

10/16

函数



例 4

若函数 $f(x) = x^2 - 3x + 1$ ，求

(1) $f(4) - f(3) = ?$ (2) $f(x + \Delta x) - f(x) = ?$ (3) $\frac{f(x + \Delta x) - f(x)}{\Delta x} = ?$

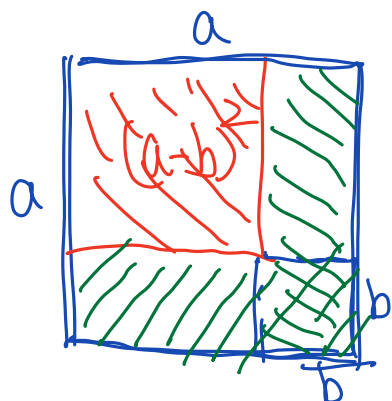
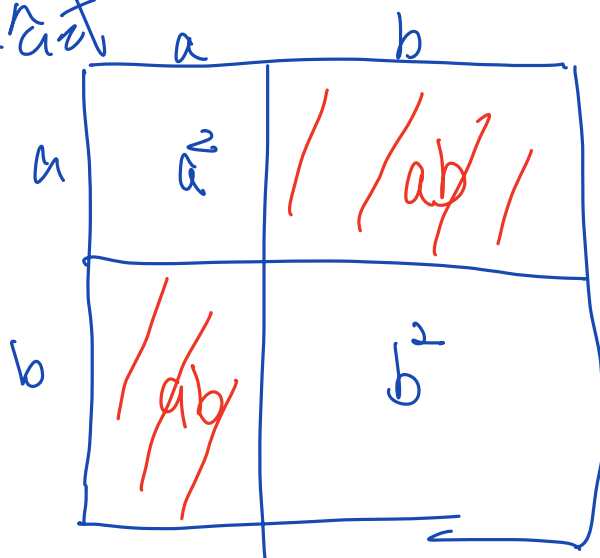


(1) $f(4) - f(3) = (4^2 - 3 \times 4 + 1) - (3^2 - 3 \times 3 + 1) = 5 - 1 = 4$

(2) $f(x + \Delta x) - f(x) = [(x + \Delta x)^2 - 3(x + \Delta x) + 1] - (x^2 - 3x + 1)$
 $= x^2 + 2x\Delta x + (\Delta x)^2 - 3x - 3\Delta x + 1 - x^2 + 3x - 1$
 $= 2x\Delta x + (\Delta x)^2 - 3\Delta x$ *合并同类项*
 $= (2x - 3)\Delta x + (\Delta x)^2$

(3) $\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{(2x - 3)\Delta x + (\Delta x)^2}{\Delta x} = (2x - 3) + \Delta x$

乘法公式



完全平方和

$(a+b)^2 = a^2 + b^2 + 2ab$

$70^2 = 41^2 + 29^2 + 2 \times 41 \times 29$

\parallel
4900.

完全平方差

$(a-b)^2 = a^2 + b^2 - 2ab$

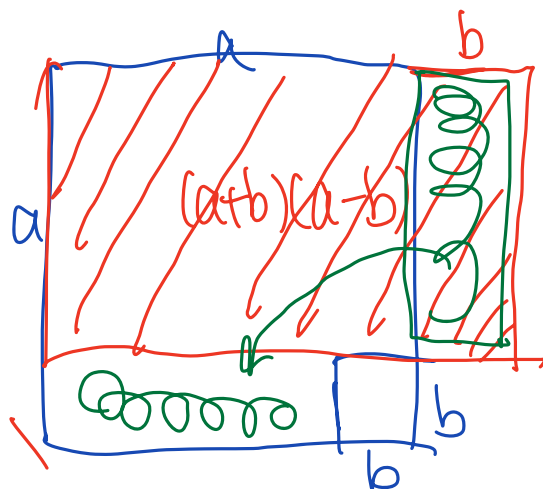
$500 = 79^2 + 29^2 - 2 \times 79 \times 29$

\parallel
500

平方差

③ $(a+b)(a-b) = a^2 - b^2$

例: $79^2 - 21^2 = 5800$



$$\begin{aligned} & \parallel \\ & (79+21)(79-21) \\ & \parallel \\ & 100 \times 58 \end{aligned}$$

代公式: 一個一個代, 一個不行就換下個

練: ① $99^2 - 1^2 = (99+1)(99-1) = 9800$

② $23^2 + 77^2 + 2 \times 23 \times 77 \stackrel{4+}{=} (23+77)^2 = 100^2 = 10000$

③ $100^2 + 50^2 - 2 \times 100 \times 50 \stackrel{4-}{=} (100-50)^2 = 50^2 = 2500$

④ $x^2 - y^2 \stackrel{2}{=} (x+y)(x-y)$ (因式分解: 寫成 $() \cdot ()$)

⑤ $x^4 - y^4 = (x^2)^2 - (y^2)^2 \stackrel{2}{=} (x^2+y^2)(x^2-y^2)$ (因)

$= (x+y)(x-y)(x^2+y^2)$

⑥ $x^8 - y^8 = (x^4)^2 - (y^4)^2 \stackrel{2}{=} (x^4+y^4)(x^4-y^4)$ (因)

⑦ $\stackrel{2}{=} (x^2+y^2)(x^2-y^2)(x^4+y^4)$

$(\square + \triangle)^2 \stackrel{3+}{=} \square^2 + \triangle^2 + 2\square\triangle$ (展開)

$$\begin{cases} 1. (a+b)(a-b) = a^2 - b^2 \\ 2. a^2 - b^2 = (a+b)(a-b) \end{cases}$$

$$\begin{cases} 3+3- (a+b)^2 = a^2 + b^2 + 2ab \\ 4+4- a^2 + b^2 + 2ab = (a-b)^2 \end{cases}$$

★ ⑧

$(x^2 + x^3)^2 \stackrel{2}{=} (x^2)^2 + (x^3)^2 + 2(x^2)(x^3)$ (展開)

$= x^{2 \cdot 2} + x^{3 \cdot 2} + 2x^{2+3} = x^4 + x^6 + 2x^5$

$a \pm b$

一次只做一件事

$\left. \begin{aligned} & 2 \times 3 = 6 \Rightarrow 2 \text{ 是 } 6 \text{ 的因數} \\ & (x+y)(x-y) = x^2 - y^2 \Rightarrow (x+y) \text{ 是 } x^2 - y^2 \text{ 的因式} \end{aligned} \right\}$

$(x+y)(x-y) = x^2 - y^2 \Rightarrow (x+y) \text{ 是 } x^2 - y^2 \text{ 的因式}$

$$\begin{aligned} ab &= a \times b = a \text{ 乘 } b \\ &= (a)(b) = a \cdot b \end{aligned}$$

合併同類項

Step 1. 判斷是哪一類 (同類才能合併, 不同類不能)

$$\underline{2} + 3 = 5 \quad \text{可}$$

$$\underline{x} + 2 \quad \text{不行}$$

$$\underline{x} + \underline{3x} = 4x \quad \text{可}$$

$$\underline{x^2} + 2x \quad \text{不行}$$

$$\underline{1 \cdot x} + \underline{ax} = (1+a)x \quad \text{可}$$

$$\underline{x} + \underline{yx} \quad \text{不行}$$

$$\underline{ax^2} + \underline{bx^2} + c$$

a : 常數 (只是不知道) • 只有 x 這個未知數

x, y : 未知數 • 有 x, y 這 2 個未知數

• 只有一個未知數 x

情況 1

類別: $1, x, x^2, x^3, x^4, \dots, x^n, \dots \Rightarrow x$ 的 多項式

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Step 2: 把同類項寫在一起、合併

$$\leadsto (1 + 1)x + (1)x^2 + (1)x^4 + \dots + (1)x^3$$

Step: 降冪排列 (大到小)

$$(1)x^n + (1) \dots + (1)x^4 + \dots + (1)x^3 + (1)$$

$$\text{例1: } (\underline{x^2+2x}) + (\underline{x^2+2x+1}) + (\underline{x^2+4x+4}) + (\underline{x^2+6x+9})$$

$$= 4x^2 + (2+2+4+6)x + (1+4+9) \\ = 4x^2 + 14x + 14$$

$$\text{例2: } (x+1)^2 + (x-1)^2$$

$$= (\underline{x^2+1+2x}) + (\underline{x^2+1-2x})$$

$$= \underline{2}x^2 + \underline{0}x + \underline{2} = 2x^2 + 2$$

情况2: 2个未知数 x, y

1	x	x^2	x^3	x^4	...
y	xy	yx^2	y^2x^3	y^3x^4	...
y^2	xy^2	x^2y^2	y^3x^3	y^4x^4	...
y^3	xy^3	y^3x^2			
y^4	y^4x				

$$\text{例3: } (x+y)^2 + (x+1)^2 + (y+1)^2 = ?$$