STAT 5361 Interim Resport

A Simple and Effective Inequality Measure

Chamundeswari Koppisetti* Shynggys Magzanov†

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Abstract

We'll write it later

Introduction

1) Examples of inequality curves and coefficients of inequality for some common income distributions and compare values of I with G.
2) We introduce empirical versions of these concepts and investigate their inferential properties, including robustness to outliers.
3) Applications to income data.

4. Application

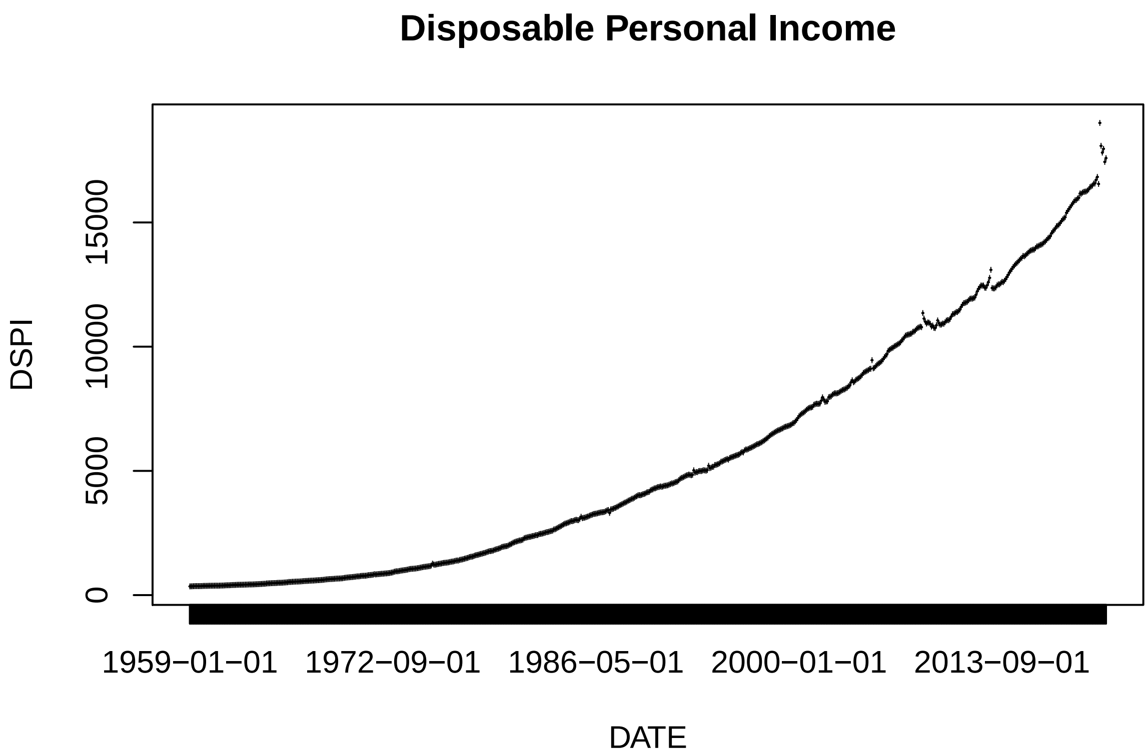
4.1. Disposable Personal Income

The data of Disposable Personal Income can be obtained at https://fred.stlouisfed.org/series/DSPI.

```
x <- read.csv("C:\\Users\\Asus\\Downloads\\DSPI.csv")
plot(x, type = "l", main = "Disposable Personal Income")
```

*chamundeswari.koppisetti@uconn.edu

†shynggys.magzanov@uconn.edu




```r
QOR.ln <- function(u){
  # QOR function for the log-normal
  q.n.0 <- 1/dnorm(qnorm(u))
  q.n.1 <- qnorm(u)*q.n.0^2
  q.n.2 <- (1 + 2*qnorm(u)^2)*q.n.0^3
  1/(q.n.0^2 + 3*q.n.1 + q.n.2/q.n.0)
}

QOR.gld <- function(u, lambda = NULL){
  # QOR function for the GLD FKML paramaterization
  if(is.null(lambda)) lambda <- fit.fkml(x)$lambda
  l3 <- lambda[3]
  l4 <- lambda[4]
  (u^(l3 - 1) + (1 - u)^(l4 - 1))/(u^(l3 - 3)*(l3 - 2)*(l3 - 1) +
                                    (1 - u)^(l4 - 3)*(l4 - 2)*(l4 - 1))
}

hatI <- function(x, J = 100, conf.level = 0.95, bw.correct = TRUE, QOR.FUN = QOR.ln, ...){

  n <- length(x)

  us <- ((1:J) - 0.5)/J
  Rs <- (xu2 <- quantile(x, us/2))/(x1u2 <- quantile(x, 1 - us/2))
```

```r
  I <- sum(1 - Rs)/J

  if(!is.null(conf.level)){
    v <- c(us/2, 1 - us/2)
    qor <- QOR.FUN(v, ...)
    bw <- 15^(1/5)*abs(qor)^(2/5)/n^(1/5)
    if (bw.correct) bw[v <= bw] <- v[v <= bw]

    kernepach <- function(u) 3/4*(1 - u^2)*(abs(u) <= 1)
    m1 <- matrix(v, nrow = 2*J, ncol = n, byrow = FALSE)
    m2 <- matrix(1:n, nrow = 2*J, ncol = n, byrow = TRUE)

    consts <- kernepach((m1 - (m2 - 1)/n)*(1/bw))*(1/bw) -
      kernepach((m1 - m2/n)*(1/bw))*(1/bw)

    x.sorted <- sort(x)
    q.hat <- c(consts%*%x.sorted)
    q.hat.1 <- q.hat[1:(length(q.hat)/2)]
    q.hat.2 <- q.hat[-(1:(length(q.hat)/2))]

    rc <- matrix(Rs, ncol = J, nrow = J, byrow = FALSE)

    covm <- ((1/x1u2)%*%t(1/x1u2))*(((us/2)%*%t(1 - us/2))*(q.hat.1%*%t(q.hat.1) + Rs%*%t(Rs)*
                                      ((us/2)%*%t(us/2))*((q.hat.1%*%t(q.hat.2))*t(rc) + (q.ha

    sigma.p2 <- (us/2)*(1 - us/2)*q.hat.1^2
    sigma.q2 <- (1 - us/2)*(us/2)*q.hat.2^2
    sigma.pq <- (us/2)^2*q.hat.1*q.hat.2
    a0 <- sigma.p2/x1u2^2
    a1 <- -2*sigma.pq/x1u2^2
    a2 <- sigma.q2/x1u2^2
    Vs <- (a0 + a1*Rs + a2*Rs^2)/n

    V <- (sum(Vs) + 2*sum(covm[row(covm) < col(covm)]))/J^2
    SE <- sqrt(V)
    conf.int <- I + c(-1, 1)*qnorm(1 - (1 - conf.level)/2)*sqrt(V)
  } else{
    V <- NULL
    SE <- NULL
    conf.int <- NULL
  }

  list(I = I, SE = SE, conf.int = conf.int)
}

hatG <- function(x, conf.level = 0.95){
```

```r
  indices <- 1:(n <- length(x))
  ordered.x <- sort(x)
  sx <- sum(ordered.x*(indices - 1/2))
  mu.hat <- mean(x)
  Gv <- 2/mu.hat/n^2*sx - 1

  Z.hat <- -(Gv + 1)*ordered.x + (2*indices - 1)/n*ordered.x - 2/n*cumsum(ordered.x)
  Z.bar <- mean(Z.hat)

  V <- 1/n^2/mu.hat^2*sum((Z.hat - Z.bar)^2)
  conf.int <- Gv + c(-1, 1)*qnorm(1 - (1 - conf.level)/2)*sqrt(V)

  list(G = Gv, SE = sqrt(V), conf.int = conf.int)
}
```

```r
y <- x$DSPI
I <- hatI(y)
I
```

```
## $I
## [1] 0.7560689
##
## $SE
## [1] 0.01064179
##
## $conf.int
## [1] 0.7352114 0.7769264
```

```r
G <- hatG(y)
G
```

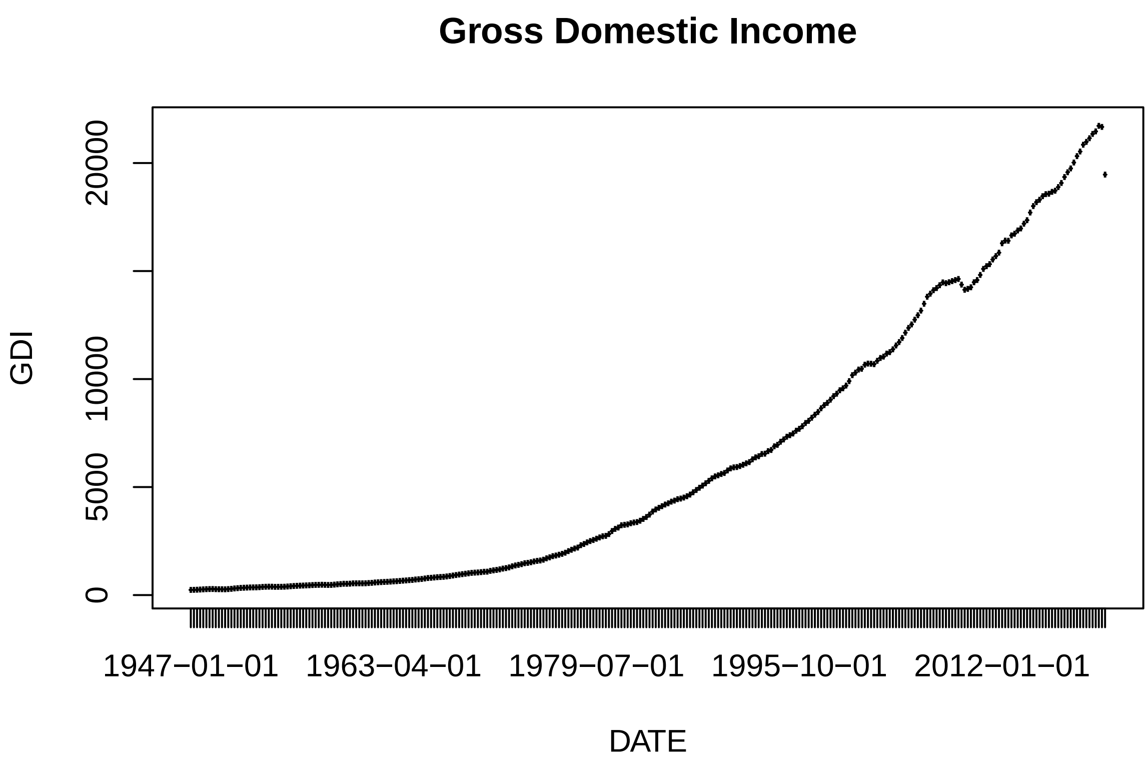
```
## $G
## [1] 0.487252
##
## $SE
## [1] 0.009500346
##
## $conf.int
## [1] 0.4686317 0.5058723
```

Firstly, I and G were calculated for the Disposable Personal Income Data. It appears that $\hat{I} = 0.7561$, which is an estimate of I, with a standard error of 0.0106. Additionally, 95% confidence interval of $\hat{I}$ was calculated to be $(0.7352, 0.7769)$. While $\hat{G} = 0.4872$ is significantly lower, with a standard error of 0.0095 and respective 95% confidence interval of $(0.4686, 0.5059)$.

4.2. Gross Domestic Income

The data on Gross Domestic Income can be found here: https://fred.stlouisfed.org/series/GDI.

```r
GDI <- read.csv("C:\\Users\\Asus\\Downloads\\GDI.csv")
plot(GDI, type = "l", main = "Gross Domestic Income")
```




```r
GDI <- GDI$GDI
I_gdi <- hatI(GDI)
I_gdi
```

```
## $I
## [1] 0.8204182
##
## $SE
## [1] 0.01624378
##
## $conf.int
## [1] 0.7885809 0.8522554
```

```r
G_gdi <- hatG(GDI)
G_gdi
```

```
## $G
## [1] 0.549877
##
## $SE
## [1] 0.01613956
```

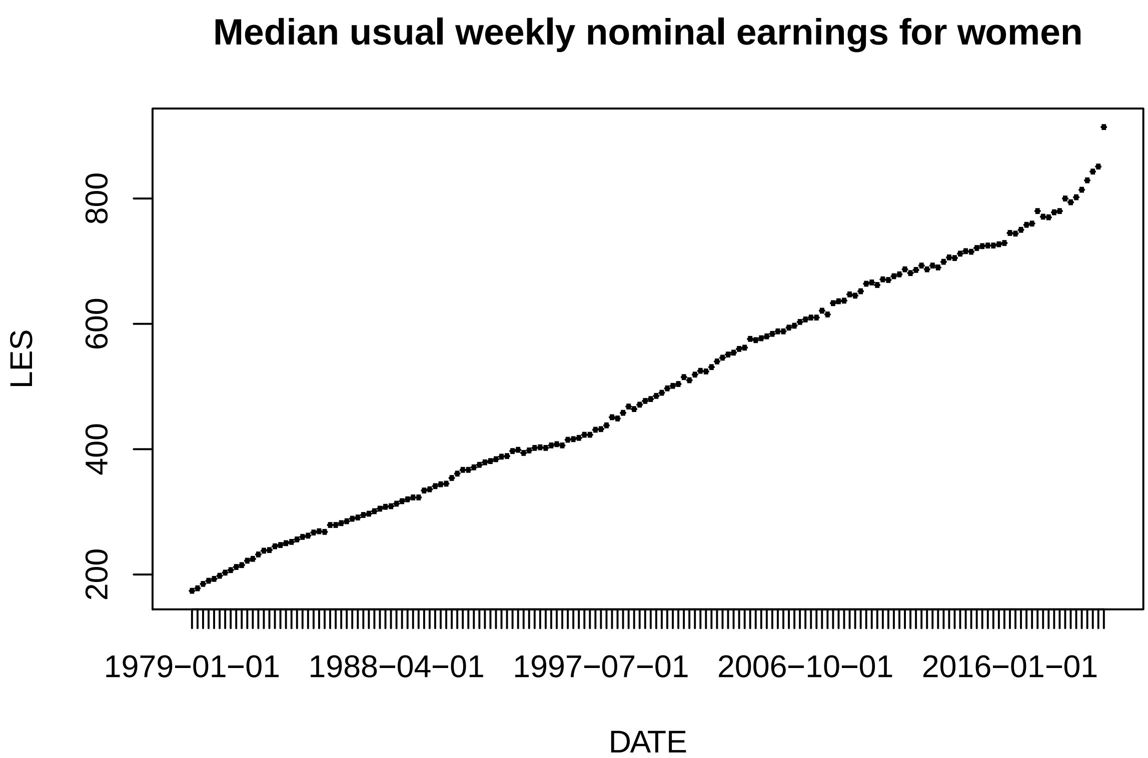
```
##
## $conf.int
## [1] 0.5182440 0.5815099
```

As a result, ratio of symmetric quantiles of the GDI is $\hat{I} = 0.8204$ with a standard error 0.0162 and confidence interval $(0.7886, 0.8523)$. At the same time $\hat{G} = 0.5499$ with a standard error of 0.0161 and CI $(0.5182, 0.5815)$.

4.3. Earnings data of women

The data on Earnings of women in the US between 1979 and 2020 can be otbained from: https://fred.stlouisfed.org/series/LES1252882700Q.

```
LES <- read.csv("C:\\Users\\Asus\\Downloads\\LES.csv")
plot(LES, type = "l", main = "Median usual weekly nominal earnings for women")
```




```
LES <- LES$LES
I_les <- hatI(LES)
I_les
```

```
## $I
## [1] 0.4665993
##
## $SE
## [1] 0.01831842
```

```
##
## $conf.int
## [1] 0.4306959 0.5025028
```

```r
G_les <- hatG(LES)
G_les
```

```
## $G
## [1] 0.2199384
##
## $SE
## [1] 0.009415175
##
## $conf.int
## [1] 0.2014850 0.2383918
```

Results reveal that $\hat{I} = 0.4666$ with a standard error 0.0183 and CI $(0.4307, 0.5025)$. While Gini coefficient is $\hat{G} = 0.2199 \pm 0.0094$ with CI $(0.2015, 0.2384)$.