

This homework is due at 11:59 pm PT on Wednesday, January 26. Upload your solutions to Gradescope by the deadline. Each problem is worth 3 points. Please be considerate of the grader by writing solutions neatly (typing solutions is preferable). At the top of the first page, please write the names of any other students with whom you discussed the assignment.

(1) Solve the following two problems.

- (a) You are rolling two dice. Let x be the sum of the faces. Find the probability for the event $x = 7$. The dice in this problem are “fair six-sided” dice.
- (b) There are two players A and B . They play a series of games, and each game is won by A with probability p , and by B with probability $1 - p$. The results of the past games do not affect the current game or the future games. They stop when the total number of wins of one of the players is two greater than that of the other player. Find the probability that a total of 6 games are played.

(2) Jane’s dog, Posie, has three puppies, and let us assume that each of the babies is equally likely to be a male (M) or a female (F). This gives us eight possible equally likely outcomes for Posie’s puppies: FFF, FFM, FMF, MFF, MMF, MFM, FMM, MMM.

- (a) Write the corresponding subset and find the probability for the each event below.
 - Event A : all three puppies are the same sex
 - Event B : there are exactly two male pups
 - Event C : there is at most one male
 - Event D : there is at least one male
- (b) Generalize observations you made in (a) to find the simplest formula (in terms of n) you can for the chance of getting at least one male in a litter of n puppies, when each puppy is equally likely to be male or female. Explain how you got your answer, and say what happens to the chance as n increases.

(3) Three candidate students (A , B , C) were signed up for a probability competition. The principal recommends one of three students at random (i.e. with the same probability). Then, the principal informs the teacher which student was chosen, and requests that the name be kept secret for a week. The next day, A tries to get the teacher to tell them who had been selected. The teacher refuses. A then asks which of B or C will be not selected. The teacher thinks for a while, then tells A that B is to be not selected. Does this help for A to increase its chance of being selected? That is, compare the probability of A being selected and the probability of A being selected given that the teacher says B is not selected. Assume that the teacher does not lie.

(4) In an experiment, a fair coin is tossed continually until heads appears, at which point the experiment stops.

- (a) What is the sample space of this experiment?
- (b) Let E_n denote the event that n tosses are necessary to complete the experiment. What points of the sample space are contained in E_n ?
- (c) What is $\mathbb{P}(\bigcup_{n=1}^{\infty} E_n)$?