2. Algebra

2.1	The basic processes of algebra	Knowledge and use of basic skills in manipulative algebra including use of the associative, commutative and distributive laws, are expected

General algebra skills. ACCURACY!

2.2	Definition of a function	Notation f (x) will be used, e.g. f $(x) = x^2 - 9$

Part of N5.

2.3	Domain and range of a function	Domain may be expressed as, for example, $x > 2$, or 'for all x, except $x = 0$' and range may be expressed as f $(x) > -1$

DOMAIN : range of values for x that make up the 'input'.

e.g. for $f(x) = \sqrt{x+3}$ domain is $x \geq -3$

RANGE : range of values that make up the 'output' of a function.

e.g. for $f(x) = (x+4)^2 - 1$ range is $f(x) \geq -1$

Pay close attention to the 'STRICTNESS' of inequalities. e.g. $<$ or $\leq$

2.4	Composite functions	The result of two or more functions, say f and g, acting in succession. fg (x) is g followed by f

If $f(x) = 2x+1$ and $g(x) = 3-x^2$ →

$fg(x)$
$= f(3-x^2)$ ← $g(x)$ is 'input' into $f(x)$
$= 2(3-x^2)+1$
$= 6-2x^2+1$
$= 7-2x^2$

2.5	Inverse functions	The inverse function of f is written f^{-1} Domains will be chosen for f to make f one-one

$$a \underset{f^{-1}}{\overset{f}{\rightleftarrows}} b$$

$f: \boxed{\times 2} \rightarrow \boxed{+1}$

e.g. if $f(x) = 2x + 1$

then $f^{-1}(x) = \dfrac{x-1}{2}$

$f^{-1}: \boxed{-1} \rightarrow \boxed{\div 2}$

A step-by-step method:

$f(x) = 4x^3 - 1$

$y = 4x^3 - 1$ # write as "$y = \dots$"

$y + 1 = 4x^3$

$\dfrac{y+1}{4} = x^3$ # Make x the subject

$\sqrt[3]{\dfrac{y+1}{4}} = x$

State in required form

$\therefore f^{-1}(x) = \sqrt[3]{\dfrac{x+1}{4}}$

2.6	Expanding brackets and collecting like terms	Expand and simplify $(y^2 - 2y + 3)(2y - 1) - 2(y^3 - 3y^2 + 4y - 2)$

Part of N5.

2.7	Expand $(a + b)^n$ for positive integer n	Expand and simplify $(5x + 2)^3$ Use Pascal's triangle to work out the coefficient of x^3 in the expansion of $(3 + 2x)^5$

PASCAL'S TRIANGLE

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

...

$(5x+2)^3$

$= 1(5x)^3(2)^0 + 3(5x)^2(2)^1 + 3(5x)^1(2)^2 + 1(5x)^0(2)^3$

$= 1(125x^3)(1) + 3(25x^2)(2) + 3(5x)(4) + 1(1)(8)$

$= 125x^3 + 150x^2 + 60x + 8$

2.8	Factorising	Factorise fully $(2x+3)^2-(2x-5)^2$ Factorise $15x^2-34xy-16y^2$ Factorise fully x^4-25x^2

Part of N5.

2.9	Manipulation of rational expressions: Use of $+\ -\ \times\ \div$ for algebraic fractions with denominators being numeric, linear or quadratic	Simplify $\frac{5}{x+2}-\frac{3}{2x-1}$ Simplify $\frac{x^3+2x^2+x}{x^2+x}$ Simplify $\frac{5x^2-14x-3}{4x^2-25} \div \frac{x-3}{4x^2+10x}$

Part of N5.

2.10	Use and manipulation of formulae and expressions	Rearrange $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$ to make v the subject

Part of N5. May require factorisation.

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v} \qquad \times v$$

$$\frac{v}{f}=\frac{v}{u}+1 \qquad \times f$$

$$v=\frac{vf}{u}+f \qquad \times u$$

$$vu=vf+uf \qquad -vf$$

$$vu-vf=uf$$

$$v(u-f)=uf \qquad \text{factorise}$$

$$v=\frac{uf}{u-f} \qquad \div(u-f)$$

2.11	Use of the factor theorem for rational values of the variable for polynomials	Factorise $x^3 - 2x^2 - 5x + 6$ Show that $2x - 3$ is a factor of $2x^3 - x^2 - 7x + 6$ Solve $x^3 + x^2 - 10x + 8 = 0$ Show that $x - 7$ is a factor of $x^5 - 7x^4 - x + 7$

If $f(a) = 0$ then :

- $(x - a)$ is a factor of $f(x)$
- $x = a$ is a solution of $f(x)$

let $f(x) = x^3 + x^2 - 10x + 8$ ← factors 1, 2, 4, 8

try $f(1) = 1^3 + 1^2 - 10(1) + 8$

$= 0 \quad \therefore \quad (x-1)$ is a factor

$x^3 + x^2 - 10x + 8 = 0$

$(x-1)(x^2 + 2x - 8) = 0$ ← algebraic division or mental approach

$(x-1)(x+4)(x-2) = 0$

$x = 1, \; x = -4, \; x = 2$

2.12	Completing the square	Work out the values of a, b and c such that $2x^2 + 6x + 7 \equiv a(x+b)^2 + c$

$\underline{2}x^2 + \underline{6}x + \underline{7} \equiv \underline{a}x^2 + \underline{2ab}x + \underline{ab^2 + c}$

$a = 2$

$2ab = 6$

$2(2)b = 6$

$4b = 6$

$b = \frac{6}{4} = \frac{3}{2}$

$ab^2 + c = 7$

$2\left(\frac{3}{2}\right)^2 + c = 7$

$\frac{9}{2} + c = 7$

$c = \frac{5}{2}$

2.13	Drawing and sketching of functions Interpretation of graphs	Graphs could be linear, quadratic, exponential and restricted to no more than 3 domains Exponential graphs will be of the form $y = ab^x$ and $y = ab^{-x}$, where a and b are rational numbers Sketch the graph of $y = x^2 - 5x + 6$ Label clearly any points of the intersection with the axes A function f is defined as $\mathrm{f}(x) = x^2 \quad 0 \leqslant x < 1$ $\quad = 1 \quad 1 \leqslant x < 2$ $\quad = 3 - x \quad 2 \leqslant x < 3$ Draw the graph of $y = \mathrm{f}(x)$ on the grid below for values of x from 0 to 3 Given a sketch of $y = ab^{-x}$, and two points, work out the values of a and b

Mostly an extension of N5 skills.

2.14	Solution of linear and quadratic equations	Solutions of quadratics to include solution by factorisation, by graph, by completing the square or by formula Problems will be set in a variety of contexts, which result in the solution of linear or quadratic equations

By completing the square:

$$x^2 - 4x - 12 = 0$$
$$(x-2)^2 - 16 = 0$$
$$(x-2)^2 = 16$$
$$x - 2 = \pm 4$$
$$x - 2 = 4, \quad x - 2 = -4$$
$$x = 6 \qquad x = -2$$

2.15	Algebraic and graphical solution of simultaneous equations in two unknowns, where the equations could both be linear or one linear and one second order	Solve $4x - 3y = 0$ and $6x + 15y = 13$ Solve $y = x + 2$ and $y^2 = 4x + 5$ Solve $y = x^2$ and $y - 5x = 6$ Solve $xy = 8$ and $x + y = 6$

N5 linear approach:

$$20x - 15y = 0$$ add
$$6x + 15y = 13$$
$$26y = 13$$

Substitution:

$$y = x + 2$$
$$y^2 = 4x + 5$$
$$(x+2)^2 = 4x + 5$$

"$y = y$":

$$y = x^2$$
$$y = 5x + 6$$
$$x^2 = 5x + 6$$

$26y = 13$

...

$(x+2)^2 = 4x+5$

...

$x^2 = 5x+6$

...

2.16	Algebraic solution of linear equations in three unknowns	Solve ① $2x - 5y + 4z = 22$ ② $x + y + 2z = 4$ ③ $x - y - 6z = -4$

From a pair in 3 unknowns, elimation to produce one in 2 unknowns.
Repeat with a different pair ↑e.g. z ↑e.g. x and y
Solve as per N5.

e.g.
$$3x + 2y - z = 8 \quad \xrightarrow{\times 2} \quad 6x + 4y - 2z = 16$$
$$2x + 3y + 2z = 9 \quad \longrightarrow \quad 2x + 3y + 2z = 9$$
$$4x - y - 3z = 11$$

add

$$8x + 7y = 25$$

$\times 3$: $6x + 9y + 6z = 27$

$\times 2$: $8x - 2y - 6z = 22$

add

$$14x + 7y = 49$$

$8x + 7y = 25$ subtract
$14x + 7y = 49$
$-6x = -24$
$x = 4$

Sub. $x = 4$
$8 \times 4 + 7y = 25$
$32 + 7y = 25$
$7y = -7$
$y = -1$

Sub. $x = 4, y = -1$
$3 \times 4 + 2 \times (-1) - z = 8$
$12 - 2 - z = 8$
$10 - z = 8$
$2 = z$

$\therefore x = 4, y = -1, z = 2$

2.17	Solution of linear and quadratic inequalities	Solve $5(x-7) > 2(x+1)$ Solve $x^2 < 9$ Solve $2x^2 + 5x \leqslant 3$

For quadratic: Rearrange to have 0 on one side.
Sketch the function given on the other.
→ State solutions.

e.g. $2x^2 + 5x \leq 3$

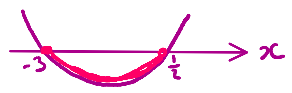
$\underline{2x^2 + 5x - 3} \leq 0$ → Roots at:

$2x^2 + 5x - 3 = 0$

$(2x-1)(x+3) = 0$

$x = \frac{1}{2}, x = -3$

Sketch



≤ 0

$-3 \leq x \leq \frac{1}{2}$ ← solution

2.18	Index laws, including fractional and negative indices and the solution of equations	Express as a single power of x $\sqrt{x^{\frac{1}{2}} \times x^{\frac{7}{2}}}$ Express as a single power of x $\sqrt{\dfrac{x^{\frac{3}{2}} \times x^{\frac{7}{2}}}{x^2}}$ Solve $x^{-\frac{1}{2}} = 3$ Solve $\sqrt{x} - \dfrac{10}{\sqrt{x}} = 3 \quad x > 0$

Part of N5.

2.19	Algebraic proof	Prove $(n+5)^2 - (n+3)^2$ is divisible by 4 for any integer value of n

$$(n+5)^2-(n+3)^2$$
$$= n^2+10n+25-(n^2+6n+9)$$
$$= 4n+16$$
$$= 4(n+4)$$

Since $n+4$ is an integer, $4(n+4)$ is a multiple of 4 for all n.

$\therefore (n+5)^2-(n+3)^2$ is divisible by 4 for all integer values of n.

2.20	Using nth terms of sequences	Work out the difference between the 16th and 6th terms of the sequence with nth term $\frac{2n}{n+4}$
	Limiting value of a sequence as $n \to \infty$	Write down the limiting value of $\frac{2n}{n+4}$ as $n \to \infty$

- For e.g. 6th term let $n=6$
- Limiting value of $\frac{ax+b}{cx+d}$ as $n \to \infty$ is $\frac{a}{c}$

2.21	nth terms of linear sequences	A linear sequence starts 180 176 172 … By using the nth term, work out which term has value -1000 Work out the nth term of the linear sequence $r+s \quad r+3s \quad r+5s \quad \ldots$

let nth term $= an+b$

1st term is 180: $180 = 1a+b$

2nd term is 176: $176 = 2a+b$

subtract

$4 = -a$

Sub. $a=-4$

$180 = -4+b$

$-1000 = -4n+184$

$-1184 = -4n$

$1184 = 4n$

Sub. $a=-4$
$180 = -4+b$
$184 = b$

$\therefore$ nth term is $-4n+184$

$-1184 = -4n$
$1184 = 4n$
$296 = n$

$\therefore$ the 296th term

2.22	nth terms of quadratic sequences	Work out the nth term of the quadratic sequence 10 16 18 16 ... Which term has the value 0?

Let nth term $= an^2+bn+c$

"Difference of differences" $=2a$ for all quadratic sequences

$$\underbrace{\underbrace{10 \quad 16}_{+6} \quad \underbrace{16 \quad 18}_{+2} \quad \underbrace{18 \quad 16}_{-2}}_{-4,\ -4}$$

$2a=-4$
$a=-2$

$\therefore \quad -2n^2+bn+c$

1st term: $10 = -2(1)^2+b(1)+c \rightarrow b+c=12$
2nd term: $16 = -2(2)^2+b(2)+c \rightarrow 2b+c=24$

subtract

$-b=-12$
$b=12$

Sub. $b=12$
$12+c=12$
$c=0$

$\therefore$ nth term $= -2n^2+12n$

$0 = -2n^2+12n$
$0 = -2n(n-6)$

$-2n=0$ or $n-6=0$

$-2n = 0$ or $n - 6 = 0$

$n = 0$ $\qquad$ $n = 6$

$\therefore$ the 6^{th} term