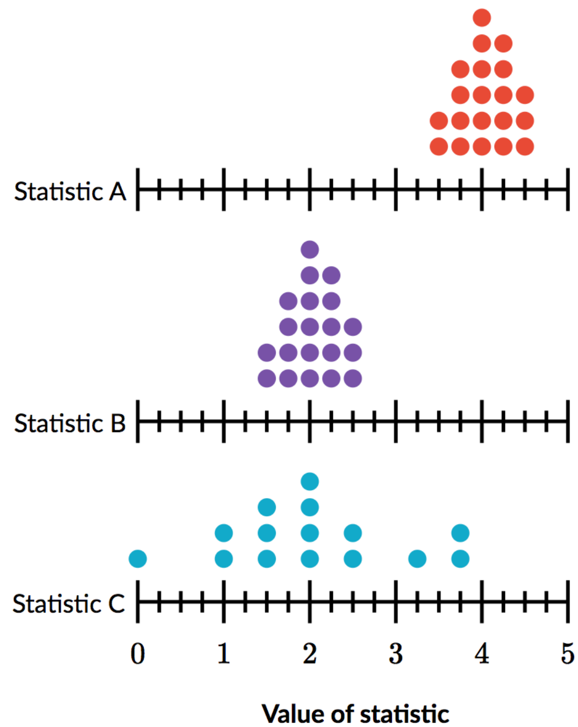
Stat 88 lec 18

Warmup 2:00-2:10

Below are 3 statistics of a parameter whose true value is 4. Which estimator has high bias and low variability?



midterm review

Dorkhan Chang

Yesterday

Extra Unbiased Estimator Questions

Question 2 and 3

Reply (8 likes)

ex The distribution of a RV, X, has an unknown parameter θ.

Suppose $E(X) = \frac{7\theta + 3}{9}$

To estimate θ you take a simple random sample $X_1, \ldots, X_n$.

Find an unbiased estimator of θ.

know

$E(\bar{X}) = E(X) = \frac{7\theta+3}{9}$

$E\left(\frac{9\bar{X}-3}{7}\right) = \theta$

unbiased estimator.

Candice Lee

Yesterday

More practice with unconditional expectation -- lecture 16 question of Tamara choosing an integer

Reply (5 likes)

You roll a die and see N dots,
and then flip a p =1/3 coin N times.
What is the expected number of heads you get?

$N \sim \text{Unif}\{1,2,3,4,5,6\}$

$X = \#\text{ heads}$

$X \mid N=n \sim \text{Binomial}(n, \tfrac{1}{3})$

$X \mid N \sim \text{Binomial}(N, \tfrac{1}{3})$.

$$E(X) = E(\underbrace{E(X \mid N)}_{= N \cdot \frac{1}{3}}) = E\left(\frac{N}{3}\right) = \frac{1}{3} E(N) = \boxed{7/6}$$

$E(N) = \frac{1+6}{2} = 7/2$

You have 2 coins in a hat. Coin A has p=1/3, Coin B has p=1/4
You randomly choose a coin. What is the expected number of flips till you get a head?

$X = \#\text{ flips till heads.}$

$$E(X) = \underbrace{E(X \mid A)}_{=\frac{1}{1/3}} \underbrace{P(A)}_{=1/2} + \underbrace{E(X \mid B)}_{=\frac{1}{1/4}} \underbrace{P(B)}_{=1/2} = \boxed{3.5}$$

Dingyue Zhang

Yesterday

Extra expectation questions please!

Reply (4 likes)

ex

In a box of tickets, 60% of the tickets are blue, 20% green, 15% yellow, and 5% purple. Find the expected number of colors that do not appear among d draws made at random with replacement from the box.

X = the # of colors (out of 4) that do not appear ...

$$I_G = \begin{cases} 1 & \text{if green doesn't appear in } d \text{ draws} \\ 0 & \text{else} \end{cases}$$

$$P = (.8)^d$$

$$X = I_B + I_G + I_Y + I_P$$

$$\boxed{E(X) = (.4)^d + (.8)^d + (.85)^d + (.95)^d}$$

Some students asked how to solve if draws were made without replacement.

Since the population size N isn't given you can assume infinite population in which case the answer above is approximately correct.

If say the problem tells you N=100 then the answer is (assuming $d \leq 40$)

$$\boxed{E(X) = \frac{\binom{60}{0}\binom{40}{d}}{\binom{100}{d}} + \frac{\binom{20}{0}\binom{80}{d}}{\binom{100}{d}} + \frac{\binom{15}{0}\binom{85}{d}}{\binom{100}{d}} + \frac{\binom{5}{0}\binom{95}{d}}{\binom{100}{d}}}.$$

Hee Soo Kim

Yesterday

How to find the minimum overlap b/w 3 or more probabilities.

Quiz 1 Q2

Bounds for union

$$\max(P(A), P(B)) \le P(A \cup B) \le P(A) + P(B)$$

Bounds for intersection

$$1 - P(A^c) - P(B^c) \le P(AB) \le \min(P(A), P(B))$$

to generalize to 3 events

$$1 - P(A^c) - P(B^c) - P(C^c) \le P(ABC) \le \min(P(A), P(B), P(C)).$$

Ben Skinner

Monday

Homework 6 question 4 + 5

Reply (2 likes)

4. Overlapping Trials

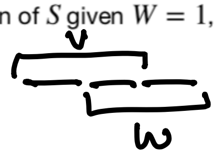
A die that has two red faces and four green faces is tossed three times. Let V be the number of times green faces show in the first two rolls, let W be the number of times green faces show in the last two rolls, and let S be the number of times green faces show in all three rolls.

a) For each $w = 0, 1, 2$, find the conditional distribution of V given $W = w$. Your answer should consist of three distribution tables (with reasoning or calculations, of course).

b) Use your answer to **a** to find $E(V \mid W = w)$ for each $w = 0, 1, 2$.

c) Find the conditional distribution of S given $W = 1$, and hence find $E(S \mid W = 1)$.

d) Find $E(W \mid S = 1)$.



ggg
ggr
grg
rgg
grr
rgr
rrg
rrr

a)

V = # green first 2

W = # green last 2

$$P(V=1|W=0) = \frac{P(V=1, W=0)}{P(W=0)} = \frac{\frac{2}{3}\left(\frac{1}{3}\right)^2}{\left(\frac{1}{3}\right)^2} = 2/3$$

$$P(V=0|W=0) = 1/3$$

← since add to 1.

$$P(V=2|W=1) = \frac{P(V=2, W=1)}{P(W=1)} = \frac{\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)}{2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right) + 2\cdot\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)} = \frac{\frac{4}{27}}{\frac{12}{27}} = 1/3$$

$$P(V=1|W=1) = \frac{P(V=1, W=1)}{P(W=1)} = \frac{1}{2}$$

$$P(V=0|W=1) = \frac{1}{6}$$

etc.

5. Sticker Collection

Each box of Cal Crunch cereal contains a sticker. The sticker is equally likely to have one of the following five pictures on it, independently of the stickers in all other boxes.

- a walking bear
- a sleeping bear
- the Campanile
- Sather Gate
- Evans Hall

Stat 88 GSIs love to snack on Cal Crunch so they buy a box each week and put the sticker in their collection. In what follows, remember that "till" means "up to and including".

a) Find the expected number of weeks till their collection has a bear sticker.

b) Find the expected number of weeks till their collection has all five kinds of stickers.

c) Find the expected number of weeks till their collection has a bear as well as a campus location.

a) X = # weeks till bear sticker

$X \sim \text{Geom}\left(\frac{2}{5}\right)$

$E(X) = \frac{5}{2}$.

etc

$$\underbrace{\underline{WB}}_{X_1 = 1}\ \underbrace{\underline{WB}\ \underline{SB}}_{X_2}\ \underbrace{\underline{SB}\ \underline{WB}\ \underline{C}}_{X_3}\ \_\ \_\ \_$$

b)

$X_1 = 1$

X_2 = # weeks till diff sticker $\sim \text{Geom}\left(\frac{4}{5}\right)$

X_3 = # week till diff sticker $\sim \text{Geom}(3/5)$

$X = X_1 + X_2 + X_3 + X_4 + X_5$

$$\boxed{E(X) = \underset{\frac{5}{5}}{\underset{\shortparallel}{E(X_1)}} + \underset{\frac{5}{4}}{\underset{\shortparallel}{E(X_2)}} + \dots + \underset{\frac{5}{1}}{\underset{\shortparallel}{E(X_5)}}}$$

c) X_1 = # weeks until bear or campus location $= 1$

X_2 = # weeks until different type sticker.

$$X = \overset{1}{\overset{\shortparallel}{X_1}} + X_2$$

$$E(X) = \underset{1}{\underset{\shortparallel}{E(X_1)}} + E(X_2)$$

B = bear in 1st week

L = location in 1st week.

$$E(X_2) = \underset{\cancel{5}/3}{\underset{\shortparallel}{E(X_2|B)}}\,\underset{\frac{2}{\cancel{5}}}{\underset{\shortparallel}{P(B)}} + \underset{\cancel{5}/2}{\underset{\shortparallel}{E(X_2|L)}}\,\underset{\frac{3}{\cancel{5}}}{\underset{\shortparallel}{P(L)}} = \frac{2}{3} + 3/2$$

$X_2|B \sim \text{Geom}\left(\frac{3}{5}\right)$

$X_2|L \sim \text{Geom}\left(\frac{2}{5}\right)$

$$\boxed{E(X) = 1 + \frac{2}{3} + \frac{3}{2}}$$

YeJin Ahn

Yesterday

Approximation with poisson (like question on quiz 2)

Edited by YeJin Ahn on Mar 3 at 7:22pm

Reply

.01

3. Xinyi is writing some code for her CS 170 project. Xinyi is a really good student so each line she codes has a 99% chance of having no bugs (errors), independent from all other lines. The project has 1000 lines of code. Assume that each line of code can only have one bug.

   a. Find an exponential approximation for the chance that at least one line from the the first 22 lines of code has a bug in it.

   X = # lines out of 22 w/ bug

   $P(X \geq 1) = 1 - P(X=0)$

   $p = P(X=0) = (1 - .01)^{22}$

   $\log p = 22 \log(1 - .01) \approx 22(-.01) \Rightarrow \boxed{p \approx e^{-.22}}$

   $P(X \geq 1) = 1 - p \approx \boxed{1 - e^{-.22}}$

   b. Use the Poisson distribution to approximate the chance that exactly 40 lines of code have a bug.

   X = # lines with bug out of 1000

   $X \sim \text{Bin}(1000, .01) \approx \text{Poisson}(10)$

   $$P(X=40) = \frac{e^{-10} 10^{40}}{40!}$$

   c. Use the Poisson distribution to approximate the chance that there are greater than 16 lines with bugs in them.

   X = # lines w/ bug out of 1000

   $X \sim \text{Binomial}(1000, .01) \approx \text{Poisson}(10)$

   $$P(X>16) = \sum_{i=17}^{\infty} P(X=i)$$

   not 1000 since we

dont leave answer as an infinite sum,

$= 1 - P(X \le 16)$

$1 - \sum_{i=0}^{16} \frac{e^{-10} 10^i}{i!}$

are approximating X as Poisson which takes values 1, 2, ...