附錄 D：微分公式與積分公式

壹、微分公式

1. $\dfrac{d}{dx}[cu] = cu'$

2. $\dfrac{d}{dx}[u \pm v] = u' \pm v'$

3. $\dfrac{d}{dx}[uv] = uv' \pm vu'$

4. $\dfrac{d}{dx}\left[\dfrac{u}{v}\right] = \dfrac{vu' - uv'}{v^2}$

5. $\dfrac{d}{dx}[c] = 0$

6. $\dfrac{d}{dx}[u^n] = nu^{n-1}u'$

7. $\dfrac{d}{dx}[x] = 1$

8. $\dfrac{d}{dx}[\,|u|\,] = \dfrac{u}{|u|}(u')$，$u \neq 0$

9. $\dfrac{d}{dx}[\ln u] = \dfrac{u'}{u}$

10. $\dfrac{d}{dx}[e^u] = e^u u'$

11. $\dfrac{d}{dx}[\sin u] = (\cos u)u'$

12. $\dfrac{d}{dx}[\cos u] = -(\sin u)u'$

13. $\dfrac{d}{dx}[\tan u] = (\sec^2 u)u'$

14. $\dfrac{d}{dx}[\cot u] = -(\csc^2 u')u'$

15. $\dfrac{d}{dx}[\sec u] = (\sec u \tan u)u'$

16. $\dfrac{d}{dx}[\csc u] = -(\csc u \cot u)u'$

貳、積分公式

一、積分型式含 u^n

1. $\displaystyle\int u^n\,du = \dfrac{u^{n+1}}{n+1} + C$，$n \neq -1$

2. $\displaystyle\int \dfrac{1}{u}\,du = \ln|u| + C$

二、積分型式含 $a + bu$

3. $\displaystyle\int \frac{u}{a+bu}\,du = \frac{1}{b^2}\left(bu - a\ln|a+bu|\right) + C$

4. $\displaystyle\int \frac{u}{(a+bu)^2}\,du = \frac{1}{b^2}\left(\frac{a}{a+bu} + \ln|a+bu|\right) + C$

5. $\displaystyle\int \frac{u}{(a+bu)^n}\,du = \frac{1}{b^2}\left[\frac{-1}{(n-2)(a+bu)^{n-2}} + \frac{a}{(n-1)(a+bu)^{n-1}}\right] + C$，$n \neq 1$，$2$

6. $\displaystyle\int \frac{u^2}{a+bu}\,du = \frac{1}{b^3}\left[-\frac{bu}{2}(2a-bu) + a^2\ln|a+bu|\right] + C$

7. $\displaystyle\int \frac{u^2}{(a+bu)^2}\,du = \frac{1}{b^3}\left(bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right) + C$

8. $\displaystyle\int \frac{u^2}{(a+bu)^3}\,du = \frac{1}{b^3}\left[\frac{2a}{a+bu} - \frac{a^2}{2(a+bu)^2} + \ln|a+bu|\right] + C$

9. $\displaystyle\int \frac{u^2}{(a+bu)^n}\,du = \frac{1}{b^3}\left[\frac{-1}{(n-3)(a+bu)^{n-3}} + \frac{2a}{(n-2)(a+bu)^{n-2}} - \frac{a^2}{(n-1)(a+bu)^{n-1}}\right] + C$，$n \neq 1$，$2$，$3$

10. $\displaystyle\int \frac{1}{u(a+bu)}\,du = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

11. $\displaystyle\int \frac{1}{u(a+bu)^2}\,du = \frac{1}{a}\left(\frac{1}{a+bu} + \frac{1}{a}\ln\left|\frac{u}{a+bu}\right|\right) + C$

12. $\displaystyle\int \frac{1}{u^2(a+bu)}\,du = -\frac{1}{a}\left(\frac{1}{u} + \frac{b}{a}\ln\left|\frac{u}{a+bu}\right|\right) + C$

13. $\displaystyle\int \frac{1}{u^2(a+bu)^2}\,du = -\frac{1}{a^2}\left[\frac{a+2bu}{u(a+bu)} + \frac{2b}{a}\ln\left|\frac{u}{a+bu}\right|\right] + C$

三、積分型式含 $\sqrt{a+bu}$

14. $\displaystyle\int u^n\sqrt{a+bu}\,du = \frac{2}{b(2n+3)}\left[u^n(a+bu)^{3/2} - na\int u^{n-1}\sqrt{a+bu}\,du\right]$

15. $\displaystyle\int \frac{1}{u\sqrt{a+bu}}\,du = \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C$，$a > 0$

16. $\displaystyle\int \frac{1}{u^n\sqrt{a+bu}}\,du = \frac{-1}{a(n-1)}\left[\frac{\sqrt{a+bu}}{u^{n-1}} + \frac{(2n-3)b}{2}\int \frac{1}{u^{n-1}\sqrt{a+bu}}\,du\right]$，$n \neq 1$

17. $\displaystyle\int \frac{\sqrt{a+bu}}{u}\,du = 2\sqrt{a+bu} + a\int \frac{1}{u\sqrt{a+bu}}\,du$

18. $\displaystyle\int \frac{\sqrt{a+bu}}{u^n}\,du = \frac{-1}{a(n-1)}\left[\frac{(a+bu)^{3/2}}{u^{n-1}} + \frac{(2n-5)b}{2}\int \frac{\sqrt{a+bu}}{u^{n-1}}\,du\right]$，$n \neq 1$

19. $\displaystyle\int \frac{u}{\sqrt{a+bu}}\,du = -\frac{2(2a-bu)}{3b^2}\sqrt{a+bu}+C$

20. $\displaystyle\int \frac{u^n}{\sqrt{a+bu}}\,du = \frac{2}{(2n+1)b}\left(u^n\sqrt{a+bu}-na\int\frac{u^{n-1}}{\sqrt{a+bu}}\,du\right)$

四、積分型式含 $u^2-a^2\ (a>0)$

21. $\displaystyle\int \frac{1}{u^2-a^2}\,du = -\int\frac{1}{a^2-u^2}\,du = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right|+C$

22. $\displaystyle\int \frac{1}{(u^2-a^2)^n}\,du = \frac{-1}{2a^2(n-1)}\left[\frac{u}{(u^2-a^2)^{n-1}}+(2n-3)\int\frac{1}{(u^2-a^2)^{n-1}}\,du\right]$ ，$n\neq1$

五、積分型式含 $\sqrt{u^2\pm a^2}\ (a>0)$

23. $\displaystyle\int \sqrt{u^2\pm a^2}\,du = \frac{1}{2}\left(u\sqrt{u^2\pm a^2}\pm a^2\ln|u+\sqrt{u^2\pm a^2}|\right)+C$

24. $\displaystyle\int u^2\sqrt{u^2\pm a^2}\,du = \frac{1}{8}\left[u(2u^2\pm a^2)\sqrt{u^2\pm a^2}-a^4\ln|u+\sqrt{u^2\pm a^2}|\right]+C$

25. $\displaystyle\int \frac{\sqrt{u^2+a^2}}{u}\,du = \sqrt{u^2+a^2}-a\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right|+C$

26. $\displaystyle\int \frac{\sqrt{u^2\pm a^2}}{u^2}\,du = \frac{-\sqrt{u^2\pm a^2}}{u}+\ln|u+\sqrt{u^2\pm a^2}|+C$

27. $\displaystyle\int \frac{1}{\sqrt{u^2\pm a^2}}\,du = \ln|u+\sqrt{u^2\pm a^2}|+C$

28. $\displaystyle\int \frac{1}{u\sqrt{u^2+a^2}}\,du = \frac{-1}{a}\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right|+C$

29. $\displaystyle\int \frac{u^2}{\sqrt{u^2\pm a^2}}\,du = \frac{1}{2}\left(u\sqrt{u^2\pm a^2}\mp a^2\ln|u+\sqrt{u^2\pm a^2}|\right)+C$

30. $\displaystyle\int \frac{1}{u^2\sqrt{u^2\pm a^2}}\,du = \mp\frac{\sqrt{u^2\pm a^2}}{a^2u}+C$

31. $\displaystyle\int \frac{1}{(u^2\pm a^2)^{3/2}}\,du = \frac{\pm u}{a^2\sqrt{u^2\pm a^2}}+C$

六、積分型式含 $\sqrt{a^2-u^2}$ ($a>0$)

32. $\displaystyle\int \frac{\sqrt{a^2-u^2}}{u}\,du = \sqrt{a^2-u^2} - a\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$

33. $\displaystyle\int \frac{1}{u\sqrt{a^2-u^2}}\,du = \frac{-1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$

34. $\displaystyle\int \frac{1}{u^2\sqrt{a^2-u^2}}\,du = \frac{-\sqrt{a^2-u^2}}{a^2 u} + C$

35. $\displaystyle\int \frac{1}{(a^2-u^2)^{3/2}}\,du = \frac{u}{a^2\sqrt{a^2-u^2}} + C$

七、積分型式含 e^u

36. $\displaystyle\int e^u\,du = e^u + C$

37. $\displaystyle\int ue^u\,du = (u-1)e^u + C$

38. $\displaystyle\int u^n e^u\,du = u^n e^u - n\int u^{n-1} e^u\,du$

39. $\displaystyle\int \frac{1}{1+e^u}\,du = u - \ln(1+e^u) + C$

40. $\displaystyle\int \frac{1}{1+e^{nu}}\,du = u - \frac{1}{n}\ln(1+e^{nu}) + C$

八、積分型式含 $\ln u$

41. $\displaystyle\int \ln u\,du = u(-1+\ln u) + C$

42. $\displaystyle\int u\ln u\,du = \frac{u^2}{4}(-1+2\ln u) + C$

43. $\displaystyle\int u^n \ln u\,du = \frac{u^{n+1}}{(n+1)^2}[-1+(n+1)\ln u] + C,\ n\neq -1$

44. $\displaystyle\int (\ln u)^2\,du = u[2-2\ln u+(\ln u)^2] + C$

45. $\displaystyle\int (\ln u)^2\,du = u(\ln u)^u - n\int (\ln u)^{n-1}\,du$

九、積分型式含 $\sin u$ 或 $\cos u$

46. $\int \sin u \, du = -\cos u + C$

47. $\int \cos u \, du = \sin u + C$

48. $\int \sin^2 u \, du = \dfrac{1}{2}(u - \sin u \cos u) + C$

49. $\int \cos^2 u \, du = \dfrac{1}{2}(u + \sin u \cos u) + C$

50. $\int \sin^n u \, du = -\dfrac{\sin^{n-1} u \cos u}{n} + \dfrac{n-1}{n}\int \sin^{n-2} u \, du$

51. $\int \cos^n u \, du = \dfrac{\cos^{n-1} u \sin u}{n} + \dfrac{n-1}{n}\int \cos^{n-2} u \, du$

52. $\int u \sin u \, du = \sin u - u \cos u + C$

53. $\int u \cos u \, du = \cos u + u \sin u + C$

54. $\int u^n \sin u \, du = -u^n \cos u + n\int u^{n-1}\cos u \, du$

55. $\int u^n \cos u \, du = u^n \sin u - n\int u^{n-1}\sin u \, du$

56. $\int \dfrac{1}{1 \pm \sin u} \, du = \tan u \mp \sec u + C$

57. $\int \dfrac{1}{1 \pm \cos u} \, du = -\cot u \mp \csc u + C$

58. $\int \dfrac{1}{\sin u \cos u} \, du = \ln|\tan u| + C$

十、積分型式含 $\tan u$、$\cot u$、$\sec u$、$\csc u$ 等

59. $\int \tan u \, du = -\ln|\cos u| + C$

60. $\int \cot u \, du = \ln|\sin u| + C$

61. $\int \sec u \, du = \ln|\sec u + \tan u| + C$

62. $\int \csc u \, du = \ln|\csc u - \cot u| + C$

63. $\int \tan^2 u \, du = -u + \tan u + C$

64. $\int \cot^2 u \, du = -u - \cot u + C$

65. $\int \sec^2 u \, du = \tan u + C$

66. $\int \csc^2 u \, du = -\cot u + C$

67. $\int \tan^n u \, du = \dfrac{\tan^{n-1} u}{n-1} - \int \tan^{n-2} u \, du$，$n \neq 1$

68. $\int \cot^n u \, du = -\dfrac{\cot^{n-1} u}{n-1} - \int \cot^{n-2} u \, du$，$n \neq 1$

69. $\int \sec^n u \, du = \dfrac{\sec^{n-2} u \tan u}{n-1} + \dfrac{n-2}{n-1} \int \sec^{n-2} u \, du$，$n \neq 1$

70. $\int \csc^n u \, du = \dfrac{\csc^{n-2} u \cot u}{n-1} + \dfrac{n-2}{n-1} \int \csc^{n-2} u \, du$，$n \neq 1$

71. $\int \dfrac{1}{1 \pm \tan u} \, du = \dfrac{1}{2} \left(u \pm \ln |\cos u \pm \sin u| \right) + C$

72. $\int \dfrac{1}{1 \pm \cot u} \, du = \dfrac{1}{2} \left(u \mp \ln |\sin u \pm \cos u| \right) + C$

73. $\int \dfrac{1}{1 \pm \sec u} \, du = u + \cot u \mp \csc u + C$

74. $\int \dfrac{1}{1 \pm \csc u} \, du = u - \tan u \pm \sec u + C$