stat 88 lec 8

warm up 2:00–2:10

Three cards are dealt from a standard 52 card deck. Find the chance:

a) the first card is red and the second two black.

R B B

$\frac{26}{52} \cdot \frac{26}{51} \cdot \frac{25}{50}$

b) exactly one of the cards dealt is red

$N = 52$

$G = 26$ (red)

$n = 3$

$X = \#$ of red out of 3

$P(X=1) = \frac{\binom{26}{1}\binom{26}{2}}{\binom{52}{3}} \approx .382$ or $\binom{3}{1} \frac{26}{52} \cdot \frac{26}{51} \cdot \frac{25}{50} = .382$

c) at least one of the cards dealt is red

$1 -$ Chance no cards red $= .882$

B B B

$\frac{26}{52}\frac{25}{51}\frac{24}{50}$

$P(X=0) = \frac{\binom{26}{0}\binom{26}{3}}{\binom{52}{3}}$

Another solution

$$\sum_{g=1}^{3} \frac{\binom{26}{g}\binom{26}{3-g}}{\binom{52}{3}}$$

All 3 solutions are equally good although some might be easier to compute by hand.

Last time

Sec 3.4 Hypergeometric distribution

Picture

$$\underbrace{\text{H H H H H H H H H}}_{G=9 \text{ good}}\ \underbrace{\text{T T T T}}_{B=4 \text{ bad}} \qquad N = G+B = 13$$

$\downarrow$ draw n w/o replacement

$$\underbrace{\text{H H H}}_{g=3}\ \underbrace{\text{T T}}_{b=2} \qquad n = b+g$$

Chance of g good in sample $= \dfrac{\binom{G}{g}\binom{B}{b}}{\binom{N}{n}}$

or in this case

$$\frac{\overset{G}{\binom{9}{3}}\overset{B}{\binom{4}{2}}}{\binom{13}{5}} = \binom{5}{3} \overset{H}{\frac{9}{13}}\ \overset{H}{\frac{8}{12}}\ \overset{H}{\frac{7}{11}}\ \overset{T}{\frac{4}{10}}\ \overset{T}{\frac{3}{9}}$$

Notice if draw with replacement this is $\binom{5}{3}\left(\frac{9}{13}\right)^3\left(\frac{4}{13}\right)^2$ binomial formula.

Today

0. Summary binomial vs hypergeom.
1. Sec 3.5 examples
2. Sec 4.1 CDF (cumulative distribution function)

⓪ Summary binomial vs hypergeom.

Binomial (n, p)

Sample with replacement

trial has two outcomes — success / failure

n independent trials

Probability of success p

X = # successes in sample

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

binomial formula

example

Pick 5 cards from a deck with replacement. What is chance you get 2 aces

HG (N, G, n) — hypergeometric

N = population size
G = # good elements in pop.
n = sample size

Sample without replacement.

trial has two outcomes — good / bad

n dependent trials

probability of good $\frac{G}{N}$

X = # good in sample

$$P(X=g) = \frac{\binom{G}{g}\binom{B}{b}}{\binom{N}{n}}$$

$B = N-G$, $b = n-g$

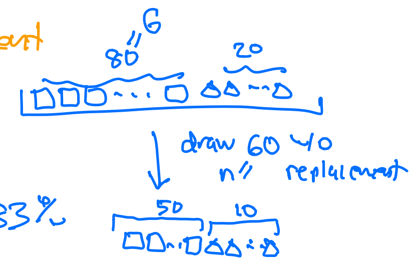
HG formula

example

Pick 5 cards from a deck without replacement. What is chance you get 2 aces

① Sec 3.5 examples

□ = recovered
△ = sick



Ex (Fisher Exact Test)

randomized controlled experiment

A RCE has 100 participants

$T = 60$ $\quad C = 40$

In T, 50 recover out of 60 — 83%

In C, 30 recover out of 40 — 75%

A ~~total~~ of 80 patients recovered out of 100.

Question Suppose the treatment isn't effective.

expect 60% of 80 = 48 recovered in T
expect 40% of 80 = 32 recovered in C

What is the chance that 50 or more of the recovered patients are randomly assigned to the treatment group?

(if the answer is really small then the treatment is probably effective).

Start with:

What is the chance that 50 of the recovered patients are randomly assigned to the treatment group?

$N = 100$ $\qquad X = \#$ good in sample (recovered)

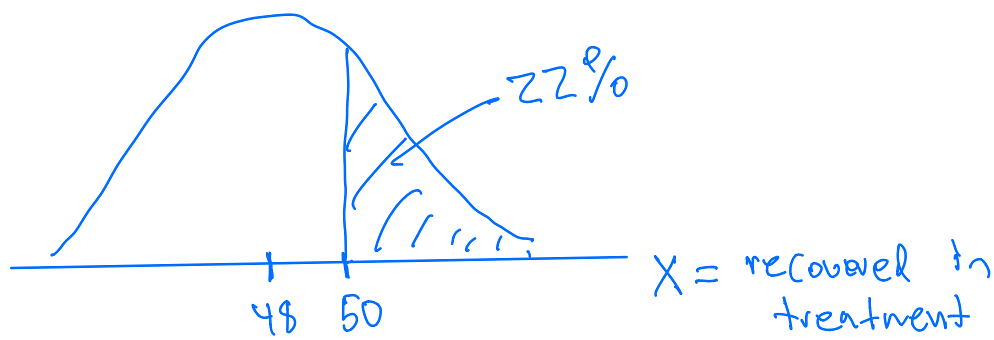
$G = 60$

$n = 60$ $\qquad P(X = 50)$?

$\qquad P(X \geq 50)$?

$$P(X=50) = \frac{\binom{80}{50}\binom{20}{10}}{\binom{100}{60}}$$

$$P(X \geq 50) = \sum_{g=50}^{60} \frac{\binom{80}{g}\binom{20}{60-g}}{\binom{100}{60}} \leftarrow 22\%$$



ex

Advisor Meetings

An advisor at a university provides guidance to 10 students. Each student has to meet with her once a month during the school year which consists of nine months.

So each month the advisor schedules one day of meetings. Each student has to sign up for one meeting that day. Students have the choice of meeting her in the morning or in the afternoon.

Assume that every month each student, independently of other students and other months, chooses to meet in the afternoon with probability 0.75.

What is the chance that she has both morning and afternoon meetings in all of the months except one?

binomial → X = # of month have both monday M and A meetings

$P(X=8)$ A M M A - - - -

$n = 9$

$p = 1 - (.75)^{10} - (.25)^{10} = .94$

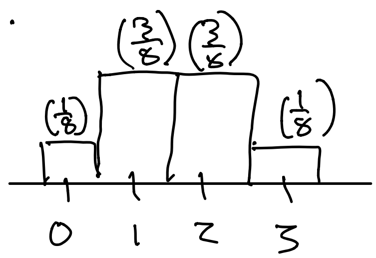
$P(X=8) = \binom{9}{8} p^8 (1-p)^1 = \boxed{\binom{9}{8}(.94)^8(.06)^1}$

② Cummulative Distribution function. (CDF).

To specify a prob. distribution we have used a probability mass function (pmf):

ex $X \sim \text{Binomial}(3, \frac{1}{2})$

x	$P(X=x)$
0	1/8
1	3/8
2	3/8
3	1/8



You can also specify a prob. distribution by giving the chance that the value of X is at most x, $F(x) = P(X \le x)$. This is called the Cummulative distribution function (CDF).

ex

x	$P(X=x)$	$F(x) = P(X \le x)$
0	1/8	1/8 ← $P(X=0)$
1	3/8	4/8 ← $P(X=0) + P(X=1)$
2	3/8	7/8 ← $P(X=0) + P(X=1) + P(X=2)$
3	1/8	1

Graph of $F(x)$:

We can define $F(x)$ on the entire x axis even though it only "jumps" at $x = 0, 1, 2, 3$.