4. Calculus

4.1	Know that the gradient function $\frac{dy}{dx}$ gives the gradient of the curve and measures the rate of change of y with respect to x	

Important to note the common applications of differentiation above.

4.2	Know that the gradient of a function is the gradient of the tangent at that point.	

As above.

4.3	Differentiation of kx^n where n is an integer, and the sum of such functions	Including expressions which need to be simplified first Given $y = (3x + 2)(x - 3)$ work out $\frac{dy}{dx}$ Given $y = \frac{5}{x^3}$ work out $\frac{dy}{dx}$

Given the simple rule above and N5 indices/brackets knowledge, a wide range of functions can be differentiating. At this point:

- Always expand brackets fully **before** differentiating.
- Always write terms in coefficient-variable form before differentiating.

e.g.

$$y = 2x(x - 3)$$
$$y = 2x^2 - 6x$$
$$\frac{dy}{dx} = 4x - 6$$

e.g.

$$y = \frac{2 - \sqrt{(x)}}{x^2}$$

$$y = \frac{2}{x^2} - \frac{\sqrt{x}}{x^2}$$

$$y = 2x^{-2} - \frac{x^{\frac{1}{2}}}{x^2}$$

$$y = 2x^{-2} - x^{-\frac{3}{2}}$$

$$\frac{dy}{dx} = -4x + \frac{3}{2}x^{-\frac{5}{2}}$$

$$\frac{dy}{dx} = -4x + \frac{3}{2\sqrt{x}^5}$$

Work carefully at all times!

4.4	The equation of a tangent and normal at any point on a curve	

A **normal** to a curve at a point is **perpendicular to its tangent**.

4.5	Increasing and decreasing functions	When the gradient is positive/negative a function is described as an increasing/decreasing function

Remember $f'(x)$ and $\frac{dy}{dx}$ are equivalent.

Increasing when $f'(x) > 0$

Decreasing when $f'(x) < 0$

4.6	Understand and use the notation $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$	Know that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ measures the rate of change of the gradient function

Differentiate $\frac{dy}{dx}$ again. If $\frac{dy}{dx}$ is rate of change of the function (ie. its gradient) then $\frac{d^2y}{dx^2}$ is the rate of change of the gradient.

4.7	Use of differentiation to find maxima and minima points on a curve	Determine the nature either by using increasing and decreasing functions or $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$

Stationary points occur when $\frac{dy}{dx} = 0$

Nature tables help us understand the **nature** of a stationary point:

- Maximum turning point
- Minimum turning point
- Rising point of inflection
- Falling point of inflection

4.8	Using calculus to find maxima and minima in simple problems	$V = 49x + \frac{81}{x}$ $\quad x > 0$ Use calculus to show that V has a minimum value and work out the minimum value of V

Finding stationary points and their nature let us sketch functions and hence state maximum and minimum values within a specified domain ($x > 0$ is common, especially for geometric problems).

4.9	Sketch/ interpret a curve with known maximum and minimum points	

As above.