# Double-debiased machine learning in Stata

Introducing ddml and pystacked†

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based on joint work with

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<sup>&</sup>lt;sup>†</sup>Preliminary version. Subject to change.

### Introduction

- ► In recent years, machine learning (ML) has increasingly been leveraged in social sciences and economics.
- One central question is how ML can be used for causal inference. Two major approaches:
  - Exploring treatment effect heterogeneity (Causal Forests, GRF, GATES)
  - 2. Robust inference in the presence of high-dimensional controls and/or instruments

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  - Exploring treatment effect heterogeneity (Causal Forests, GRF, GATES)
  - 2. Robust inference in the presence of high-dimensional controls and/or instruments ← Today's focus
- ► We introduce ddml for Double-debiased machine learning and pystacked for Stacking (a meta-learning algorithm).
- ► Requirement for fast ML implementations: Stata's Python integration means that we can utilize Python's ML modules.

Motivating example. The partial linear model:

$$y_i = \underbrace{\frac{\theta d_i}{\text{causal part}}}_{\text{causal part}} + \underbrace{\frac{g(\mathbf{x}_i)}{\text{nuisance}}}_{\text{nuisance}} + \varepsilon_i$$

How do we account for confounding factors  $x_i$ ? — The standard approach is to assume linearity  $g(x_i) = x_i'\beta$  and consider alternative combinations of controls.

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#### Problems:

- ► Non-linearity & unknown interaction effects
- ► High-dimensionality: we might have "many" controls
- ► We don't know which controls to include

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Both "double" approaches rely on the *sparsity assumption* and use two auxiliary lasso regressions:  $y_i \rightsquigarrow \mathbf{x}_i$  and  $d_i \rightsquigarrow \mathbf{x}_i$ . Tasso PDS

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Related approaches exist for *optimal IV* estimation (Belloni et al., 2012) and/or *IV with many controls* (Chernozhukov, Hansen, and Spindler, 2015).

These methods have been implemented for Stata in pdslasso (Ahrens, Hansen, and Schaffer, 2019), dsregress (StataCorp) and R (hdm; Chernozhukov, Hansen, and Spindler, 2016).

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### A quick example using AJR (2001):

Example 1 (pdslasso) allows for high-dimensional controls. Example 2 (ivlasso) treats avexpr as endogenous and exploits logem4 as an instrument. (More details in the pds/ivlasso help file.)

### **Motivation**

There are *advantages* of relying on lasso:

- ▶ intuitive assumption of (approximate) sparsity
- computationally relatively cheap (due to plugin lasso penalty; no cross-validation needed)
- ► Linearity has its advantages (e.g. extension to fixed effects; Belloni et al., 2016)

### **Motivation**

There are advantages of relying on lasso:

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- computationally relatively cheap (due to plugin lasso penalty; no cross-validation needed)
- ► Linearity has its advantages (e.g. extension to fixed effects; Belloni et al., 2016)

#### But there are also *drawbacks*:

- What if the sparsity assumption is not plausible?
- ► There is a wide set of machine learners at disposable—lasso might not be the best choice.
- ► Lasso requires careful feature engineering to deal with non-linearity & interaction effects.

## Our contribution

Double-debiased Machine Learning (DDML) due to Chernozhukov, Chetverikov, Demirer, Duflo, Hansen, Newey, and Robins (2018) has been suggested as an extension to Post-double selection. DDML allows for a broad set of machine learners.

## Our contribution

Double-debiased Machine Learning (DDML) due to Chernozhukov, Chetverikov, Demirer, Duflo, Hansen, Newey, and Robins (2018) has been suggested as an extension to Post-double selection. DDML allows for a broad set of machine learners.

#### We introduce ddml for Stata:

- ► Compatible with various ML programs in Stata (e.g. lassopack, pylearn, randomforest).
  - → Any program with the classical "reg y x" syntax and post-estimation predict will work.
- ► Short (one-line) and flexible multi-line version
- ► Uses *Stacking Regression* as the default machine learner (implemented via separate program pystacked)
- ► 5 models supported: partial linear model, interactive model, LATE, partial IV model, optimal IV.

### Review of DDML

### The partial linear model:

$$y_i = \theta d_i + g(\mathbf{x}_i) + \varepsilon_i$$
  
 $d_i = m(\mathbf{x}_i) + v_i$ 

Naive idea: We estimate conditional expectations  $\ell(\mathbf{x}_i) = E[y_i|\mathbf{x}_i]$  and  $m(\mathbf{x}_i) = E[d_i|\mathbf{x}_i]$  using ML and partial out the effect of  $\mathbf{x}_i$  (in the style of Frisch-Waugh-LovelI):

$$\hat{\theta}_{DDML} = \left(\frac{1}{n}\sum_{i}\hat{v}_{i}^{2}\right)^{-1}\frac{1}{n}\sum_{i}\hat{v}_{i}(y_{i}-\hat{\ell}),$$

where  $\hat{v}_i = d_i - \hat{m}_i$ .

### Review of DDML

Yet, there is a problem: The estimation error  $\ell(\mathbf{x}_i) - \hat{\ell}$  and  $v_i$  may be correlated due to over-fitting, leading to poor performance.

DDML, thus, relies on *cross-fitting*. Cross-fitting is sample splitting with swapped samples.

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## DDML with the partial linear model

We split the sample in K random folds of equal size denoted by  $I_k$ :

- ▶ For k = 1, ..., K, estimate  $\ell(\mathbf{x}_i)$  and  $m(\mathbf{x}_i)$  using sample  $I_k^c$  and form out-of-sample predictions  $\hat{\ell}_i$  and  $\hat{m}_i$  for all i in  $I_k$ .
- ightharpoonup Construct estimator  $\hat{\theta}$  as

$$\left(\frac{1}{n}\sum_{i}\hat{v}_{i}^{2}\right)^{-1}\frac{1}{n}\sum_{i}\hat{v}_{i}(y_{i}-\hat{\ell}),$$

where  $\hat{v}_i = d_i - \hat{m}_i$ .  $\hat{m}_i$  and  $\hat{\ell}_i$  are the cross-fitted predicted values.

The DDML framework can be applied to other models (all implemented in ddml):

#### Interactive model

$$y_i = g(d_i, \mathbf{x}_i) + u_i$$
  $E[u_i | \mathbf{x}_i, d_i] = 0$   
 $z_i = m(\mathbf{x}_i) + v_i$   $E[u_i | \mathbf{x}_i] = 0$ 

As in the Partial Linear Model, we are interested in the ATE, but do not assume that  $d_i$  (a binary treatment variable) and  $\mathbf{x}_i$  are separable.

We estimate the conditional expectations  $E[y_i|\mathbf{x}_i,d_i=0]$  and  $E[y_i|\mathbf{x}_i,d_i=1]$  as well as  $E[d_i|\mathbf{x}_i]$  using a supervised machine learner.

The DDML framework can be applied to other models (all implemented in ddml):

#### Partial linear IV model

$$y_i = d_i \theta + g(\mathbf{x}_i) + u_i$$
  $E[u_i | \mathbf{x}_i, z_i] = 0$   
 $z_i = m(\mathbf{x}_i) + v_i$   $E[v_i | \mathbf{x}_i] = 0$ 

where the aim is to estimate the average treatment effect  $\theta$  using observed instrument  $z_i$  in the presence of controls  $\mathbf{x}_i$ . We estimate the conditional expectations  $E[y_i|\mathbf{x}_i]$ ,  $E[d_i|\mathbf{x}_i]$  and  $E[z_i|\mathbf{x}_i]$  using a supervised machine learner.

The DDML framework can be applied to other models (all implemented in ddml):

## Optimal IV model

$$y_i = d_i\theta + g(\mathbf{x}_i) + u_i$$
  
 $d_i = h(\mathbf{z}_i) + m(\mathbf{x}_i) + v_i$ 

where the estimand of interest is  $\theta$ . The instruments and controls enter the model through unknown functions g(), h() and f().

We estimate the conditional expectations  $E[y_i|\mathbf{x}_i]$ ,  $E[\hat{d}_i|\mathbf{x}_i]$  and  $\hat{d}_i := E[d_i|\mathbf{z}_i,\mathbf{x}_i]$  using a supervised machine learner. The instrument is then formed as  $\hat{d}_i - \hat{E}[\hat{d}_i|\mathbf{x}_i]$  where  $\hat{E}[\hat{d}_i|\mathbf{x}_i]$  denotes the estimate of  $E[\hat{d}_i|\mathbf{x}_i]$ .

The DDML framework can be applied to other models (all implemented in ddml):

#### LATE model

$$y_i = \mu(\mathbf{x}_i, \mathbf{z}_i) + u_i$$
  $E[u_i | \mathbf{x}_i, z_i] = 0$   
 $d_i = m(z_i, \mathbf{x}_i) + v_i$   $E[v_i | \mathbf{x}_i, z_i] = 0$   
 $z_i = p(\mathbf{x}_i) + \xi_i$   $E[\xi_i | \mathbf{x}_i] = 0$ 

where the aim is to estimate the local average treatment effect.

We estimate, using a supervised machine learner, the following conditional expectations:  $E[y_i|\mathbf{x}_i,z_i=0]$  and  $E[y_i|\mathbf{x}_i,z_i=1]$ ;  $E[D|\mathbf{x}_i,z_i=0]$  and  $E[D|\mathbf{x}_i,z_i=1]$ ;  $E[z_i|\mathbf{x}_i]$ .

## **Stacking regression**

#### Which machine learner should we use?

ddml supports a range of ML programs: pylearn, lassopack, randomforest. — Which one should we use?

We don't know whether we have a sparse or dense problem; linear or non-linear; etc.

## Stacking regression

Which machine learner should we use?

We suggest *Stacking regression* (Wolpert, 1992) as the *default* machine learner, which we have implemented in the separate program pystacked using Python's scikit learn.

Stacking is an ensemble method that combines multiple base learners into one model. As the default, we use *non-negative least squares*:

$$\mathbf{w} = \arg\min_{w_j \ge 0} \sum_{i=1}^n \left( y_i - \sum_{m=1}^M w_m \hat{f}_m^{-i}(\mathbf{x}_i) \right)^2,$$

where  $\hat{f}_m^{-i}(\mathbf{x}_i)$  are cross-validated predictions of base learner m.

## pystacked

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### Syntax:

```
pystacked depvar [indepvars] [if] [in] [type(string)
methods(string) finalest(string) folds(integer) voting ...]
```

methods() list of ML algorithms in any order separated by spaces reg(ress) for regression problems or class(ify) for classification problems.

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pystacked supports lasso, elastic net, random forest, gradient boosting and support vector machines.

## A brief pystacked example

```
. insheet using https://statalasso.github.io/dta/housing.csv, comma
(14 vars, 506 obs)
.
. pystacked medv crim-lstat, ///
> type(regress) pyseed(243) method(lassoic rf gradboost)

Method Weight

lassoic 0.0768684
rf 0.0000000
gradboost 0.9231316
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The method() argument can be used to specify the base learners. Here, we use lasso with AIC (lassoic), random forest (rf) and gradient boosting (gradboost).

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Due to the non-negativity constraint some base learners are assigned a weight of zero. Also note that weights are standardized to sum to  $1. \,$ 

### Back to DDML

#### The one-line syntax:

```
qddml depvar [indepvars] [(controls)] [(endog=instruments)] [if]
[in] [weight], model(string) [cmd(string) cmdopt(string)
kfolds(integer) ...]
```

cmd() selects the machine learner (default pystacked). cmdopt()
allows to pass options to the ML program.

## Examples:



## qddml example: Partial linear model

qddml is the one-line ('quick') version of ddml and uses a syntax similar to pds/ivlasso.

```
. use https://statalasso.github.io/dta/AJR.dta, clear
.
. global Y logpgp95
. global X lat_abst edes1975 avelf temp* humid* steplow-oilres
. global D avexpr
.
. qddml $Y $D ($X), model(partial) cmdopt(method(rf gradboost))
DML with Y=mO_y and D=mO_d1:
```

m	о_у	Coef.	Std. Err.	z	P> z	[95% Conf	. Interval]
mO.	_d1	.3391897	.0621291	5.46	0.000	.217419	.4609604

Step 1: Initialise ddml and select model:

ddml init model

where  $\emph{model}$  is either 'partial', 'iv', 'interactive', 'optimaliv', 'late'.

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where model is either 'partial', 'iv', 'interactive', 'optimaliv', 'late'.

*Step 2:* Add supervised ML programs for estimating conditional expectations:

 $\label{eq:delta} \texttt{ddml} \ \, \textit{eq newvarname} \ \, \big[ \ \, , \ \, \textit{eqopt} \, \big] \colon \ \, \textit{command depvar indepvars} \, \big[ \ \, , \\ \ \, \textit{cmdopt} \, \big]$ 

where *eq* selects the conditional expectations to be estimated. *command* is a ML program that supports the standard reg y x-type syntax. *cmdopt* are specific to that program.

Multiple estimation commands per equation are allowed.

```
Step 3: Cross-fitting
ddml crossfit [, kfolds(integer) ...]
This allows to set the number of folds.
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Step 4: Estimation of causal effects
ddml estimate [, robust ...]
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# Extended ddml syntax

```
Step 3: Cross-fitting

ddml crossfit [, kfolds(integer) ...]

This allows to set the number of folds.

Step 4: Estimation of causal effects

ddml estimate [, robust ...]
```

#### Additional auxiliary commands:

ddml describe (describe current model set up), ddml save, ddml use, ddml export (export in csv format ).

# Extended ddml syntax: Example

Equation successfully added.

```
. global Y logpgp95
. global X lat_abst edes1975 avelf temp* humid* steplow-oilres
. global D avexpr
. *** initialise ddml and select model:
. ddml init partial
. *** specify supervised machine learners for E[Y|X] ("yeq") and E[D|X] ("deq")
. * v-equation:
. ddml yeq, gen(pyy): pystacked $Y $X, type(reg) method(rf gradboost)
Equation successfully added.
. * d-equation:
. ddml deq, gen(pyd): pystacked $D $X, type(reg) method(rf gradboost)
```

# Extended ddml syntax: Example (cont'd.)

. \*\*\* cross-fitting and display mean-squared prediction error

. ddml crossfit Model: partial Number of Y estimating equations: 1 Number of D estimating equations: 1 Cross-fitting equation 1 2 Mean-squared error for v|X: Name Orthogonalized Command N MSPE. logpgp95 pystacked 64 0.573751 wvg 0m Mean-squared error for D|X: Name Orthogonalized Command N MSPE 64 1.648804 avexpr m0 pyd pystacked

. \*\*\* estimation of parameter of interest

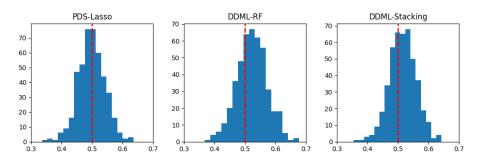
. ddml estimate

DML with Y=mO\_pyy and D=mO\_pyd (N=):

mO_pyy	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]
m0_pyd	.3794184	.0569073	6.67	0.000	.2678821	.4909546

## Simulation example

How does DDML perform compared to PDS-lasso?



The simulation is based on an approximate sparse design with  $\beta_j = (1/j)^2$  and p = 100 (which should favor the lasso). DGP

# Replication & transparency

Stata's Python integration allows for fast computing and to make use of existing ML programs.

But there are also additional set-up cost and, in particular, *challenges for replications*.

We have to set the seed in both Stata (due to randomization of folds) and Python (due to randomization specific to ML algorithm).

Randomization is involved in the estimation of conditional expectations. To facilitate transparency and replication, estimated conditional expectations can be saved for later inspection or use.

We also need to keep track of package versions of all programs involved (ddml, pystacked, Python, scikit-learn, etc.)

### **Summary**

- ddml implements Double/Debiased Machine Learning for Stata:
  - ► Compatible with various ML programs in Stata
  - ► Short (one-line) and flexible multi-line version
  - ► Uses Stacking Regression as the default machine learner; implemented via separate program pystacked
  - ► 5 models supported
- The advantage to pdslasso is that we can make use of almost any machine learner.
- ▶ But which machine learner should we use? We suggest Stacking as it combines multiple ML methods into one prediction.
- ▶ More testing & de-bugging needed; hopefully we can make ddml available soon (following your feedback)

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# qddml example: Interactive model

. webuse cattaneo2. clear

```
(Excerpt from Cattaneo (2010) Journal of Econometrics 155: 138-154)
.
. global Y bweight
. global D mbsmoke
. global X c.(mmarried mage fbaby medu)#c.(mmarried mage fbaby medu)
.
. qddml $Y $D ($X), model(interactive) cmdopt(method(rf gradboost))
DML with Y0=m0_y, Y1=m0_y and D=m0_d:
```

	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]
mbsmoke	-1533.568	45.91291	-33.40	0.000	-1623.556	-1443.581

Back

## qddml example: IV model

```
. use https://statalasso.github.io/dta/AJR.dta, clear
.
. global Y logpgp95
. global X edes1975 avelf temp* humid* steplow-oilres
. global D avexpr
. global Z logem4
.
. *** now, using one-line command:
. qddml $Y ($X) ($D = $Z), model(iv) cmdopt(method(rf gradboost))
DML with Y=mO_y and D=mO_d1, Z=mO_z1:
```

mO_y	Coef.	Std. Err.	z	P> z	[95% Conf.	<pre>Interval]</pre>
m0_d1	.8975094	. 2329957	3.85	0.000	.4408462	1.354173



## qddml example: LATE

```
. use "http://fmwww.bc.edu/repec/bocode/j/jtpa.dta",clear
.
. gen lnearnings = log(earnings)
(1,332 missing values generated)
. global Y lnearnings
. global D training
. global Z assignmt
. global X sex-age4554
```

. \*\*\* now, do the same using one-line command

. qddml \$Y (\$X) (\$D=\$Z), model(late) cmdopt(method(rf gradboost))

DML with  $Y0=m0_y$ ,  $Y1=m0_y$ ,  $D0=m0_d$ ,  $D1=m0_d$ :

	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]
training	13.17984	.1269613	103.81	0.000	12.931	13.42868



### Simulation example

#### Approximate sparsity:

$$y_i = 0.5d_i + \mathbf{x}_i'(c_y\beta) + \varepsilon_i,$$
  $\varepsilon_i \sim N(0,1)$   
 $d_i = \mathbf{x}_i'(c_d\beta) + \nu_i,$   $\nu_i \sim N(0,1)$ 

where  $\beta_j$  is approximately sparse:  $\beta_j = (1/j)^2$ .  $c_y$  and  $c_d$  are chosen such that  $R_y^2$  and  $R_d^2$  are 0.8 and 0.2, respectively. N = 500, number of cross-fitting folds=5, p = 100.

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### The LASSO

The LASSO (Least Absolute Shrinkage and Selection Operator, Tibshirani, 1996), " $\ell_1$  norm".

Minimize: 
$$\frac{1}{n} \sum_{i=1}^{n} (y_i - \mathbf{x}_i' \boldsymbol{\beta})^2 + \lambda \sum_{j=1}^{p} |\beta_j|$$

There's a cost to including lots of regressors, and we can reduce the objective function by throwing out the ones that contribute little to the fit.

The effect of the penalization is that LASSO sets the  $\hat{\beta}_j$ s for some variables to zero. In other words, it does the *model selection* for us.

In contrast to  $\ell_0$ -norm penalization (AIC, BIC) computationally feasible. Path-wise coordinate descent ('shooting') algorithm allows for fast estimation.

## Choosing controls: Post-Double-Selection LASSO

Our model is

$$y_i = \underbrace{\frac{\alpha d_i}{\text{aim}}}_{\text{nuisance}} + \underbrace{\frac{\beta_1 x_{i,1} + \ldots + \beta_p x_{i,p}}{\text{nuisance}}}_{\text{nuisance}} + \varepsilon_i.$$

► Step 1: Use the LASSO to estimate

$$y_i = \beta_1 x_{i,1} + \beta_2 x_{i,2} + \ldots + \beta_i x_{i,i} + \ldots + \beta_p x_{i,p} + \varepsilon_i,$$

i.e., without  $d_i$  as a regressor. Denote the set of LASSO-selected controls by A.

► Step 2: Use the LASSO to estimate

$$d_i = \beta_1 x_{i,1} + \beta_2 x_{i,2} + \ldots + \beta_j x_{i,j} + \ldots + \beta_p x_{i,p} + \varepsilon_i,$$

i.e., where the causal variable of interest is the dependent variable. Denote the set of LASSO-selected controls by B.

► Step 3: Estimate using OLS

$$y_i = \alpha d_i + \mathbf{w}_i' \beta + \varepsilon_i$$

where  $\mathbf{w}_i = A \ \cup \ B$ , i.e., the union of the selected controls from Steps 1 and 2