Mapping Finite State Machines to zk-SNARKs Using Category Theory

A talk for DEVCON V

Fabrizio Genovese, Andre Knispel, Joshua Fitzgerald

Statebox Team

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What is important is to know that if you have a **boolean circuit**, then you can turn it into a zk-SNARK.

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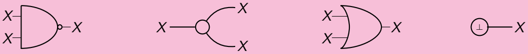

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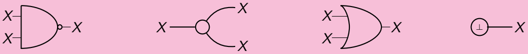

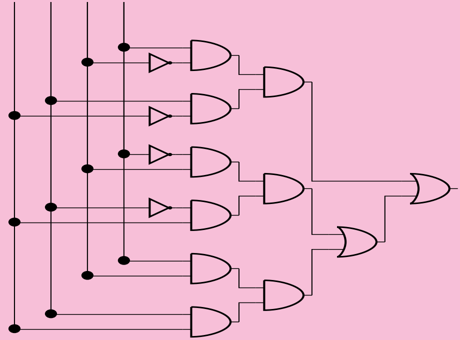

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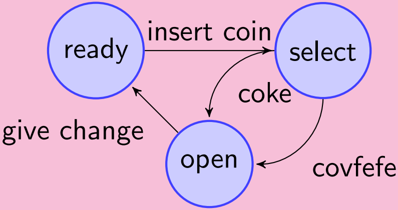

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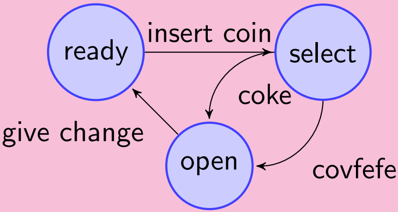

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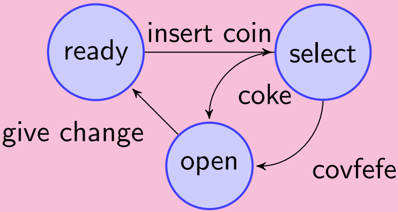

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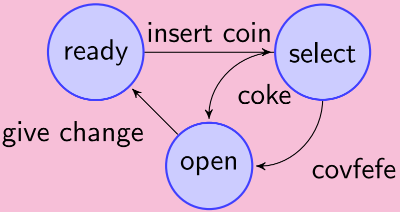

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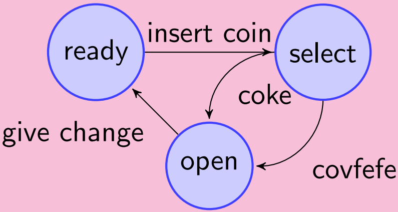

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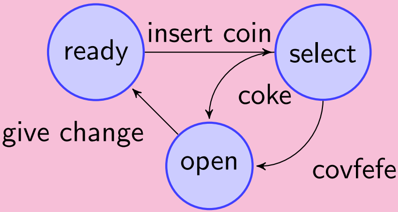

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- An arc between two vertexes represents an operation that mutates the state of the FSM;
- A computation that follows the rules of the FSM is just a path on the graph.

If we can map graph paths to boolean circuits, then we can get a zk-SNARK verifying that a given computation followed the FSM rules!

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Category theory allows to relate different mathematical structures compositionally: That is, in a way that respects the structure of the things we are relating.

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Moreover, once categorified concepts are automatically related to all mathematics: If you have a categorical way to map what you care about to graphs, then you automatically have a way to map that thing to boolean circuits (and hence to get a zk-SNARK for it!)

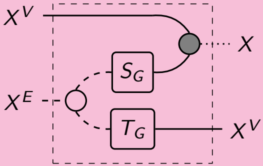
The category of paths for a graph

Each graph G can be used to build a category $\mathfrak{F}(G)$, which represents all the possible paths in the graph:

$$\begin{tikzcd}
G \arrow[r, mapsto] \arrow[d, "f"'] & \mathfrak{F}(G) \arrow[d, "\mathfrak{F}(f)"] \\
G' \arrow[r, mapsto] & \mathfrak{F}(G')
\end{tikzcd}$$

Circuits from graphs

We can use the adjacency matrix of a graph G to build a circuit:



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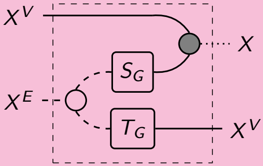

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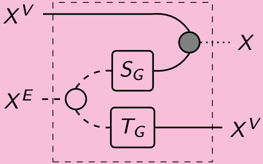

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The wire up right returns 1 if the vertex is the source of the edge, 0 otherwise. The bottom right wire returns the target vertex of the edge.

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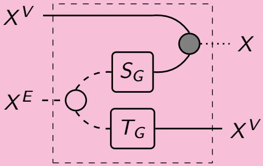

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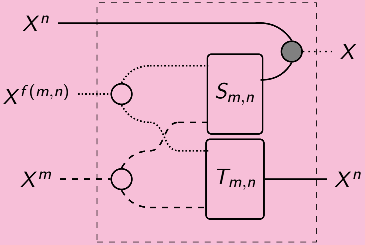
We can clearly compose these circuits by piping them one into the other. For each category of paths $\mathfrak{F}(G)$, we are are able to get a functor – a structure preserving map – to the category of boolean circuits.

Generalizing over graphs

We can abstract over the adjacency matrix obtaining the following circuit:



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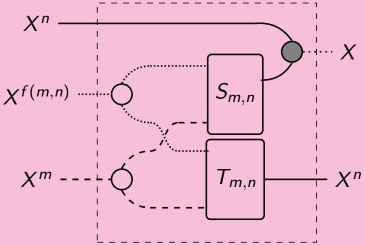

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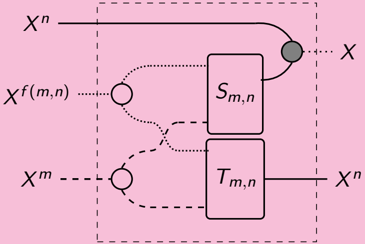

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Again the circuits are composable and we obtain a functor from the category of graphs to the category of boolean circuits.

The power of category theory

Everything we built is compositional!!

$$
\begin{tikzcd}
 & & & \mathbb{B}^{f(m,n)}_{\mathbf{ZKP}} \arrow[rrr, equal] \arrow[dd] & & & \mathbb{B}^{f(m,n)}_{\mathbf{ZKP}} \arrow[dd] \\
 & & \mathbf{Count} \arrow[ur] \arrow[rrr] \arrow[ddd, equal] & & & \mathbb{B}^{G}_{\mathbf{Graph}} \arrow[ur] \arrow[ddd] & \\
 & & & \mathbb{B}^{f(m',n')}_{\mathbf{ZKP}} \arrow[rrr, equal] & & & \mathbb{B}^{f(m',n')}_{\mathbf{ZKP}} \\
G \arrow[r] \arrow[dd] & \mathfrak{F}(G) \arrow[uur] \arrow[rrr] \arrow[dd] & & & \mathbb{B}^{G}_{\mathbf{path}} \arrow[uur] \arrow[dd] & & \\
 & & \mathbf{Count} \arrow[uur] \arrow[rrr] & & & \mathbb{B}^{G'}_{\mathbf{Graph}} \arrow[uur] & \\
G' \arrow[r] & \mathfrak{F}(G') \arrow[uur] \arrow[rrr] & & & \mathbb{B}^{G'}_{\mathbf{path}} \arrow[uur] & &
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Category theory seems difficult in the beginning, but it's just a better method to do software engineering!


We are finished
THANKS
QUESTIONS PLEASE