Assignment 8

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10/30/2020

(1) Non-Homogeneous poisson process parameter

$$N(t) - Poisson(\lambda t)$$

,

$$Pr(N(t) = k) = e^{-\lambda t}\frac{(\lambda t^k)}{k!}$$

They constitute a Poisson process with rate λ(t) > 0 Mean Value is theoretically equal to:\

$$\int_0^5 \lambda(t)dt$$

which is equal to:\

$$\begin{aligned}
&= \frac{2}{3}t^{3/2} + \int_0^5 \sqrt{t} + e^{-t}sin(2\pi t)dt \\
&= \frac{2}{3}t^{3/2} + \frac{1}{2\pi}e^{-t} - cos(2\pi t) - \int_0^5 \sqrt{t} + e^{-t} - cos(2\pi t)dt \\
&= \frac{2}{3}t^{3/2} + \frac{1}{2\pi}e^{-t}(cos(2\pi t) + sin(2\pi t)) - \int_0^5 e^{-t}sin(2\pi t)dt
\end{aligned}$$

Then :\

$$\int_0^5 \lambda(t)dt = \frac{2}{3}t^{3/2} - \frac{1}{4\pi}(cos(2\pi t) + sin(2\pi t))|_0^5$$

=7.4536\ the parameter is 7.4536

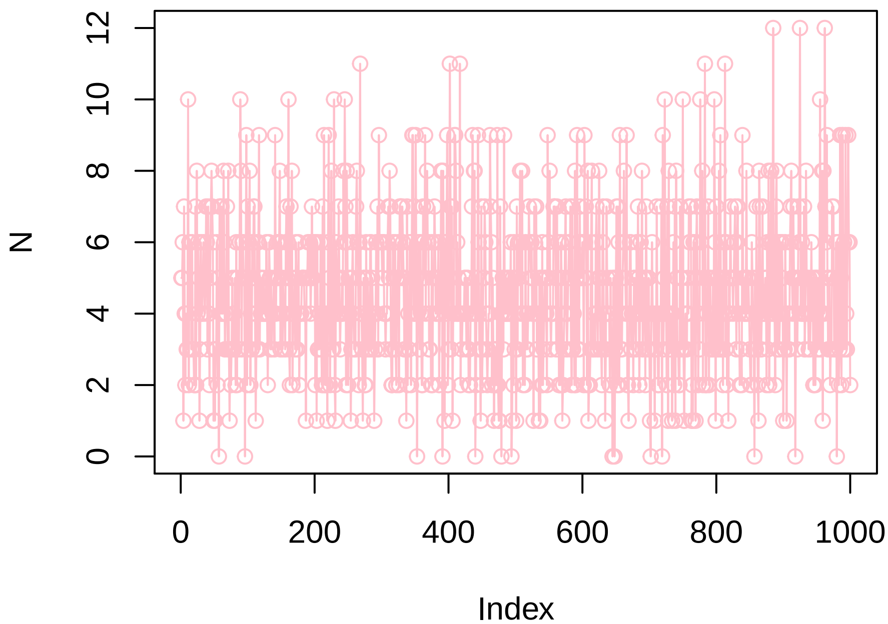
(2)function

1.Let t= 0 , I = 0
2.generate the random data (U)
3.t=t-log U/ λ if t>T break.
4.if U $\leq \lambda(t)/\lambda$ let I=I+1, S(I)=t;
5.or go back to step 2.

$\lambda(t) \leq \lambda = 1$

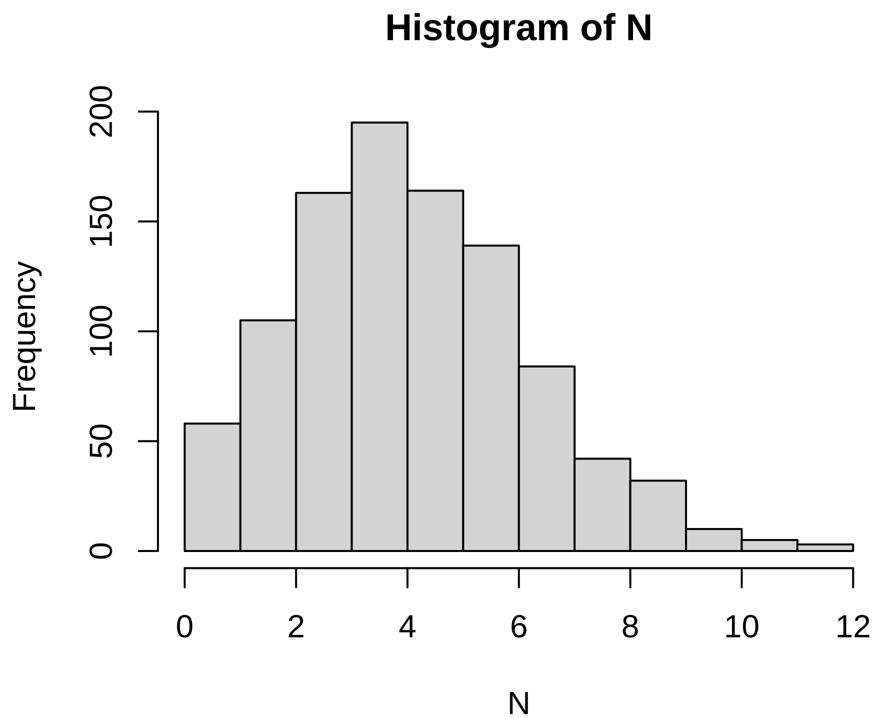
```
N<-0
for(i in 1:1000){
t<-0
```

```
I<-0
while(t<=5){
    u<-runif(1)
    t<-t-log(u)
    if(t<=5){
    u1<-runif(1)
    if(u1<=(t^.5 +exp(-t)*sin(2* pi* t))) I<-I+1
     }
     }
N[i]<-I
}
plot(N,type = "o", col = "pink")
```



simulation

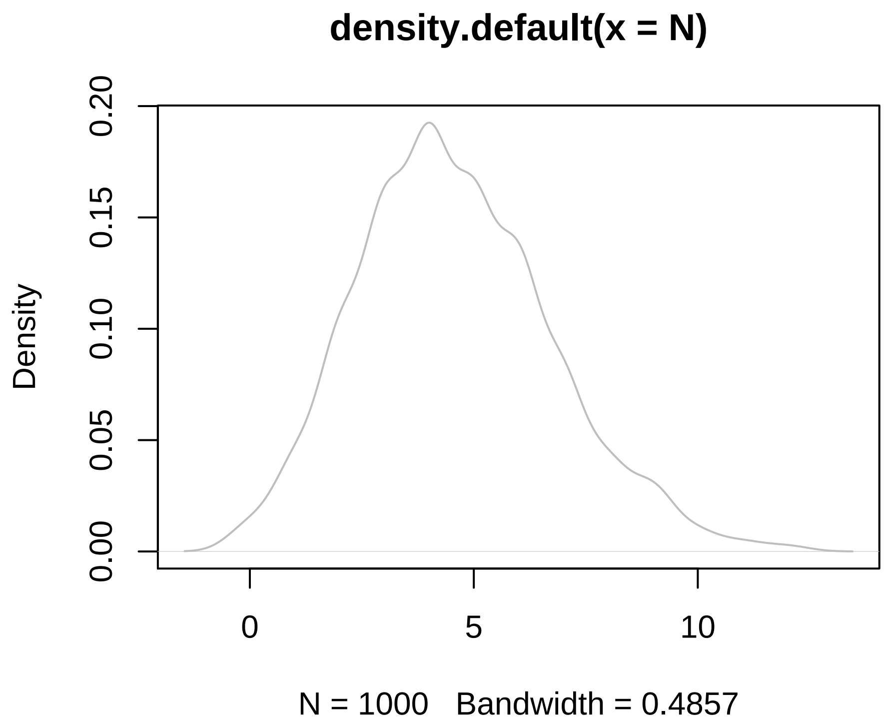
```
hist(N)
```




```
mean(N)
```

```
## [1] 4.58
```

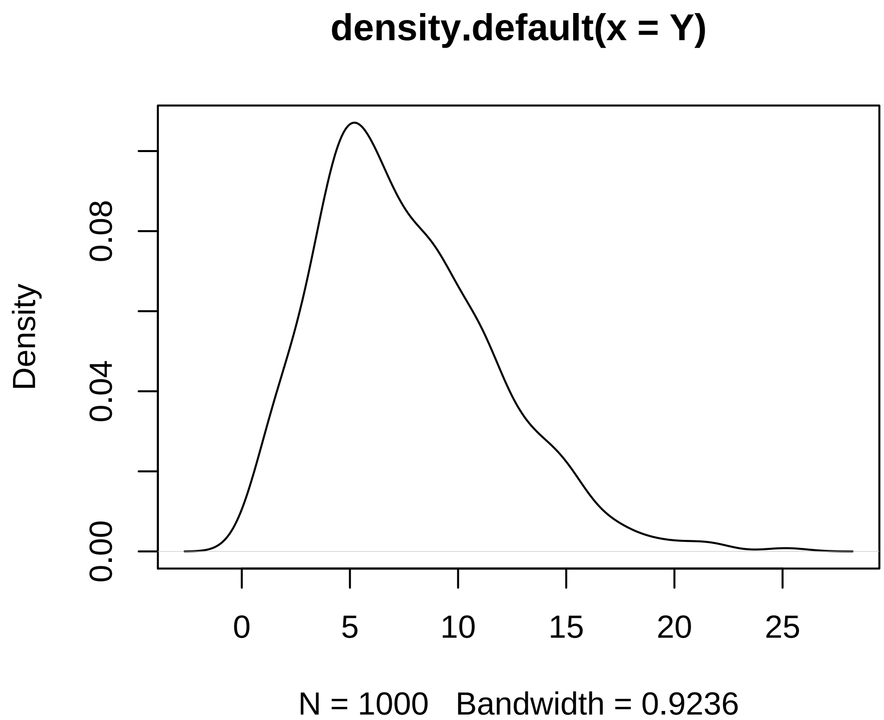
```
plot(density(N),col = "grey")
```




```
#kernel estimate
Y<-0
for (i in 1:1000){
N<-rpois(1,7.4536)
X<-rexp(N,1)
Y[i]<-sum(X)
}
mean(Y)
```

```
## [1] 7.584919
```

```
sp=spline(Y,n=1000)
plot(density(Y))
```



comparing the real simulation data of possion process and the kernel density above, we could see that the mean value of the times that events happened is around 4.749