# RNG Project

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# Find the Value of C & Identify weights of component

$$\int_0^\infty (2x^{\theta-1} + x^{\theta-\frac{1}{2}})e^{-x}dx = 2\int_0^\infty x^{\theta-1}e^{-x}dx + \int_0^\infty x^{\theta-\frac{1}{2}}e^{-x}dx = 2\Gamma(\theta) + \Gamma(\theta+\frac{1}{2})$$

–>

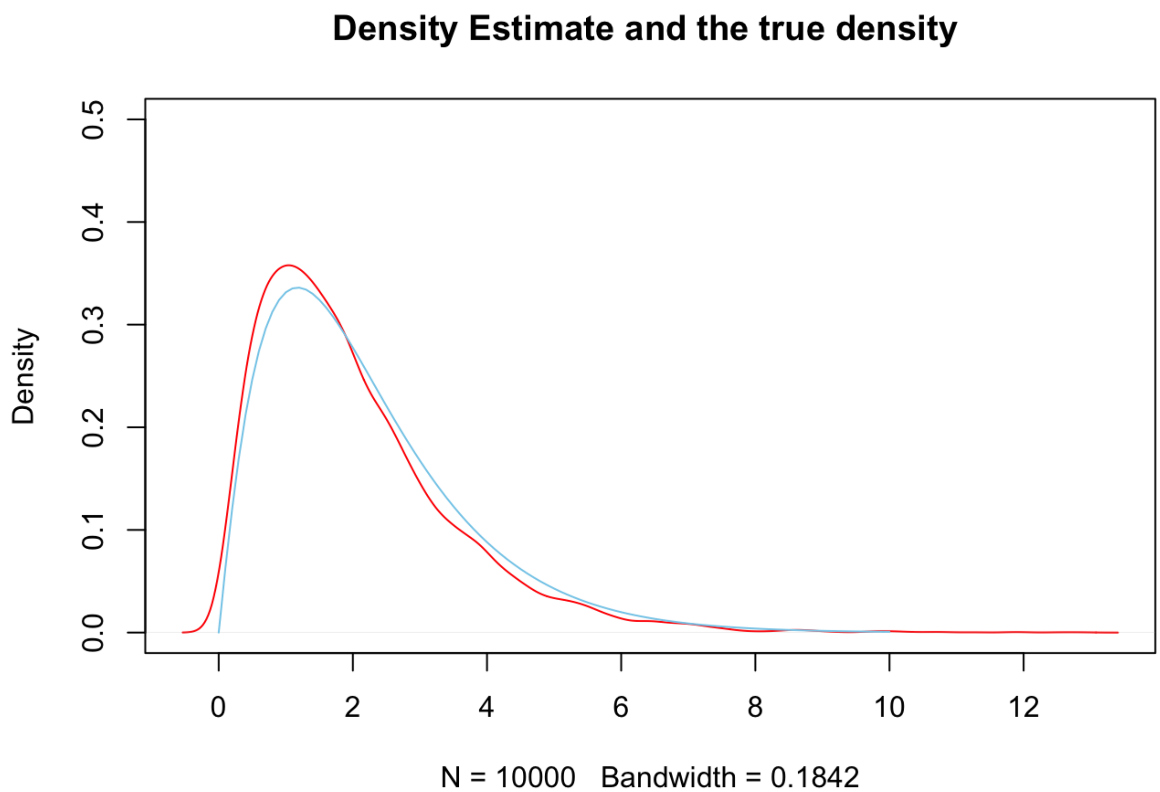
$$C = \frac{1}{2\Gamma(\theta) + \Gamma(\theta+\frac{1}{2})}$$

–>

$$g(x) = \frac{1}{2\Gamma(\theta) + \Gamma(\theta+\frac{1}{2})}(2x^{\theta-1} + x^{\theta-\frac{1}{2}})e^{-x} = \frac{2\Gamma(\theta)}{2\Gamma(\theta) + \Gamma(\theta+\frac{1}{2})}\frac{1}{\Gamma(\theta)}x^{\theta-1}e^{-x} + \frac{\Gamma(\theta+\frac{1}{2})}{2\Gamma(\theta) + \Gamma(\theta+\frac{1}{2})}\frac{1}{\Gamma(\theta+\frac{1}{2})}x^{\theta-\frac{1}{2}}e^{-x}$$

# Design a Procedure

```
h1 <- function(n, theta){
  U <- runif(1)
  X <- rep(0,n)
  weight <- 2*gamma(2) / (2*gamma(2) + gamma(2.5))
  for(i in 1:n){
    if(U < weight){
      X[i] <- rgamma(1,theta,1)
    }else{
      X[i] <- rgamma(1,theta+0.5,1)
    }
  }
  X
}
theta <- 2
sample <- h1(10000, 2)
C <- 1/(2*gamma(2) + gamma(2.5))
plot(density(sample),ylim = c(0,0.5),col="red", main="Density Estimate and the true density ")
curve((2*x^(theta-1)+x^(theta-1/2))*exp(-x)*C, from=0, to=10, add=TRUE, col="skyblue")
```




# Use rejection sample

In order to find $\alpha$ such that

$$q(x) = \sqrt{4 + x} x^{\theta - 1} e^{-x} <= \alpha g(x)$$

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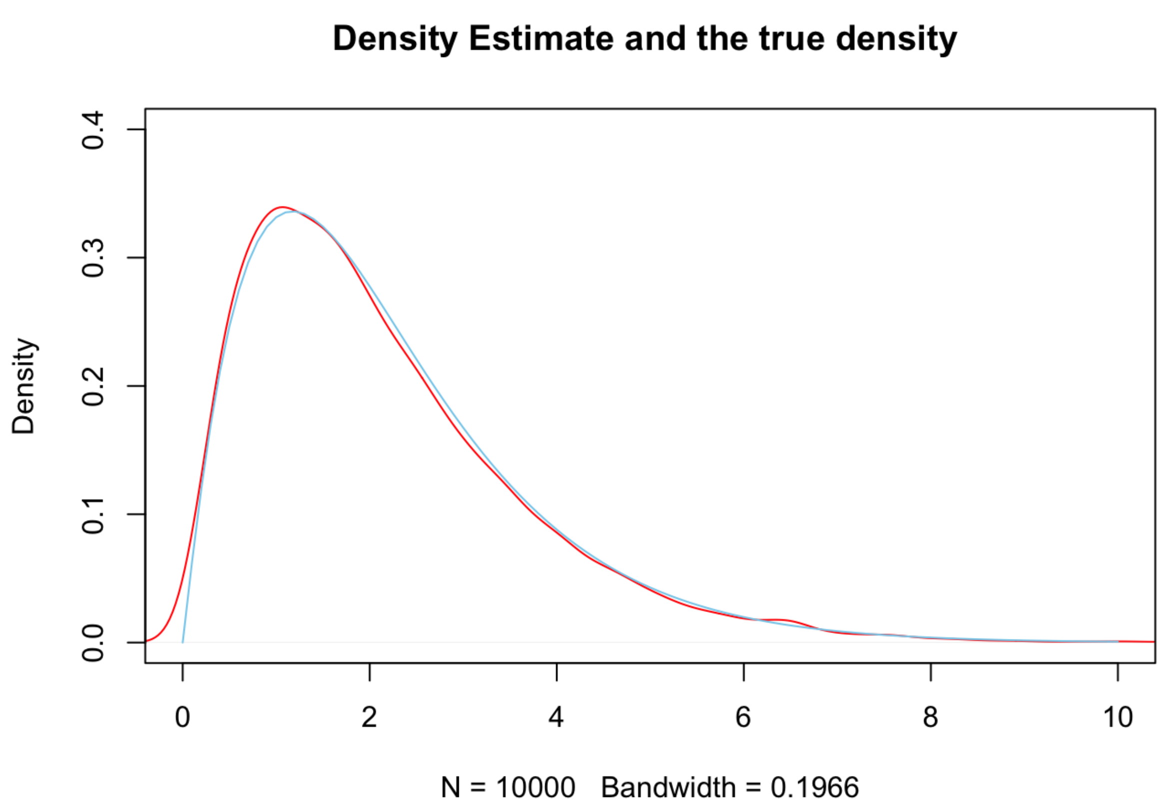
$$\alpha = \sup_{x > 0} \frac{q(x)}{g(x)} = \frac{1}{C}$$

$$q(x) = \frac{1}{C} g(x) = (2x^{\theta - 1} + x^{\theta - 0.5}) e^{-x}$$

Generate T with density m. Generate U, uniform on [0,1] and independent of T. If M(T)*U ≤ ƒ(T), then let X = T (accept T). Otherwise, go to Step 1 (Reject T).

```
h2 <- function(n, theta){
  X <- rep(0,n)
  i <- 1
  while(i <= n){
    U <- runif(1)
    x <- h1(1, theta)
    if(U <= sqrt(4+x)/(2+sqrt(x))){
      X[i] <- x
      i <- i+1
    }
  }
  X
}

theta <- 2
samplea <- h2(10000, 2)
C <- 1/(2*gamma(2) + gamma(2.5))
plot(density(samplea),ylim = c(0,0.4), xlim = c(0,10), col="red", main="Density Estimate and the
 true density ")
curve((2*x^(theta-1)+x^(theta-1/2))*exp(-x)*C, from=0, to=10, add=TRUE, col="skyblue")
```




# Design a procedure to F

$$q(x) = \frac{x^{\theta-1}}{1+x^2} + \sqrt{2+x^2}(1-x)^{\beta-1} \le x^{\theta-1} + 2(1-x)^{\beta-1}$$

choose

$$\lambda = \frac{\theta}{\theta+2\beta}$$

$$g(x) = \lambda Beta(\theta, 1) + (1-\lambda)Beta(1, \beta) = \frac{\lambda}{\theta}x^{\theta-1} + \frac{1-\lambda}{\beta}(1-x)$$

so, when

$$\alpha = \theta + 2\beta$$

, it satisfy

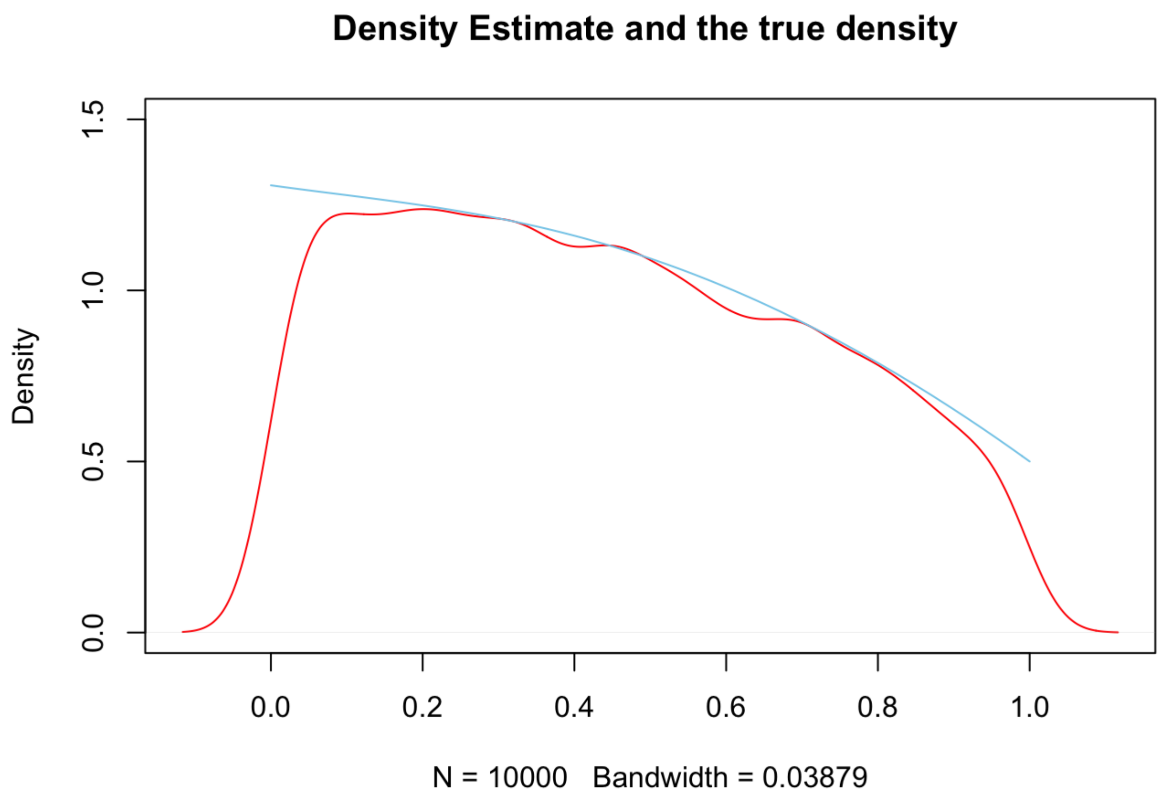
$$q(x) \leq \alpha g(x)$$

```
n <- 10000
U <- runif(n)
X <- rep(0,n)


s1 <- function(n, theta, beta){
  lambda <- theta/(theta+2*beta)
  g <- rep(0, n)
  for(i in 1:n){
    u <- runif(1)
    if(u < lambda){
      g[i] <- rbeta(1, theta, 1)
    }else{
      g[i] <- rbeta(1, 1, beta)
    }
  }
  g
}


s2 <- function(n, theta, beta){
  s <- rep(0, n)
  lambda <- theta/(theta + 2*beta)
  alpha <- theta+2*beta
  i <- 1
  while(i <= n){
    u <- runif(1)
    x <- s1(1, theta, beta)
    if(u <= (x^(theta-1)/(1+x^2)+sqrt(2+x^2)*(1-x)^(beta-1))/
        (alpha*(lambda*dbeta(x, theta, 1)+(1-lambda)*dbeta(x, 1, beta)))){
      s[i] <- x
      i <- i + 1
    }
  }
  s
}

#theta = 2, beta = 2
theta <- 2
beta <- 2
c <- integrate(function(x) {x^(theta-1)/(1+x^2)+sqrt(2+x^2)*(1-x)^(beta-1)}, 0, 1)$value
sampleb <- s2(10000, 2, 2)
plot(density(sampleb),ylim=c(0,1.5),col="red", main="Density Estimate and the true density ")
curve(x^(theta-1)/(1+x^2)+sqrt(2+x^2)*(1-x)^(beta-1)/c, from=0, to=1, add=TRUE, col="skyblue")
```




## Assume

$$q_1(x) = \frac{x^{\theta-1}}{1+x^2} \leq x^{\theta-1} = \theta Beta(\theta, 1) = \alpha_1 g_1(x)$$

$$q_2(x) = \sqrt{2+x^2}(1-x)^{\beta-1} \leq 2(1-x)^{\beta-1} = 2\beta Beta(1, \beta) = \alpha_2 g_2(x)$$

$$\alpha_1 = \theta$$

$$\alpha_2 = 2\beta$$

```
s3 <- function(n, theta, beta){
  s <- rep(0,n)
  lambda <- theta/(theta+2*beta)
  i <- 1
  while (i <= n) {
    u <- runif(1)
    if(u < lambda){
      x <- rbeta(1, theta, 1)
      u1 <- runif(1)
      if(u1 <= 1/(1+x^2)){
        s[i] <- x
        i <- i+1
      }
    }else{
      x <- rbeta(1, 1, beta)
      u2 <- runif(1)
      if(u2 <= sqrt(2+x^2)/2){
        s[i] <- x
        i <- i +1
      }
    }
  }
  s
}
#theta=2, beta=2
samplec <- s3(10000, 2, 2)
c <- integrate(function(x) {x^(theta-1)/(1+x^2)+sqrt(2+x^2)*(1-x)^(beta-1)}, 0, 1)$value
plot(density(samplec),ylim=c(0,1.5),col="red", main="Density Estimate and the true density ")
curve(x^(theta-1)/(1+x^2)+sqrt(2+x^2)*(1-x)^(beta-1)/c, from=0, to=1, add=TRUE, col="skyblue")
```