5. Matrix transformations

5.1	Multiplication of matrices	Multiplying a 2 × 2 matrix by a 2 × 2 matrix or by a 2 × 1 matrix Multiplication by a scalar

Scalar x matrix produces a **matrix**

$$k\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

Matrix x vector produces a **vector**

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

Matrix x matrix produces a **matrix**

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

The above looks more complicated than it is in reality - **rows** from the first matrix are "multiplied by" **rows** from the second vector.

Note that in general the order matters - matrix multiplication is **not commutative:** $AB \neq BA$

5.2	The identity matrix **I**	2 × 2 only

Identity matrix $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

When a matrix or vector is multiplied by the identity matrix, it does not change. e.g. $AI = IA = A$

5.3	Transformations of the unit square in the $x-y$ plane	Representation by a 2 × 2 matrix Transformations restricted to rotations of 90°, 180° or 270° about the origin, reflections in the lines $x = 0$, $y = 0$, $y = x$, $y = -x$ and enlargements centred on the origin

Since $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$ this means the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is associated with a reflection in the line $y = x$, where a coordinate such as $(3, -4)$ becomes $(-4,3)$.

Similarly:

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ is a reflection in the x-axis, where (x, y) becomes $(x, -y)$.

$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ is a reflection in the y-axis, where (x, y) becomes $(-x, y)$.

$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ is an enlargement from the origin where (x, y) becomes $(2x, 2y)$.

$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is a rotation of angle θ about the origin.

e.g. for a rotation of 90° use $\begin{bmatrix} \cos 90° & -\sin 90° \\ \sin 90° & \cos 90° \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

5.4	Combination of transformations	Using matrix multiplications Use of **i** and **j** notation is not required

To combine two or more transformations into a single composite transformation matrix, the **order is important**. Using the matrices above, if you want to **reflect in the y-axis** and then **rotate by 90 degrees:**

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$