ddml: Double/debiased machine learning in Stata

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Package website: https://statalasso.github.io/ Latest version available here

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Introduction

A rich and growing literature exploits machine learning to facilitate causal inference.

A central focus: *high-dimensional* controls and/or instruments, which can arise if

- ▶ we observe many controls/instruments
- ► controls/instruments enter through an unknown function

Belloni, Chernozhukov, and Hansen (2014) and Belloni et al. (2012) propose estimators *relying on the Lasso* that allow for high-dimensional controls/instruments.

 \Rightarrow Available via pdslasso in Stata (Ahrens, Hansen, and Schaffer, 2020)

Introduction

What if we don't want to use the lasso?

- ► The Lasso might not be the *best-performing machine learner* for a particular problem.
- ► The Lasso relies on the *approximate sparsity assumption*, which might not be appropriate in some settings.

Chernozhukov et al. (2018) propose *Double/Debiased Machine Learning* (DDML) which allow to exploit machine learners other than the Lasso.

Our contribution:

- ▶ We introduce ddml, which implements DDML for Stata.
- ► We provide simulation evidence on the finite sample performance of DDML.
- Our recommendation is to use DDML in combination with Stacking.

Motivating example. The partial linear model:

$$y_i = \underbrace{\frac{\theta d_i}{\text{causal part}}}_{\text{causal part}} + \underbrace{\frac{g(\mathbf{x}_i)}{\text{nuisance}}}_{\text{nuisance}} + \varepsilon_i$$

How do we account for confounding factors x_i ? — The standard approach is to assume linearity $g(x_i) = x_i'\beta$ and consider alternative combinations of controls.

Problems:

- ► Non-linearity & unknown interaction effects
- ► High-dimensionality: we might have "many" controls
- ► We don't know which controls to include

Motivating example. The partial linear model:

$$y_i = \underbrace{\frac{\theta d_i}{\text{causal part}}} + \underbrace{\frac{g(\mathbf{x}_i)}{\text{nuisance}}} + \varepsilon_i$$

Post-double selection (Belloni, Chernozhukov, and Hansen, 2014) and *post-regularization* (Chernozhukov, Hansen, and Spindler, 2015) provide data-driven solutions for this setting.

Both "double" approaches rely on the *sparsity assumption* and use two auxiliary lasso regressions: $y_i \rightsquigarrow \mathbf{x}_i$ and $d_i \rightsquigarrow \mathbf{x}_i$. [asso] PDS

Related approaches exist for *optimal IV* estimation (Belloni et al., 2012) and/or IV with many controls (Chernozhukov, Hansen, and Spindler, 2015).

These methods have been implemented for Stata in pdslasso (Ahrens, Hansen, and Schaffer, 2020), dsregress (StataCorp) and R (hdm; Chernozhukov, Hansen, and Spindler, 2016).

A quick example using AJR (2001):

Example 1 (pdslasso) allows for high-dimensional controls.

Example 2 (ivlasso) treats avexpr as endogenous and exploits logem4 as an instrument. (More details in the pds/ivlasso help file.)

There are advantages of relying on lasso:

- ▶ intuitive assumption of (approximate) sparsity
- computationally relatively cheap (due to plugin lasso penalty; no cross-validation needed)
- ► Linearity has its advantages (e.g. extension to fixed effects; Belloni et al., 2016)

But there are also drawbacks:

- ► What if the sparsity assumption is not plausible?
- ► There is a wide set of machine learners at disposable—Lasso might not be the best choice.
- ► Lasso requires careful feature engineering to deal with non-linearity & interaction effects.

Review of DDML

The partial linear model:

$$y_i = \theta d_i + g(\mathbf{x}_i) + \varepsilon_i$$

 $d_i = m(\mathbf{x}_i) + v_i$

Naive idea: We estimate conditional expectations $\ell(\mathbf{x}_i) = E[y_i|\mathbf{x}_i]$ and $m(\mathbf{x}_i) = E[d_i|\mathbf{x}_i]$ using ML and partial out the effect of \mathbf{x}_i (in the style of Frisch-Waugh-LovelI):

$$\hat{\theta}_{DDML} = \left(\frac{1}{n}\sum_{i}\hat{v}_{i}^{2}\right)^{-1}\frac{1}{n}\sum_{i}\hat{v}_{i}(y_{i}-\hat{\ell}),$$

where $\hat{v}_i = d_i - \hat{m}_i$.

Review of DDML

Yet, there is a problem: The estimation error $\ell(\mathbf{x}_i) - \hat{\ell}$ and v_i may be correlated due to **over-fitting**, leading to poor performance.

DDML, thus, relies on **cross-fitting** (sample splitting with swapped samples).

DDML for the partial linear model (DML 2)

We split the sample in K random folds of equal size denoted by I_k :

- ▶ For k = 1, ..., K, estimate $\ell(\mathbf{x}_i)$ and $m(\mathbf{x}_i)$ using sample I_k^c and form out-of-sample predictions $\hat{\ell}_i$ and \hat{m}_i for all i in I_k .
- ightharpoonup Construct estimator $\hat{\theta}$ as

$$\left(\frac{1}{n}\sum_{i}\hat{v}_{i}^{2}\right)^{-1}\frac{1}{n}\sum_{i}\hat{v}_{i}(y_{i}-\hat{\ell}),$$

where $\hat{v}_i = d_i - \hat{m}_i$. \hat{m}_i and $\hat{\ell}_i$ are the cross-fitted predicted values.

The DDML framework can be applied to other models (all implemented in ddml):

Interactive model

$$y_i = g(d_i, \mathbf{x}_i) + u_i$$
 $E[u_i | \mathbf{x}_i, d_i] = 0$
 $z_i = m(\mathbf{x}_i) + v_i$ $E[u_i | \mathbf{x}_i] = 0$

As in the Partial Linear Model, we are interested in the ATE, but do not assume that d_i (a binary treatment variable) and \mathbf{x}_i are separable.

We estimate the conditional expectations $E[y_i|\mathbf{x}_i,d_i=0]$ and $E[y_i|\mathbf{x}_i,d_i=1]$ as well as $E[d_i|\mathbf{x}_i]$ using a supervised machine learner.

The DDML framework can be applied to other models (all implemented in ddml):

Partial linear IV model

$$y_i = d_i \theta + g(\mathbf{x}_i) + u_i$$
 $E[u_i | \mathbf{x}_i, z_i] = 0$
 $z_i = m(\mathbf{x}_i) + v_i$ $E[v_i | \mathbf{x}_i] = 0$

where the aim is to estimate the average treatment effect θ using observed instrument z_i in the presence of controls \mathbf{x}_i . We estimate the conditional expectations $E[y_i|\mathbf{x}_i]$, $E[d_i|\mathbf{x}_i]$ and $E[z_i|\mathbf{x}_i]$ using a supervised machine learner.

The DDML framework can be applied to other models (all implemented in ddml):

High-dimensional IV model

$$y_i = d_i\theta + g(\mathbf{x}_i) + u_i$$

$$d_i = h(\mathbf{z}_i) + m(\mathbf{x}_i) + v_i$$

where the parameter of interest is θ . The instruments and controls enter the model through unknown functions g(), h() and f().

We estimate the conditional expectations $E[y_i|\mathbf{x}_i]$, $E[\hat{d}_i|\mathbf{x}_i]$ and $\hat{d}_i := E[d_i|\mathbf{z}_i,\mathbf{x}_i]$ using a supervised machine learner. The instrument is then formed as $\hat{d}_i - \hat{E}[\hat{d}_i|\mathbf{x}_i]$ where $\hat{E}[\hat{d}_i|\mathbf{x}_i]$ denotes the estimate of $E[\hat{d}_i|\mathbf{x}_i]$.

The DDML framework can be applied to other models (all implemented in ddml):

Interactive IV model

$$y_i = \mu(\mathbf{x}_i, \mathbf{z}_i) + u_i$$
 $E[u_i | \mathbf{x}_i, z_i] = 0$
 $d_i = m(z_i, \mathbf{x}_i) + v_i$ $E[v_i | \mathbf{x}_i, z_i] = 0$
 $z_i = p(\mathbf{x}_i) + \xi_i$ $E[\xi_i | \mathbf{x}_i] = 0$

where the aim is to estimate the local average treatment effect.

We estimate, using a supervised machine learner, the following conditional expectations: $E[y_i|\mathbf{x}_i,z_i=0]$ and $E[y_i|\mathbf{x}_i,z_i=1]$; $E[D|\mathbf{x}_i,z_i=0]$ and $E[D|\mathbf{x}_i,z_i=1]$; $E[z_i|\mathbf{x}_i]$.

The ddml package

We introduce ddml for Stata:

- ► Compatible with various ML programs in Stata (e.g. lassopack, pylearn, randomforest).
 - → Any program with the classical "reg y x" syntax and post-estimation predict will work.
- ► Short (one-line) and flexible multi-line version
- ► 5 models supported: partial linear model, interactive model, interactive IV model, partial IV model, optimal IV.

Stacking regression

Which machine learner should we use?

ddml supports a range of ML programs: pylearn, lassopack, randomforest. — Which one should we use?

We don't know whether we have a sparse or dense problem; linear or non-linear. We don't know whether, e.g., lasso or random forests will perform better.

Stacking, as implemented in pystacked, provides a solution: We use an 'optimal' combination of base learners.

Stacking regression

Which machine learner should we use?

We don't know whether we have a sparse or dense problem; linear or non-linear; etc.

Stacking is an ensemble method that combines multiple base learners into one model. As the default, we use *non-negative least* squares: $\frac{n}{2} \left(\frac{J}{J} \right)^{2}$

 $\mathbf{w} = \arg\min_{w_j \ge 0} \sum_{i=1}^n \left(y_i - \sum_{j=1}^J w_j \hat{y}_i^{(j)} \right)^2,$

where $\hat{y}_i^{(j)}$ are cross-validated predictions of base learner j.

Voting regression is a special case with unweighted (or user-specified) weights.

Extended ddml syntax

Step 1: Initialise ddml and select model:

```
ddml init model [, kfolds(integer) reps(integer) ...] where model is either 'partial', 'iv', 'interactive', 'ivhd', 'late'.
```

Step 2: Add supervised ML programs for estimating conditional expectations:

```
\label{eq:delta} \texttt{ddml} \  \  \, \textit{eq newvarname} \  \, \big[ \  \, , \  \, \textit{eqopt} \, \big] \colon \  \, \textit{command depvar indepvars} \, \big[ \  \, , \\ \  \, \textit{cmdopt} \, \big]
```

where *eq* selects the conditional expectations to be estimated. *command* is a ML program that supports the standard reg y x-type syntax. *cmdopt* are specific to that program.

Multiple estimation commands per equation are allowed.

Extended ddml syntax

```
Step 3: Cross-fitting
ddml crossfit
Step 4: Estimation of causal effects
ddml estimate [, robust ...]
```

Additional auxiliary commands:

ddml describe (describe current model set up), ddml save & ddml use (to import/save ddml objects), ddml extract (to retrieve objects), ddml export (export in csv format).

Extended ddml syntax: Example

We demonstrate the use of ddml using the partially linear model by extending the analysis of 401(k) eligibility and total financial wealth of Poterba, Venti, and Wise (1995). The data consists of n = 9915 households from the 1991 SIPP.

Step 0: Load data, define globals

```
. insheet using "restatw.dat", tab clear
(75 vars, 9,915 obs)
. global Y tw
. global X i2 i3 i4 i5 i6 i7 a2 a3 a4 a5 ///
> fsize hs smcol col marr twoearn db pira hown
. global D e401
. set seed 123
```

Step 1: Initialise ddml and select model:

. ddml init partial

Extended ddml syntax: Example (cont'd.)

Step 2: Add supervised ML programs for estimating conditional expectations. We use OLS, Lasso and Random Forest.

```
. *** add learners for E[Y|X]
. ddml E[Y|X]: reg $Y $X
Learner Y1_reg added successfully.
. ddml E[Y|X]: pystacked $Y c.($X)#c.($X), m(lassocv)
Learner Y2_pystacked added successfully.
. ddml E[Y|X]: pystacked $Y $X, m(rf)
Learner Y3 pystacked added successfully.
. *** add learners for E[D|X]
. ddml E[D|X]: reg $D $X
Learner D1 reg added successfully.
. ddml E[D|X]: pystacked $D c.($X)#c.($X), m(lassocv)
Learner D2 pystacked added successfully.
. ddml E[D|X]: pystacked $D $X, m(rf)
Learner D3_pystacked added successfully.
```

Extended ddml syntax: Example (cont'd.)

Step 3: Cross-fitting with 5 folds

```
. ddml crossfit
Fold IDs: m0_fid_1
Y eqn learners (3): Y1_reg Y2_pystacked Y3_pystacked
D equations (1): e401
        D equation e401:
            learners: D1_reg D2_pystacked D3_pystacked
Cross-fitting E[Y|X] equation: tw
Cross-fitting fold 1 2 3 4 5 ...completed cross-fitting
Cross-fitting E[D|X] equation: e401
Cross-fitting fold 1 2 3 4 5 ...completed cross-fitting
```

Extended ddml syntax: Example (cont'd.)

Step 4: Estimation of causal effects

. ddml estimate, robust

```
Summary DDML estimation results:
       Y learner D learner
spec r
                                      b
                                              SF.
       Y1_reg D1_reg 6446.073(2121.497)
    1 Y1_reg D2_pystacked 6353.115(2143.219)
            Y1_reg D3_pystacked 4736.845(1877.325)
  4 1 Y2_pystacked D1_reg 6540.954(2048.532)
  5 1 Y2_pystacked D2_pystacked 6770.558(2069.652)
  6 1 Y2_pystacked D3_pystacked 4517.166(1817.041)
  7 1 Y3_pystacked D1_reg 5035.011(2278.751)
  8 1 Y3 pystacked D2 pystacked 5024.630(2290.085)
    1 Y3 pystacked D3 pystacked 5562.772(2026.662)
Min MSE DDML model, specification 4
y-E[y|X] = Y2 \text{ pystacked } 1
                                            Number of obs = 9915
D-E[D|X,Z] = D1_{reg_1}
        tw | Coef. Std. Err. z P>|z| [95% Conf. Interval]
      e401 | 6540.954 2048.532 3.19 0.001 2525.906
```

qddml example: Partial linear model

qddml is the one-line ('quick') version of ddml and uses a syntax similar to pds/ivlasso.

Simulation set-up

We consider a *linear DGP* and a *non-linear DGP*, and compare performance of OLS, PDS-Lasso and various machine learners, including stacking.

We would expect that stacking performs well under both settings, while linear approaches only perform well if the DGP is linear.

- 1. Use the full sample OLS estimate to obtain $\hat{\tau}_{OLS}$. Construct the partial residuals $y_i^{(r)} = y_i \hat{\tau}_{OLS}d_i$.
- 2. Fit a supervised learning estimator that aims to predict $y_i^{(r)}$ with the controls x_i and d_i with x_i , respectively. Denote the fitted values as \tilde{g} and \tilde{h} . We either use
 - ► linear regression (*Linear DGP*)
 - ▶ gradient boosting (*Non-linear DGP*)
- 3. Sample from the empirical distribution of x_i by bootstrapping n_s observations from the original data. Denote the bootstrapped sample by \mathcal{D}_b .
- 4. Generate the treatment and outcome variable over the bootstrap sample:

$$\tilde{d}_i^{(b)} = \mathbb{1}\{\tilde{h}(x_i) + \nu_i \ge 0.5\}$$
 (1)

$$\tilde{\mathbf{y}}_{i}^{(b)} = \tau_{0} \tilde{\mathbf{d}}_{i}^{(b)} + \tilde{\mathbf{g}}(\mathbf{x}_{i}) + \varepsilon_{i}, \forall i \in \mathcal{D}_{b}$$
 (2)

where $\nu_i \stackrel{iid}{\sim} \mathcal{N}(0, \kappa_1)$ and $\varepsilon_i \stackrel{iid}{\sim} \mathcal{N}(0, \kappa_2)$, $\tau_0 = 6,000$.

We consider the following base learners:

- 1.-2. CV-Lasso & Ridge with interactions and 2nd order polynomials
- 3.-4. CV-Lasso & Ridge with 10th order polynomials and no interactions
 - 5. Random forest with low regularization (max_features(8) min_samples_leaf(1) max_samples(.7))
 - Random forest with high regularization (max_features(5) min_samples_leaf(10) max_samples(.7))
 - Gradient boosted trees with low regularization (n_estimators(500) learning_rate(0.01))
 - Gradient boosted trees with high regularization (n_estimators(250) learning_rate(0.01))
 - Feed-forward neural nets with 3 hidden layers of size 5 (hidden_layer_sizes(5 5 5))

Table: Average Stacking Weights

	Stacking		Single-Best	
Panel (A): Linear DGP	Y X	D X	Y X	D X
OLS	.646	.492	.813	.643
Lasso with CV (2nd order poly)	.111	.158	.161	.267
Ridge with CV (2nd order poly)	.063	.061	.018	.019
Lasso with CV (10th order poly)	.032	.08	.003	.049
Ridge with CV (10th order poly)	.03	.047	.005	.016
Random forest (low regularization)	.013	.012	0	0
Random forest (high regularization)	.017	.027	0	0
Gradient boosting (low regularization)	.028	.043	0	.006
Gradient boosting (high regularization)	.024	.074	0	.002
Neural net	.036	.005	0	0

Stacking assigns the highest weight to OLS if the DGP is linear. . .

Table: Average Stacking Weights

	Stacking		S	Single-Best	
	(1)	(2)	(3)	(4)
Panel (B): Non-Linear DGP	Y X	D X	Y	X	D X
OLS	.037	.021		0	0
Lasso with CV (2nd order poly)	.039	.067	.0	83	.149
Ridge with CV (2nd order poly)	.177	.23		12	.125
Lasso with CV (10th order poly)	.052	.077	.0	88	.061
Ridge with CV (10th order poly)	.078	.068	.0	19	.044
Random forest (low regularization)	.041	.01		0	0
Random forest (high regularization)	.028	.069	.0	01	0
Gradient boosting (low regularization)	.517	.213	.6	78	.359
Gradient boosting (high regularization)	.02	.239	.0	11	.262
Neural net	.012	.005		0	0

... and the highest weight to gradient boosting if *the DGP is non-linear*.

Table: Bias and Coverage Rates in the Linear DGP

	$n_s = 9915$			ns	$n_s = 99150$			
Panel (A): Linear DGP	Bias	MAB	Rate	Bias	MAB	Rate		
Full sample:								
OLS	100.99	918.03	.95	-22.61	255.52	.94		
PDS-Lasso	101.83	913.18	.95	-19.9	257.29	.94		
DDML methods:								
Base learners								
OLS	105.07	906.96	.94	-23.05	256.51	.94		
Lasso with CV (2nd order poly)	104.33	907.84	.94	-22.45	257.23	.94		
Ridge with CV (2nd order poly)	103.22	898.56	.94	-23.27	255.54	.94		
Lasso with CV (10th order poly)	49.56	1120.59	.93	37.98	260.53	.95		
Ridge with CV (10th order poly)	1066	1342.38	.9	15.85	260.41	.95		
Random forest (low regularization)	-59.63	1083.64	.91	-59.29	343.46	.86		
Random forest (high regularization)	105.58	952.35	.94	-46.54	275.56	.91		
Gradient boosting (low regularization)	53.97	930.93	.94	-41.84	252.14	.94		
Gradient boosting (high regularization)	162.75	923.08	.95	48.31	259.12	.95		
Neural net	-3594.99	5380.31	.17	-2165.41	3212.12	.16		
Meta learners								
Stacking: NNLS	100.01	935.27	.94	-22.7	254.01	.94		
Single best	92.79	944.07	.95	-25.03	255.75	.94		

The bias of stacking is similar to OLS if the DGP is linear. . .

Table: Bias and Coverage Rates in the Non-Linear DGP

	$n_s = 9915$			$n_s = 99150$			
Panel (B): Non-Linear DGP	Bias	MAB	Rate	Bias	MAB	Rate	
Full sample:							
OLS	-2496.16	2477.19	.63	-2658.04	2636.31	0	
PDS-Lasso	-2507.47	2489.77	.62	-2657.5	2635.94	0	
DDML methods:							
Base learners							
OLS	-2522.98	2540.36	.62	-2660.54	2640.98	0	
Lasso with CV (2nd order poly)	767.2	1078.29	.91	691.67	695.3	.64	
Ridge with CV (2nd order poly)	825.21	1091.19	.9	702.55	707.28	.64	
Lasso with CV (10th order poly)	-4214.09	1895.22	.92	-10.06	294.34	.94	
Ridge with CV (10th order poly)	-2123.59	2095.56	.91	4.42	288.37	.94	
Random forest (low regularization)	-104.54	1019.55	.92	-28.83	332.87	.87	
Random forest (high regularization)	-110.06	959.96	.95	-21.52	280.36	.94	
Gradient boosting (low regularization)	69.44	890.94	.95	7.28	263.62	.95	
Gradient boosting (high regularization)	213.04	895.47	.95	174.14	291.63	.93	
Neural net	-4706.76	5831.79	.17	-3216.85	3837.37	.15	
Meta learners							
Stacking: NNLS	-62.97	1068.87	.84	18.36	269.02	.95	
Single best	-135.15	1035.41	.89	7.94	263.06	.95	

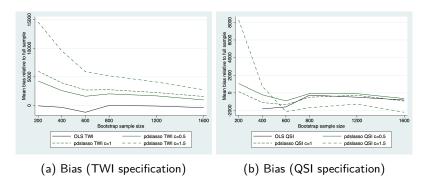
... but substantially smaller in absolute size if *the DGP is* non-linear.

Wüthrich and Zhu (2021, henceforth WZ) demonstrate that PDS-Lasso suffers from a large finite sample bias and tends to underselect; again using the application of Poterba, Venti, and Wise (1995) and Belloni et al. (2017).

They use two specifications:

- ▶ two-way interactions (TWI) (as in Chernozhukov and Hansen, 2004); p = 167
- ▶ quadratic splines & interactions (QSI) (as in Belloni et al., 2017); p = 272

WZ run their simulations on bootstrap samples of the data $(n_b = \{200, 400, 800, 1600\})$ and calculate the bias as the mean difference to the full sample estimate (N = 9915).



Notes: The figures report the mean bias calculated as the mean difference to the full sample estimates. Following WZ, we draw 600 bootstrap samples of size $n_b = \{200, 400, 600, 800, 1200, 1600\}$. 'TWI' indicates that the predictors have been expanded by two-way interactions. 'QSI' refers to the quadratic spline & interactions specification of Belloni et al. (2017).

Figure: Replication of Figure 8 in WZ

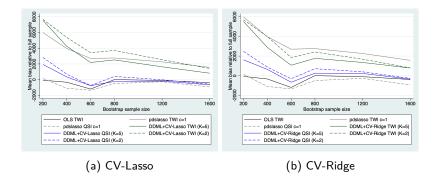


Figure: Mean bias relative to full sample

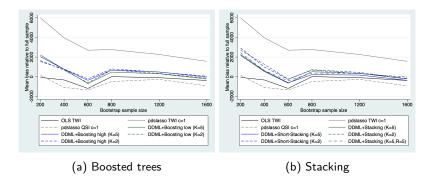


Figure: Mean bias relative to full sample

The small sample bias of stacking stabilizes for $n_b > 600$, suggesting that stacking may perform well for 'moderate' sample sizes.

Summary

- ddml implements Double/Debiased Machine Learning for Stata:
 - ► Compatible with various ML programs in Stata
 - ► Short (one-line) and flexible multi-line version
 - ► Uses Stacking Regression as the default machine learner; implemented via separate program pystacked
 - ► 5 models supported
- The advantage to pdslasso is that we can make use of almost any machine learner.
- ► But which machine learner should we use?
 - ► We suggest stacking. We don't know which learner is best suited for a particular problem.
 - ► Stacking allows to consider multiple learners in a joint framework, and thus reduces the risk of misspecification.
- ► We are in the final phase of development; hopefully we can make ddml available soon (following your feedback)

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