3.1 Actuarial Pure Premium

Consider a person facing a total possible loss of S during a period of one year. The actuarial pure premium, or fair premium, is the expected value of S, $\mathbb{E}[S]$.

3.2 Actuarial Pure Premium in a Segmented Environment

This is best described through an example. Let X be a binary variable specifying the location of the home of a customer such that, $X \in \{\text{village}, \text{city}\}$. If the loss S is dependent on the value of X, then it is reasonable to suppose that the people living in cities and villages should pay different premiums. Therefore the premium π would be:

- People living in a village: $\pi(\text{village}) = \mathbb{E}[S|X = \text{village}]$
- People living in a city: $\pi(\text{city}) = \mathbb{E}[S|X = \text{city}]$

3.3 Annual Result

For a given year, let π_i denote the premium paid by the insured $i \in \{1, \cdots, n\}$. Further, let s_i denote the sum of all losses related to the insured i during the period covered:

- if $\sum_{i=1}^{n} \pi_i > \sum_{i=1}^{n} s_i$ then the insurance company has a positive result.
- if $\sum_{i=1}^{n} \pi_i < \sum_{i=1}^{n} s_i$ then the insurance company has a negative result and is losing money.

4 Mimicking an Insurance Market

After the deadline for stage 2, the market simulation can begin!

4.1 Generating all prices

Before any analysis, after everyone has submitted the necessary files (see section 5), your submitted models will be used to generate premium prices for all the $\approx 700,000$

contracts available in the third year. These will be used to represent **your model** in the final market competition alongside **other player models**.

Therefore, each player j will offer each customer i a premium π_i^j.

4.2 Behaviour of customers

Consider customer i. In a given market, this customer will have a to choose a value among premiums $\Pi_i = \{\pi_i^1, \cdots, \pi_i^N\}$ as offered by all the player models. Let π_i^j denote the ordered premium values:

$$\pi_i^1 \leq \pi_i^2 \leq \cdots \leq \pi_i^N.$$

Given that we are using data from home insurance, we will use a purely price-drive customer model. Specifically, we will use the **lowest affectation rule** where the customer i will always choose the company offering the cheapest price, π_i^1.

4.3 Metrics for insurers

Let $\mathcal{C}_j$ denote the set of customers $i \in \mathcal{C}_j$ that chose insurer j.

Definition 4.1. The annual **earned premium** of insurer j is:

$$P_j = \sum_{i \in \mathcal{C}_j} \pi_i^j$$

Definition 4.2. The annual **total loss** of insurer j, calculated at the end of the year and not known in advance, is:

$$L_j = \sum_{i \in \mathcal{C}_j} s_i$$

Here, s_i is the amount that customer i has cost their insurer. For example, if in one year, Ali's house burns down[5] causing damage worth $\$150,000$, then $s_{\text{Ali}} = 150,000$[6].

Definition 4.3. The annual **earned profit** of insurer j is:

$$EP_j = P_j - L_j.$$

Definition 4.4. The **final score** of insurer j in the market is the sum of their profits from stage 1, $EP_j^{\text{stage 1}}$ and stage 2, $EP_j^{\text{stage 2}}$. Therefore:

$$FS_j = EP_j^{\text{stage 1}} + EP_j^{\text{stage 2}}$$

Note that in each market of N players, **the winner** is the player with the maximum total profit over **both stages** of the game.

[5] Hopefully it won't.

[6] Assuming his insurer covers all costs.