Stat 88 lec 6

Warm up 2:00–2:10

a) You flip a coin 8 times. What is the chance you get all heads? $\left(\frac{1}{2}\right)^8 = \frac{1}{256}$

b) Everyone in a class of 100 people flip a coin 8 times. What is the chance at least one person gets all heads?

$1 - P(\text{no one gets all heads}) = 1 - \left(1 - \frac{1}{256}\right)^{100}$

$\text{no one gets all heads} = \text{everyone gets at least one tail}$

$P(\text{you get at least one tail}) = 1 - \frac{1}{256}$

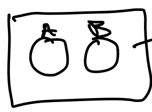
last time

Sec 2.5 Independence

$$P(B|A) = P(B)$$

then $P(AB) = P(A)P(B|A) = P(A)P(B)$

If A and B are independent, non empty sets then they must overlap (i.e. $P(AB) = P(A)P(B) > 0$).



mutually exclusive and dependent sets.

Today

1. review concept tests from lecture
2. sec 3.2 Random Variable
3. Sec 3.3 Binomial Distribution.

(1) Concept tests from last time

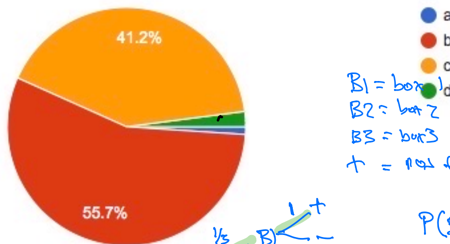
. There are three boxes, each of which contains two coins. One box has two gold coins, one has two silver coins, and one has a gold coin and a silver coin. A box is picked at random and then a coin is picked at random from the box. Given that the coin is gold, what is the chance that the other coin in the box is silver?

a $\frac{1}{4}$

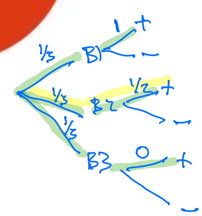
b $\frac{1}{3}$

c $\frac{1}{2}$

d none of the above



$B1$ = box 1 (GG)
$B2$ = box 2 (GS)
$B3$ = box3 (SS)
$+$ = pos for gold in box



$P(B2|+)$

$= \frac{1/3 \cdot 1/2}{\frac{1}{3}\cdot\frac{1}{2} + \frac{1}{3}\cdot 1 + \frac{1}{3}\cdot 0}$

$= \frac{1/6}{1/2} = \boxed{1/3}$

b	(1/3)(1/2) / ((1/3)(1/2)+(1/3)(1)) = 1/3

b	There are three possibilities where you get gold (two from the first box and one from the third box), and out of those only one (the third box) satisfies the criteria that the other coin be silver.

The sample space is

G_1G_2 G_2G_1
S_1S_2 S_2S_1
G_1S_1 S_1G_2

and you pick the left coin first then the right coin second giving 1/3.

c	It's given that it's gold so the probability that the box contains silver is 1/2

correct answer if same sample space as above but problem says "if least one coin is gold, what is chance the other coin is silver?" Then answer is 1/2.

Suppose A and B are two events with

$$P(A) = 0.5 \text{ and } P(A \cup B) = 0.8.$$

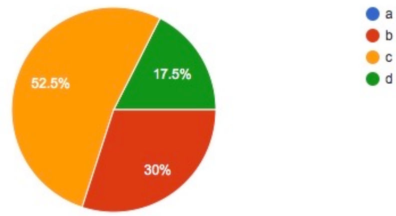
For what value of $P(B)$ would A and B be independent?

a P(B)=0

b P(B)=0.3

c P(B)=0.6

d none of the above



d	you cannot figure it out

b	If P(A) and P(B) are independent, then P(AUB) is P(A) + P(B)

c	Since a and b are independent, p(a and b) = p(a)p(b), so using addition rule: .8 = .5 + p(b) - (.5 * p(b)), Which simplifies out to .6

② Sec 3.2 <u>Random Variables</u>

Random Variables (RV) help reduce the amount of writing involved in phrases like "the chance that there are no more than 1 head in three tosses of a coin".

You can write instead:

let X = # heads in three coin tosses

Find $P(X \leq 1)$

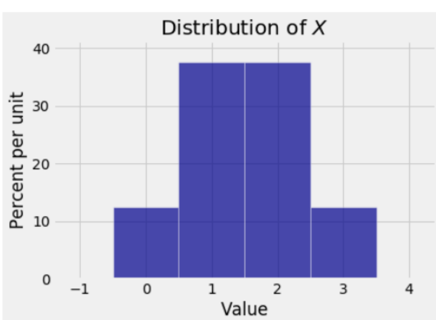
X is a function from the outcome space to the real numbers

$\Omega =$

outcome	X(outcome)	Probability
HHH	3	1/8
HHT	2	1/8
HTH	2	1/8
THH	2	1/8
HTT	1	1/8
THT	1	1/8
TTH	1	1/8
TTT	0	1/8

$$P(X \leq 1) = \underbrace{P(X=0)}_{= 1/8} + \underbrace{P(X=1)}_{= \frac{1}{8}+\frac{1}{8}+\frac{1}{8}} = \boxed{1/2}$$

The distribution or probability mass function (pmf) allows us to visualize the probability for each value of X



<u>Equality of RVs</u> Two RVs can have the same distribution but not be equal. Suppose you and I both flip a coin 3 times. I get $X_{Adam} = 2$ and you get $X_{you} = 1$ then $X_{Adam} \neq X_{you}$ but both RV have the same distribution.

③ Binomial Distribution

A binomial distribution (written binomial(n,p)) is the distribution for the number of successes in n independent trials where the probability of success for each trial is p.

eg $X = \#$ heads out of 5 coin tosses of a $\underbrace{p = 1/4 \text{ coin}}_{\text{chance } 1/4 \text{ coin lands head.}}$

lets find $P(X=2)$:

here $n = 5$ independent coin tosses

$p = 1/4$ is chance for heads

$k = 2$

First what is chance you get HHTTT ? $\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$

HTHTT ?

How many permutations of 5 letters abcde? ← 5!

In case of HHTTT we must divide by

$2!\,3!$ giving $\binom{5}{2} = \frac{5!}{2!\,3!}$

$\Rightarrow P(X=2) = \binom{5}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$ ← binomial formula.

what values does X take?

More generally

$X \sim \text{Binomial}(n,p)$

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$X = \#$ success in n independent trials each having probability p for success.

ex (exercise 3.6.3) $P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$

3. Yi likes to bet on "red" at roulette. Each time she bets, her chance of winning is 18/38 independently of all other times. Suppose she bets repeatedly on red. Find the chance that:

Let X = # wins in first 10 bets.

$n = ?$
$p = ?$
$k = ?$

a) she wins four of the first 10 bets

b) she wins at most four of the first 10 bets

a) $n=10$
$p = 18/38$
$k = 4$

$P(X=4) = \binom{10}{4}\left(\frac{18}{38}\right)^4\left(\frac{20}{38}\right)^6$

b) $P(X \le 4) = \sum_{k=0}^{4} \binom{10}{k}\left(\frac{18}{38}\right)^k\left(\frac{20}{38}\right)^{10-k}$

c) the third time she wins is on the 10th bet

$\underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_}_{\text{wins 2 out of 9}}\ \_$ W — $\left(\frac{18}{38}\right)$

$\binom{9}{2}\left(\frac{18}{38}\right)^2\left(\frac{20}{38}\right)^7$

so answer: $\binom{9}{2}\left(\frac{18}{38}\right)^2\left(\frac{20}{38}\right)^7 \cdot \left(\frac{18}{38}\right) = \binom{9}{2}\left(\frac{18}{38}\right)^3\left(\frac{20}{38}\right)^7$

d) she needs more than 10 bets to win five times

$\underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_}_{\text{wins} \le 4 \text{ in } 10}$

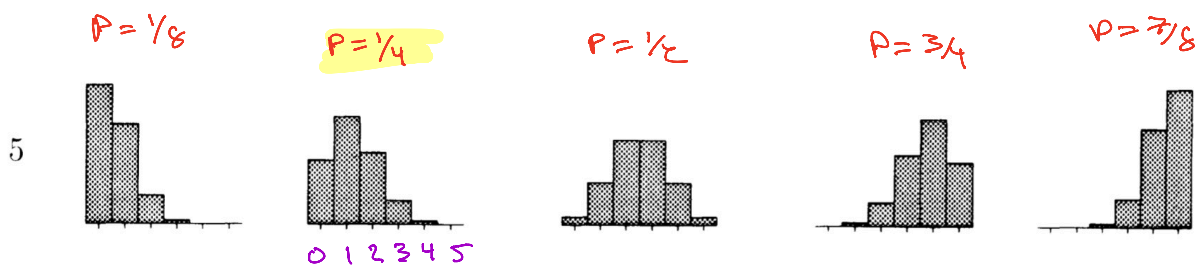
$P(X \le 4) = \sum_{k=0}^{4} \binom{10}{k}\left(\frac{18}{38}\right)^k\left(\frac{20}{38}\right)^{10-k}$

Note A student asked if you can use the complement rule here. The complement is that you have at least 5 wins in 10 bets. Answer is yes but it isn't easier since it is a sum with $k=5,6,7,8,9,10$.

Why does $\sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k} = 1$? (so Binomial(n,p) is a distribution)

$$\underbrace{(p + (1-p))^n}_{= 1} = \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k}$$ binomial theorem

Binomial $(5, p)$ for different p :



In python :

```
In [6]: from scipy import stats
        import numpy as np

In [7]: stats.binom.pmf(2, 5, 1/4)
Out[7]: 0.26367187499999994
```

$\leftarrow \binom{5}{2} (1/4)^2 (3/4)^3$

```
In [11]: stats.binom.pmf(np.arange(6), 5, 1/4)
Out[11]: array([0.23730469, 0.39550781, 0.26367187, 0.08789062, 0.01464844,
        0.00097656])
```

tinyurl.com / feb3 -pt1

Ten cards are dealt off the top of a well shuffled deck. The binominal formula doesn't apply to find the chance of getting exactly three diamonds because:

a The probability of a trial being successful changes

b The trials aren't independent

c There isn't a fixed number of trials

d more than one of the above

The i^{th} trial is whether the i^{th} card is an ace. By symmetry, this always has probability 1/4. Hence the probability of a trial being successful doesn't change (i.e not choice a.)

However, the trials are dependent because the cards are dealt without replacement, For example chance the second card is an ace depends on what the first card is,

This is an example where we have 10 dependent trials all probability 1/4.