Experimental Design for Estimating Burn Rate Parameters and Predicting Chamber Pressure of a 5-Grain Solid Rocket Motor

1. Summary and Objective

This document presents a comprehensive plan for conducting static tests of a sugar-based solid propellant (KNSB, 65% KNO_3, 35% sorbitol) to determine burn rate parameters—specifically the burn rate coefficient a and the pressure exponent n—using Richard Nakka's simplified static test method. These values will then be used to accurately estimate the chamber pressure of a 5-grain BATES-configured motor and inform material selection for safe motor casing construction.

2. Theoretical Background

2.1 Burn Rate Model (Saint Robert's Law)

Solid rocket propellants generally obey the empirical relationship known as Saint Robert's Law:

$$r = a \cdot P^n$$

Where:

- r is the linear burn rate (mm/s),
- P is the chamber pressure (MPa),
- a is the burn rate coefficient (unit depends on pressure units),
- n is the pressure exponent (dimensionless).

The coefficient a and exponent n are dependent on the specific chemical composition and physical characteristics of the propellant. These parameters govern how aggressively the propellant burns under different pressures and are critical for predicting motor behavior.

2.2 Pressure Determination and Kn Ratio

The internal pressure of a solid rocket motor at steady state is governed by:

$$P_c \propto \left(\frac{A_b}{A_t}\right)^{\frac{1}{1-n}} = Kn^{\frac{1}{1-n}}$$

Where:

- A_b is the burning surface area of the propellant,
- A_t is the nozzle throat area,
- Kn is the ratio A_b/A_t, also called the "Kn ratio".

By changing the nozzle throat diameter (and therefore A_t), while keeping the propellant geometry constant, different chamber pressures can be achieved. This enables solving for n through multiple pressure–burn rate data points.

3. Methodological Plan

3.1 Static Test Campaign Overview

Two static tests will be conducted using single BATES grains of identical dimensions. By using two different nozzle throat diameters, different chamber pressures will be achieved. The burn rate r will be calculated from the burned web thickness and burn time, while pressure data will be collected using a calibrated pressure transducer.

3.2 Grain Design

Each test grain will be a single BATES grain with:

- Outer diameter: 86 mm
- Core diameter: 33 mm
- Length: 145 mm

This geometry ensures a nearly neutral burn profile, and simplifies burn area calculations.

3.3 Nozzle Design

Two nozzles with different throat diameters will be used:

- Test 1: 26 mm throat (lower pressure)
- Test 2: 21 mm throat (higher pressure)

These diameters are selected to achieve a pressure ratio of $1.8\times$, sufficient for regression analysis of a and n, while remaining within safety limits of 6061-T6 aluminum casings.

4. Experimental Apparatus

- **Motor casing:** Aluminum 6063-T5, 94 mm OD, 3 mm wall thickness.
- **Liner:** Epoxy-bonded paper (8 layers) for insulation and structural protection.
- **Bulkheads:** Machined 6061-T6 with O-ring grooves for sealing.
- **Nozzles:** Mild steel, machined to precise throat sizes (erosion-resistant).

- **Pressure Transducer:** Rated 0–200 Bar, with threaded tap and high-temp fittings.
- **Thrust Sensor:** Load cell (0–8000 N) for thrust-time curve validation.
- **Ignition:** Nichrome wire + pyrogen, triggered remotely.
- **Test Stand:** Welded steel frame with mounting brackets and shielding.

5. Test Procedure

5.1 Preparation

1. Fabricate liner by winding epoxy-coated paper inside the casing; allow to cure 3 days.
2. Cast three BATES grains from a single well-mixed batch of KNSB propellant.
3. Machine and inspect two nozzles with throat diameters of 26 mm and 21 mm.

5.2 Test Execution

1. Mount the grain, nozzle, and bulkheads into the motor casing.
2. Install motor on test stand with pressure transducer and thrust sensor connected to a microcontroller.
3. Perform static firings at a secure outdoor location (e.g., remote test range).
4. Record pressure and thrust data throughout the burn.

6. Data Analysis Plan

6.1 Data to Collect Per Test

- Pressure vs. time
- Thrust vs. time
- Burn time (ignition to burnout)
- Grain web thickness (before and after burn)
- Throat diameter changes (to monitor erosion)

6.2 Calculations

1. Compute linear burn rate:
$$r = \frac{\text{web thickness}}{\text{burn time}}$$
2. Use chamber pressure at mid-burn as corresponding P_c

3. Apply log–log plot:

$$\log(r) = \log(a) + n\log(P_c)$$

4. Use linear regression to solve for a and n

6.3 Use of Tools

- **Burn Rate Calculator:** Automates a, n regression
- **SRM.XLS:** Computes burning surface area A_b, Kn, and regression vs. time
- **OpenMotor:** Simulates pressure/thrust profile of scaled 5-grain motor using a, n

7. Extrapolation to 5-Grain Motor

Once a and n are determined, they can be used to simulate the chamber pressure of a 5-grain motor because:

- Grain geometry and propellant formulation remain unchanged
- Burning surface area A_b increases linearly with number of grains
- Throat area A_t remains the same
- Therefore, new Kn can be calculated
- Using:

$$P_c = \left(\frac{r}{a}\right)^{1/n}$$

- SRM.XLS and OpenMotor can then model the full pressure profile and validate design

8. Safety Considerations

- Testing will be conducted in a remote, open-air location with no bystanders.
- Protective gear: flame-resistant clothing, safety goggles, gloves.
- All fittings will be leak-tested and pressure-rated.
- Pressure sensors and load cells will be calibrated before use.
- Follow Nakka's pressure transducer installation recommendations.

9. Conclusion

This experimental plan provides a robust, cost-effective framework for characterizing the burn rate behavior of sugar-based solid propellants using well-established amateur rocketry methods. By executing two carefully controlled static tests with varying nozzle throat diameters, the team can derive accurate burn rate parameters, simulate the 5-grain motor behavior, and safely proceed with motor design and casing material selection. The use of SRM.XLS and OpenMotor provides high-fidelity prediction tools validated by experimental input.