

# Robust covariance estimation for partially observed functional data

2021-07-23

## 1. Delaigle et al.(2020) setting

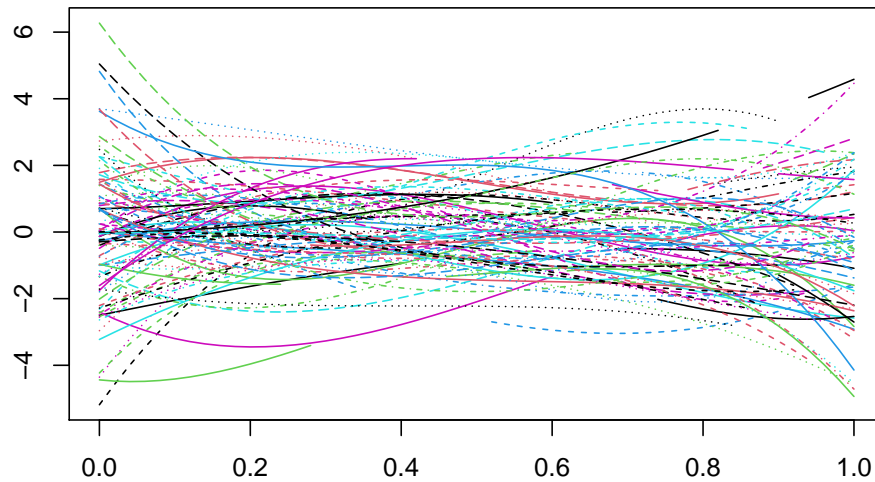


Figure 1: Sample trajectories of Delaigle et al.(2020) setting

Table 1: 4 FPCs from 50 repetitions

Type	Method	4 FPCs				95% PVE		
		PVE	Reconstruction	Completion	Eigenfunction	K	Reconstruction	Completion
Not contaminated	Yao	1.00	<b>0.01 (0.01)</b>	<b>0.03 (0.03)</b>	<b>0.06 (0.04)</b>	3.94	<b>0.01 (0.02)</b>	<b>0.04 (0.04)</b>
	Huber	0.84	0.08 (0.04)	0.27 (0.13)	0.06 (0.09)	15.40	0.11 (0.07)	0.38 (0.19)
	Kraus			0.23 (0.21)				0.23 (0.21)
	Kraus-M			0.43 (0.25)				0.43 (0.25)
	Kraus-M(sm)			1.16 (0.45)				1.16 (0.45)
	Boente	1.00	0.19 (0.21)	0.56 (0.65)	0.13 (0.04)	<b>3.34</b>	0.23 (0.18)	0.54 (0.50)
	M-est	0.95	0.07 (0.04)	0.23 (0.16)	0.13 (0.08)	4.56	0.07 (0.04)	0.25 (0.16)
	M-est-noise	0.95	0.04 (0.02)	0.14 (0.07)	0.13 (0.08)	4.52	0.04 (0.02)	0.15 (0.07)
	M-est(smooth)	0.99	0.36 (0.74)	1.07 (2.12)	0.08 (0.04)	3.68	0.39 (0.74)	1.11 (2.11)
	M-est(smooth)-noise	0.99	0.03 (0.01)	0.10 (0.04)	0.08 (0.04)	3.68	0.07 (0.06)	0.16 (0.10)
Contaminated 1	Yao	0.82	1.51 (0.25)	2.10 (0.50)	1.69 (0.16)	7.06	1.31 (0.28)	2.01 (0.52)
	Huber	0.84	0.09 (0.04)	0.33 (0.15)	<b>0.07 (0.08)</b>	15.18	0.12 (0.08)	0.44 (0.21)
	Kraus			2.64 (0.64)				2.64 (0.64)
	Kraus-M			1.35 (0.47)				1.35 (0.47)
	Kraus-M(sm)			0.46 (0.29)				0.46 (0.29)
	Boente	1.00	0.11 (0.13)	0.31 (0.46)	0.15 (0.05)	3.78	0.13 (0.13)	0.32 (0.37)
	M-est	0.68	0.16 (0.12)	0.46 (0.39)	0.21 (0.13)	18.80	0.18 (0.16)	0.62 (0.59)
	M-est-noise	0.71	0.10 (0.04)	0.26 (0.12)	0.21 (0.13)	15.80	<b>0.06 (0.02)</b>	<b>0.23 (0.10)</b>
	M-est(smooth)	0.99	2.58 (3.84)	8.32 (11.93)	0.09 (0.05)	3.80	2.56 (3.79)	8.14 (11.68)
	M-est(smooth)-noise	0.99	<b>0.06 (0.02)</b>	<b>0.19 (0.08)</b>	0.09 (0.05)	<b>3.60</b>	0.11 (0.07)	0.26 (0.13)
Contaminated 2	Yao	0.94	0.69 (0.53)	1.20 (0.70)	1.26 (0.59)	4.38	0.66 (0.58)	1.15 (0.73)
	Huber	0.84	0.10 (0.04)	0.38 (0.17)	<b>0.07 (0.09)</b>	15.40	0.14 (0.09)	0.49 (0.23)
	Kraus			1.95 (0.89)				1.95 (0.89)
	Kraus-M			1.04 (0.42)				1.04 (0.42)
	Kraus-M(sm)			0.49 (0.27)				0.49 (0.27)
	Boente	1.00	0.21 (0.55)	0.60 (1.42)	0.14 (0.05)	3.84	0.23 (0.54)	0.60 (1.39)
	M-est	0.76	0.20 (0.40)	0.66 (1.42)	0.19 (0.13)	16.90	0.26 (0.46)	0.90 (1.65)
	M-est-noise	0.82	0.07 (0.02)	0.24 (0.10)	0.19 (0.13)	11.40	<b>0.06 (0.02)</b>	<b>0.24 (0.10)</b>
	M-est(smooth)	0.99	0.93 (1.50)	3.06 (5.12)	0.10 (0.07)	3.70	0.95 (1.46)	3.01 (4.98)
	M-est(smooth)-noise	0.99	<b>0.06 (0.02)</b>	<b>0.20 (0.09)</b>	0.10 (0.07)	<b>3.48</b>	0.12 (0.07)	0.29 (0.14)
	Kraus-M(sm)			0.24 (0.13)				
	M-est(smooth)	0.97	0.07 (0.05)	0.24 (0.14)	0.14 (0.10)			
	M-est(smooth)-noise	0.97	0.06 (0.02)	0.20 (0.09)	0.14 (0.10)			

## 2. Kraus(2015) setting

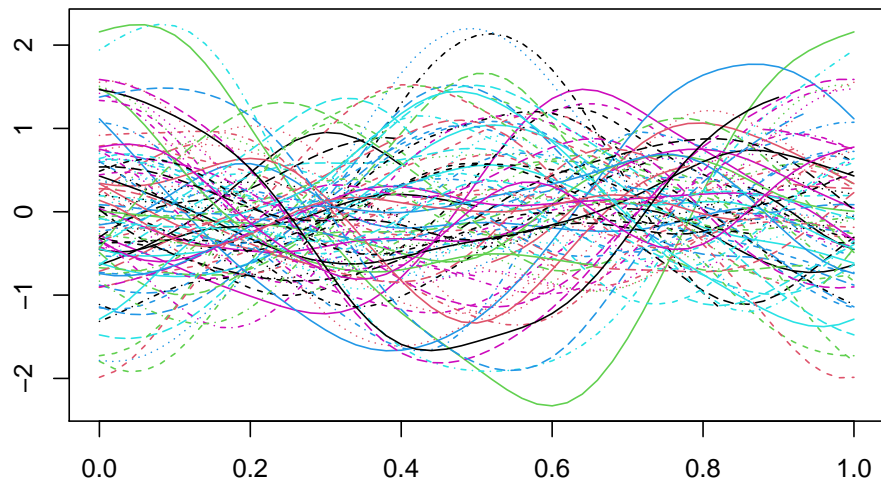


Figure 2: Sample trajectories of Kraus(2015) setting

Table 2: 5 FPCs from 50 repetitions

Type	Method	5 FPCs				95% PVE		
		PVE	Reconstruction	Completion	Eigenfunction	K	Reconstruction	Completion
Not contaminated	Yao	1.00	<b>0.01 (0.00)</b>	<b>0.02 (0.01)</b>	<b>0.12 (0.12)</b>	3.04	<b>0.02 (0.00)</b>	<b>0.03 (0.01)</b>
	Huber	0.85	0.06 (0.03)	0.18 (0.08)	1.42 (0.07)	15.20	0.08 (0.04)	0.22 (0.09)
	Kraus			<b>0.02 (0.01)</b>				0.02 (0.01)
	Kraus-M			0.06 (0.02)				0.06 (0.02)
	Kraus-M(sm)			0.09 (0.04)				0.09 (0.04)
	Boente	1.00	0.05 (0.03)	0.14 (0.08)	0.74 (0.13)	<b>2.94</b>	0.07 (0.02)	0.13 (0.06)
	M-est	0.94	0.02 (0.01)	0.05 (0.02)	0.34 (0.19)	6.18	0.02 (0.01)	0.06 (0.02)
	M-est-noise	0.94	0.02 (0.00)	0.04 (0.01)	0.34 (0.19)	5.94	0.01 (0.00)	0.04 (0.02)
	M-est(smooth)	0.99	0.06 (0.07)	0.17 (0.19)	0.45 (0.19)	3.18	0.05 (0.04)	0.09 (0.07)
	M-est(smooth)-noise	0.99	0.02 (0.00)	0.04 (0.02)	0.45 (0.19)	3.16	0.03 (0.01)	0.04 (0.01)
Contaminated 1	Yao	0.89	0.44 (0.08)	0.59 (0.14)	1.64 (0.15)	6.72	0.42 (0.08)	0.59 (0.14)
	Huber	0.85	0.09 (0.04)	0.25 (0.09)	1.42 (0.08)	15.28	0.10 (0.04)	0.29 (0.10)
	Kraus			1.01 (0.29)				1.01 (0.29)
	Kraus-M			0.17 (0.06)				0.17 (0.06)
	Kraus-M(sm)			0.06 (0.02)				0.06 (0.02)
	Boente	1.00	0.10 (0.14)	0.33 (0.40)	0.69 (0.12)	<b>2.92</b>	0.11 (0.11)	0.24 (0.26)
	M-est	0.70	0.05 (0.04)	0.10 (0.09)	0.72 (0.12)	20.22	0.08 (0.09)	0.26 (0.31)
	M-est-noise	0.77	0.03 (0.01)	<b>0.05 (0.01)</b>	0.72 (0.12)	14.18	<b>0.02 (0.00)</b>	<b>0.05 (0.01)</b>
	M-est(smooth)	0.99	0.17 (0.17)	0.52 (0.58)	0.49 (0.14)	3.52	0.14 (0.15)	0.34 (0.46)
	M-est(smooth)-noise	1.00	<b>0.02 (0.00)</b>	<b>0.05 (0.01)</b>	<b>0.44 (0.15)</b>	3.06	0.03 (0.01)	<b>0.05 (0.01)</b>
Contaminated 2	Yao	0.97	0.29 (0.18)	0.35 (0.22)	1.61 (0.26)	3.90	0.29 (0.18)	0.36 (0.22)
	Huber	0.86	0.10 (0.10)	0.25 (0.14)	1.42 (0.09)	13.94	0.12 (0.11)	0.30 (0.16)
	Kraus			0.62 (0.66)				0.62 (0.66)
	Kraus-M			0.15 (0.05)				0.15 (0.05)
	Kraus-M(sm)			0.05 (0.02)				0.05 (0.02)
	Boente	1.00	0.07 (0.07)	0.22 (0.20)	0.76 (0.10)	<b>2.98</b>	0.09 (0.05)	0.17 (0.13)
	M-est	0.73	0.04 (0.03)	0.08 (0.06)	0.67 (0.11)	19.32	0.07 (0.08)	0.22 (0.25)
	M-est-noise	0.80	0.03 (0.01)	0.05 (0.01)	0.67 (0.11)	13.54	<b>0.02 (0.01)</b>	<b>0.04 (0.01)</b>
	M-est(smooth)	0.99	0.09 (0.09)	0.27 (0.28)	0.46 (0.15)	3.36	0.08 (0.07)	0.17 (0.20)
	M-est(smooth)-noise	1.00	<b>0.02 (0.01)</b>	<b>0.04 (0.01)</b>	<b>0.41 (0.15)</b>	3.06	0.03 (0.01)	0.05 (0.01)
	Kraus-M(sm)			0.04 (0.01)				
	M-est(smooth)	0.98	0.02 (0.01)	0.05 (0.02)	0.34 (0.12)			
	M-est(smooth)-noise	0.98	0.02 (0.00)	0.04 (0.01)	0.34 (0.12)			