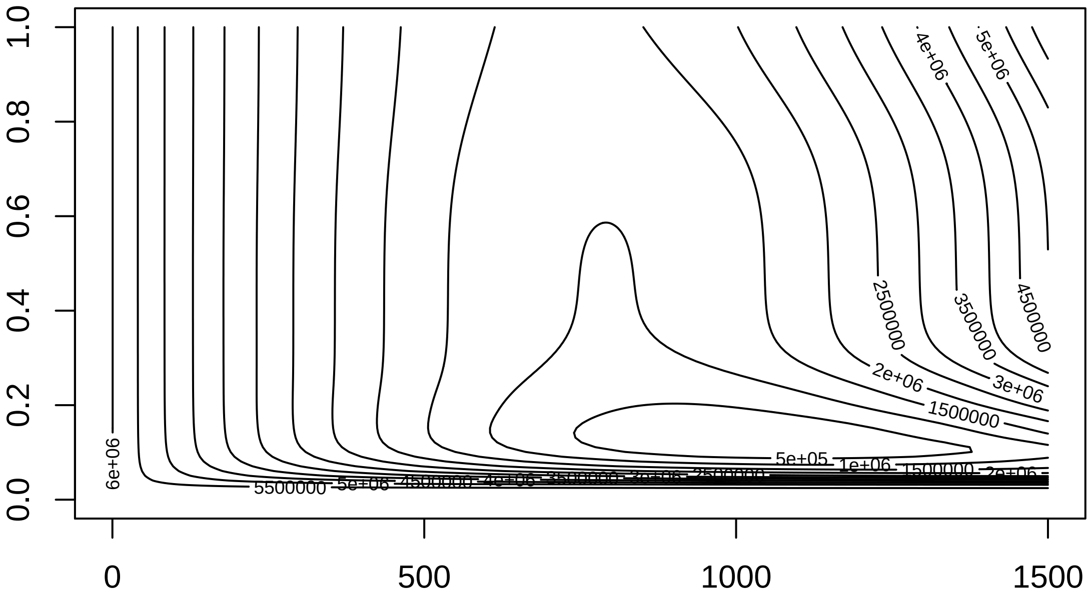
3.3.3 Modeling beetle data

```
beet <- data.frame(
    days    = c(0,  8,  28,  41,  63,  69,   97, 117,  135,  154),
    beetles = c(2, 47, 192, 256, 768, 896, 1120, 896, 1184, 1024))

growth <- function(x) {
  k <- x[1]; r <- x[2]
  k*beet$beetles[1]/(beet$beetles[1] + (k-beet$beetles[1])*exp(-r*beet$days))
}

growth.contour <- function(k,r) {
  k*beet$beetles[1]/(beet$beetles[1] + (k-beet$beetles[1])*exp(-r*beet$days))
}
growth.error <- function(k, r) {
  sum( (beet$beetles - growth.contour(k, r))^2 )
}
```

Plot the contour plot to visually check:



Gauss-Newton method

Multivariate method based on log-normal model MLE

(a) Use self method: Steepest Descent(Gradient Descent)

(b) Validate using R function optim: use L-BFGS-B method

```
growth.error.optim <- function(x) {
  sum( (beet$beetles - growth(x))^2 )
}
optim(c(1, 1), growth.error.optim, lower=c(0,0), upper=c(Inf, 1), method = "L-BFGS-B")
```

```
## $par
## [1] 1049.3480876    0.1182881
##
## $value
## [1] 73419.77
##
## $counts
## function gradient
##       54       54
##
## $convergence
## [1] 0
##
## $message
## [1] "CONVERGENCE: REL_REDUCTION_OF_F <= FACTR*EPSMCH"
```