Stat 88 lec 2

Warm up 2:00–2:10

Based on historical averages

P(A) = Chance you catch bus to school : 70%

P(B) = Chance it rains : 50%

P(C) = Chance you make it to class on time : 10%

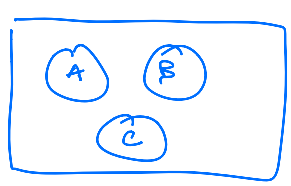
What is the chance at least one of these events occurs (making no assumptions). If it can't be found exactly, find the best lower bound and upper bound that you can.

Soln: We are asked to find the union of the 3 events.

We can't find the answer exactly because we don't know if events A, B, C overlap.

Upperbound: $\boxed{100\%}$ An upperbound for the probability of the union is always the sum of the individual probabilities. But in this case it is 100% since the sum > 100%.

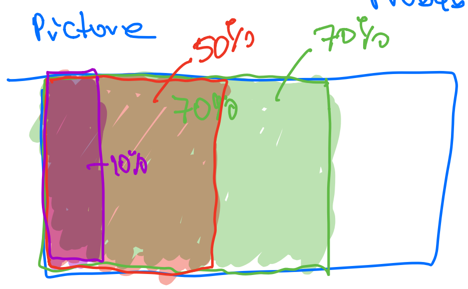
Picture



union is biggest when events have no overlap.

Lowerbound: $\boxed{70\%}$ A lower bound for the probability of the overlap is always the largest individual probability.

Picture



To be in the union it is possible you catch the bus to school. That is why the answer isn't 10% since you can be anywhere in the green set.

last time

Probability allows you to learn about a random sample from a known population

ex Population: deck of cards
sample: poker hand
Find chance of getting 4 of a kind.

Sec 1.1 Probability as proportions

We call the set of all outcomes of an experiment Ω, the outcome space or the sample space

let $A \subseteq \Omega$ event

For equally likely outcomes, $P(A) = \frac{\#A}{\#\Omega}$

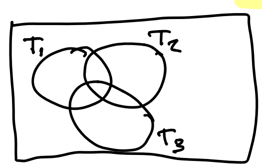
Sec 1.2 Probability bounds

When we dont know how much events overlap we sometimes need to give upper and lower bound for what a probability is.

ex $P(T_1) = .1$, $P(T_2) = .05$, $P(T_3) = .01$

$P(T_1 \cup T_2 \cup T_3) \leq P(T_1) + P(T_2) + P(T_3) = \boxed{.16}$ Boole/Bonferroni inequality

$P(T_1 \cup T_2 \cup T_3) \geq P(T_1) = \boxed{.1}$



Today

1. Sec 1.3 Fundamental Rules and probability bounds
2. Sec 2.1 The chance of an intersection

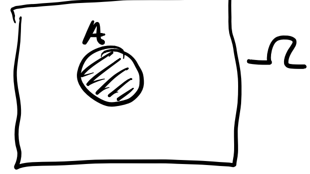
Sec 1.3 Fundamental Rules and probability bounds

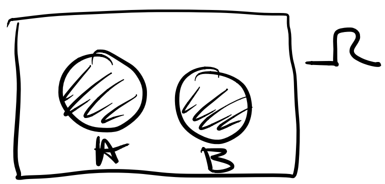
probability is a numerical function on events satisfying 3 axioms

Axioms

1) $P(A) \geq 0$ for all $A \subseteq \Omega$
2) $P(\Omega) = 1$
3) If A and B are mutually exclusive sets then $P(A \cup B) = P(A) + P(B)$ (addition rule)

Venn Diagram

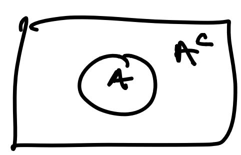




Consequences of 3 axioms:

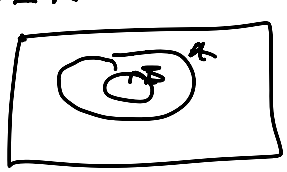
1) Complement rule

$P(A^c) = 1 - P(A)$



2) Difference rule

If $B \subseteq A$ then $P(B) \leq P(A)$



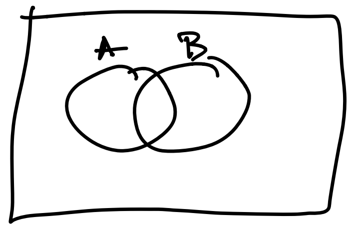
3) Boole/Bonferroni inequality

$$P\left(\bigcup_{i=1}^{n} A_i\right) \le \sum_{i=1}^{n} P(A_i)$$

equal if mutually exclusive events (addition rule)

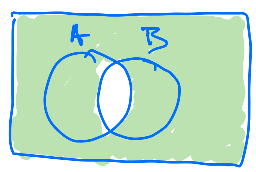
ex

$P(A \cup B) \le P(A) + P(B)$



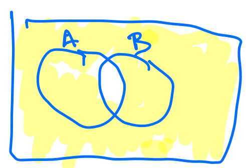
De Morgan's rule

$$(A \cap B)^c = A^c \cup B^c$$



$(A \cap B)^c$

$\parallel$ — the shadings are the same.



$A^c \cup B^c$

This generalizes to 3 or more events

$$(A \cap B \cap C)^c = A^c \cup B^c \cup C^c$$

Another form of De Morgan's rule is

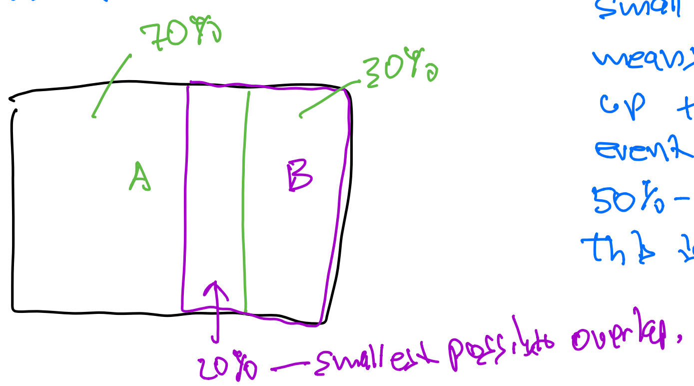
$$(A \cup B)^c = A^c \cap B^c$$

eg

$P(A)$ = Chance you catch bus to school = 70%
$P(B)$ = Chance it rains = 50%
Find the best lower bound for $P(A \cap B)$ you can without making any assumptions.

WARNING: I am asking about $A \cap B$ not $A \cup B$ as before.

Visual solution: we want the intersection to be as small as possible, this means event B must take up the 30% not used by event A. This leaves $50\% - 30\% = 20\%$ as overlap. This is smallest overlap you can have.



so $P(A \cap B) \geq 20\%$

Here is a solution involving complements and De Morgan's rule that gives us a formula.

First find an upperbound for $P(A^c \cup B^c)$:

$P(A^c) = 30\% \Rightarrow P(A^c \cup B^c) \leq P(A^c) + P(B^c) = 80\%$
$P(B^c) = 50\%$

By De Morgan's rule

$$(A^c \cup B^c)^c = \underbrace{(A^c)^c}_{A} \cap \underbrace{(B^c)^c}_{B} \qquad \text{so } A \cap B = (A^c \cup B^c)^c$$

$P(A^c) + P(B^c) = 80\%$
$\vee\!\!|$

and $P(A \cap B) = P((A^c \cup B^c)^c) = 1 - P(A^c \cup B^c) \geq \boxed{20\%}$

So we get formula $P(A \cap B) \geq 1 - P(A^c) - P(B^c)$

ex

$P(A)$ = Chance you catch bus to school = 70%

$P(B)$ = Chance it rains = 50%

$P(C)$ = Chance you make it to class on time = 10%

Find the best lower bound for $A \cap B \cap C$ you can without making any assumptions.

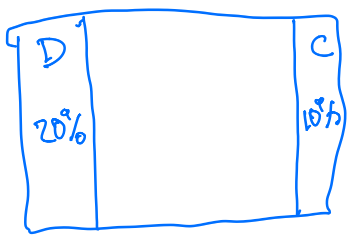
Visual solution:

We know from above $P(A \cap B) \geq 20\%$

To find a lower bound for $A \cap B \cap C$, $A \cap B$ must be at least 20%. Lets suppose $P(A \cap B) = 20\%$

let $D = A \cap B$ with $P(D) = 20\%$
$P(C) = 10\%$.

Picture



It is possible for D and C not to have any overlap.

So $P(A \cap B \cap C) = P(D \cap C) \geq 0\%$

Alternatively using a generalization of the formula above and De Morgan's rule $(A \cap B \cap C)^c = A^c \cup B^c \cup C^c$ we have

$$P(A \cap B \cap C) \geq 1 - \underbrace{\underbrace{P(A^c)}_{30\%} - \underbrace{P(B^c)}_{50\%} - \underbrace{P(C^c)}_{90\%}}_{100\%} = \boxed{0\%}$$