

Portfolio Analysis and Optimisation

The aim of this project is to analyse, evaluate and optimise three portfolios, each consisting of five randomly selected financial assets, using key performance and risk measures such as annual return, volatility, VaR, CVaR and the Sharpe ratio. The project further applies portfolio optimisation techniques to identify optimal asset allocations that maximise risk-adjusted returns under standard investment constraints.

Portfolio Construction

Returns are calculated using historical data which was sourced from Yahoo Finance API. The historical data is set at a period of 5 years and the portfolios are equally weighted. The historical data chosen consists of adjusted closing price for the past 5 years. The adjusted closing price reflects a stocks closing price adjusted for dividends and stock splits. For analysis, returns and log returns were used.

The assets for each portfolio are as follows:

Portfolio 1:

1. NVIDIA
2. Ford
3. Plug Power
4. SoundHound
5. Nu Holdings

Portfolio 2:

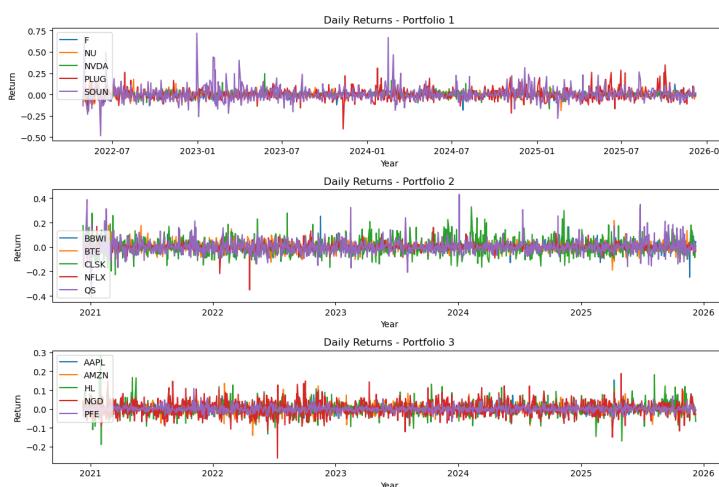
1. Netflix
2. QuantumScape
3. Baytex Energy Corp
4. Bath and Body Works
5. CleanSpark Inc

Portfolio 3:

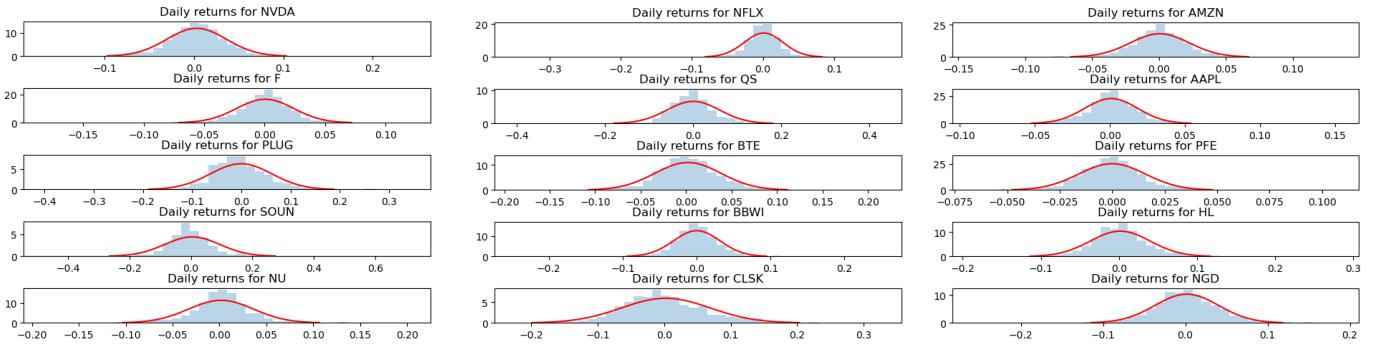
1. Amazon
2. Apple
3. Pfizer
4. Hecla Mining Company
5. New Gold Inc

Portfolio Analysis

Below is a graph visualising the daily simple returns for each three portfolios. The daily return plots illustrate the price movements of the assets in each portfolio. While the mean daily return is near zero for all, portfolio 2 exhibits the most significant daily price movements, implying the highest short-term volatility risk from its constituent holdings and portfolio 1 has the sharpest spikes in price movement. Conversely, portfolio 3 appears to have the most stable daily return profile with movements more tightly constrained around the mean, suggesting comparatively lower daily volatility.



Below are histograms overplayed with a normal distribution for portfolios 1, 2 and 3 respectively.

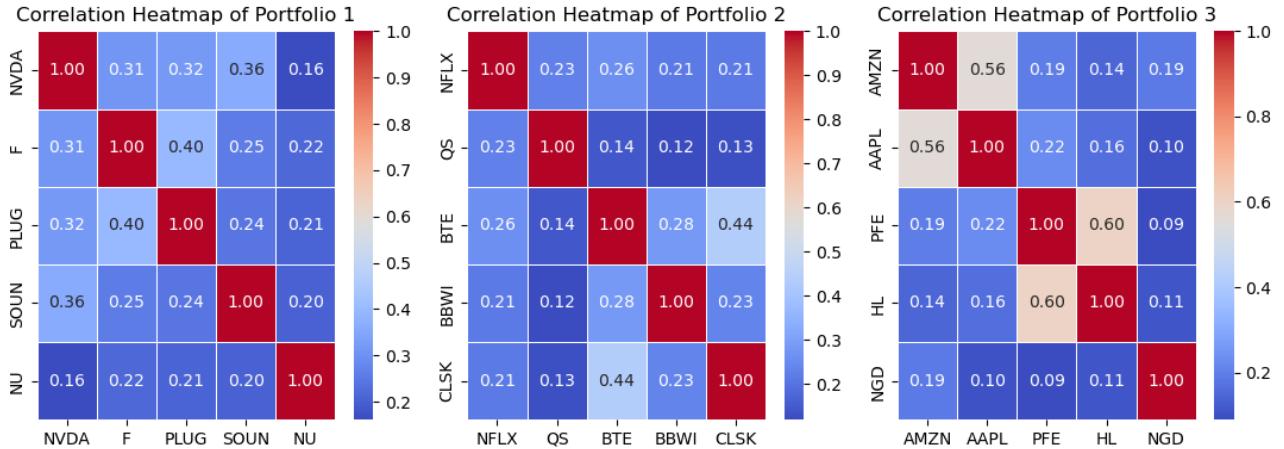


The daily return histograms for portfolio 1 stocks confirm a diverse risk profile. PLUG and SOUN are the primary contributors to the portfolio's volatility, frequently exhibiting the largest daily price movements. In contrast, F and NU provide a lower-risk foundation due to their tighter return distribution and lower daily volatility.

The distribution analysis for portfolio 2 highlights the significant risk contribution from the high-growth, high-volatility stock CLSK, which frequently experiences large daily fluctuations, indicating a non-negligible tail risk. Conversely, BTE and BBWI act as stabilising components, exhibiting relatively lower daily volatility while NFLX exhibits the least volatility.

The distribution analysis of portfolio 3 reveals a clear contrast in risk. AAPL and PFE serve as a low-volatility for the portfolio, characterised by tight return distributions. Conversely, HL, NGD introduce substantial daily volatility and tail risk, with significantly wider return ranges.

The following image displays the correlation between the assets for each portfolio. For all three portfolios, the correlations are positive and fall in the low to moderate range of 0.1- 0.4 with a few exceptions where the correlation range to a maximum of 0.6. Portfolio 1 and 2 have the least amount of correlation whereas portfolio 3 consists of assets with relatively moderate-high correlation. This suggests the portfolio assets generally move in the same direction.



Annual returns, Volatility and Sharpe Ratios

Using equal weightings for the three portfolios, the annual returns for portfolio 1, 2 and 3 are 55.8158%, 32.448% and 28.8211% respectively. The annual volatility for portfolios 1, 2 and 3 are 48.296%, 43.7015% and 26.6353% respectively.

Portfolio 1 has the highest annual return and also exhibits the most volatility. From portfolio 1 to portfolio 2, there is a substantial jump between their annual returns, however there is no significant difference between their volatilities. Portfolio 3 shows the lowest return and volatility.

The U.S. 3-month Treasury bill is used as the risk-free rate as it is highly liquid, has minimal default and interest rate risk and is widely accepted in practice as a standard benchmark for risk-free returns. The Sharpe ratio measures an investment's risk-adjusted performance by indicating how much excess return is

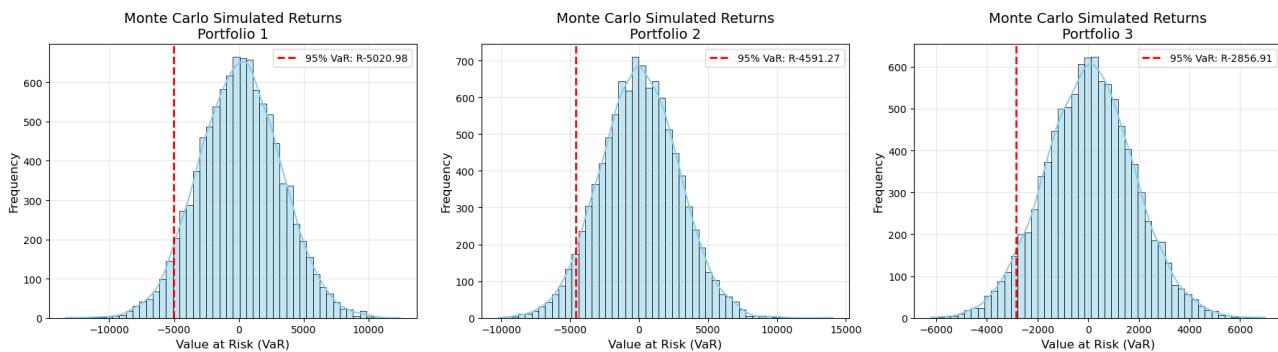
earned per unit of volatility. The annual Sharpe ratios for portfolios 1, 2, and 3 are 0.807, 0.541 and 0.777 respectively. This indicates that although portfolio 1 is the most volatile, it still provides the highest excess return per unit of risk, while Portfolio 3, despite having the lowest return, offers a relatively strong risk-adjusted performance.

Risk Measures

After converting simple returns to log returns, we estimate the portfolio risk using both the variance–covariance VaR and Monte Carlo simulation VaR. We also compute the Conditional Value at Risk (CVaR) to assess the average loss in the tail beyond the VaR level, providing a more comprehensive view of extreme downside risk.

Using a 5% confidence level and a portfolio value of \$100 000, the variance–covariance VaR estimates are \$4985.2218, \$4571.9144 and \$2806.8671 for portfolios 1, 2 and 3 respectively. The Monte Carlo VaR estimates are \$5011.7971, \$4736.2541 and \$2832.3258 for each portfolio respectively.

The figure below displays the distribution of simulated portfolio returns generated using the Monte Carlo method for each of the three portfolios. Each histogram shows the range and frequency of potential outcomes based on 10000 simulated returns. The red dashed line in each plot marks the 95% VaR, representing the 5th percentile of the simulated return distribution. This point indicates the level of loss that is exceeded in only 5% of the worst-case scenarios. Comparing the three panels, portfolio 1 shows the largest left-tail losses, followed by portfolio 2, while portfolio 3 has the smallest downside risk. The visualisation clearly highlights the different risk profiles across the portfolios and how their respective VaR values relate to the shape and spread of their simulated return distributions.



The VaR estimates provide a clear indication of the downside risk associated with each portfolio at the 5% confidence level. For a portfolio of \$100000, the results show that portfolio 1 has the highest potential loss with a 5% chance of losing approximately more than \$5000 a day. Portfolio 2 exhibits slightly less risk with losses expected to exceed roughly \$4,600 a day. Portfolio 3 is the least risky with a potential to lose approximately more than \$2800 a day.

The CVaR, also known as Expected Shortfall, provides a measure of the average loss in the worst 5% of outcomes, offering deeper insight into tail risk beyond the VaR threshold. The CVaR results for the three portfolios reveal distinct risk characteristics. Portfolio 1 has a CVaR of \$353.1117, indicating that when losses exceed the 5% VaR level, the average loss is relatively small compared to the other portfolios. Portfolio 2 shows a higher CVaR of \$416.8178, reflecting moderately larger tail losses. Portfolio 3, with a CVaR of \$994.395, exhibits the largest expected loss in extreme scenarios.

These results suggest that although portfolio 3 has the lowest volatility and the smallest VaR among the three portfolios, its tail losses are proportionally larger once the VaR threshold is breached. In contrast, portfolios 1 and 2 exhibit more controlled behaviour in the extreme left tail. This highlights an important distinction between VaR and CVaR, VaR captures the threshold of extreme loss, while CVaR reveals how severe those extreme losses can become, providing a more comprehensive understanding of downside risk.

The three portfolios exhibit clear trade-offs between risk and returns. In summary portfolio 1 has the highest annual return, accompanied by the highest Sharpe ratio, indicating the best risk-adjusted performance. Portfolio 1 has the highest VaR using both methods, variance-covariance method and Monte Carlo method, however it has the lowest CVaR, indicating losses beyond the VaR threshold are relatively less severe compared to other portfolios. Portfolio 2 has the second highest annual return and volatility with the lowest Sharpe ratio, meaning the returns are not being compensated for the amount of risk in the investment.

Portfolio 2 has the second highest VaR using both methods, variance-covariance method and Monte Carlo method, with has the second highest CVaR, implying neither strong upside potential nor effective downside risk. Portfolio 3 has the lowest annual return and volatility, yet it achieves the second highest Sharpe ratio, reflecting relatively efficient risk-adjusted performance. Portfolio 3 has the lowest VaR using both methods, variance-covariance method and Monte Carlo method, however it has the highest CVaR. This tells us that we can expect losses under normal market conditions as well as implying greater exposure to extreme tail losses during adverse market scenarios.

For risk-seeking investors, portfolio 1 appears to be the most attractive investment option, as it offers the highest annual return and adequately compensates investors for the level of risk undertaken, as reflected by its superior Sharpe ratio. In contrast, portfolio 2 appears to be the least attractive, as indicated by its lowest Sharpe ratio, suggesting that it provides the weakest risk-adjusted return among the three portfolios. For risk-averse investors who prioritise stability over returns, portfolio 3 is the preferred choice, as it exhibits the lowest volatility and the lowest VaR under both methods, indicating smaller losses under normal market conditions, despite its higher exposure to extreme tail losses as reflected by its CVaR.

Portfolio Optimisation

To construct an optimal portfolio, a Sharpe ratio maximisation approach subject to standard portfolio constraints was applied. The asset weights were restricted to sum to one and each weight was bounded between 0 and 1 to prevent short-selling. The optimisation identified the set of weights that yielded the highest Sharpe ratio.

- For portfolio 1, the optimal allocation placed the majority of capital in the third asset, Plug Power, with an allocation of 79.55%, and smaller allocations to the second asset, Ford, with an allocation of 9.581% and fifth asset, Nu Holdings, with an allocation of 10.872%, resulting in a maximum annualised Sharpe ratio of 1.45.
- For portfolio 2, the optimal allocation placed the majority of capital in the second asset, QuantumScape, with an allocation of 58.97%, followed by the 4th asset, Bath and Body Works, with an allocation of 28.4%, and third asset, Baytex Energy Corp, with an allocation of 12.633%, resulting in a maximum annualised Sharpe ratio of 0.94.
- For portfolio 3, the optimal allocation placed the majority of capital in the first asset , Amazon, with an allocation of 59.082%, followed by the 4th asset , Hecla Mining Company, with an allocation of 23.6% and third asset , Pfizer, with an allocation of 17.321%, resulting in a maximum annualised Sharpe ratio of 0.87.

These results show that portfolio 1 offers the strongest risk-adjusted performance under optimal weighting, while portfolios 2 and 3 converge to similar allocations with lower overall Sharpe ratios.

Using the optimised weights that maximise the Sharpe ratio, the annual returns and volatilities of the portfolios improved considerably. Portfolio 1 shows the strongest performance, achieving an annual return of 110.8399% with an annual volatility of 49.9971%, reflecting a substantial increase in risk-adjusted performance. Portfolio 2 generates an annual return of 51.6301% with a volatility of 41.7813%, while Portfolio 3 achieves an annual return of 36.884% and a volatility of 29.6442%.

There is a substantial increase in the Sharpe ratio when optimal weights are applied, with annual returns more than doubling compared to the equal-weighted portfolios. Under the optimal weighting scheme, portfolio 2 now attains the second-highest Sharpe ratio, while portfolio 3 records the lowest Sharpe ratio. Annual returns for portfolios 2 and 3 increase more conservatively relative to portfolio 1. Changes in annual volatility are modest, volatility increases by approximately 1% for portfolio 1, rises by about 2% for portfolio 2, and decreases by roughly 3% for portfolio 3.

Overall, the application of optimal portfolio weights significantly enhances risk-adjusted performance compared to equal weighting. The improvement in Sharpe ratios, particularly for portfolio 1, demonstrates the effectiveness of the optimisation process in efficiently balancing risk and return. While portfolios 2 and 3 experience more conservative gains, changes in volatility remain relatively small, indicating that higher returns are not achieved through disproportionate increases in risk. Consequently, the results confirm that portfolio optimisation is a valuable tool for improving performance, with portfolio 1 emerging as the most attractive option for investors seeking superior risk-adjusted returns.