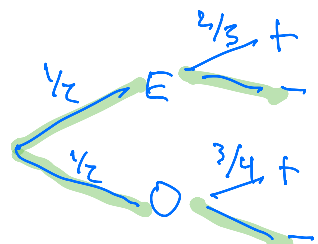
Stat 88 lec 5

Warm up 2:00 – 2:10

There are two boxes, the odd box containing 1 black marble and 3 white marbles, and the even box containing 2 black marbles and 4 white marbles. A box is selected at random, and a marble is drawn at random from the selected box.

a) What is the probability that the marble is black?

b) Given that the marble is white, what is the probability that it came from the even box?

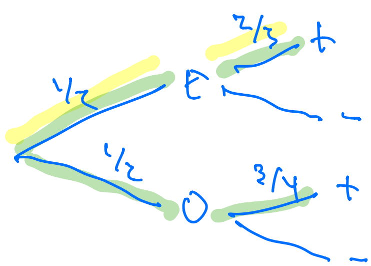


a) E = even box

+ = positive for white

$P(-) = \frac{1}{2}\cdot\frac{1}{3} + \frac{1}{2}\cdot\frac{1}{4}$

b)



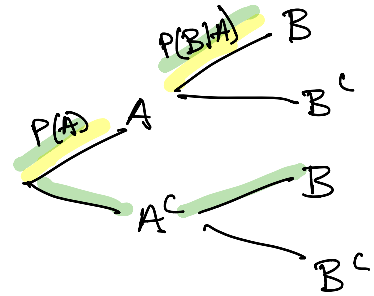
$P(E|+) = \dfrac{\frac{1}{2}\cdot\frac{2}{3}}{\frac{1}{2}\cdot\frac{2}{3} + \frac{1}{2}\cdot\frac{3}{4}} = \boxed{.47}$

↑ posterior

See Piazza announcement about University sanctioned accommodations for Quizzes/Midterm.

last time

Sec 2.4 Use and interpretation of Bayes rule



Bayes rule:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

Prior knowledge about B, P(A) is called prior probability

given knowledge about B, P(A|B) is called posterior probability

base rate fallacy — according to Baye's rule you can't ignore the base rate (i.e prior probability) when computing the posterior probability.

Today

1. **Sec 2.4** Use and interpretation of Baye's rule continued
2. **Sec 2.5** A closer look at independence.

① Sec 2.4

tinyurl.com / jan29-pt1

Chapter 2 Monday January 27 2020

1. There are three boxes, each of which contains two coins. One box has two gold coins, one has two silver coins, and one has a gold coin and a silver coin. A box is picked at random and then a coin is picked at random from the box. Given that the coin is gold, what is the chance that the other coin in the box is silver?

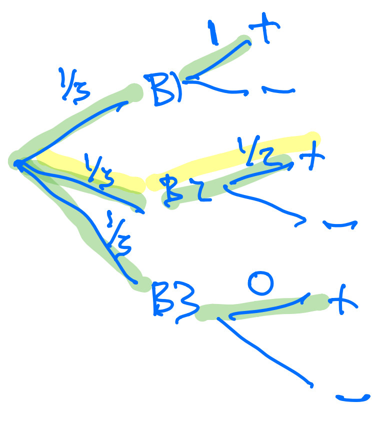
B1 = box 1 (GG)
B2 = box 2 (GS)
B3 = box 3 (SS)
+ = pos for gold in box

a $\frac{1}{4}$

b $\frac{1}{3}$

c $\frac{1}{2}$

d none of the above



$P(B2|+)$

$= \dfrac{1/3 \cdot 1/2}{\frac{1}{3}\cdot\frac{1}{2} + 1/3 \cdot 1 + 1/3 \cdot 0}$

$= \dfrac{1/6}{1/2} = \boxed{1/3}$

$P(B1|+) = 2/3$

$P(B3|+) = 0$

Sec 2.4 A closer look at independence

Events A and B are **independent** if the information that one of them occured doesn't change the chance that the other occured,

i.e $P(B|A) = P(B)$.

ex deal 2 cards from a deck

A = 1st card is an ace

B = 2nd card is an ace

$P(B|A) = 3/51$

$P(B) = 4/52$

so A and B not indep.

However if deal 2 cards w/ replacement then A, B indep.

Special case of multiplication rule:

If A, B are indep

$$P(AB) = \underbrace{P(B|A)P(A)}_{\text{mult rule}} = \underbrace{P(B)P(A)}_{\text{indep.}}$$

$P(AB)$: both cards are ace

ex Around 2003, Sally Clarke, in a famous murder trial had two babies mysteriously die.

Sally Clarke's defence was that the babies both died of Sudden Infant Death Syndrome (SIDS).

A = event the first infant dies of SIDS

B = event the second infant dies of SIDS

$P(A) = P(B) = \frac{1}{8543}$

A medical expert witness said the chance of two babies dying of SIDS is $\frac{1}{7.3M} = \left(\frac{1}{8543}\right)^2$ and hence Sally Clarke must have murdered her babies.

Problem with arguement? — the assumption of independence is hard to justify.

There may be genetic or environmental factors that predispose families to SIDS, so that the second case within the family becomes much more likely.

$$P(B|A) < \frac{1}{8543}.$$

$$P(AB) = P(A)P(B|A) < \left(\frac{1}{8543}\right)^2 = \frac{1}{7.3M}$$

Moral of the story:

dont just assume independance without justification.

ex A population consists of equal numbers of students in 4 categories: freshman, sophomore, junior, and senior.

4 people are drawn with replacement from the population. What is the chance that a member from each category is chosen?

$$\frac{4}{4}\,\frac{3}{4}\,\frac{2}{4}\,\frac{1}{4} = \frac{4!}{4^4}$$

ex Exercise 2.6.5

5. There are n students in a class. Assume that each student's birthday is equally likely to be any of 365 days of the year, regardless of the birthdays of others.

a) What is the chance that at least one of the students was born on January 1?

b) What is the chance that at least two students have the same birthday?

a) 1 − P(no one born Jan 1)

$1 - \left(\frac{364}{365}\right)^n$

b) 1 − P(no student same B-day)

$1 - \frac{365}{365} \cdot \frac{364}{365} \cdot \ldots \cdot \frac{365-n+1}{365}$

tinyurl.com / jan31 -pt1

$P(AB) = P(A)P(B)$

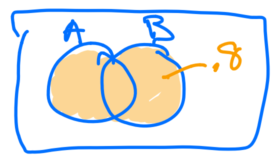
Stats 88

Chapter 2 Friday January 29 2020

1. Suppose A and B are two events with

$$P(A) = 0.5 \text{ and } P(A \cup B) = 0.8.$$

For what value of $P(B)$ would A and B be independent?



a P(B)=0

b P(B)=0.3

c P(B)=0.6

d none of the above

$$\underbrace{P(A \cup B)}_{.8} = \underbrace{P(A)}_{.5} + P(B) - \underbrace{P(AB)}_{P(A)P(B),\ P(A) = .5}$$ inclusion exclusion

$$.3 = P(B) - .5P(B) = P(B)\underbrace{(1 - .5)}_{.5}$$

$$\Rightarrow \boxed{P(B) = .6}$$

Note: If A and B are mutually exclusive then A, B are dependent since mutually exclusive means if you have A you don't have B which is a dependency between A and B.