# Homework 3

## 2. Given probability densities f(x) and g(x)

a) Find normailizing constant C, and show g(x) is a mixture of two gamma functions and determine weights

```
Set theta =1 to find constant C
fun <- function(x) {((x^.5)*exp(-x) + 2*exp(-x))} # distribute exp(-x) to separate functions
integrate(fun, lower = 0, upper = Inf)
#receive 2.886227
C     <- 1 / 2.886227        #normalizing constant
```

As g(x) can be separated into $exp(-x) * (2 * x^{(\theta-1)}$$ and

$$x^(\theta - .5) * e^{(-x)}$$

we can clearly see g is a composition of gamma functions where

$$r = 1$$

and

$$a = \theta$$

```
fun.1     <- function(x) {2*exp(-x)}
Fun.1     <- integrate(fun.1, lower = 0, upper = Inf)
fun.2     <- function(x) {x^(.5)*exp(-x)}
Fun.2     <- integrate(fun.2, lower = 0, upper = Inf)

weightfun.1 <- print((Fun.1$$value)*C)        #weighted component 1
weightfun.2 <- print((Fun.2$$value)*C)        #weighted component 2
```

b)

```
probdenG <- function(x, theta) {
            (2 * x ^ (theta-1) + x ^ (theta - .5)) * exp(-x)
}
```

2) a) As

$$g(x) = (2x^{\theta-1} + x^{\theta-1/2})e^{-x}$$

we find normalizing constant C such that

$$2C * \Gamma(\theta) + C * \Gamma(\theta + \frac{1}{2} = 1)$$

$$\Rightarrow C = \frac{1}{2\Gamma(\theta) + \Gamma(\theta + \frac{1}{2})}$$

Therefore we can change $g(x)$ to

$$g(x) = \frac{1}{2\Gamma(\theta) + \Gamma(\theta + \frac{1}{2})} 2x^{\theta-1}e^{-x} + \frac{1}{2\Gamma(\theta) + \Gamma(\theta + \frac{1}{2})} 2x^{\theta-1/2}e^{-x}$$

$$\Rightarrow g(x) = \Gamma(\theta, 1)$$

weighted by

$$2 * \Gamma(\theta)/(2\Gamma(\theta) + \Gamma(\theta + .5))$$

and

$$\Gamma(.5 * \theta, 1)$$

weighted by and

$$\Gamma(\theta + .5)/(2 * \Gamma(\theta) + \Gamma(\theta + .5))$$

b) "' fun.g <- function(theta) { n <- 10000 weight <- 2*gamma(theta) / (2*gamma(theta)+gamma(theta+1/2)) iter <- 0 niter <- c() rand <- runif(n) for (i in 1:n) { if (rand[i] < weight) { x <- rgamma91, theta, 1) iter <- iter + 1 niter <- c(niter, x) } } return(niter) }

graph.g <- fun.g(1) hist(graph.g) "'