Power-law relations

Log-log regressions

- Several biological relations take the form of power-law relations

- We can linearize these relations using a log-log transformation

- In this model, the slope is and estimate of the exponent of the power-law

- The interpretation of the slope is that a 1% increase in x leads to a $\beta\,\%$ increase in y

$$\boxed{\begin{gathered} log(y_i) \sim N(\mu_i, \sigma) \\ \mu_i = \alpha + \beta\ log(x_i) \\ \Downarrow \\ \%\,\Delta y \approx \beta\,\%\,\Delta x \end{gathered}}$$

West, G. B., Brown, J. H. & Enquist, B. J. A general model for the origin of allometric scaling laws in biology. *Science* **276**, 122–126 (1997).

Quick reference for log transformations

Model	Dependent variable	Independent Variable	Interpretation of β
Level-level	y	x	$\Delta y \approx \beta \Delta x$
Level-log	y	log(x)	$\Delta y \approx (\frac{\beta}{100})\,\%\,\Delta x$
log-level	log(y)	x	$\%\,\Delta y \approx (100\ \beta)\Delta x$
Log-log	log(y)	log(x)	$\%\,\Delta y \approx \beta\,\%\,\Delta x$