Log transforming the response

Multiplicative increments

- The **logarithmic transformation** of the response is a common transformation when the effect of the predictor on the response is thought to be multiplicative
  - A change of a unit in x is associated with a constant **percentage** change in y
- Many processes benefit from log transformation:
  - Growth is proportional do previous size
  - Any multiplicative process

$$y_i \sim N(\mu_i, \sigma)$$
$$\mu_i = \alpha + \beta x_i$$
$$\downarrow$$
$$\Delta y \approx \beta \Delta x$$

$$log(y_i) \sim N(\mu_i, \sigma)$$
$$\mu_i = \alpha + \beta x_i$$
$$\downarrow$$
$$\% \, \Delta y \approx (100 \times \beta) \Delta x$$

Houle, D., Pélabon, C., Wagner, G. P. & Hansen, T. F. Measurement and meaning in biology. Q. *Rev. Biol.* 86, 3–34 (2011).

Biomass by diameter

Example of non-linear relation

Option 1: log-transform y

```
df = data.frame(diameter, biomass)
stan_fit = stan_glm(log(biomass) ~ diameter,
                    data = df)
```

Option A: Assign y a log-normal likelihood

```
rt_fit = ulam(alist(
            biomass ~ lognormal(mu, sigma),
            mu <- a + b*diameter,
            a ~ normal(0, 2),
            b ~ normal(0, 1),
            sigma ~ exponential(1)),
        data = df, chains = 4, cores = 4)
```