Statistical Computing Homework 6, Chapter 5

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Contents

Rejection Sampling

(a)

It's obvious that first component is Gamma(θ, 1), second component is Gamma($\theta + 0.5$, 1). In order to make sure the integral of pdf is 1, $C = \left(2\Gamma(\theta) + \Gamma(\theta + 0.5)\right)^{-1}$. Weights for two components are $2\Gamma(\theta)C$ and $\Gamma(\theta + 0.5)C$

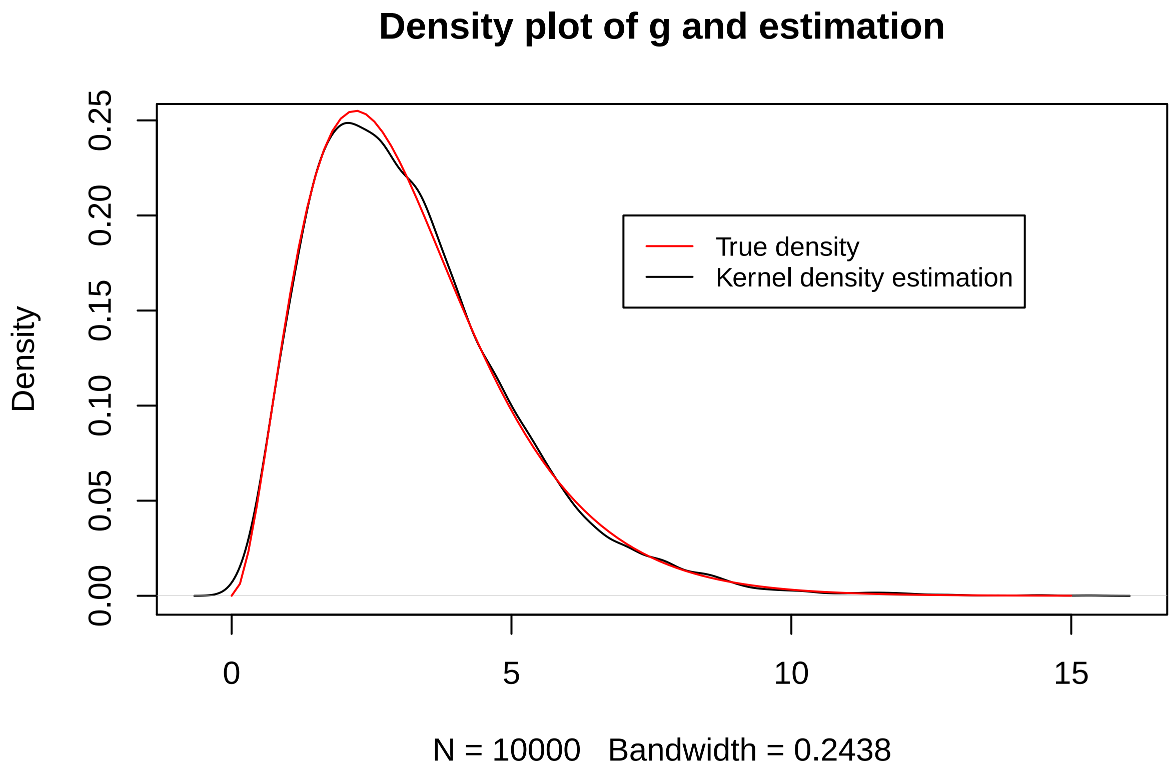
(b)

```
theta <- 3
C <- function(theta) {
  ( 2 * gamma(theta) + gamma(theta+0.5) )^(-1)
}
cmp1 <- function(theta) {
  2 * gamma(theta) * C(theta)
}

#cmp2 <- function(theta) {
#  gamma(theta+0.5) * C(theta)
#}

G <- function(x) {
  cmp1(theta) * dgamma(x, shape = theta, scale = 1) + (1-cmp1(theta)) * dgamma(x, shape = theta + 0.5, s
}
#curve(G, 0, 10)
```

Draw 10,000 samples from g: mixture gamma

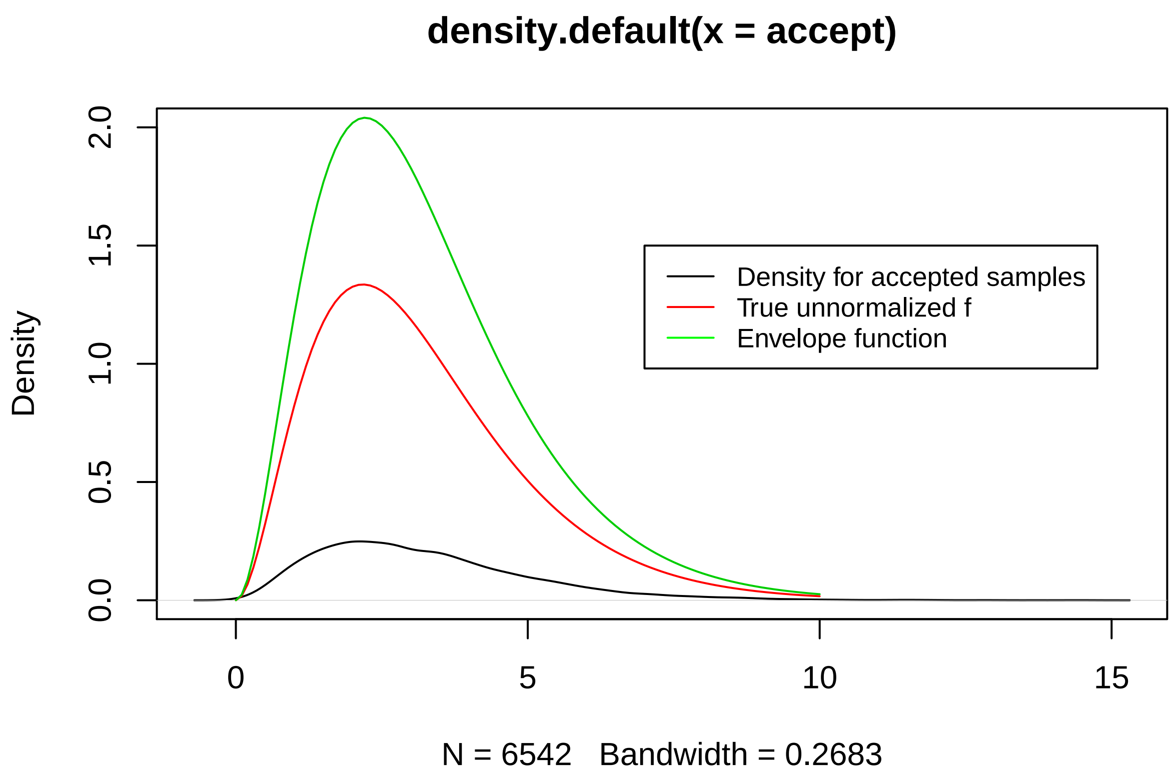
```
p1 <- cmp1(theta)
#p2 <- cmp2(theta)
n <- 10000;  mixture_g <- numeric(n)


for (i in 1:n) {
  u <- runif(1, min = 0, max = 1)
  if (u < p1) {
    mixture_g[i] <- rgamma(1, shape = theta, scale = 1)
  } else {
    mixture_g[i] <- rgamma(1, shape = theta + 0.5, scale = 1)
  }
}
```

```r
#curve(G, 0, 10, col = 2, add = T)
plot(density(mixture_g), main = "Density plot of g and estimation")
curve(G, 0, 15, col = 2, add = T)
legend(7, 0.2, legend=c("True density", "Kernel density estimation"),
       col=c("red", "black"), lty = 1, cex=0.8)
```



(c) Choose $\alpha = 8$, this can be proved that envelope function is greater than unnormalized f on positve real line.

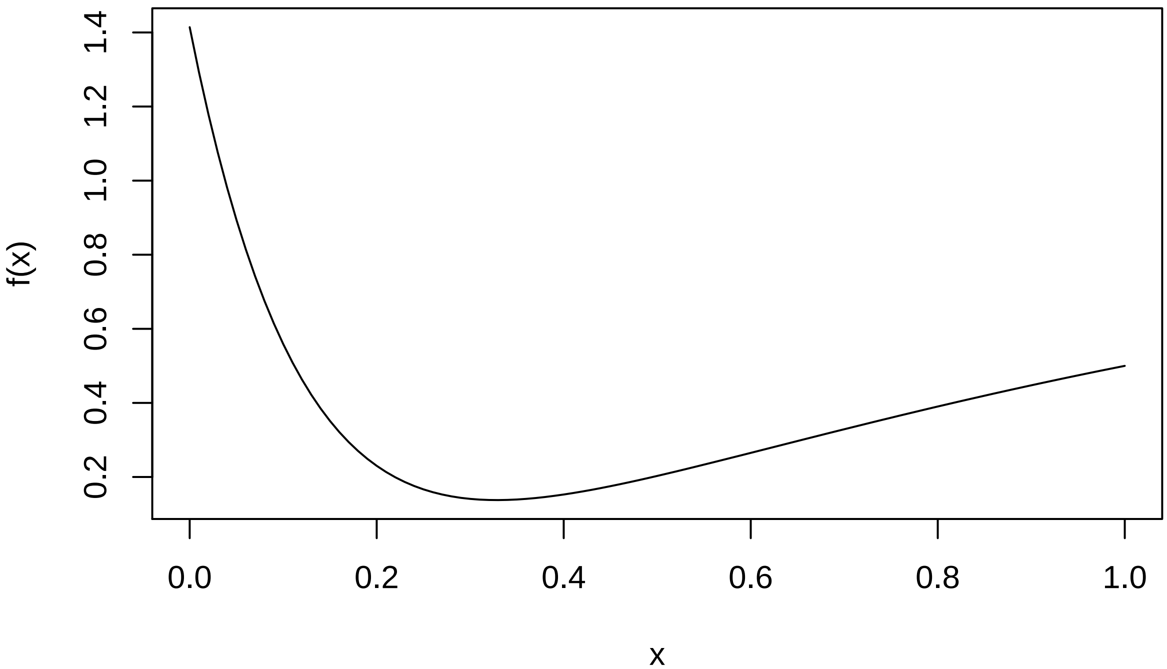
```r
f <- function(x) {
  sqrt(4 + x) * x^(theta - 1) * exp(-x)
}
#curve(8*G(x), 0, 10, add = T, col = 2)

u <- runif(10000, min = 0, max = 1)
accept <- mixture_g[u < f(mixture_g)/(8*G(mixture_g))]

plot(density(accept), ylim = c(0, 2))
curve(f, 0, 10, add = T, col =2 )
curve(8*G(x), 0, 10, add = T, col = 3)
legend(7, 1.5, legend=c("Density for accepted samples", "True unnormalized f", "Envelope function"),
       col=c("black", "red", "green"), lty = 1, cex=0.8)
```



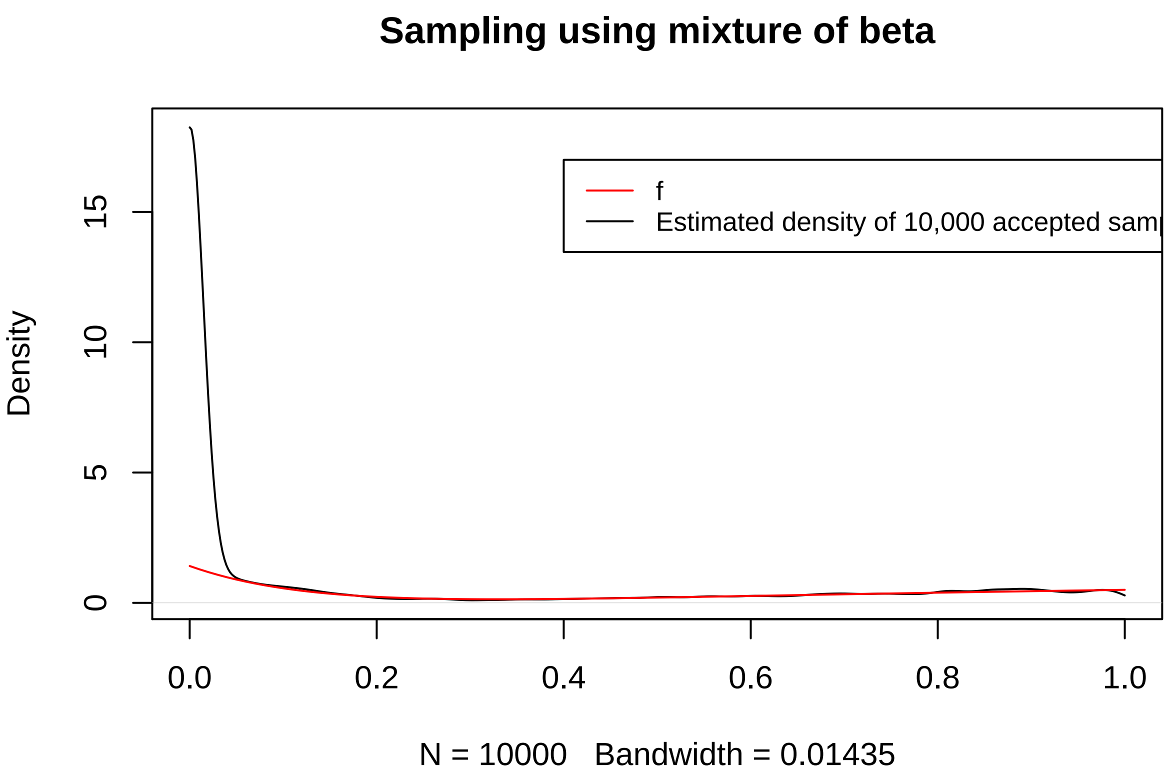
Mixture Proposal

(a) Mixture beta as envelope function: set $\theta = 3$, $\beta = 10$

```
theta <- 3
bet <- 10

f <- function(x) {
  x^(theta - 1)/(1+x^2) + sqrt(x^2+2)*(1-x)^(bet-1)
}
curve(f, 0, 1)
```



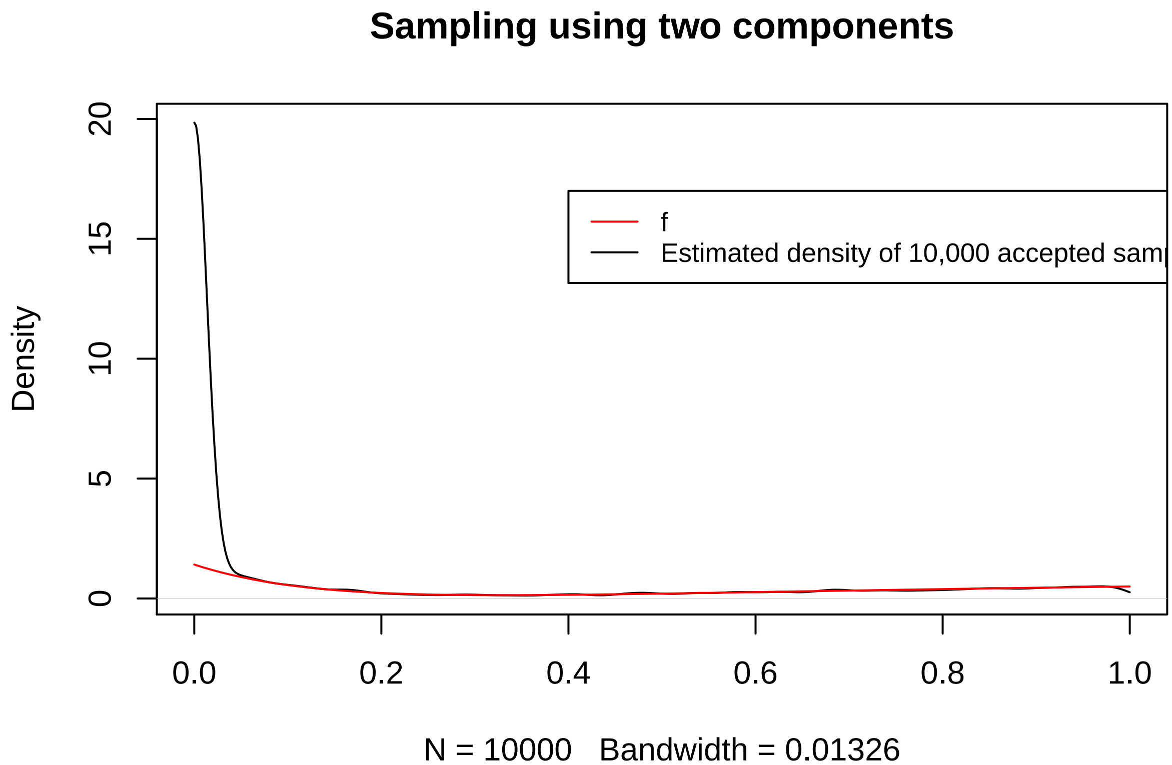

```r
### mixture beta with component probability 0.5 and 0.5
mixture_beta_fun <- function(x) {
  0.5*dbeta(x, shape1 = theta, shape2 = 1) + 0.5*dbeta(x, shape1 = 1, shape2 = bet)
}

accepted_f_1 <- numeric(10000)
n <- 0

### Draw the sample based on mixture beta envelope:
while(n <= 10000) {
  # First sample from mixture beta with component probability 0.5 and 0.5
  u <- runif(1, min = 0, max = 1)
  if (u < 0.5) {
    mixture_b <- rbeta(1, shape1 = theta, shape2 = 1)
  } else {
    mixture_b <- rbeta(1, shape1 = 1, shape2 = bet)
  }
  u <- runif(1, min = 0, max = 1)
  if (u <= f(mixture_b)/mixture_beta_fun(mixture_b)) {accepted_f_1[n] <- mixture_b}
  n <- n + 1
}

plot(density(accepted_f_1, from = 0, to = 1), main = "Sampling using mixture of beta")
curve(f, 0, 1, col = 2, add = T)
legend(0.4, 17, legend=c("f", "Estimated density of 10,000 accepted samples"),
       col=c("red", "black"), lty = 1, cex=0.8)
```



(b)

```
#curve(x^(theta-1)/(1+x^2), 0, 1)
#curve(theta*x^(theta-1)*0.6, 0, 1, add = T, col = 2)

#curve( sqrt(2+x^2)*(1-x)^(bet-1), 0, 1)
#curve( bet*(1-x)^(bet-1)*0.4, 0, 1, add = T, col = 2)

alpha1 <- 0.6
alpha2 <- 0.4

q1 <- function(x) {
  x^(theta-1)/(1+x^2)
}

q2 <- function(x) {
  sqrt(2+x^2)*(1-x)^(bet-1)
}

accepted_f_2 <- numeric(10000)
n <- 0
while (n <= 10000) {
  k <- sample(c(1, 2), size = 1, prob = c(alpha1, alpha2))
  u <- runif(1, min = 0, max = 1)
  if (k == 1) {
    propose <- rbeta(1, shape1 = theta, shape2 = 1)
    if (u <= q1(propose)/(alpha1*dbeta(propose, shape1 = theta, shape2 = 1))) {accepted_f_2[n] <- propos


  } else {
```

```
    propose <- rbeta(1, shape1 = 1, shape2 = bet)
    if (u <= q2(propose)/(alpha2*dbeta(propose, shape1 = 1, shape2 = bet))) {accepted_f_2[n] <- propose}

  }

  n <- n + 1
}
plot(density(accepted_f_2, from = 0, to = 1), main = "Sampling using two components")
curve(f, 0, 1, col = 2, add = T)
legend(0.4, 17, legend=c("f", "Estimated density of 10,000 accepted samples"),
       col=c("red", "black"), lty = 1, cex=0.8)
```



α_1 should be greater than $\theta^{-1} = 0.333$, $\alpha_2$should be greater than $\sqrt{3}\beta^{-1} = 0.173$, because p_1, p_2 proportional to α_1 and α_2, so we choose $\alpha_1 = 0.6$, $\alpha_2 = 0.4$