5361_stat_hw_Qi_You

Question 1b

Equations

$$
\begin{align}
& l(\theta) \\
&= \ln \prod_{i=1}^{n} \frac{1}{\pi[1+(x-\theta)^2]} \\
&= \sum_{i=1}^{n} \ln \frac{1}{\pi[1+(x-\theta)^2]} \\
&= \sum_{i=1}^{n} [\ln \frac{1}{\pi} + \ln \frac{1}{1+(x-\theta)^2}] \\
&= -n \ln \pi - \sum_{i=1}^{n} \ln[1+(x-\theta)^2] \\
&
\end{align}
$$

$$
\begin{align}
& l'(\theta) \\
&= 0 - \sum_{i=1}^{n} \frac{2(\theta - x_i)}{1+(\theta-x_i)^2} \\
&= -2 \sum_{i=1}^{n} \frac{\theta - x_i}{1+(\theta-x_i)} \\
&
\end{align}
$$

$$
\begin{align}
& l''(\theta) \\
&= -2 \sum_{i=1}^{n} \frac{1+(\theta-x_i)^2 - 2(\theta-x_i)(\theta-x_i)}{[1+(\theta-x_i)^2]^2} \\
&= -2 \sum_{i=1}^{n} \frac{1-(\theta-x_i)^2}{[1+(\theta-x_i)^2]^2} \\
&
\end{align}
$$

We can set x to be tant(x). In this way we can compute the value of I easiliy

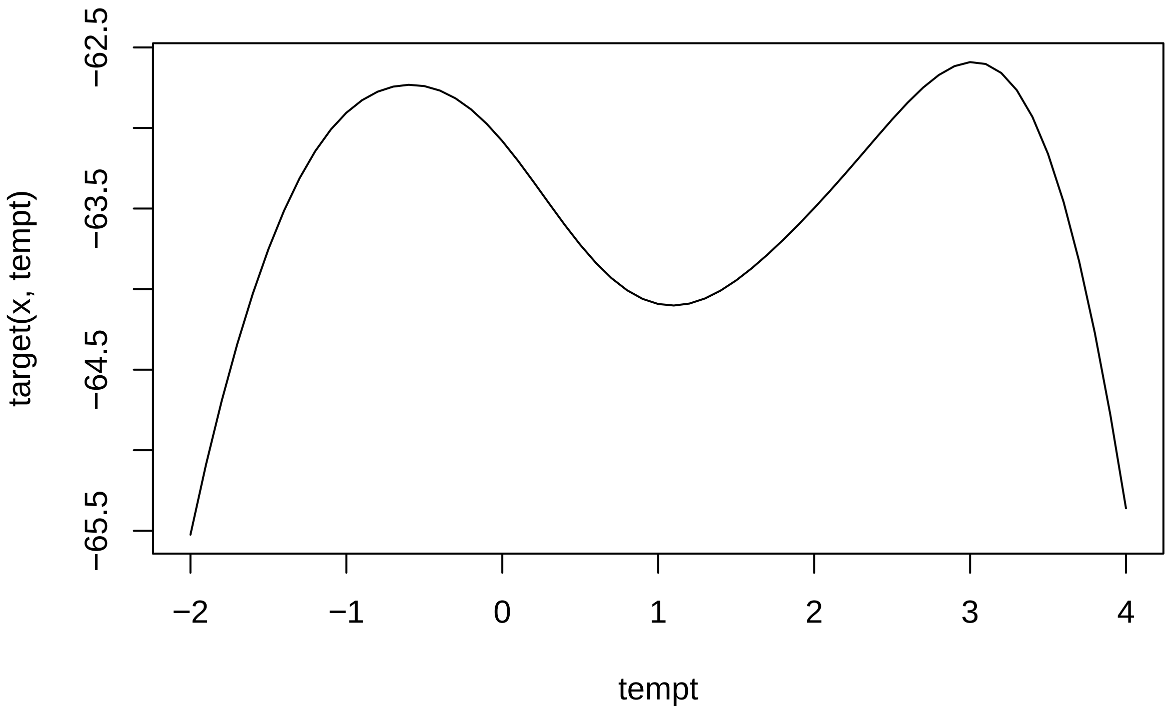
$$
\begin{aligned}
& I(\theta) && (15)\\
&= n\int \frac{\{p'(x)\}^2}{p(x)}dx && (16)\\
&= n\int \frac{4(x-\theta)^2}{\pi[1+(x-\theta)^2]^4} * \pi[1+(x-\theta)^2]dx && (17)\\
&= \frac{4n}{\pi}\int_{-\infty}^{\infty} \frac{(x-\theta)^2}{[1+(x-\theta)^2]^3}dx && (18)\\
&= \frac{4n}{\pi}\int_{-\infty}^{\infty} \frac{x^2}{(1+x^2)^3}dx && (19)\\
&= \frac{4n}{\pi}\int_{-\infty}^{\infty} [(\frac{1}{(1+x^2)^2} - \frac{1}{(1+x^2)^3})]dx && (20)\\
&= \frac{4n}{\pi}(\int_{-\infty}^{\infty} \frac{1}{(1+x^2)^2}dx - \int_{-\infty}^{\infty} \frac{1}{(1+x^2)^3}dx) && (21)\\
&= \frac{4n}{\pi}[\int_{-\infty}^{\infty} \frac{1}{(1+x^2)^2}dx - (\frac{x}{4(x^2+1)^2}|_{-\infty}^{\infty} + \frac{3}{4}\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2})] && (22)\\
&= \frac{4n}{\pi}(\int_{-\infty}^{\infty} \frac{1}{4(1+x^2)^2}dx - \frac{x}{4(x^2+1)^2}|_{-\infty}^{\infty}) && (23)\\
&= \frac{4n}{\pi}[\frac{1}{4}(\frac{x}{2(x^2+1)}|_{-\infty}^{\infty} + \frac{1}{2}\int_{-\infty}^{\infty} \frac{1}{1+x^2}dx) - \frac{x}{4(x^2+1)^2}|_{-\infty}^{\infty}] && (24)\\
&= \frac{4n}{\pi}(\frac{x(x^2-1)}{8(x^2+1)^2}|_{-\infty}^{\infty} + \frac{1}{8}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sec^2 t}{1+\tan^2 t}dt) && (25)\\
&= \frac{4n}{\pi}(0+\frac{\pi}{8}) && (26)\\
&= \frac{n}{2} && (27)\\
& && (28)
\end{aligned}
$$

Functions I setup

```
derivitive1 <- function(x,sita){
  value <- 0
  for (i in 1:length(x)){
    value <- value-2*(sita-x[i])/(1+(sita-x[i])^2)
  }
  return(value)
}

derivitive2 <- function(s,sita){
  value <- 0
  for (i in 1:length(x)){
    value <- value-2*((1-(sita-x[i])^2)/(1+(sita-x[i])^2)^2)
  }
  return(value)
}
```

Plots(1b)

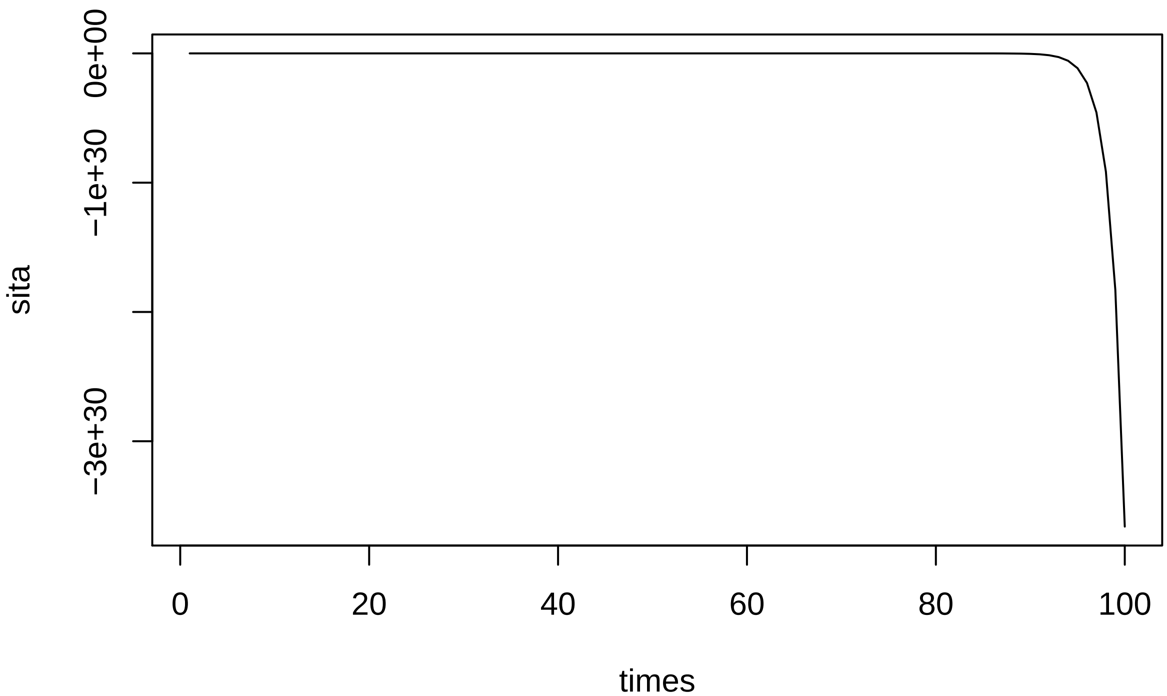
Here are the plots of my result



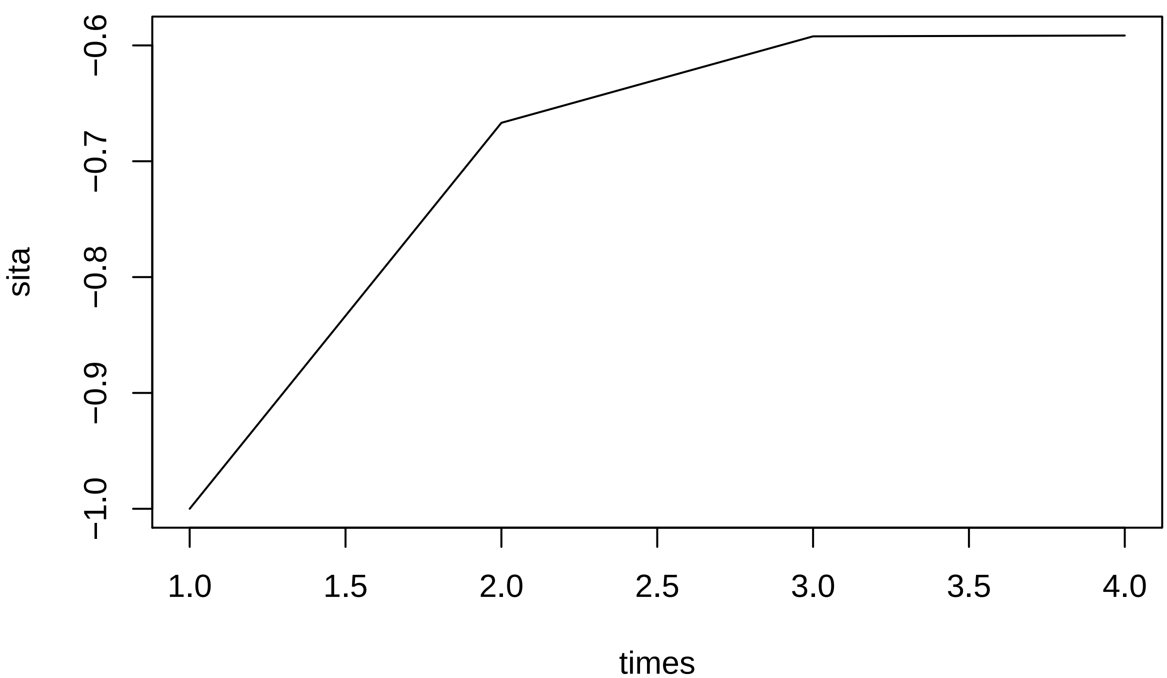
Here are the plots of mutiple start points

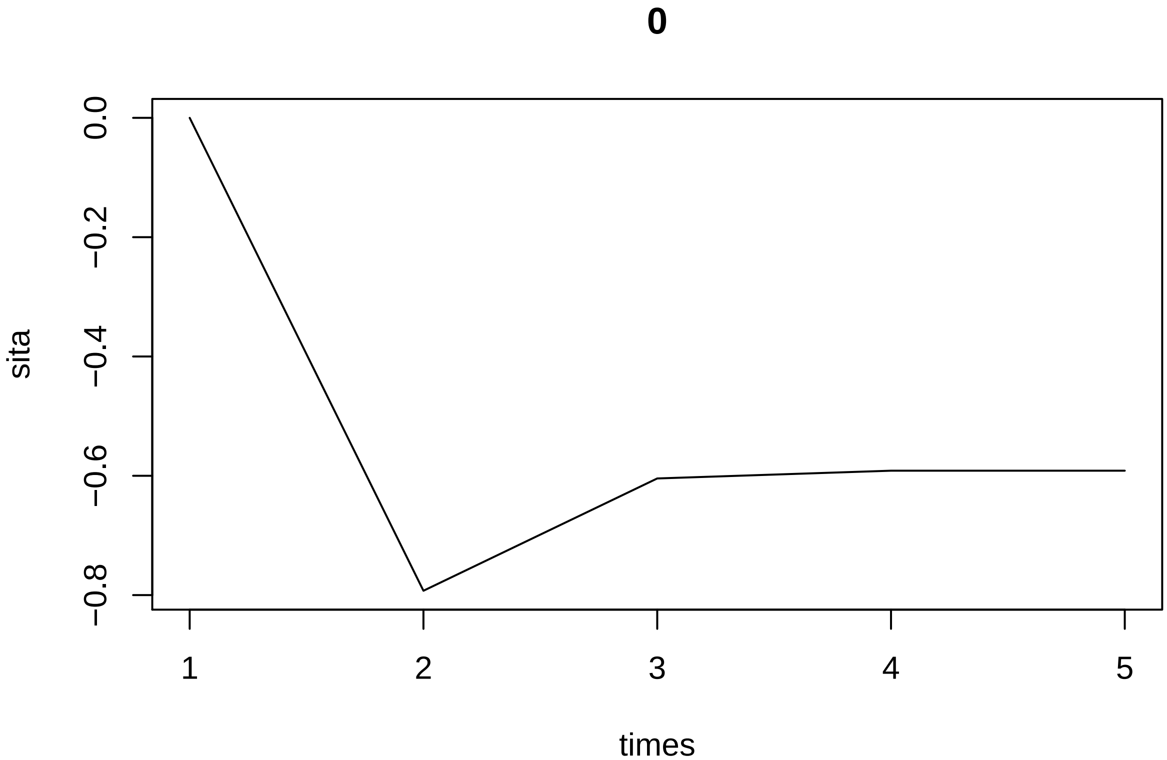
```
for(i in 1:length(start_points)){
  estimate(start_points[i])
}
```

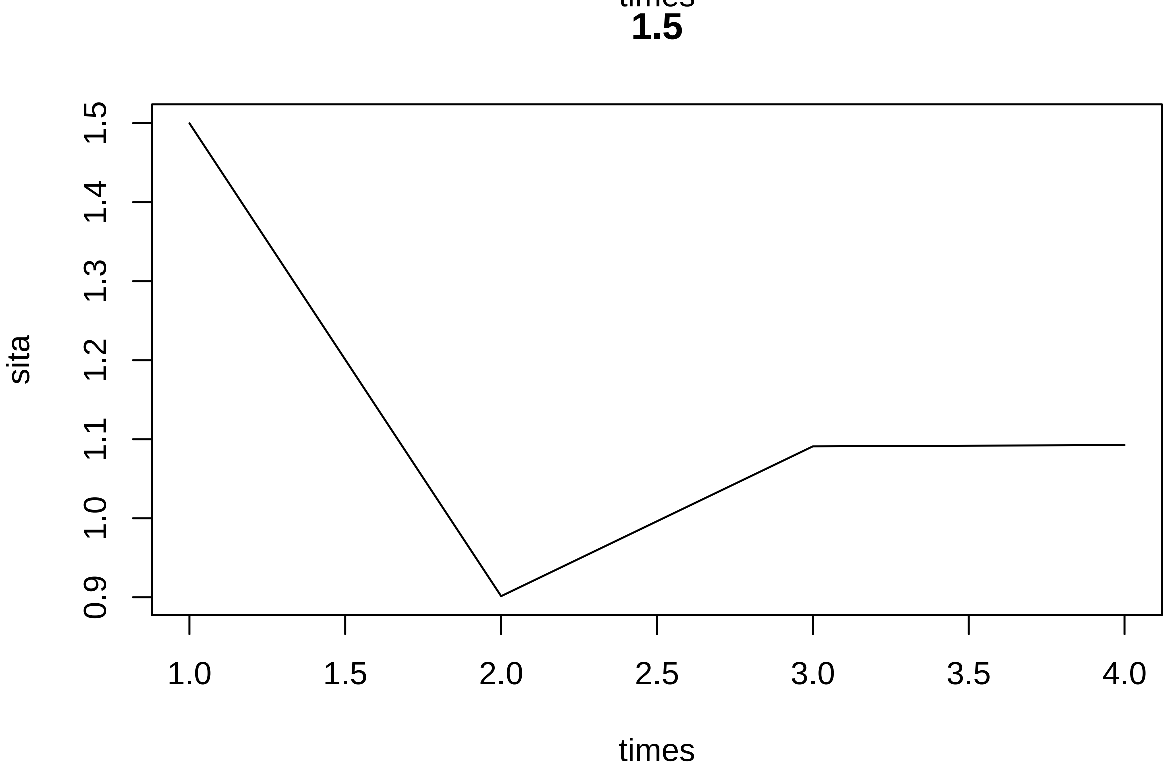
−11

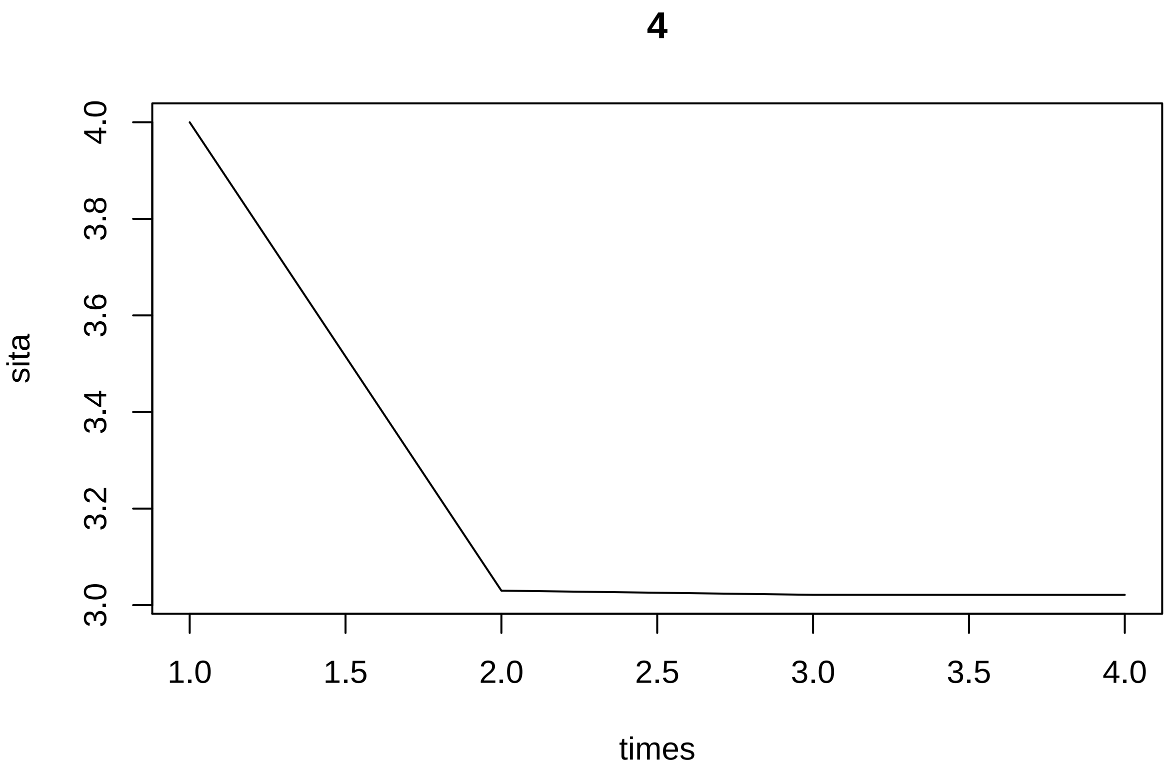

0e+00
−1e+30
−3e+30
sita
0
20
40
60
80
100
times

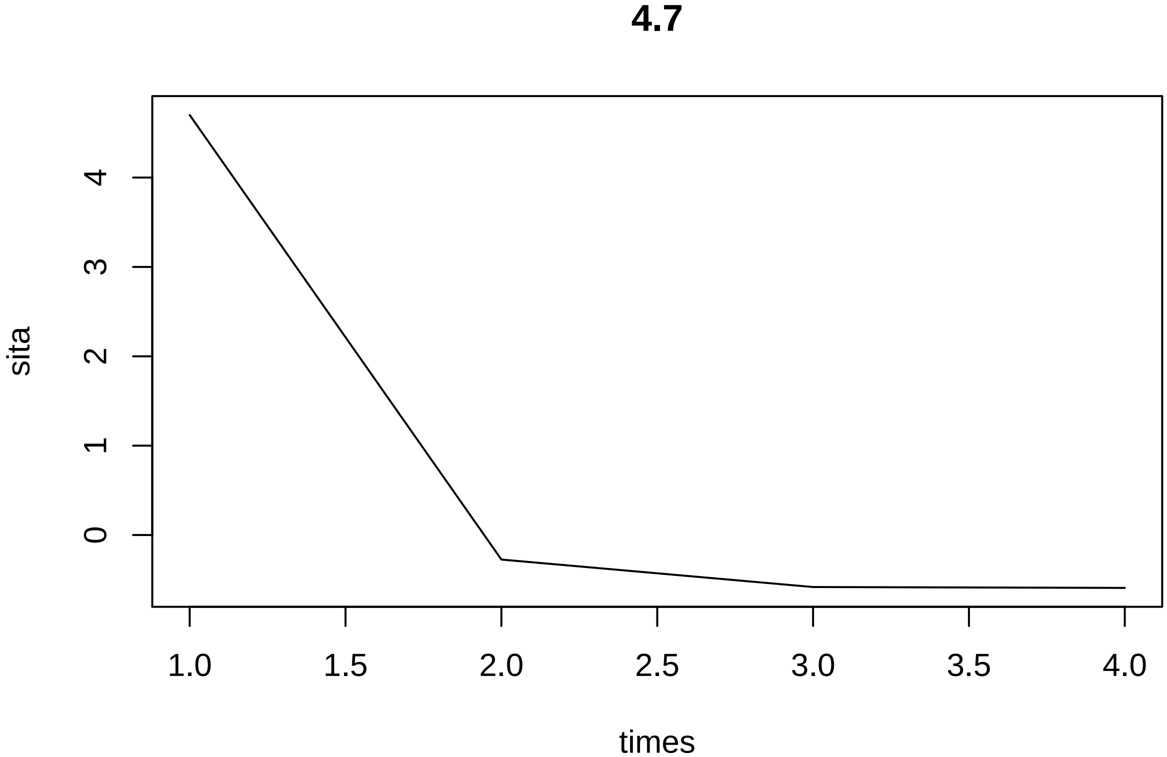
−1

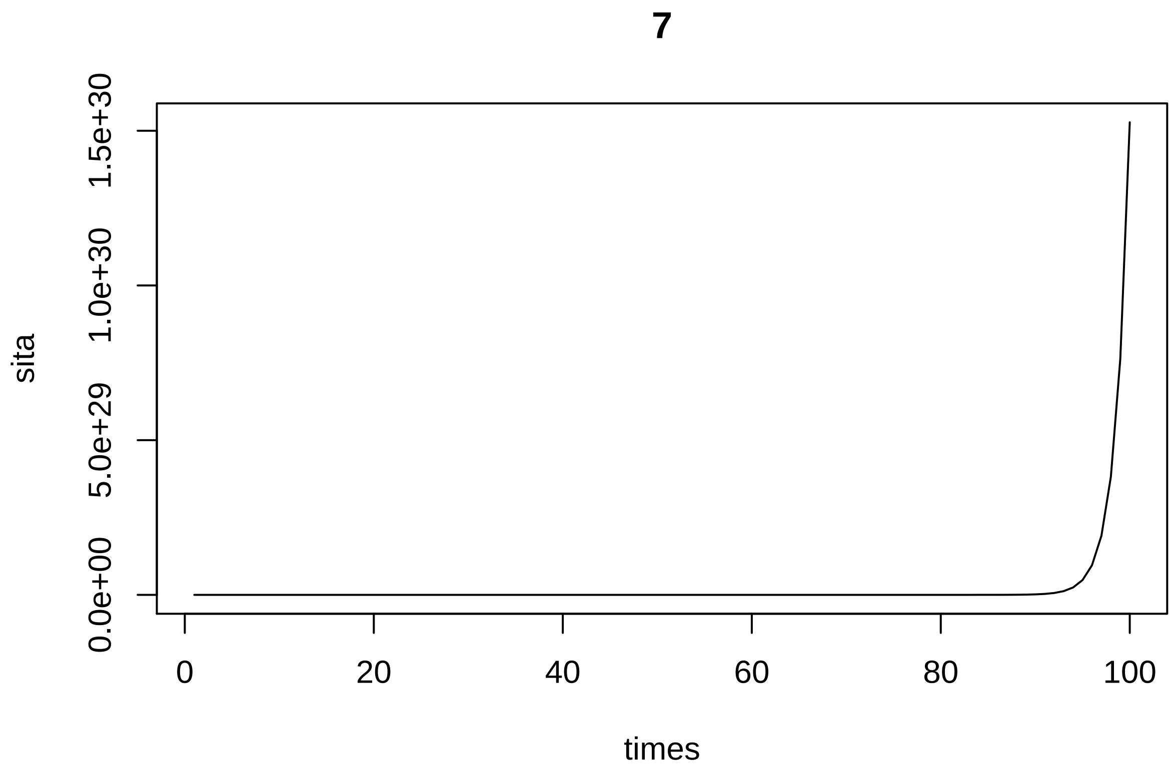

−0.6
−0.7
−0.8
−0.9
−1.0
sita
1.0
1.5
2.0
2.5
3.0
3.5
4.0
times

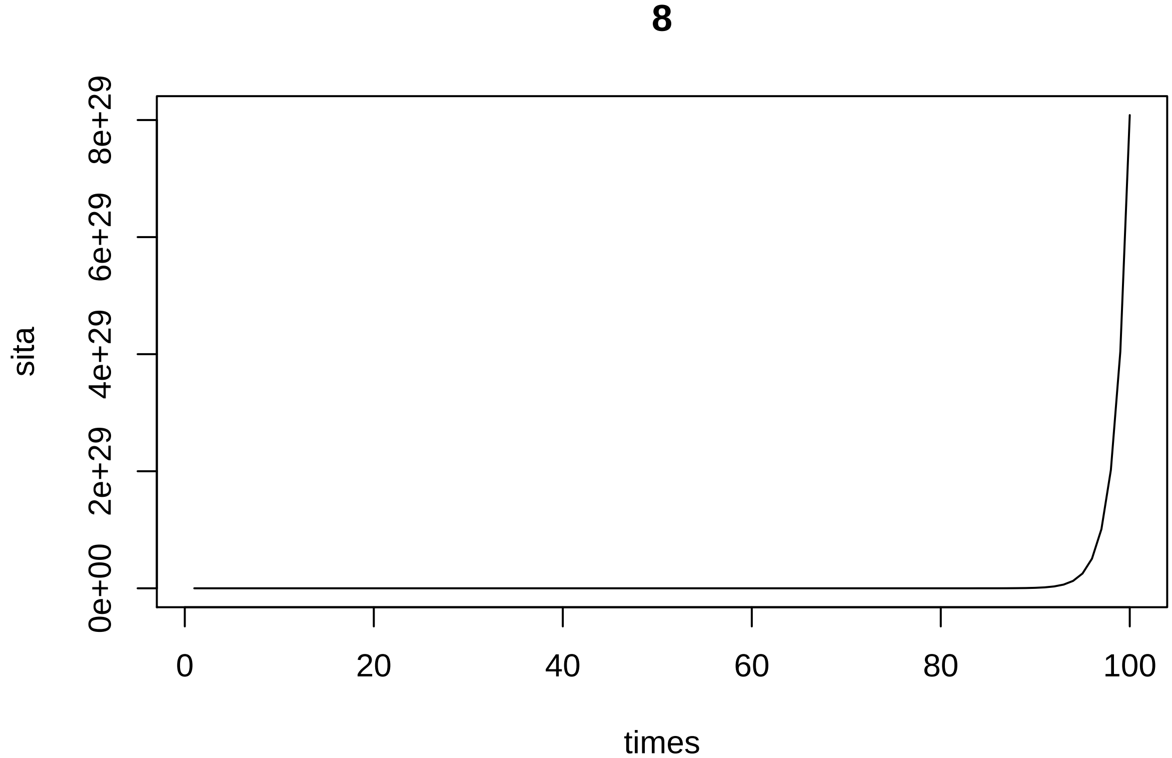

0
sita
0.0
−0.2
−0.4
−0.6
−0.8
1
2
3
4
5
times


1.5
sita
1.5
1.4
1.3
1.2
1.1
1.0
0.9
1.0
1.5
2.0
2.5
3.0
3.5
4.0
times


4
sita
4.0
3.8
3.6
3.4
3.2
3.0
1.0
1.5
2.0
2.5
3.0
3.5
4.0
times

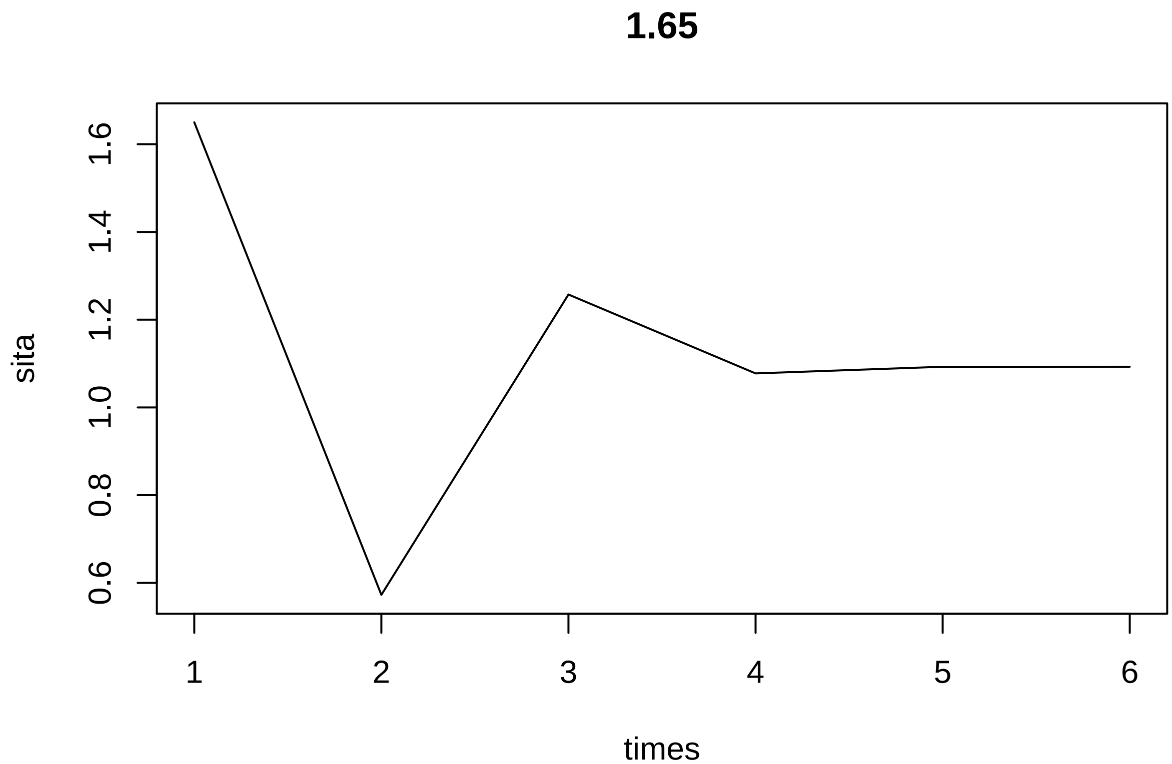

4.7
sita
4
3
2
1
0
1.0
1.5
2.0
2.5
3.0
3.5
4.0
times





This is the plot of mean

```
estimate(mean(start_points))
```



Question 1(c)

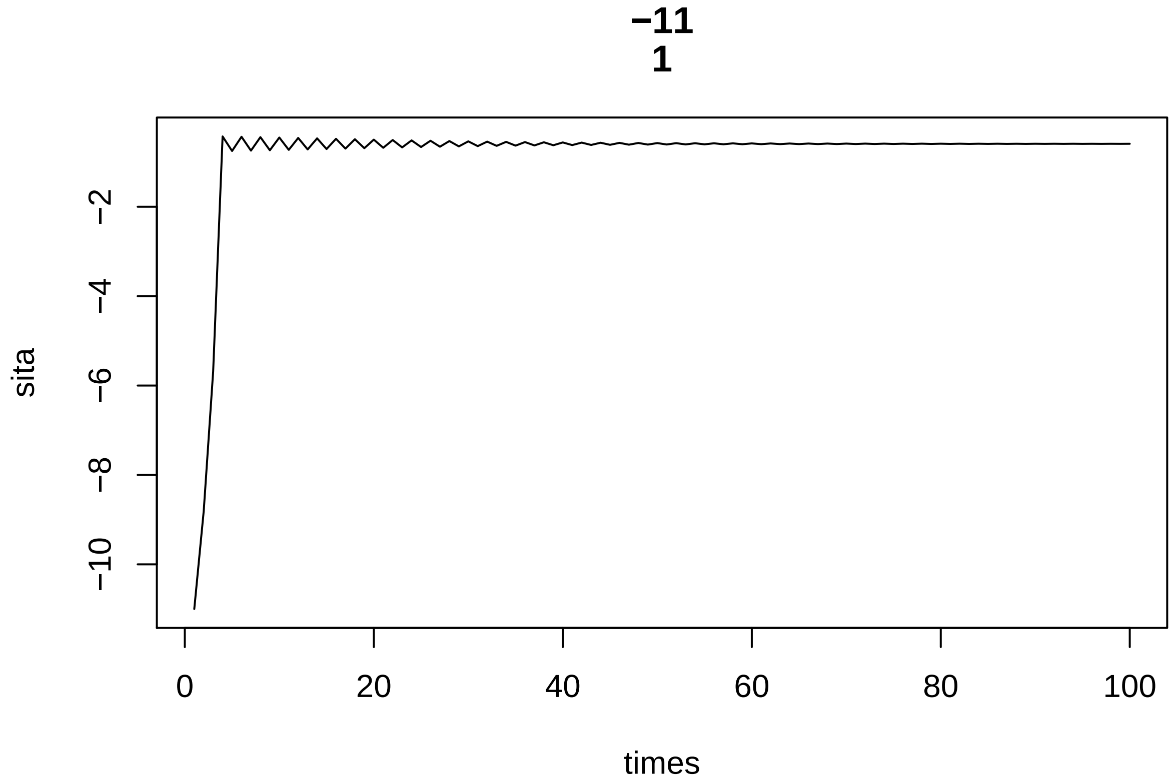
This time we use fixed points

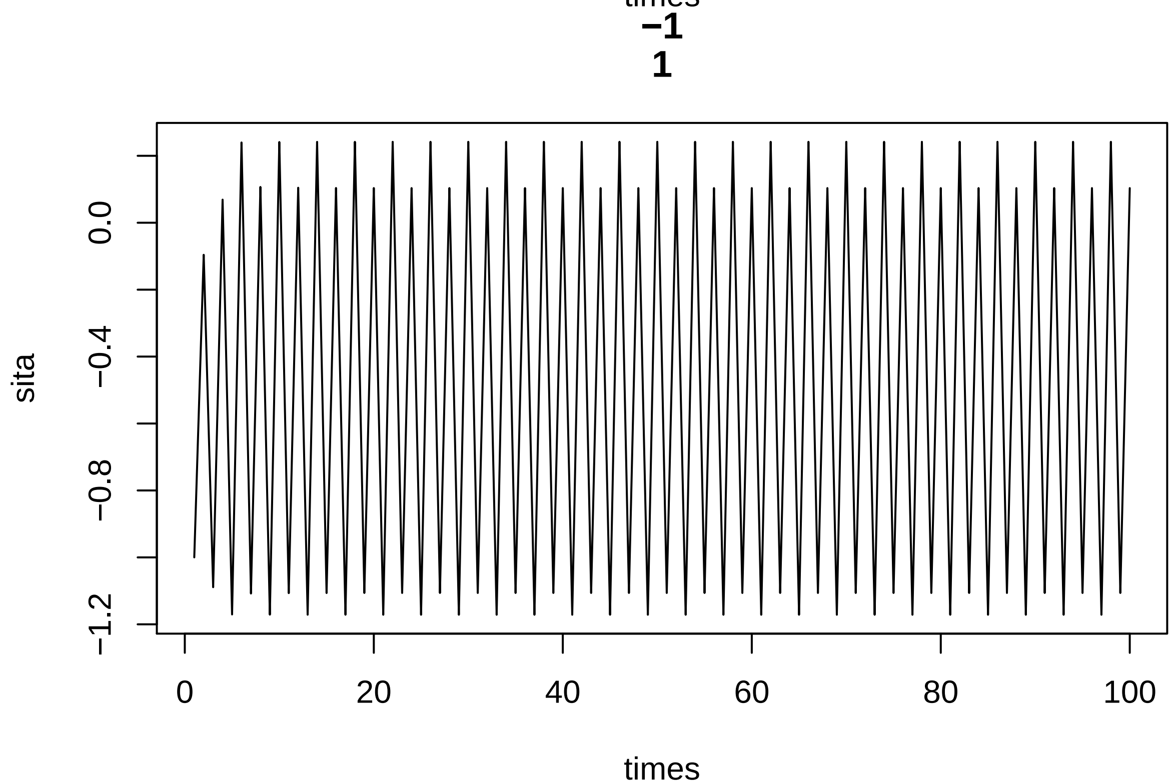
These are functions I established to compute derivition

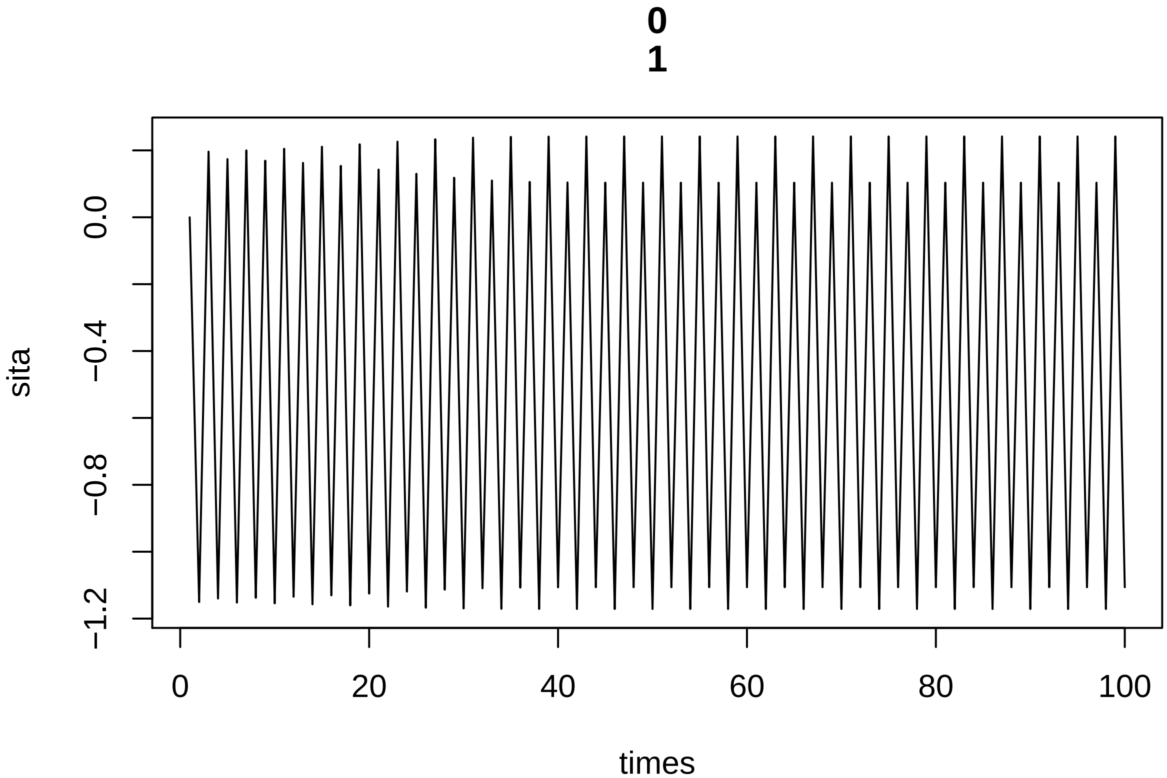
```
derivitive1 <- function(x,sita){
  value <- 0
  for (i in 1:length(x)){
    value <- value-2*(sita-x[i])/(1+(sita-x[i])^2)
  }
  return(value)
}

G <- function(alpha,x,sita){
  value <- alpha*derivitive1(x,sita)+sita
  return(value)
}
```

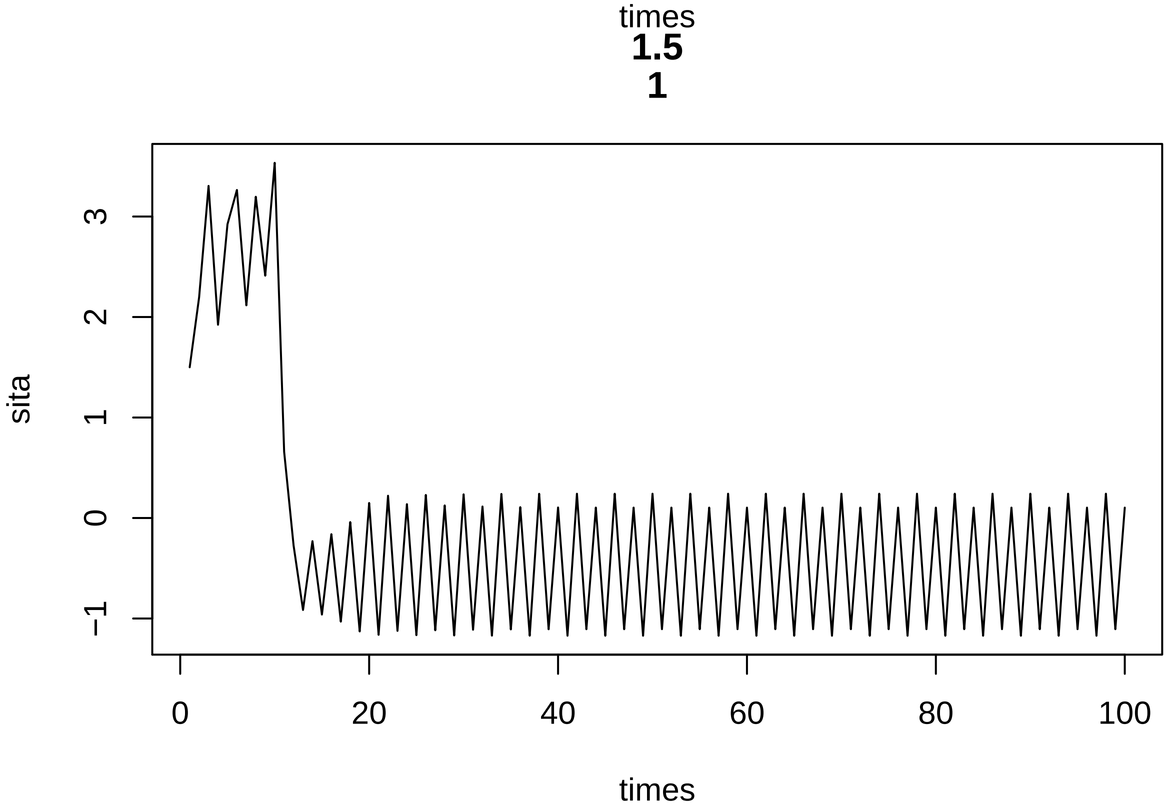
Here are plots1(c)

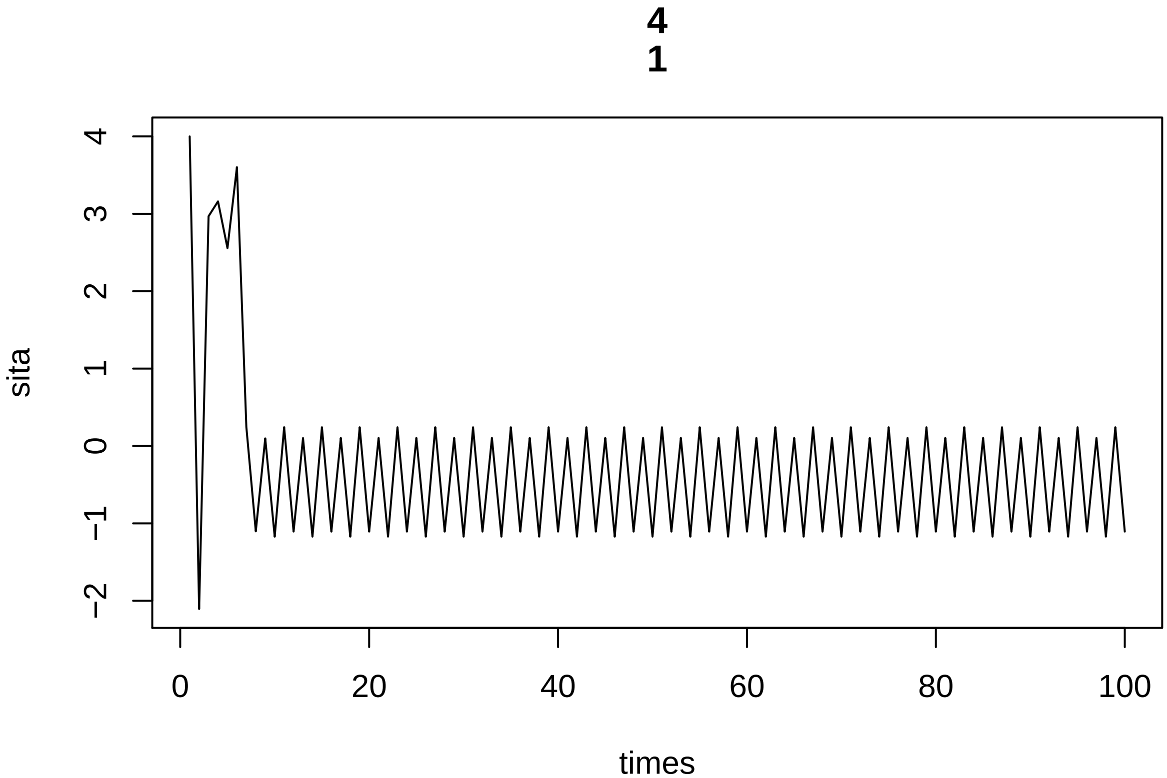
These plots are using different alphas and start points

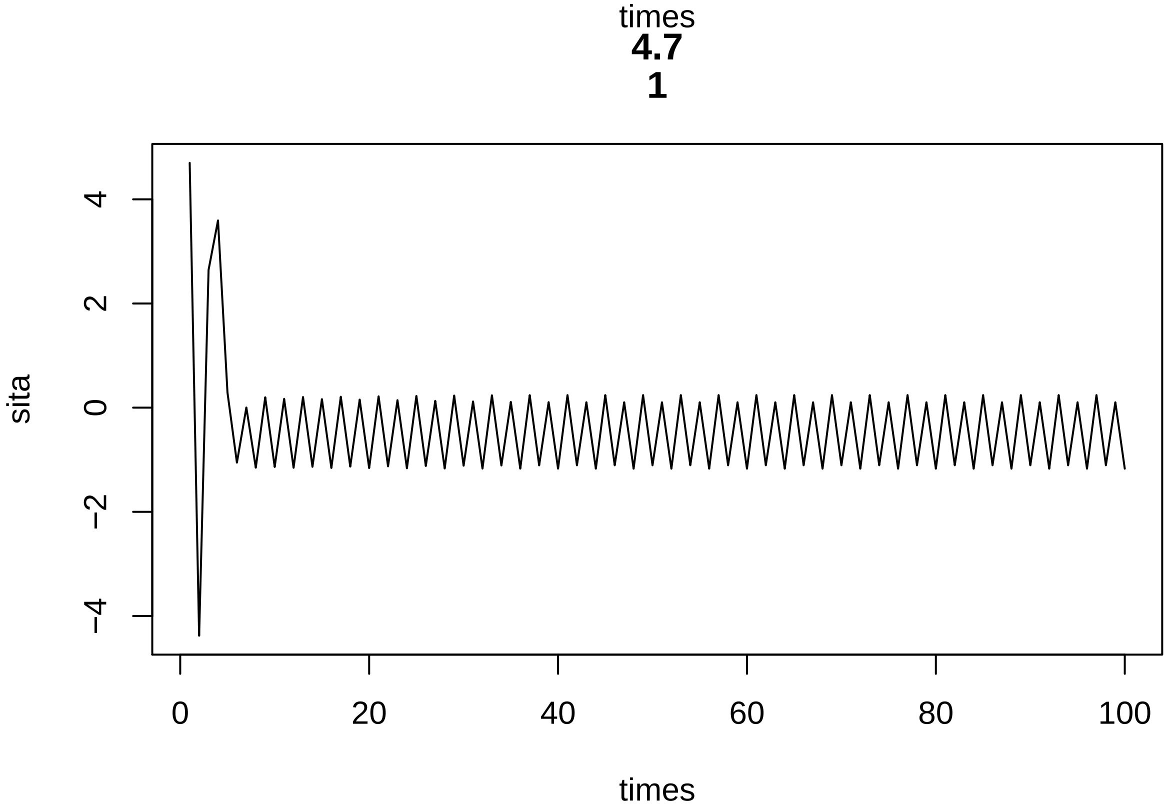

−11
1
sita
−2
−4
−6
−8
−10
0
20
40
60
80
100
times

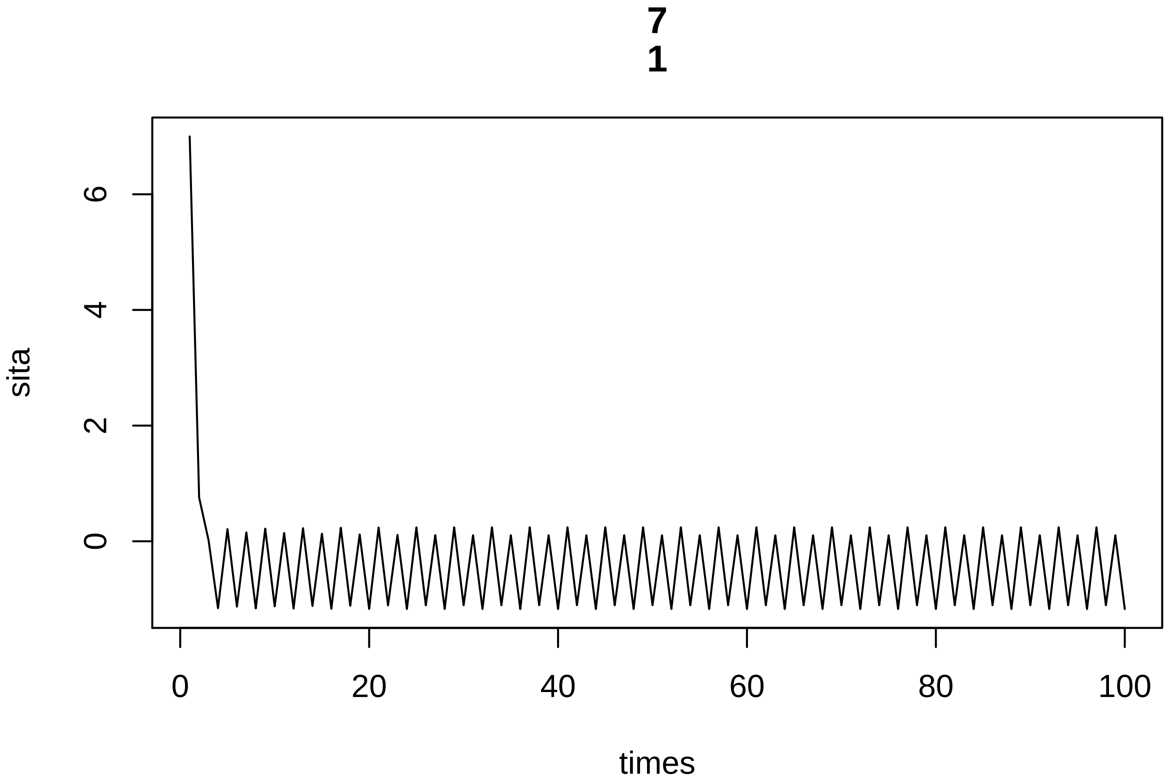

−1
1
sita
0.0
−0.4
−0.8
−1.2
0
20
40
60
80
100
times

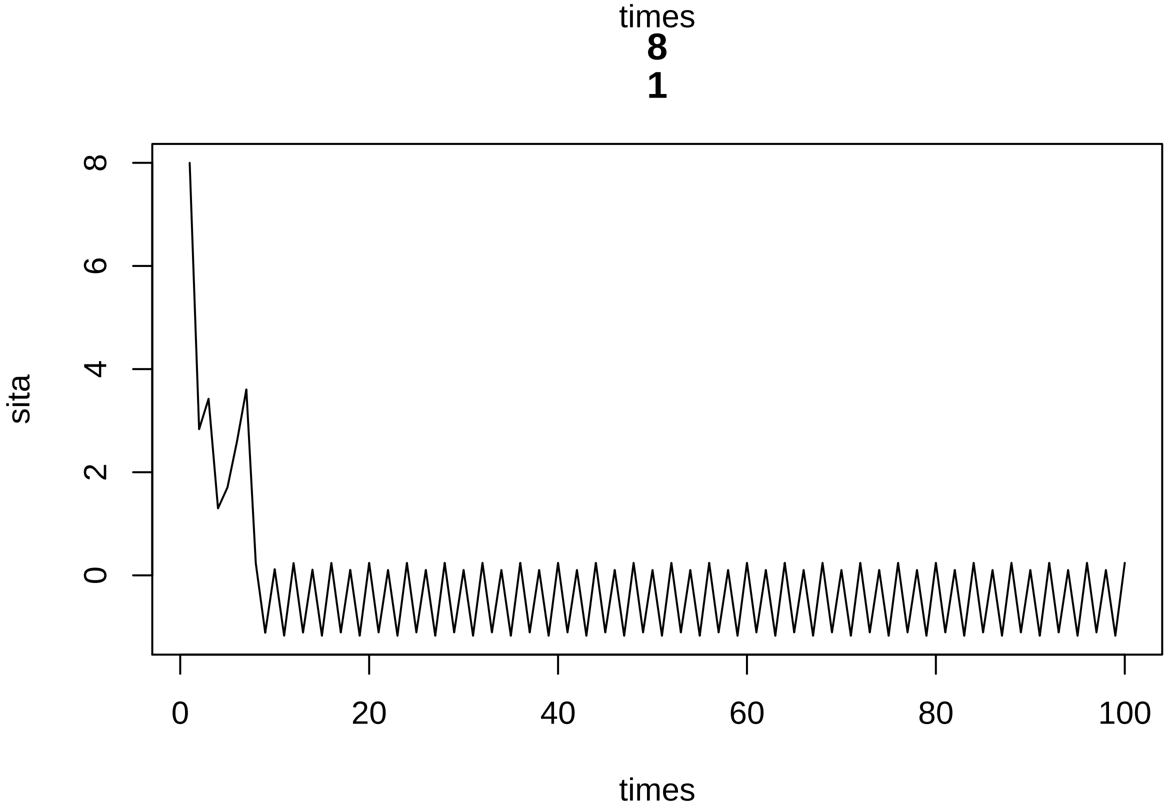

0
1
sita
0.0
−0.4
−0.8
−1.2
0
20
40
60
80
100
times


1.5
1
sita
3
2
1
0
−1
0
20
40
60
80
100
times

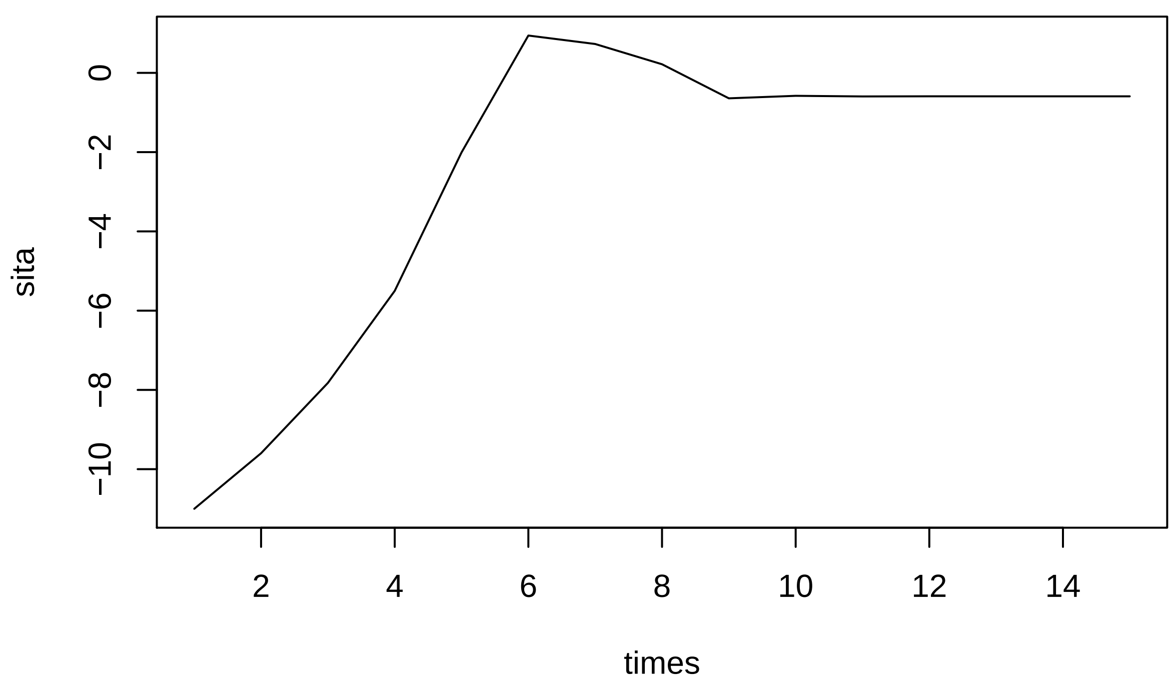

4
1
sita
times
4
3
2
1
0
−1
−2
0
20
40
60
80
100


4.7
1
sita
times
4
2
0
−2
−4
0
20
40
60
80
100

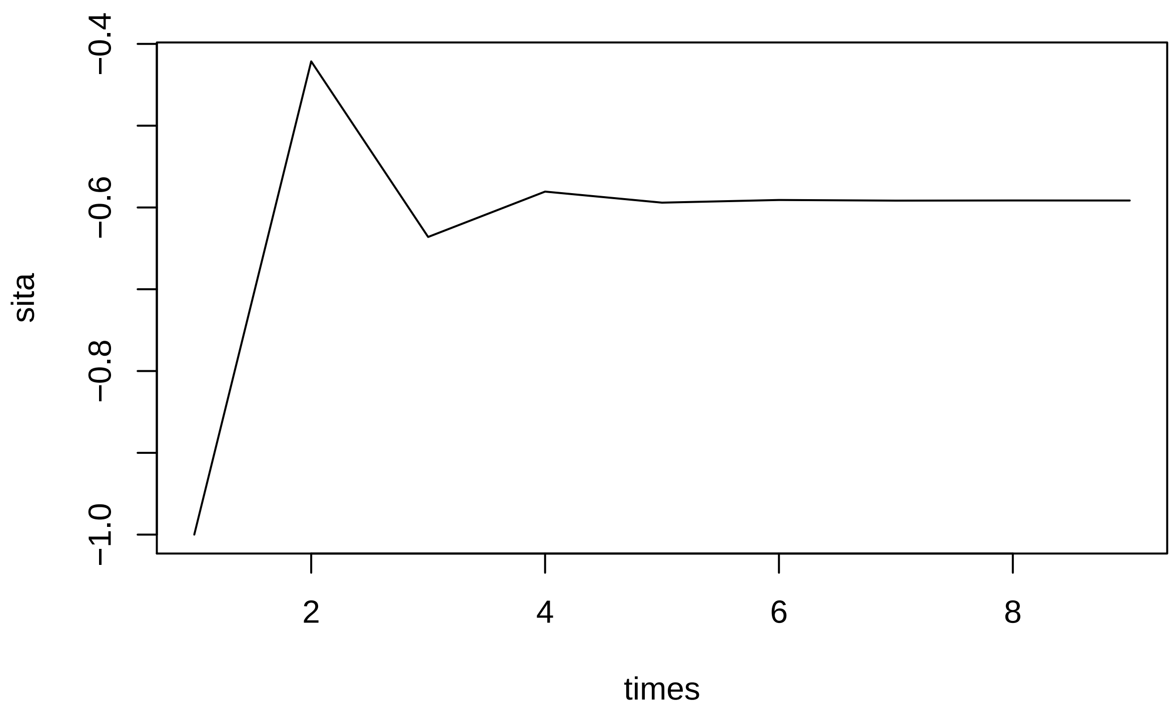

7
1
sita
0
2
4
6
0
20
40
60
80
100
times

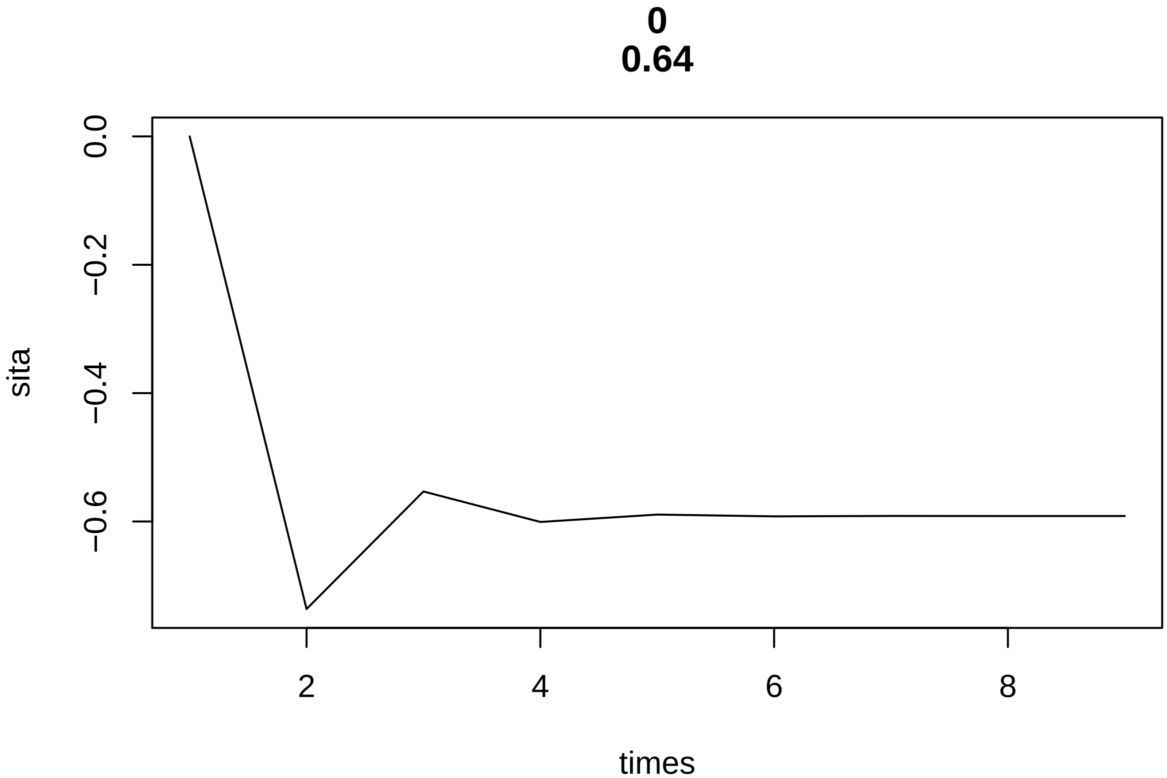

8
1
sita
0
2
4
6
8
0
20
40
60
80
100
times

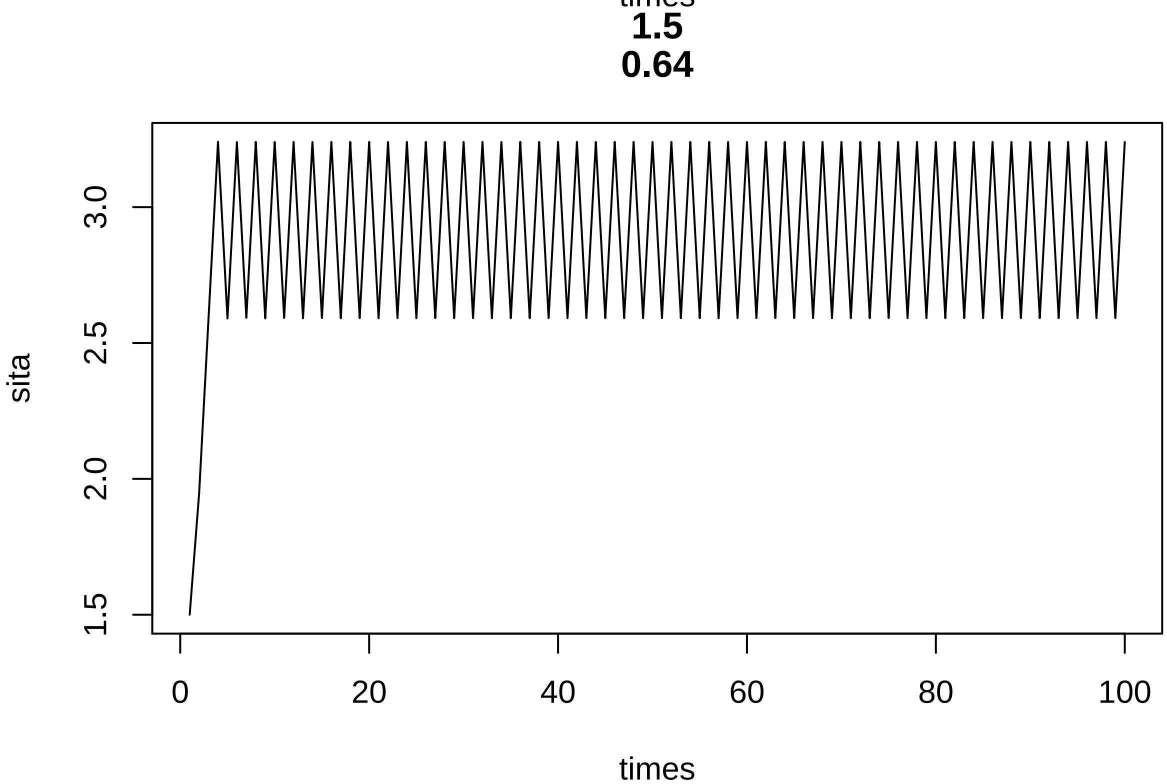
−11
0.64

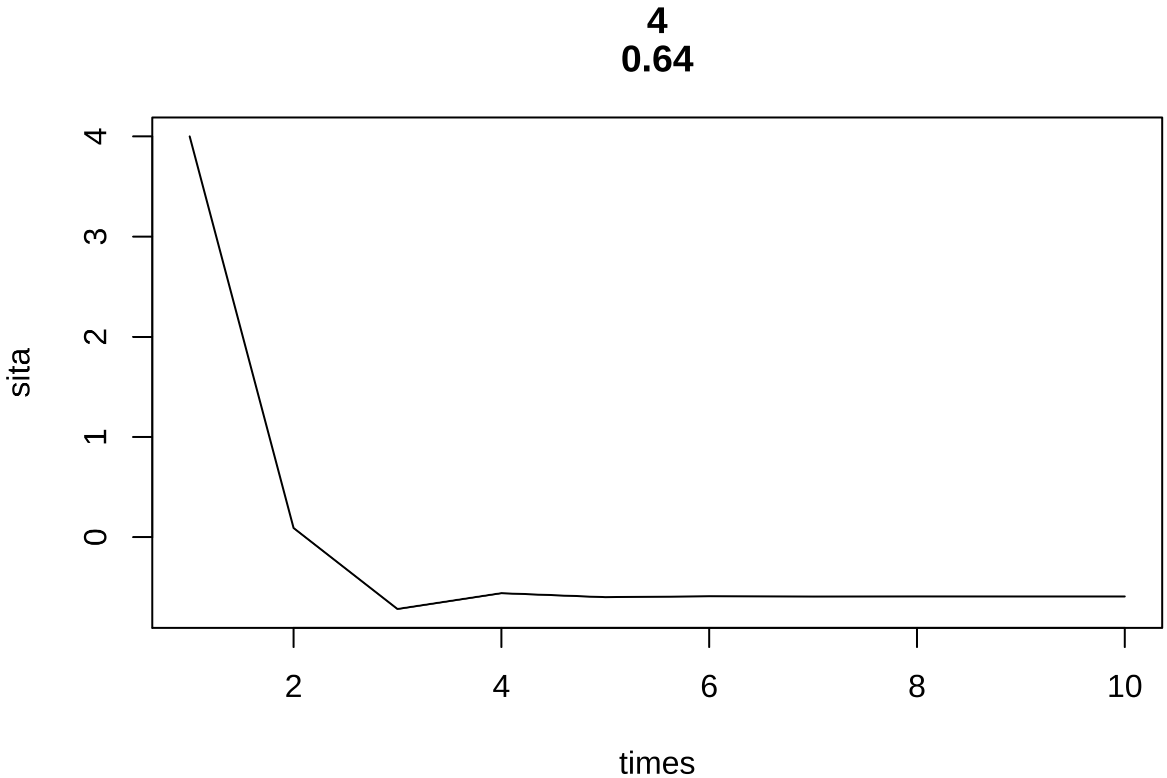


−1
0.64

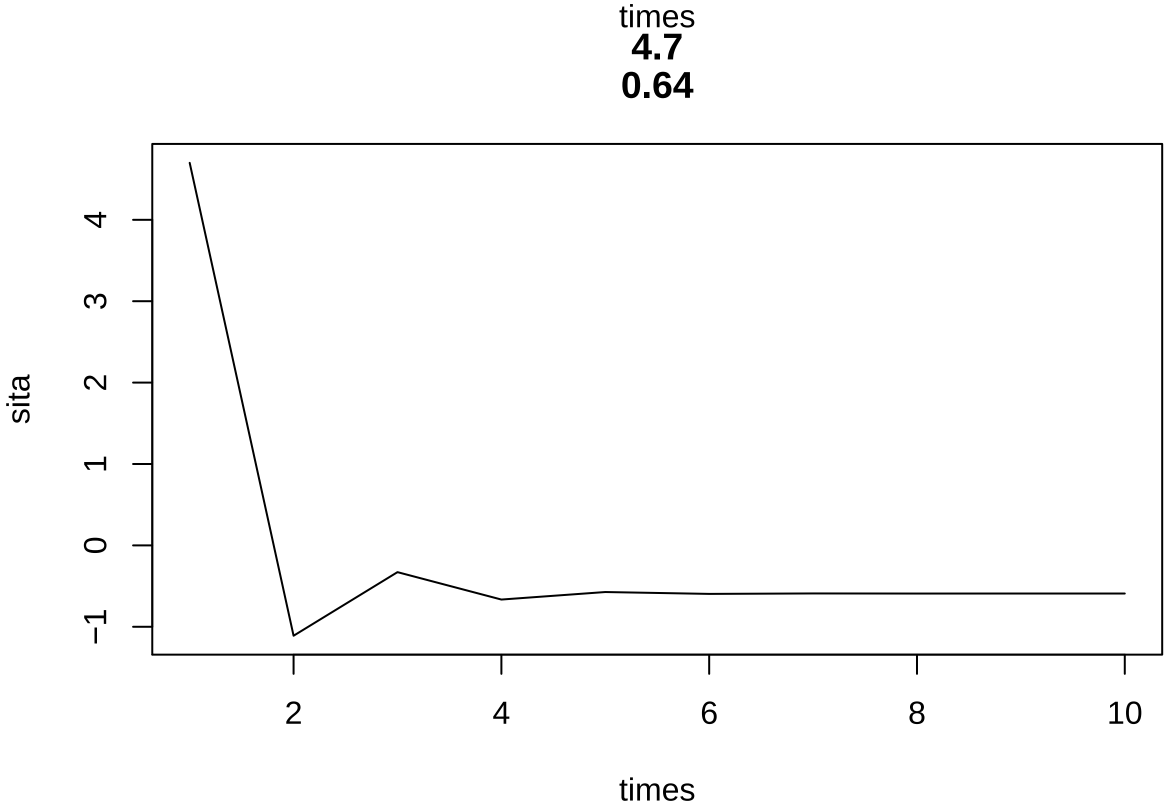



0
0.64
sita
0.0
−0.2
−0.4
−0.6
2
4
6
8
times


1.5
0.64
sita
3.0
2.5
2.0
1.5
0
20
40
60
80
100
times

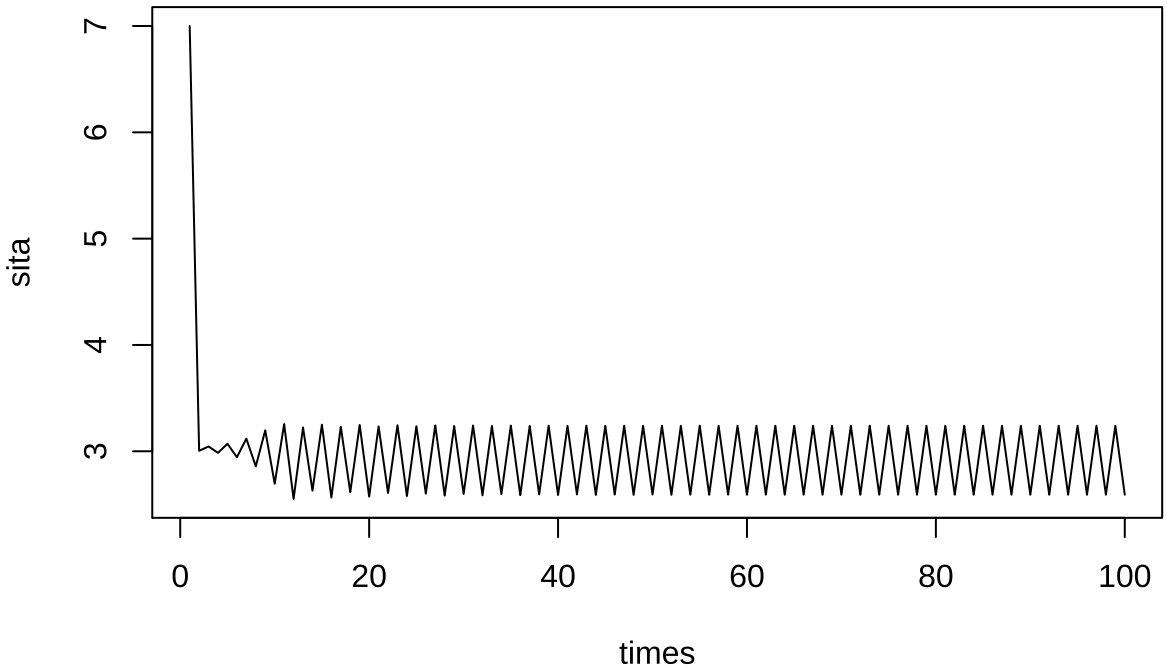


4
0.64
sita
times

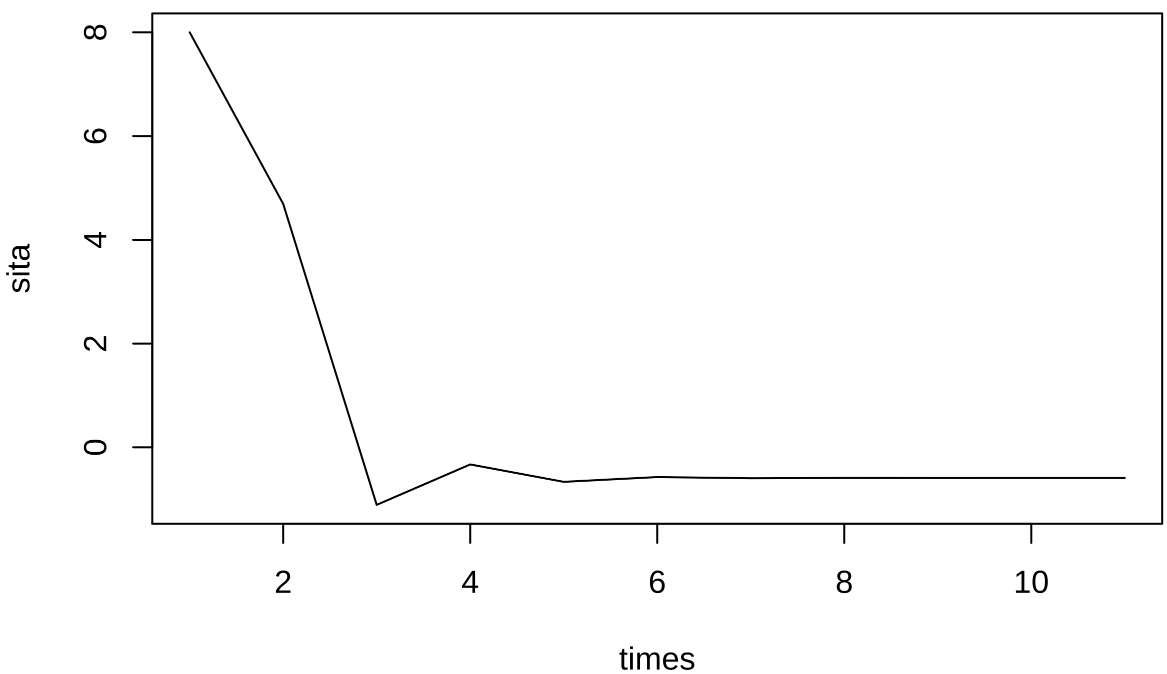


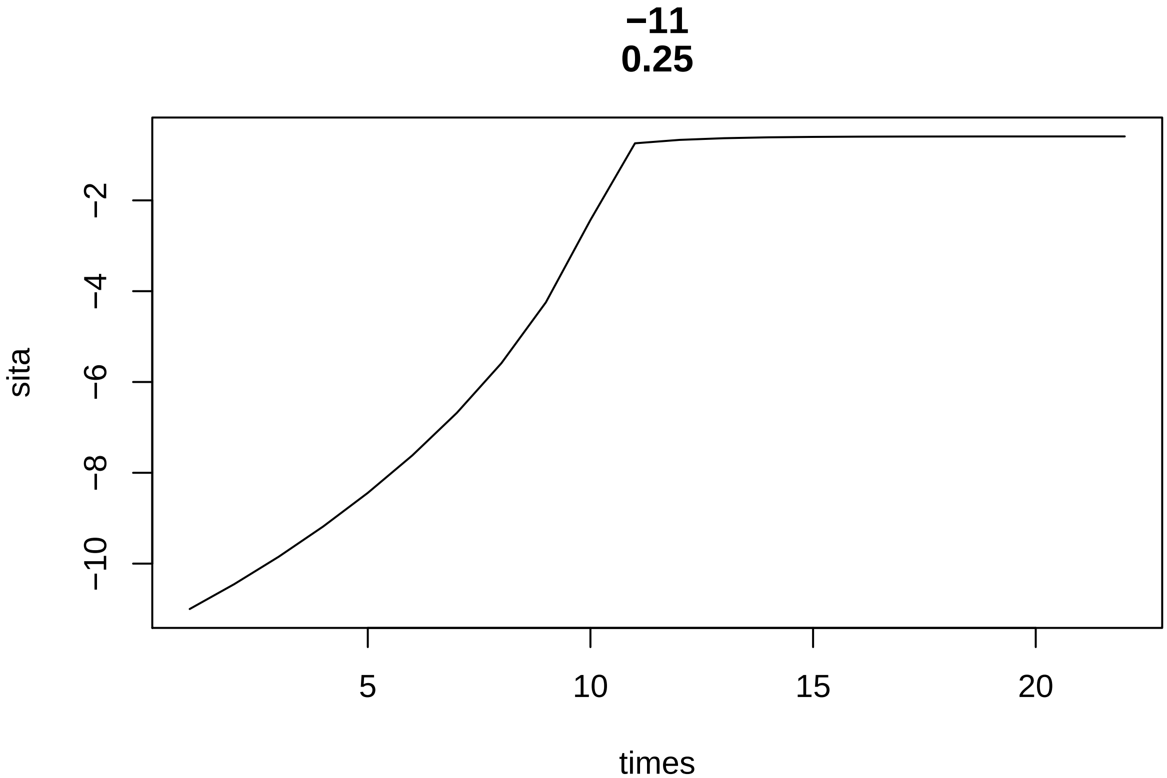
4.7
0.64
sita
times

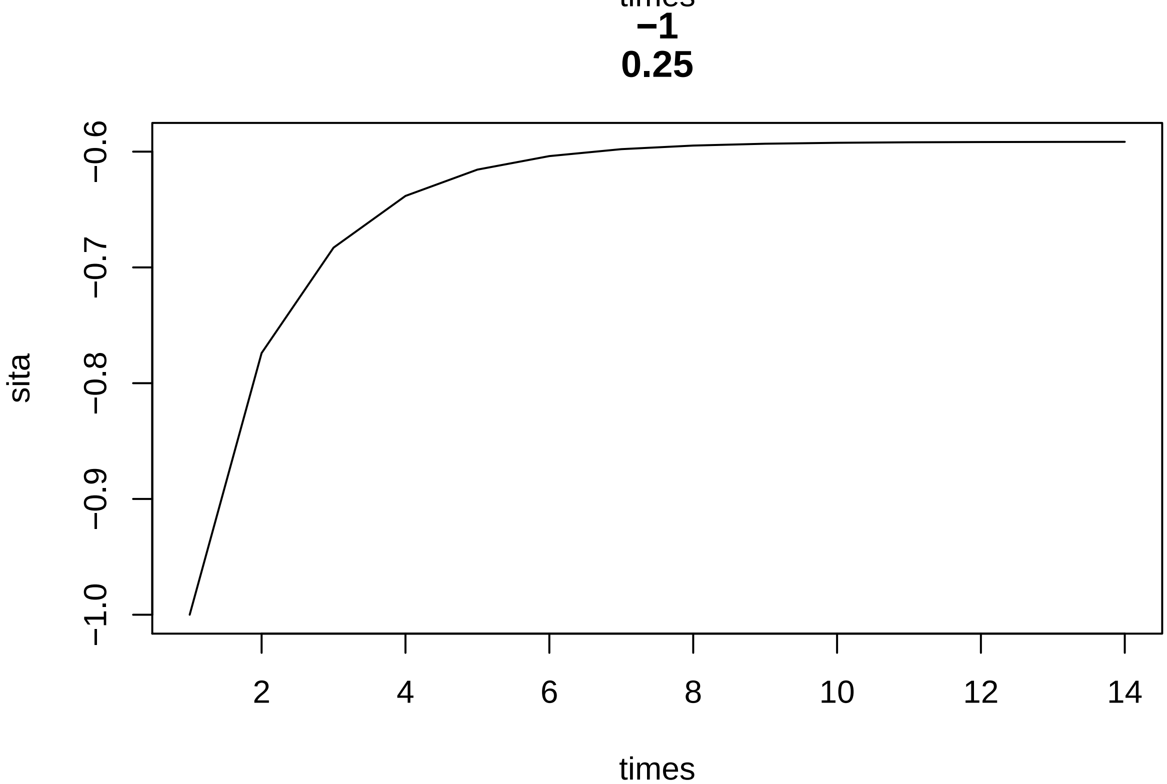
7
0.64

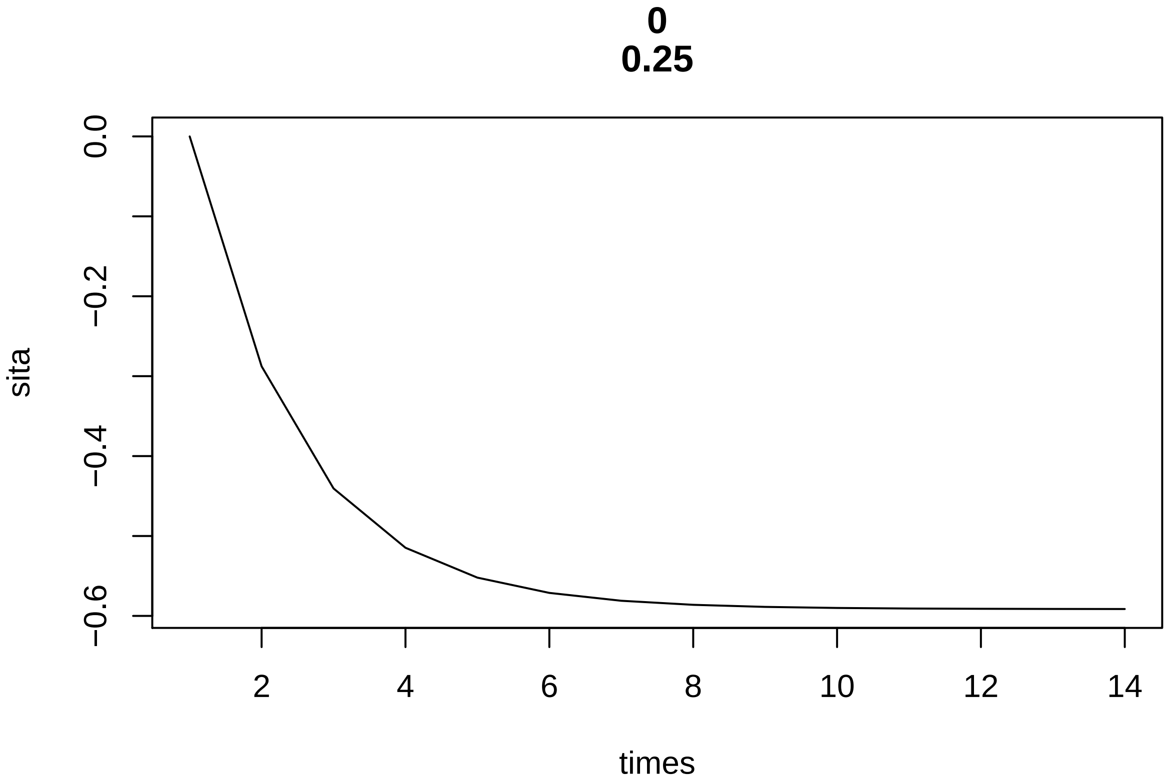


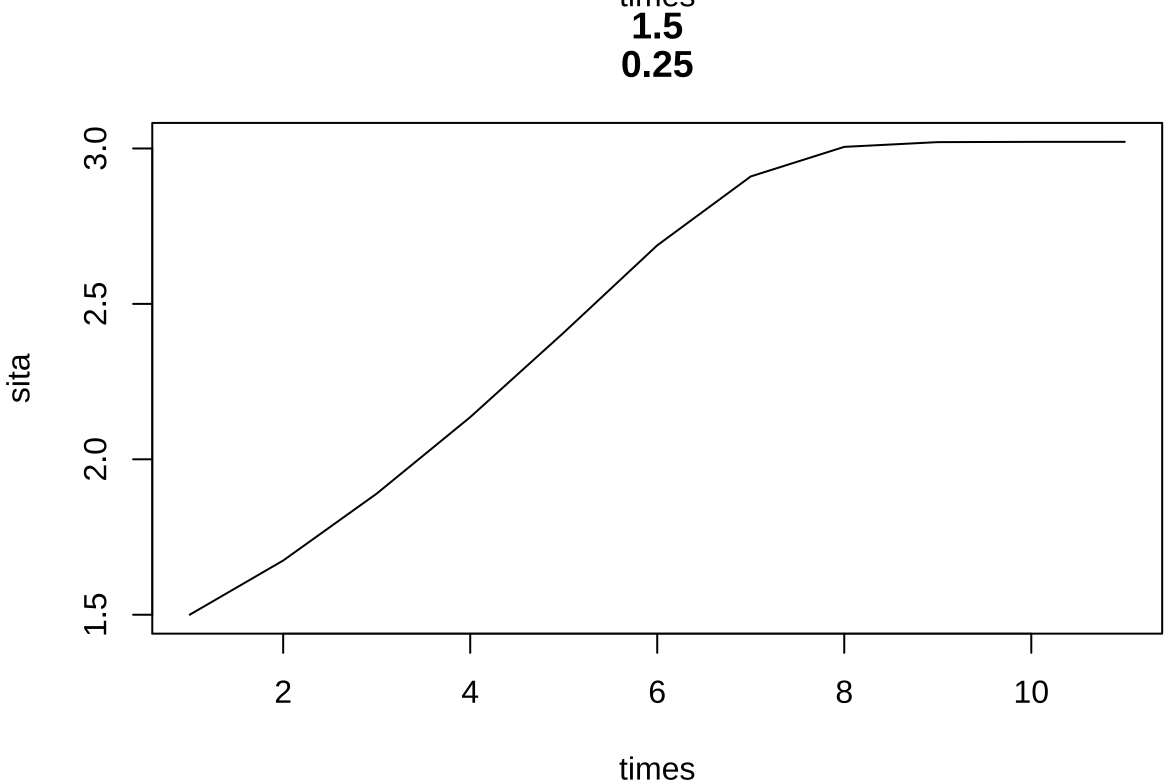
8
0.64

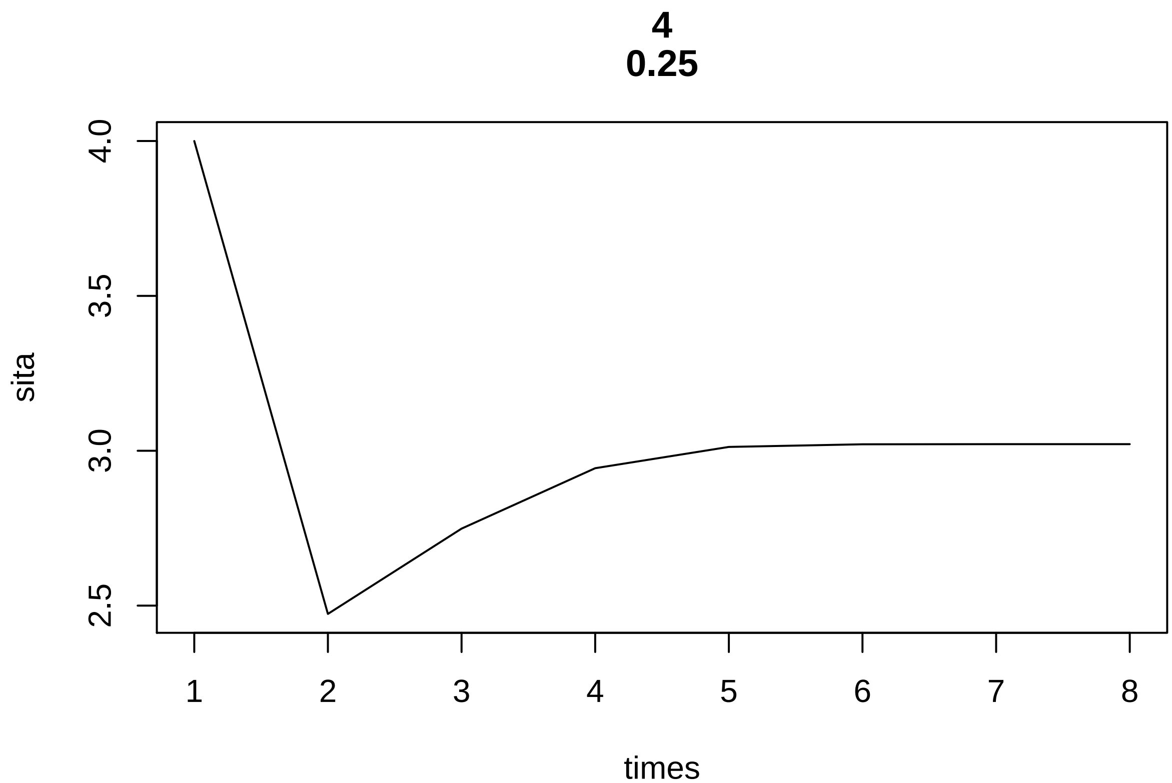


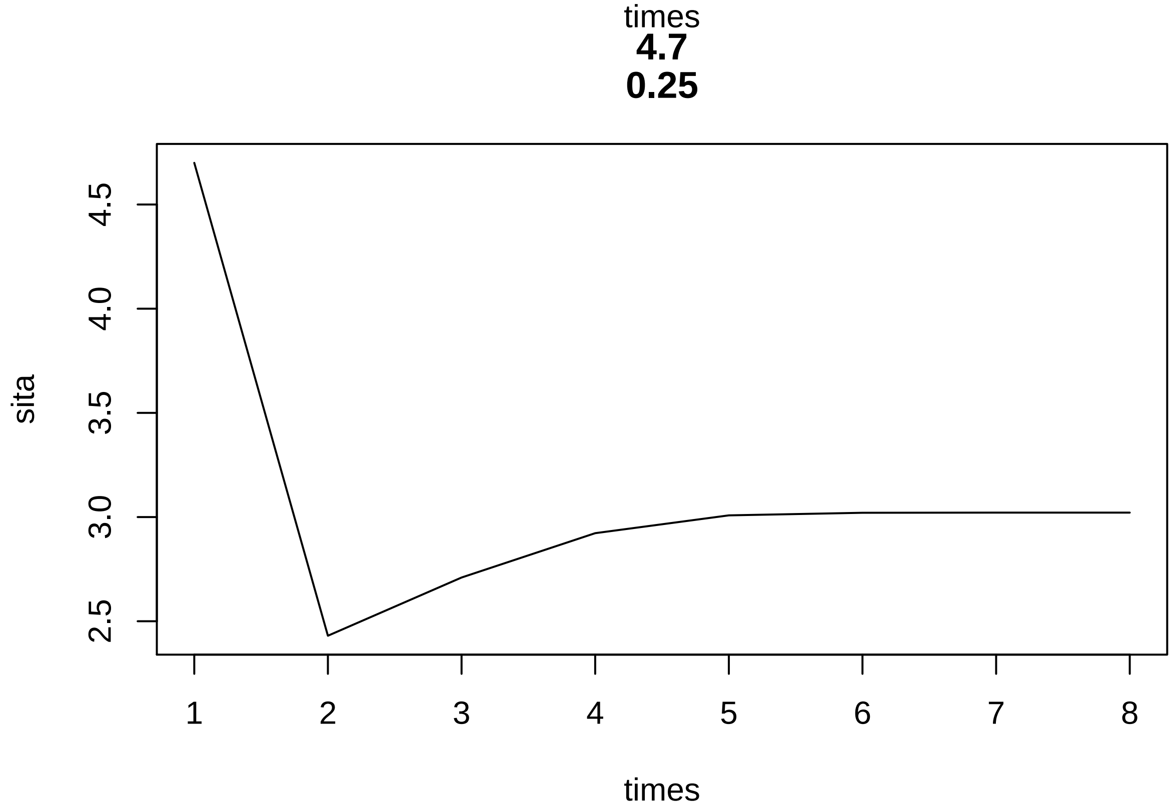

−11
0.25
sita
−2
−4
−6
−8
−10
5
10
15
20
times

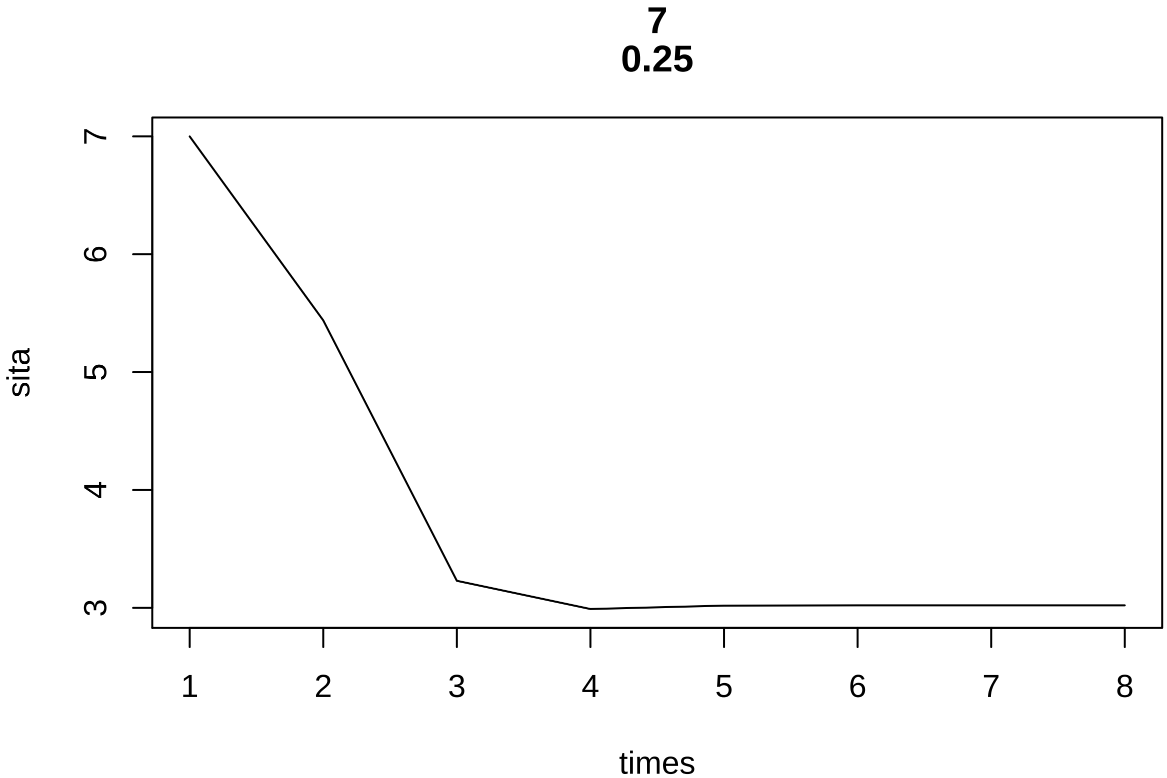

−1
0.25
sita
−0.6
−0.7
−0.8
−0.9
−1.0
2
4
6
8
10
12
14
times

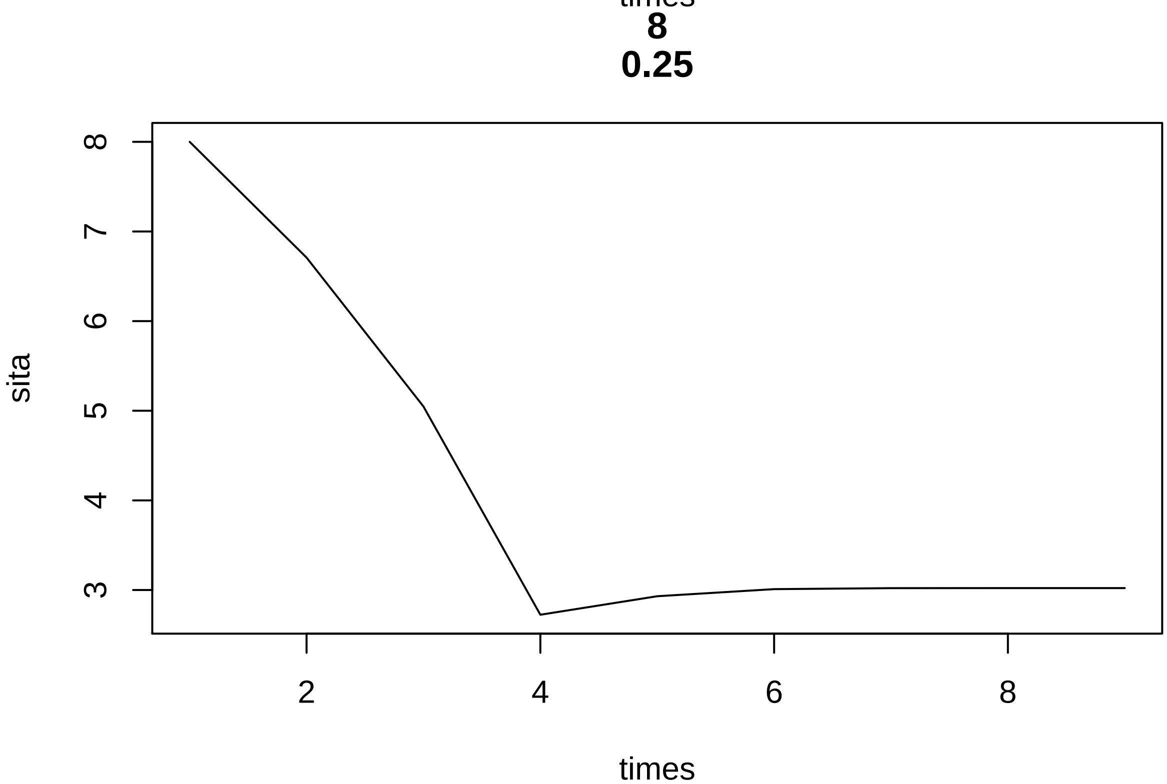

0
0.25
sita
0.0
−0.2
−0.4
−0.6
2
4
6
8
10
12
14
times


1.5
0.25
sita
3.0
2.5
2.0
1.5
2
4
6
8
10
times


4
0.25
sita
4.0
3.5
3.0
2.5
1
2
3
4
5
6
7
8
times


4.7
0.25
sita
4.5
4.0
3.5
3.0
2.5
1
2
3
4
5
6
7
8
times





Question 1(d)

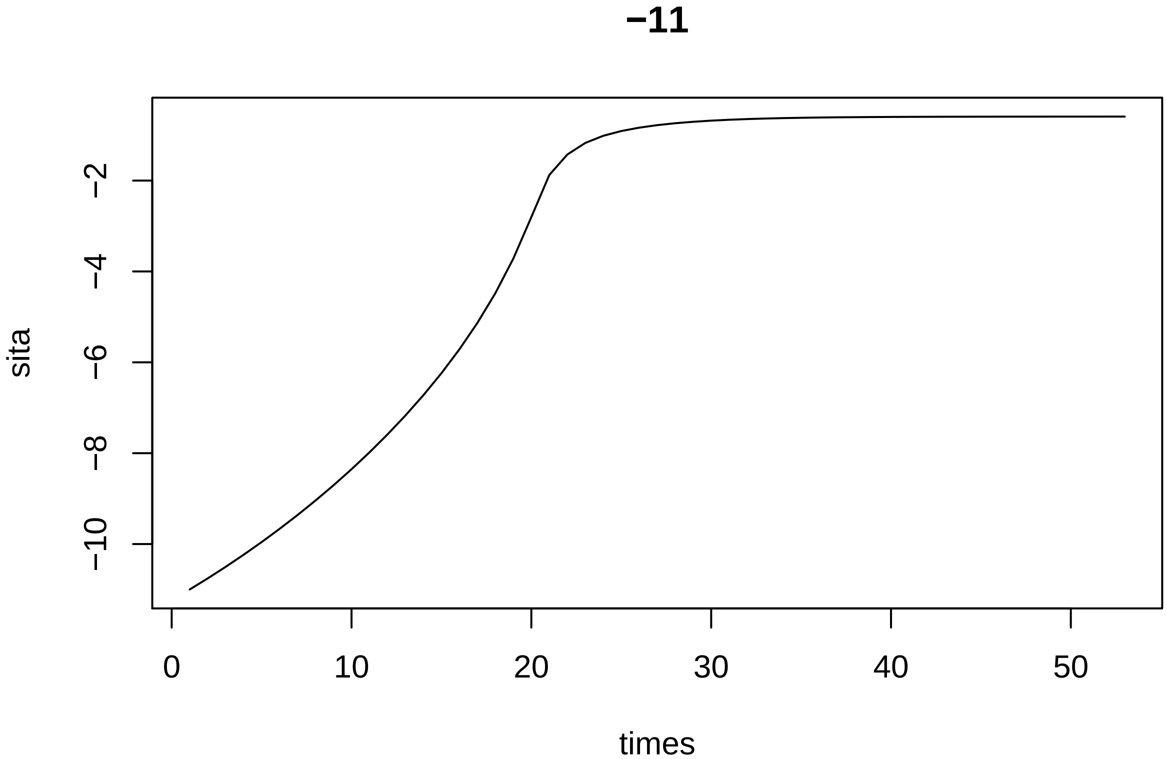
This section we use Fisher Scoring to find MLE, with I(theta) being constant of n/2

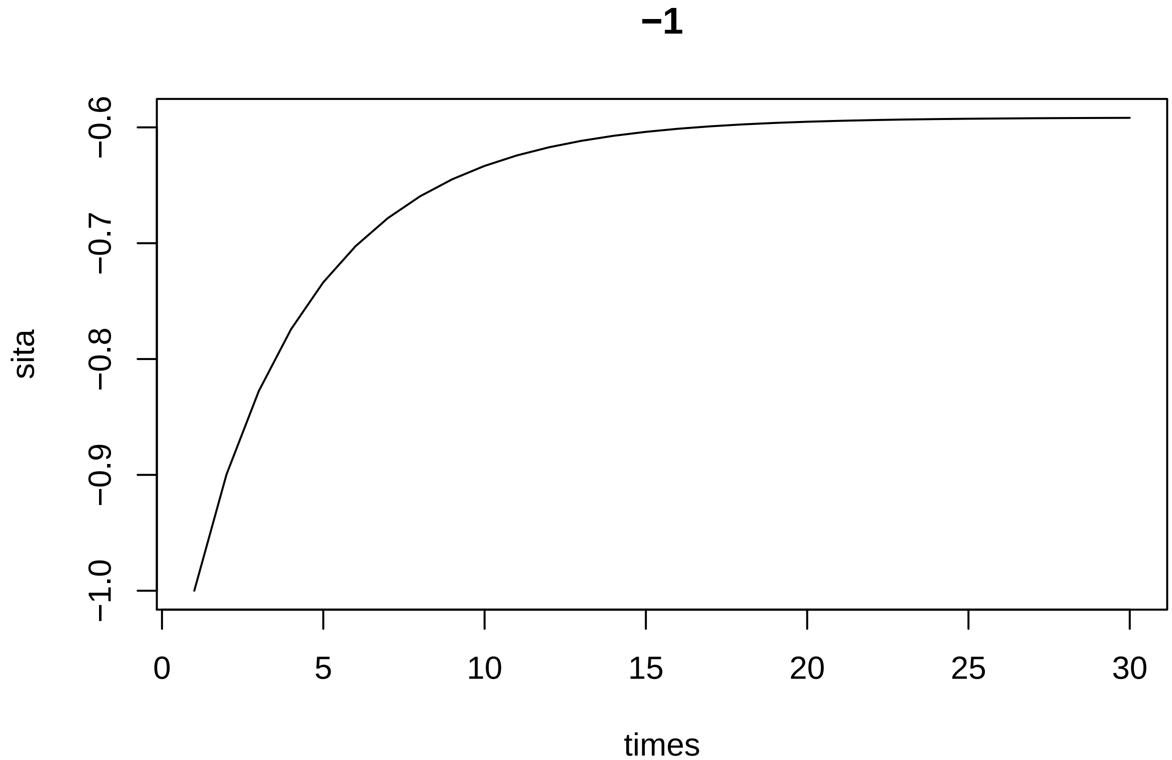
The functions we set up are similar with previous functions, and we compute I

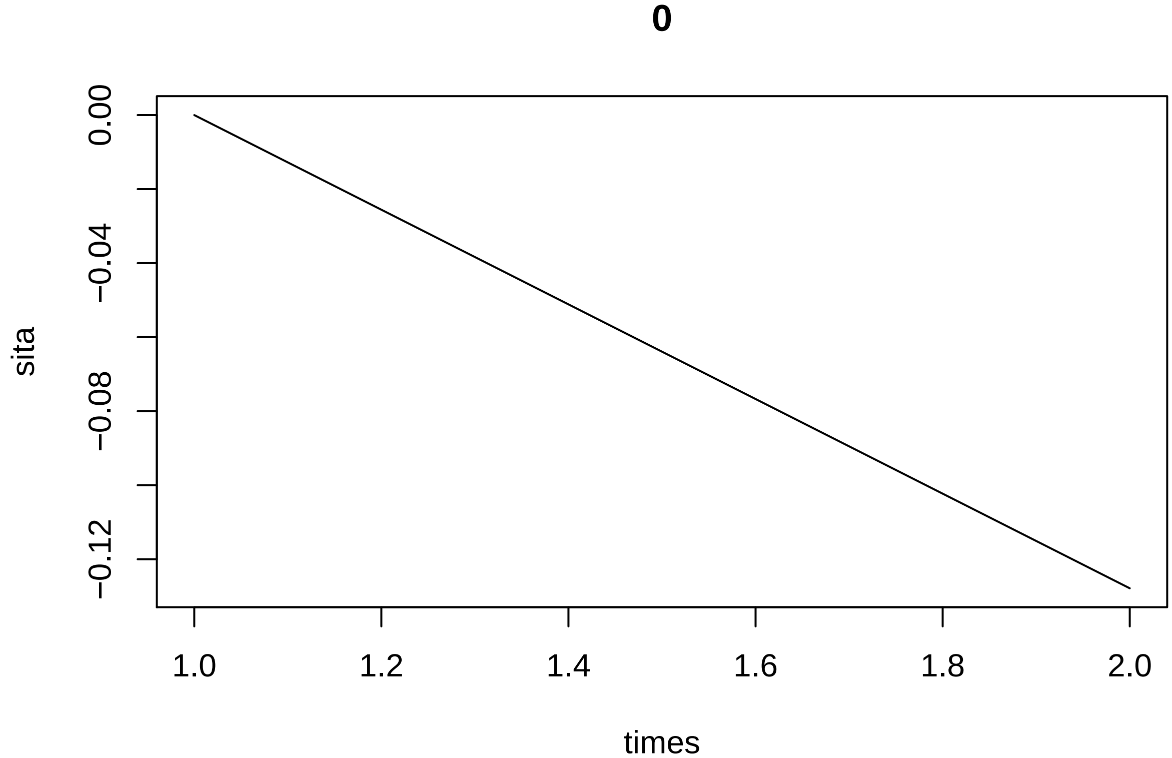
```
derivitive1 <- function(x,sita){
  value <- 0
  for (i in 1:length(x)){
    value <- value-2*(sita-x[i])/(1+(sita-x[i])^2)
  }
  return(value)
}

I <- length(x)/2
```

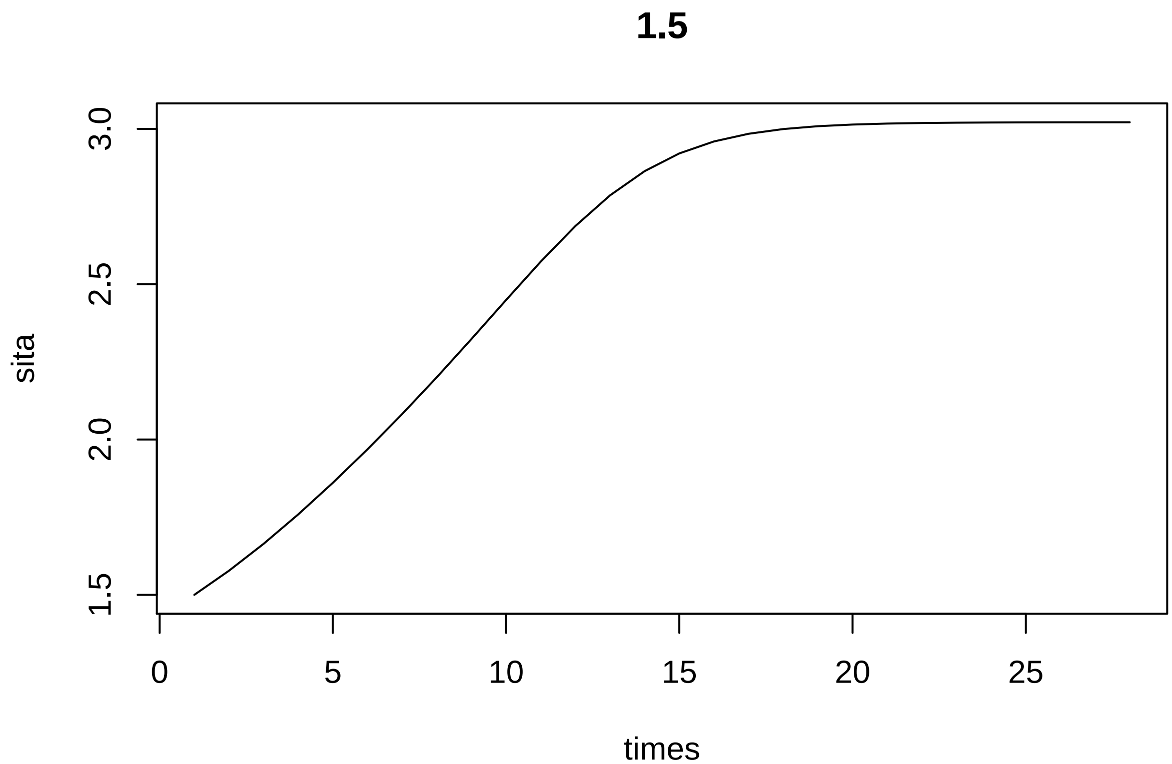
Plots1(d)

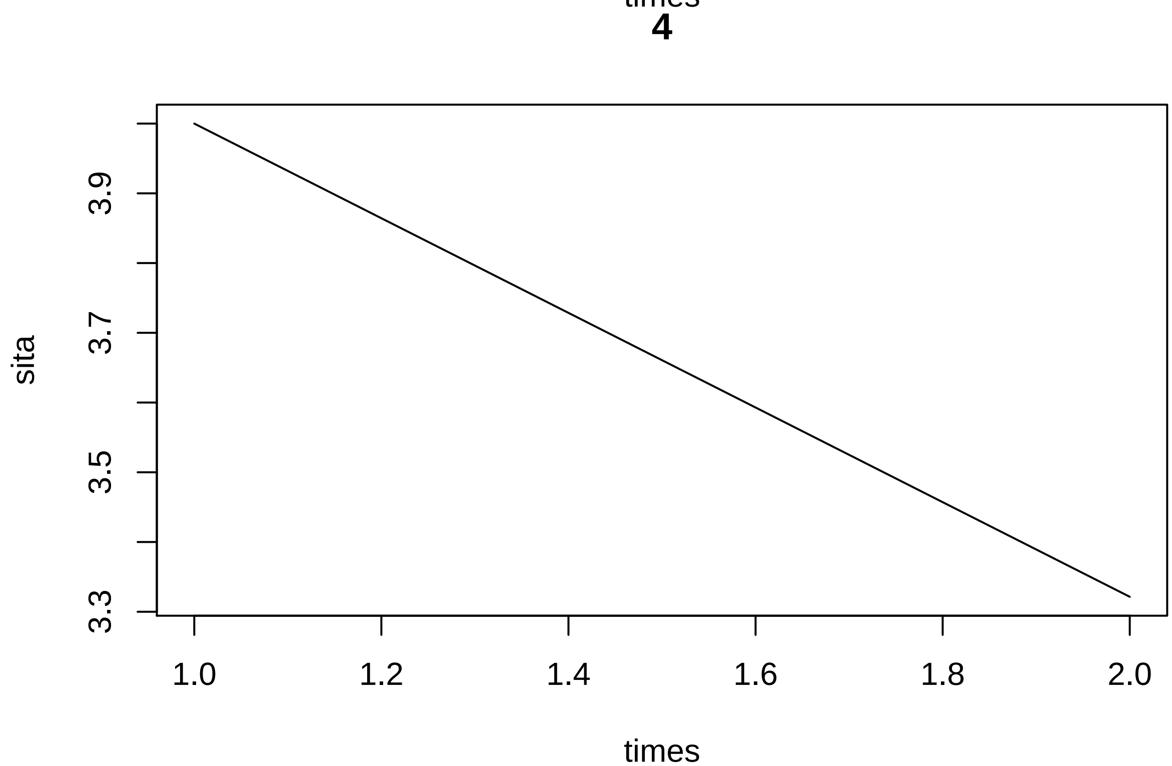
These are plots that I use multiple start points

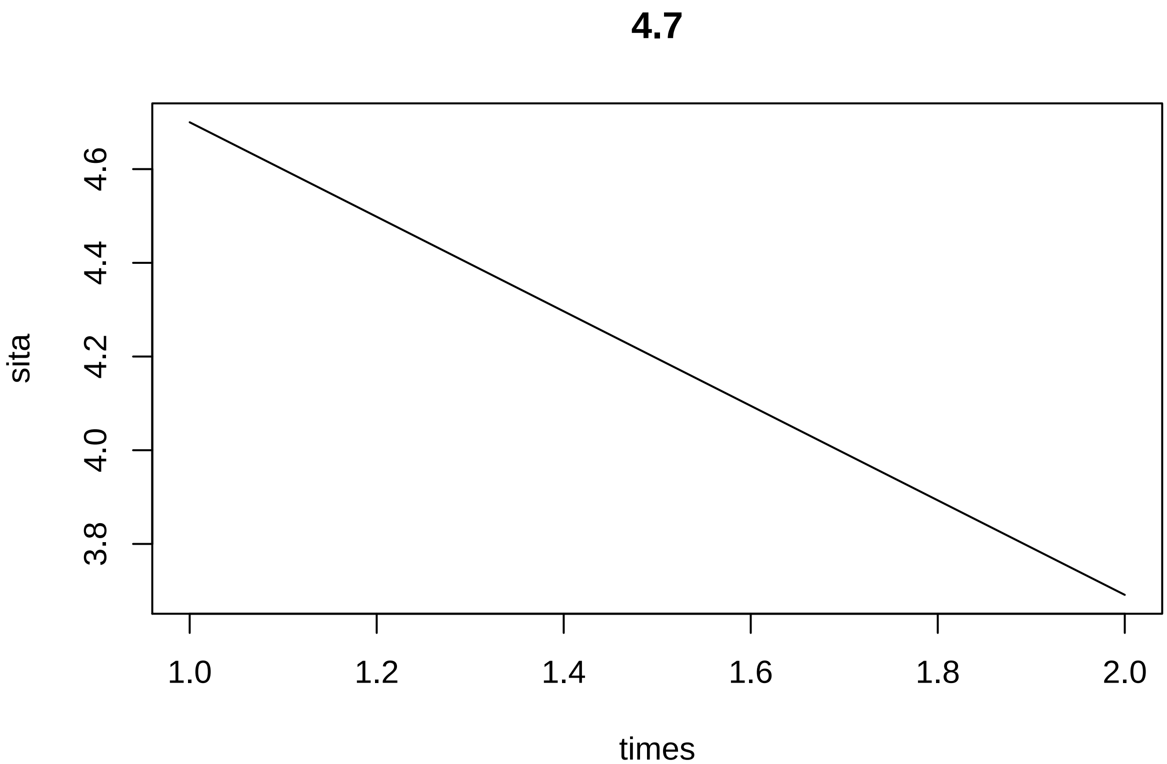


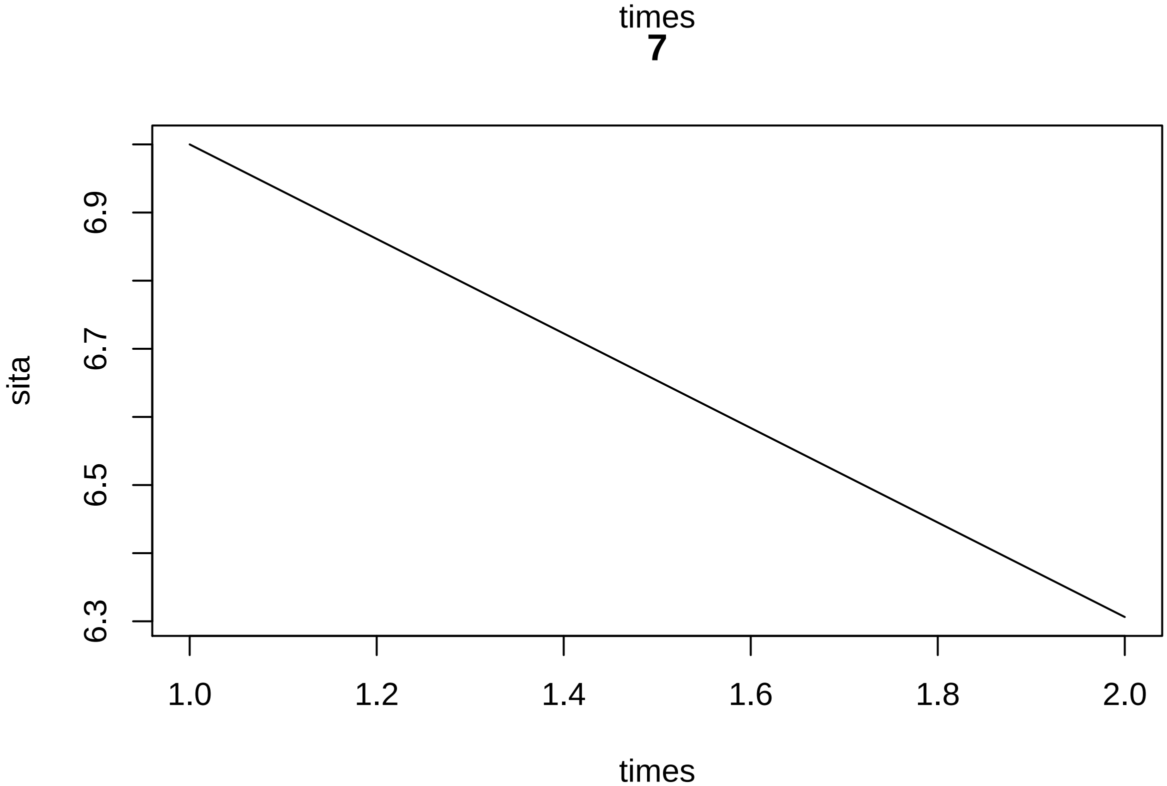

−1
sita
−0.6
−0.7
−0.8
−0.9
−1.0
0
5
10
15
20
25
30
times

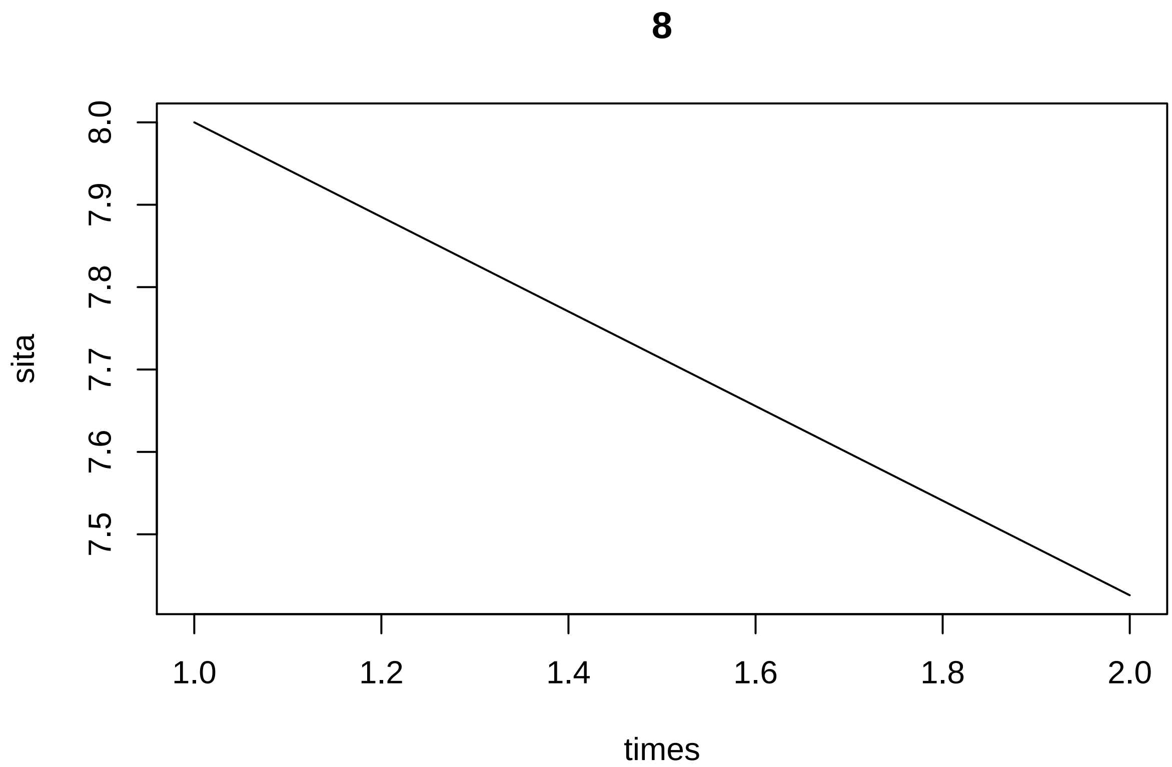

0
sita
0.00
−0.04
−0.08
−0.12
1.0
1.2
1.4
1.6
1.8
2.0
times


1.5
sita
3.0
2.5
2.0
1.5
0
5
10
15
20
25
times


4
sita
3.9
3.7
3.5
3.3
1.0
1.2
1.4
1.6
1.8
2.0
times


4.7
sita
4.6
4.4
4.2
4.0
3.8
1.0
1.2
1.4
1.6
1.8
2.0
times


7
sita
6.9
6.7
6.5
6.3
1.0
1.2
1.4
1.6
1.8
2.0
times



Qusetion 2

The likelihood function

$$E[X|\theta] = \pi + \sin(\theta) \tag{29}$$

two points

```
target <- asin(mm-pi)
print(target)
```

```
## [1] 0.09539407
```

```
target1 <- pi-target
print(target1)
```

```
## [1] 3.046199
```

The sequence of same converge start points

```
loglikelihood <- function(x,theta){
  value <- 0
  for (i in 1:length(x)){
    value <- value+log((1-cos(x[i]-theta))/(2*pi))
  }
  return(value)
```

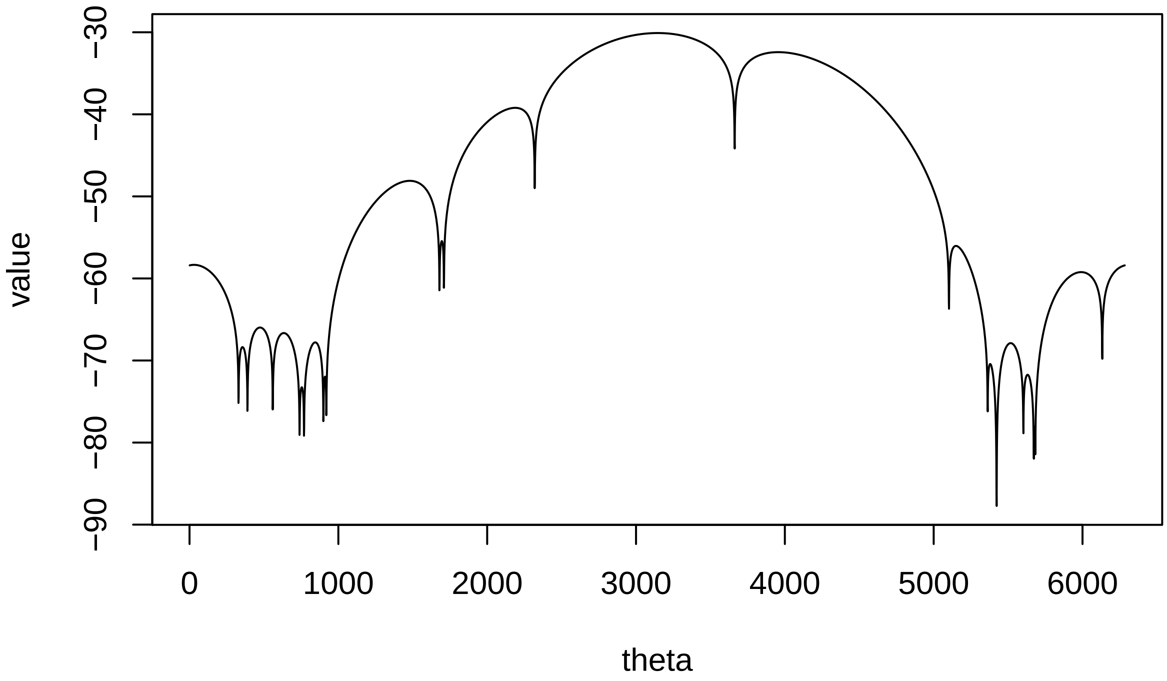
```r
}
for (i in 1:length(result)){
  if (abs(result[i]-result[j])>10^(-4)){
    j <- i
    num <- append(num,j-1)
   }
}
num <- unique(num)
print(unique(num[1:length(num)-1]))
```

```
## [[1]]
## [1] 11
##
## [[2]]
## [1] 13
##
## [[3]]
## [1] 18
##
## [[4]]
## [1] 24
##
## [[5]]
## [1] 25
##
## [[6]]
## [1] 29
##
## [[7]]
## [1] 30
##
## [[8]]
## [1] 54
##
## [[9]]
## [1] 55
##
## [[10]]
## [1] 74
##
## [[11]]
## [1] 116
##
## [[12]]
## [1] 162
##
## [[13]]
## [1] 170
##
## [[14]]
## [1] 172
##
## [[15]]
## [1] 178
```

```
##
## [[16]]
## [1] 180
##
## [[17]]
## [1] 195
```

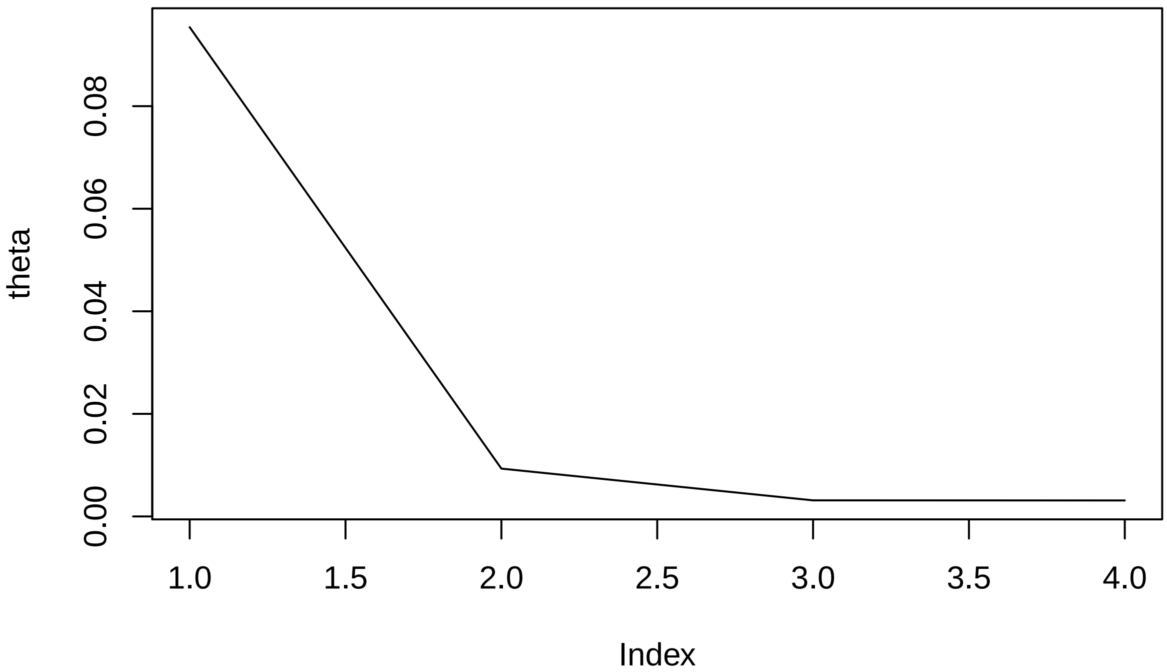
Plots2

We plot the value of our target



we also plot the picture with strat point being 'target'

```
MLE <- function(theta){
  diff <- 1
  i <- 1
  while(abs(diff)>10^(-4)){
    theta[i+1] <- theta[i]-derivetive1(x,theta[i])/derivetive2(x,theta[i])
    diff <- theta[i+1]-theta[i]
    i <- i+1
  }
  plot(theta,type='l')
  return(theta)
}

theta <- array()
theta[1] <- target
s <- MLE(theta)
```



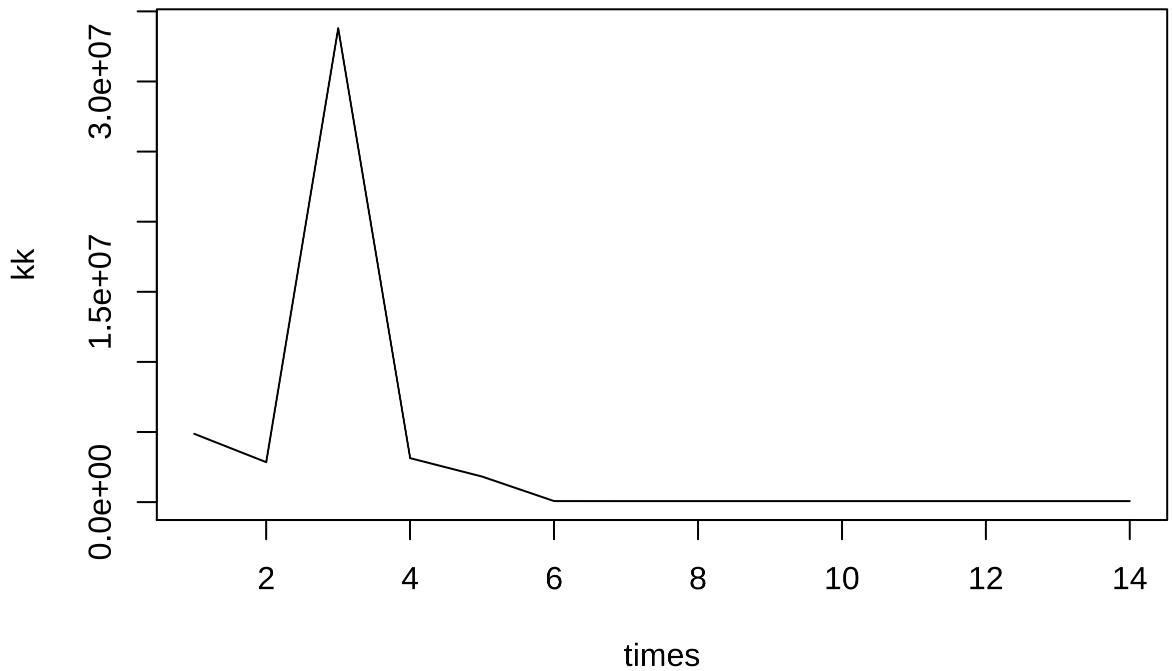
Qustion 3(b)

Functions

Here are matrix we need to establish to build our function

```
A_matrix <- function(r,k,times){
  A <- matrix(nrow=times,ncol=2)
  for (i in 1:times){
    A[i,1] <- (N0^2-N0^2*exp(-1*r*beetles$days[i]))/((N0+(k-N0)*exp(-1*r*beetles$days[i]))^2)
    A[i,2] <- beetles$days[i]*(k-N0)*exp(-1*r*beetles$days[i])*k*N0/((N0+(k-N0)*exp(-1*r*beetles$days[i
  }
  A
}


Z_matrix <- function(r,k,times){
  z <- matrix(nrow=times,ncol=1)
  for (i in 1:times){
    z[i,1] <- beetles$beetles[i]-k*2/(2+(k-2)*exp(-r*beetles$days[i]))
  }
  z
}
```

Sum of squared errors



Question 3(c)

This time we need to optimize the value of r, K, sigma

Partial Derivative

```
d_sigma <- as.expression(D(u,'sigma'))
dd_sigma <- as.expression(D(d_sigma,'sigma'))

d_k <-as.expression(D(u,'k'))
dd_k <- as.expression(D(d_k,'k'))

d_r <- as.expression(D(u,'r'))
dd_r <- as.expression(D(d_r,'r'))
```

3c

I use Newton method to optimize the r,K, and square of sigma. These numbers are

Times we use

```
## [1] 37
```

The results

```
print(kF)
```

```
## [1] 820.38
```

```r
print(rF)
```

```
## [1] 0.1926401
```

```r
print(sigmaF)
```

```
## [1] 0.9108726
```