HW2-Q2

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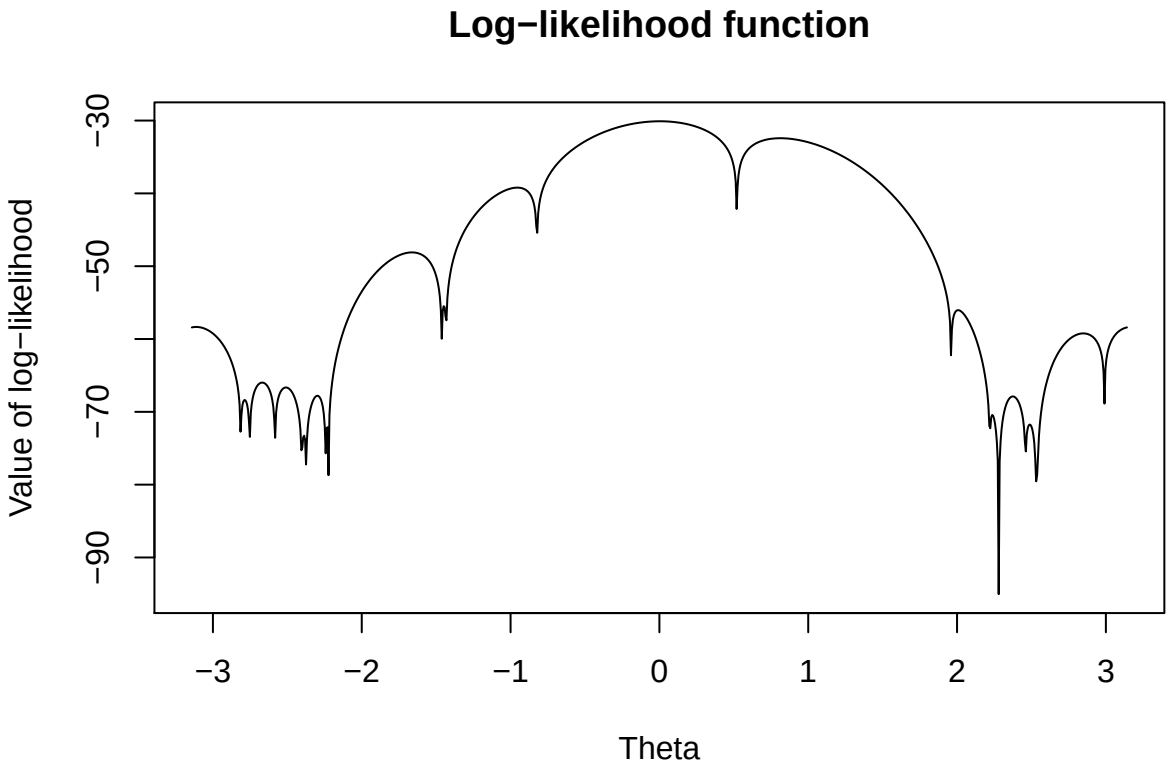
Q2 a



Q2 b

$$E[x|\theta] = \int_0^{2\pi} x\frac{1-cos(x-\theta)}{2\pi}$$

$$E[x|\theta] = \frac{1}{2\pi}[\int_0^{2\pi} xdx - \int_0^{2\pi} xcos(x-\theta)dx)]$$

$$E[x|\theta] = \pi - \frac{1}{2\pi}(xsin(x-\theta) + cos(x-\theta))|0^{2\pi}$$

$$E[x|\hat{\theta}moment] = \pi + sin(\hat{\theta}moment)$$

To solve for θ is equivalent to find a numerical solution to $\pi + sin(\hat{\theta}moment) - \bar{x} = 0$.

```
## [1] 0.09539388
```

```
## [1] 3.046199
```

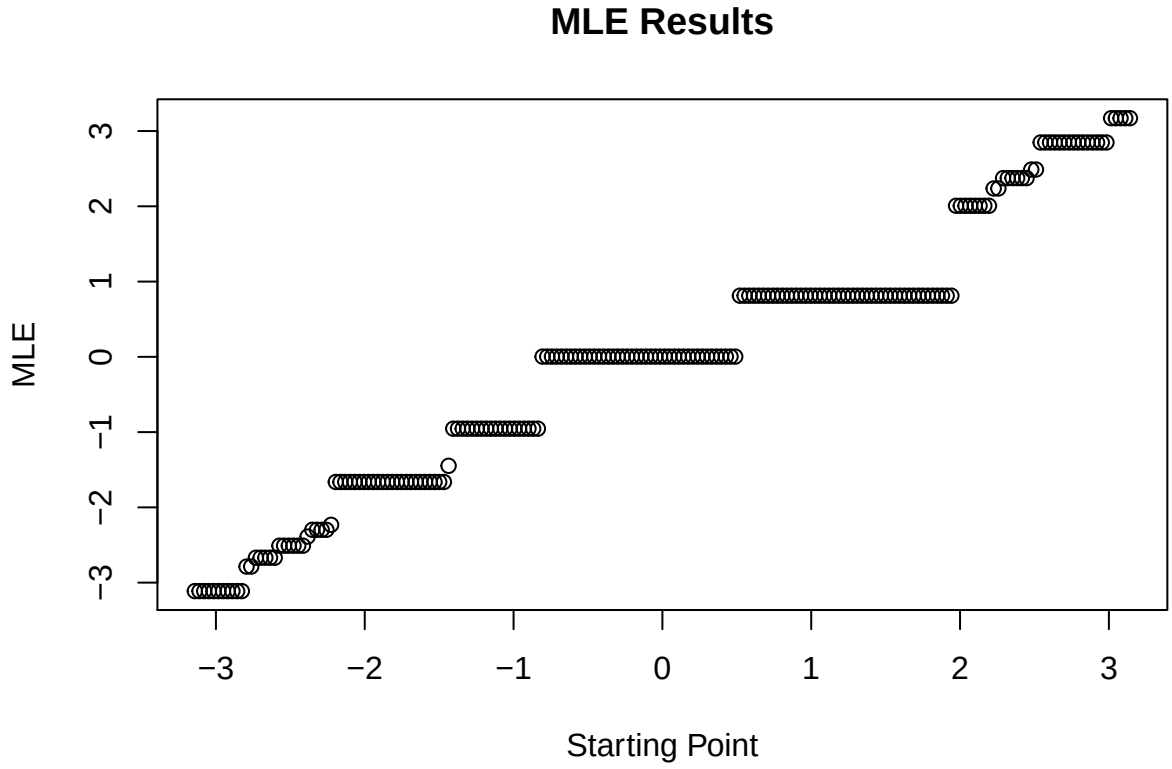
Q2 c

```
## [[1]]
## [1] 0.003118157
##
## [[2]]
## [1] 3.170715
```

Q2 d

```
## [[1]]
## [1] -2.668857
##
## [[2]]
## [1] 2.848415
```

Q2 e




```
##        Sequence Starting    MLE
##   [1,] 1        -3.141593   -3.112471
##   [2,] 2        -3.110019   -3.112471
##   [3,] 3        -3.078445   -3.112471
##   [4,] 4        -3.046871   -3.112471
```

```
##   [5,] 5        -3.015297   -3.112471
##   [6,] 6        -2.983724   -3.112471
##   [7,] 7        -2.95215    -3.112471
##   [8,] 8        -2.920576   -3.112471
##   [9,] 9        -2.889002   -3.112471
##  [10,] 10       -2.857428   -3.112471
##  [11,] 11       -2.825855   -3.112471
##  [12,] 12       -2.794281   -2.786557
##  [13,] 13       -2.762707   -2.786557
##  [14,] 14       -2.731133   -2.668857
##  [15,] 15       -2.69956    -2.668857
##  [16,] 16       -2.667986   -2.668857
##  [17,] 17       -2.636412   -2.668857
##  [18,] 18       -2.604838   -2.668857
##  [19,] 19       -2.573264   -2.509356
##  [20,] 20       -2.541691   -2.509356
##  [21,] 21       -2.510117   -2.509356
##  [22,] 22       -2.478543   -2.509356
##  [23,] 23       -2.446969   -2.509356
##  [24,] 24       -2.415395   -2.509356
##  [25,] 25       -2.383822   -2.388267
##  [26,] 26       -2.352248   -2.297926
##  [27,] 27       -2.320674   -2.297926
##  [28,] 28       -2.2891     -2.297926
##  [29,] 29       -2.257526   -2.297926
##  [30,] 30       -2.225953   -2.232192
##  [31,] 31       -2.194379   -1.662712
##  [32,] 32       -2.162805   -1.662712
##  [33,] 33       -2.131231   -1.662712
##  [34,] 34       -2.099657   -1.662712
##  [35,] 35       -2.068084   -1.662712
##  [36,] 36       -2.03651    -1.662712
##  [37,] 37       -2.004936   -1.662712
##  [38,] 38       -1.973362   -1.662712
##  [39,] 39       -1.941788   -1.662712
##  [40,] 40       -1.910215   -1.662712
##  [41,] 41       -1.878641   -1.662712
##  [42,] 42       -1.847067   -1.662712
##  [43,] 43       -1.815493   -1.662712
##  [44,] 44       -1.783919   -1.662712
##  [45,] 45       -1.752346   -1.662712
##  [46,] 46       -1.720772   -1.662712
##  [47,] 47       -1.689198   -1.662712
##  [48,] 48       -1.657624   -1.662712
##  [49,] 49       -1.62605    -1.662712
##  [50,] 50       -1.594477   -1.662712
##  [51,] 51       -1.562903   -1.662712
##  [52,] 52       -1.531329   -1.662712
```

```
##  [53,] 53       -1.499755   -1.662712
##  [54,] 54       -1.468181   -1.662712
##  [55,] 55       -1.436608   -1.447503
##  [56,] 56       -1.405034   -0.9544058
##  [57,] 57       -1.37346    -0.9544058
##  [58,] 58       -1.341886   -0.9544058
##  [59,] 59       -1.310313   -0.9544058
##  [60,] 60       -1.278739   -0.9544058
##  [61,] 61       -1.247165   -0.9544058
##  [62,] 62       -1.215591   -0.9544058
##  [63,] 63       -1.184017   -0.9544058
##  [64,] 64       -1.152444   -0.9544058
##  [65,] 65       -1.12087    -0.9544058
##  [66,] 66       -1.089296   -0.9544058
##  [67,] 67       -1.057722   -0.9544058
##  [68,] 68       -1.026148   -0.9544058
##  [69,] 69       -0.9945746  -0.9544058
##  [70,] 70       -0.9630008  -0.9544058
##  [71,] 71       -0.931427   -0.9544058
##  [72,] 72       -0.8998532  -0.9544058
##  [73,] 73       -0.8682794  -0.9544058
##  [74,] 74       -0.8367056  -0.9544058
##  [75,] 75       -0.8051318  0.003118157
##  [76,] 76       -0.773558   0.003118158
##  [77,] 77       -0.7419842  0.003118157
##  [78,] 78       -0.7104104  0.003118157
##  [79,] 79       -0.6788366  0.003118164
##  [80,] 80       -0.6472628  0.003118157
##  [81,] 81       -0.615689   0.003118157
##  [82,] 82       -0.5841152  0.003118165
##  [83,] 83       -0.5525414  0.003118157
##  [84,] 84       -0.5209676  0.003118157
##  [85,] 85       -0.4893938  0.003118157
##  [86,] 86       -0.45782    0.003118157
##  [87,] 87       -0.4262462  0.003118157
##  [88,] 88       -0.3946724  0.003118157
##  [89,] 89       -0.3630986  0.003118157
##  [90,] 90       -0.3315249  0.003118157
##  [91,] 91       -0.2999511  0.003118175
##  [92,] 92       -0.2683773  0.003118159
##  [93,] 93       -0.2368035  0.003118157
##  [94,] 94       -0.2052297  0.003118157
##  [95,] 95       -0.1736559  0.003118157
##  [96,] 96       -0.1420821  0.003118157
##  [97,] 97       -0.1105083  0.003118157
##  [98,] 98       -0.07893449 0.003118157
##  [99,] 99       -0.04736069 0.003118157
## [100,] 100      -0.0157869  0.003118167
```

```
## [101,] 101      0.0157869   0.00311816
## [102,] 102      0.04736069  0.003118157
## [103,] 103      0.07893449  0.003118157
## [104,] 104      0.1105083   0.003118158
## [105,] 105      0.1420821   0.003118167
## [106,] 106      0.1736559   0.003118157
## [107,] 107      0.2052297   0.003118157
## [108,] 108      0.2368035   0.003118157
## [109,] 109      0.2683773   0.003118157
## [110,] 110      0.2999511   0.003118161
## [111,] 111      0.3315249   0.003118157
## [112,] 112      0.3630986   0.003118157
## [113,] 113      0.3946724   0.003118173
## [114,] 114      0.4262462   0.003118157
## [115,] 115      0.45782     0.003118157
## [116,] 116      0.4893938   0.003118157
## [117,] 117      0.5209676   0.8126374
## [118,] 118      0.5525414   0.8126374
## [119,] 119      0.5841152   0.8126374
## [120,] 120      0.615689    0.8126374
## [121,] 121      0.6472628   0.8126374
## [122,] 122      0.6788366   0.8126374
## [123,] 123      0.7104104   0.8126374
## [124,] 124      0.7419842   0.8126374
## [125,] 125      0.773558    0.8126374
## [126,] 126      0.8051318   0.8126374
## [127,] 127      0.8367056   0.8126374
## [128,] 128      0.8682794   0.8126374
## [129,] 129      0.8998532   0.8126374
## [130,] 130      0.931427    0.8126374
## [131,] 131      0.9630008   0.8126374
## [132,] 132      0.9945746   0.8126374
## [133,] 133      1.026148    0.8126374
## [134,] 134      1.057722    0.8126374
## [135,] 135      1.089296    0.8126374
## [136,] 136      1.12087     0.8126374
## [137,] 137      1.152444    0.8126374
## [138,] 138      1.184017    0.8126374
## [139,] 139      1.215591    0.8126374
## [140,] 140      1.247165    0.8126374
## [141,] 141      1.278739    0.8126374
## [142,] 142      1.310313    0.8126374
## [143,] 143      1.341886    0.8126374
## [144,] 144      1.37346     0.8126374
## [145,] 145      1.405034    0.8126374
## [146,] 146      1.436608    0.8126374
## [147,] 147      1.468181    0.8126374
## [148,] 148      1.499755    0.8126374
```

```
## [149,] 149      1.531329    0.8126374
## [150,] 150      1.562903    0.8126374
## [151,] 151      1.594477    0.8126374
## [152,] 152      1.62605     0.8126374
## [153,] 153      1.657624    0.8126374
## [154,] 154      1.689198    0.8126374
## [155,] 155      1.720772    0.8126374
## [156,] 156      1.752346    0.8126374
## [157,] 157      1.783919    0.8126374
## [158,] 158      1.815493    0.8126374
## [159,] 159      1.847067    0.8126374
## [160,] 160      1.878641    0.8126374
## [161,] 161      1.910215    0.8126374
## [162,] 162      1.941788    0.8126374
## [163,] 163      1.973362    2.007223
## [164,] 164      2.004936    2.007223
## [165,] 165      2.03651     2.007223
## [166,] 166      2.068084    2.007223
## [167,] 167      2.099657    2.007223
## [168,] 168      2.131231    2.007223
## [169,] 169      2.162805    2.007223
## [170,] 170      2.194379    2.007223
## [171,] 171      2.225953    2.237013
## [172,] 172      2.257526    2.237013
## [173,] 173      2.2891      2.374712
## [174,] 174      2.320674    2.374712
## [175,] 175      2.352248    2.374712
## [176,] 176      2.383822    2.374712
## [177,] 177      2.415395    2.374712
## [178,] 178      2.446969    2.374712
## [179,] 179      2.478543    2.48845
## [180,] 180      2.510117    2.48845
## [181,] 181      2.541691    2.848415
## [182,] 182      2.573264    2.848415
## [183,] 183      2.604838    2.848415
## [184,] 184      2.636412    2.848415
## [185,] 185      2.667986    2.848415
## [186,] 186      2.69956     2.848415
## [187,] 187      2.731133    2.848415
## [188,] 188      2.762707    2.848415
## [189,] 189      2.794281    2.848415
## [190,] 190      2.825855    2.848415
## [191,] 191      2.857428    2.848415
## [192,] 192      2.889002    2.848415
## [193,] 193      2.920576    2.848415
## [194,] 194      2.95215     2.848415
## [195,] 195      2.983724    2.848415
## [196,] 196      3.015297    3.170715
```

```
## [197,] 197      3.046871    3.170715
## [198,] 198      3.078445    3.170715
## [199,] 199      3.110019    3.170715
## [200,] 200      3.141593    3.170715
```

```
##       Sequence Starting  MLE
##  [1,] 1        -3.141593 -3.112471
##  [2,] 2        -3.110019 -3.112471
##  [3,] 3        -3.078445 -3.112471
##  [4,] 4        -3.046871 -3.112471
##  [5,] 5        -3.015297 -3.112471
##  [6,] 6        -2.983724 -3.112471
##  [7,] 7        -2.95215  -3.112471
##  [8,] 8        -2.920576 -3.112471
##  [9,] 9        -2.889002 -3.112471
## [10,] 10       -2.857428 -3.112471
## [11,] 11       -2.825855 -3.112471
```

```
##      Sequence Starting  MLE
## [1,] 12       -2.794281 -2.786557
## [2,] 13       -2.762707 -2.786557
```

```
##      Sequence Starting  MLE
## [1,] 14       -2.731133 -2.668857
## [2,] 15       -2.69956  -2.668857
## [3,] 16       -2.667986 -2.668857
## [4,] 17       -2.636412 -2.668857
## [5,] 18       -2.604838 -2.668857
```

```
##      Sequence Starting  MLE
## [1,] 19       -2.573264 -2.509356
## [2,] 20       -2.541691 -2.509356
## [3,] 21       -2.510117 -2.509356
## [4,] 22       -2.478543 -2.509356
## [5,] 23       -2.446969 -2.509356
## [6,] 24       -2.415395 -2.509356
```

```
##      Sequence Starting  MLE
## [1,] 25       -2.383822 -2.388267
```

```
##      Sequence Starting  MLE
## [1,] 26       -2.352248 -2.297926
## [2,] 27       -2.320674 -2.297926
## [3,] 28       -2.2891   -2.297926
## [4,] 29       -2.257526 -2.297926
```

```
##      Sequence Starting  MLE
## [1,] 30       -2.225953 -2.232192
```

```
##       Sequence Starting  MLE
##  [1,] 31       -2.194379 -1.662712
##  [2,] 32       -2.162805 -1.662712
```

```
##  [3,] 33       -2.131231 -1.662712
##  [4,] 34       -2.099657 -1.662712
##  [5,] 35       -2.068084 -1.662712
##  [6,] 36       -2.03651  -1.662712
##  [7,] 37       -2.004936 -1.662712
##  [8,] 38       -1.973362 -1.662712
##  [9,] 39       -1.941788 -1.662712
## [10,] 40       -1.910215 -1.662712
## [11,] 41       -1.878641 -1.662712
## [12,] 42       -1.847067 -1.662712
## [13,] 43       -1.815493 -1.662712
## [14,] 44       -1.783919 -1.662712
## [15,] 45       -1.752346 -1.662712
## [16,] 46       -1.720772 -1.662712
## [17,] 47       -1.689198 -1.662712
## [18,] 48       -1.657624 -1.662712
## [19,] 49       -1.62605  -1.662712
## [20,] 50       -1.594477 -1.662712
## [21,] 51       -1.562903 -1.662712
## [22,] 52       -1.531329 -1.662712
## [23,] 53       -1.499755 -1.662712
## [24,] 54       -1.468181 -1.662712
```

```
##      Sequence Starting  MLE
## [1,] 55       -1.436608 -1.447503
```

```
##       Sequence Starting   MLE
##  [1,] 56       -1.405034  -0.9544058
##  [2,] 57       -1.37346   -0.9544058
##  [3,] 58       -1.341886  -0.9544058
##  [4,] 59       -1.310313  -0.9544058
##  [5,] 60       -1.278739  -0.9544058
##  [6,] 61       -1.247165  -0.9544058
##  [7,] 62       -1.215591  -0.9544058
##  [8,] 63       -1.184017  -0.9544058
##  [9,] 64       -1.152444  -0.9544058
## [10,] 65       -1.12087   -0.9544058
## [11,] 66       -1.089296  -0.9544058
## [12,] 67       -1.057722  -0.9544058
## [13,] 68       -1.026148  -0.9544058
## [14,] 69       -0.9945746 -0.9544058
## [15,] 70       -0.9630008 -0.9544058
## [16,] 71       -0.931427  -0.9544058
## [17,] 72       -0.8998532 -0.9544058
## [18,] 73       -0.8682794 -0.9544058
## [19,] 74       -0.8367056 -0.9544058
```

```
##       Sequence Starting    MLE
##  [1,] 75       -0.8051318  0.003118157
```

```
##  [2,] 76       -0.773558   0.003118158
##  [3,] 77       -0.7419842  0.003118157
##  [4,] 78       -0.7104104  0.003118157
##  [5,] 79       -0.6788366  0.003118164
##  [6,] 80       -0.6472628  0.003118157
##  [7,] 81       -0.615689   0.003118157
##  [8,] 82       -0.5841152  0.003118165
##  [9,] 83       -0.5525414  0.003118157
## [10,] 84       -0.5209676  0.003118157
## [11,] 85       -0.4893938  0.003118157
## [12,] 86       -0.45782    0.003118157
## [13,] 87       -0.4262462  0.003118157
## [14,] 88       -0.3946724  0.003118157
## [15,] 89       -0.3630986  0.003118157
## [16,] 90       -0.3315249  0.003118157
## [17,] 91       -0.2999511  0.003118175
## [18,] 92       -0.2683773  0.003118159
## [19,] 93       -0.2368035  0.003118157
## [20,] 94       -0.2052297  0.003118157
## [21,] 95       -0.1736559  0.003118157
## [22,] 96       -0.1420821  0.003118157
## [23,] 97       -0.1105083  0.003118157
## [24,] 98       -0.07893449 0.003118157
## [25,] 99       -0.04736069 0.003118157
## [26,] 100      -0.0157869  0.003118167
## [27,] 101      0.0157869   0.00311816
## [28,] 102      0.04736069  0.003118157
## [29,] 103      0.07893449  0.003118157
## [30,] 104      0.1105083   0.003118158
## [31,] 105      0.1420821   0.003118167
## [32,] 106      0.1736559   0.003118157
## [33,] 107      0.2052297   0.003118157
## [34,] 108      0.2368035   0.003118157
## [35,] 109      0.2683773   0.003118157
## [36,] 110      0.2999511   0.003118161
## [37,] 111      0.3315249   0.003118157
## [38,] 112      0.3630986   0.003118157
## [39,] 113      0.3946724   0.003118173
## [40,] 114      0.4262462   0.003118157
## [41,] 115      0.45782     0.003118157
## [42,] 116      0.4893938   0.003118157
```

```
##       Sequence Starting  MLE
##  [1,] 117      0.5209676 0.8126374
##  [2,] 118      0.5525414 0.8126374
##  [3,] 119      0.5841152 0.8126374
##  [4,] 120      0.615689  0.8126374
##  [5,] 121      0.6472628 0.8126374
```

```
##  [6,] 122      0.6788366 0.8126374
##  [7,] 123      0.7104104 0.8126374
##  [8,] 124      0.7419842 0.8126374
##  [9,] 125      0.773558  0.8126374
## [10,] 126      0.8051318 0.8126374
## [11,] 127      0.8367056 0.8126374
## [12,] 128      0.8682794 0.8126374
## [13,] 129      0.8998532 0.8126374
## [14,] 130      0.931427  0.8126374
## [15,] 131      0.9630008 0.8126374
## [16,] 132      0.9945746 0.8126374
## [17,] 133      1.026148  0.8126374
## [18,] 134      1.057722  0.8126374
## [19,] 135      1.089296  0.8126374
## [20,] 136      1.12087   0.8126374
## [21,] 137      1.152444  0.8126374
## [22,] 138      1.184017  0.8126374
## [23,] 139      1.215591  0.8126374
## [24,] 140      1.247165  0.8126374
## [25,] 141      1.278739  0.8126374
## [26,] 142      1.310313  0.8126374
## [27,] 143      1.341886  0.8126374
## [28,] 144      1.37346   0.8126374
## [29,] 145      1.405034  0.8126374
## [30,] 146      1.436608  0.8126374
## [31,] 147      1.468181  0.8126374
## [32,] 148      1.499755  0.8126374
## [33,] 149      1.531329  0.8126374
## [34,] 150      1.562903  0.8126374
## [35,] 151      1.594477  0.8126374
## [36,] 152      1.62605   0.8126374
## [37,] 153      1.657624  0.8126374
## [38,] 154      1.689198  0.8126374
## [39,] 155      1.720772  0.8126374
## [40,] 156      1.752346  0.8126374
## [41,] 157      1.783919  0.8126374
## [42,] 158      1.815493  0.8126374
## [43,] 159      1.847067  0.8126374
## [44,] 160      1.878641  0.8126374
## [45,] 161      1.910215  0.8126374
## [46,] 162      1.941788  0.8126374
```

```
##      Sequence Starting MLE
## [1,] 163      1.973362 2.007223
## [2,] 164      2.004936 2.007223
## [3,] 165      2.03651  2.007223
## [4,] 166      2.068084 2.007223
## [5,] 167      2.099657 2.007223
```

```
## [6,] 168      2.131231 2.007223
## [7,] 169      2.162805 2.007223
## [8,] 170      2.194379 2.007223
```

```
##      Sequence Starting MLE
## [1,] 171      2.225953 2.237013
## [2,] 172      2.257526 2.237013
```

```
##      Sequence Starting MLE
## [1,] 173      2.2891   2.374712
## [2,] 174      2.320674 2.374712
## [3,] 175      2.352248 2.374712
## [4,] 176      2.383822 2.374712
## [5,] 177      2.415395 2.374712
## [6,] 178      2.446969 2.374712
```

```
##      Sequence Starting MLE
## [1,] 179      2.478543 2.48845
## [2,] 180      2.510117 2.48845
```

```
##       Sequence Starting MLE
##  [1,] 181      2.541691 2.848415
##  [2,] 182      2.573264 2.848415
##  [3,] 183      2.604838 2.848415
##  [4,] 184      2.636412 2.848415
##  [5,] 185      2.667986 2.848415
##  [6,] 186      2.69956  2.848415
##  [7,] 187      2.731133 2.848415
##  [8,] 188      2.762707 2.848415
##  [9,] 189      2.794281 2.848415
## [10,] 190      2.825855 2.848415
## [11,] 191      2.857428 2.848415
## [12,] 192      2.889002 2.848415
## [13,] 193      2.920576 2.848415
## [14,] 194      2.95215  2.848415
## [15,] 195      2.983724 2.848415
```

```
##      Sequence Starting MLE
## [1,] 196      3.015297 3.170715
## [2,] 197      3.046871 3.170715
## [3,] 198      3.078445 3.170715
## [4,] 199      3.110019 3.170715
## [5,] 200      3.141593 3.170715
```