Stat 88 lec 13

Warmup 2:00 – 2:10

a) (X_1, X_2) has joint distribution:

	$\frac{1}{6}$ $X_2=0$	$\frac{5}{6}$ $X_2=1$
$\frac{5}{6}$ $X_1=0$	$\frac{5}{36}$	$\frac{25}{36}$
$\frac{1}{6}$ $X_1=1$	$\frac{1}{36}$	$\frac{5}{36}$

Is X_1, X_2 independent?

$\frac{5}{36} = \frac{5}{6} \cdot \frac{1}{6}$ ✓

$P(X_1=0, X_2=0) = P(X_1=0) \cdot P(X_2=0)$

b) A die is rolled 10 times. Find the expectation of the number of times an odd number of spots appears

$10\left(\frac{1}{2}\right) = 5$

Quiz 2 chapters 3, 4, 5.1

Last Time

Expectation = average or center of distribution

Recall (Indicator RV)

$$X = \begin{cases} 1 \xleftarrow{\text{success}} & \text{with prob } p \\ 0 \xleftarrow{\text{failure}} & \text{with prob } 1-p \end{cases}$$

$$E(X) = 1 \cdot p + 0 \cdot (1-p) = p$$

Additivity of Expectation $E(X_1+X_2) = E(X_1) + E(X_2)$

sec 5.3 Method of indicator to find $E(X)$

Counting the number of successful trials is the same as adding zeros and ones.

ex A success is blue, and failure non blue.

B R R G B R B B

1 0 0 0 1 0 1 1

blue = 1 + 0 + 0 + 0 + 1 + 0 + 1 + 1 = 4

Suppose a trial is blue with probability p

Find the expected # blue in n trials.

Step 1 Write down what X is.

X = # trials out of n that are blue

Step 2 Find I_2 (second indicator)

p

$$I_2 = \begin{cases} 1 & \text{if } 2^{nd} \text{ trial is blue} \\ 0 & \text{else} \end{cases}$$

Nothing special about second indicator.

Step 3 Find p

Step 4 Write X as sum of indicators

$X = I_1 + \cdots + I_n$

Step 5 Find $E(X)$

$$E(X) = E(I_1 + \cdots + I_n) = E(I_1) + \cdots + E(I_n)$$
$$= nE(I_1)$$
$$= \boxed{np}$$

Conclusion

If $X \sim \text{Binomial}(n, p)$

$E(X) = np$.

Today ① review concept test last time
② sec 5.3 method of indicators

① review concept test last time

A joint distribution for two random variables M and S is given below.

1/3 2/3

	$M=2$	$M=3$
$S=2$	0	$\frac{1}{3}$
$S=1$	$\frac{1}{3}$	$\frac{1}{3}$

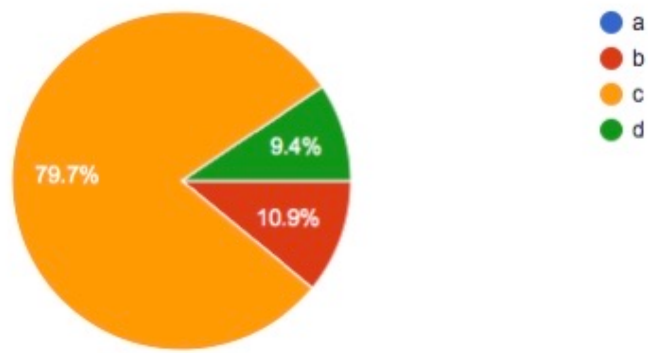
E(M) equals:

a 1.67

b 2.33

c 2.67

d none of the above



c	Use marginal expectation and add up
	2(1/3)+3(2/3)

c	E(M) = 2(0)+2(1/3)+3(1/3)+3(1/3)

② sec 5.3 method of indicators

ex Let X be the number of spades in 7 cards dealt with replacement from a well shuffled deck of 52 cards containing 13 spades.

Find $E(X)$.

$X \sim \text{Binomial}(7, \frac{1}{4})$

Step 1 describe X = # trials out of 7 that is a spade

Step 2 $I_2 = \begin{cases} 1 & \text{if } 2^{nd} \text{ trial is spade} \\ 0 & \text{else.} \end{cases}$

Step 3 $p = \frac{1}{4}$

Step 4 $X = I_1 + \dots + I_7$

Step 5 $E(X) = 7\left(\frac{1}{4}\right)$.

ex

Let X be the number of spades in 7 cards dealt from a well-shuffled deck of 52 cards containing 13 spades. Find $E(X)$.

X = # cards out of 7 that are spade

$I_2 = \begin{cases} 1 & \text{if } 2^{nd} \text{ card is spade} \\ 0 & \text{else} \end{cases}$ p

$p = \frac{1}{4}$

$X = I_1 + \cdots + I_7$

$E(X) = \boxed{7\left(\frac{1}{4}\right)}$

~~$X \sim HG(52, 13, 7)$~~

$\boxed{E(X) = n\frac{G}{N}}$

If X isn't binomial or hypergeometric be thoughtful how define your indicator. You want each indicator to have same p.

exercise 5.7.6

6. A die is rolled 12 times. Find the expectation of

c) the number of faces that don't appear

Step 1 describe X = # of faces (out of 6) that don't appear.

Step 2 $I_2 = \begin{cases} 1 & \text{if } 2^{nd} \text{ face doesn't appear} \\ 0 & \text{else} \end{cases}$

p

Step 3 $P = \left(\frac{5}{6}\right)^{12}$

Step 4 $X = I_1 + \dots + I_6$

Step 5 $E(X) = \boxed{6\left(\frac{5}{6}\right)^{12}}$

Tinyurl.com/feb21-pt1

Stats 88

Friday February 21 2020

1. n people with hats have had a bit too much to drink at a party. As they leave the party, each person randomly grabs a hat. A match occurs if a person gets his or her own hat.

   a The expected number of matches depends on n

   b The expected number of matches is 1

   c The number of matches is hypergeometric

   d more than one of the above

X = # people (out of n) who get a match,

$$I_2 = \begin{cases} 1 & \text{if 2nd person gets match} \\ 0 & \text{else} \end{cases}$$

$$p = \frac{1}{n}$$

$$X = I_1 + \dots + I_n$$

$$E(X) = n\left(\frac{1}{n}\right) = \boxed{1}$$

indicators dependent but X not HG

For HG you must be able to tell if element in pop is good before you draw. Here you can only tell after you draw.

ex

A building has 10 floors above the basement. If 12 people get into an elevator at the basement, and each chooses a floor at random to get out, independently of the others, at how many floors do you expect the elevator to make a stop to let out one or more of these 12 people?

X = # floors (out of 10) that's chosen

$I_2 = \begin{cases} 1 & \text{if 2nd floor is chosen} \\ 0 & \text{else} \end{cases}$ $\rightarrow p$

$p = 1 - \left(\frac{9}{10}\right)^{12}$

$X = I_1 + \dots + I_{10}$

$$\boxed{E(X) = 10\left(1 - \left(\frac{9}{10}\right)^{12}\right)}$$